Abstract
A modified ACO algorithm that derives from specific definition of pheromone and cooperation mechanism between ants was applied for solving topology optimization problem of structure. Mesh topology of finite element model for structure was treated as possible paths for ant’s movement. A tour on mesh topology map for seeking food finished by ant is transformed into a structure and the finite element method was applied to analyze the structure for calculating pheromone deposited on the path. The amount of accumulated pheromone deposited on every element by different ants was used to find optimum structural design. From the results studied in this paper, the purposed ACO algorithm provides as alternate optimization method that has high potential in finding the best design for topology optimization of structure successfully and efficiently.

Categories and Subject Descriptors
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Numerical Analysis – Optimization – Stochastic programming

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1. Introduction
Lightweight structure and small size are the trend of design requirements for many products. How to design a lightweight structure with sufficient strength is a difficult problem in the development of the product. The objective of the structural optimization is to find the best structure that fits the design requirements [1]. Topology optimization can achieve significant weight savings in structural design through modification of the arrangement of the material (the topology of the structure) during the conceptual design phase. It is a method more general than the parametric and shape optimization methods, where only some dimensions or the nodal coordinates of the structure are optimized respectively. Topology optimization is a powerful tool which can help designer to select suitable structure and decrease the trial-and-error effort [2-4] in the concept design stage.

Structural optimization has been studied in the last two decades [2]. A lot of methods were presented to find optimal structure for fitting design requirements such as boundary-based shape optimization, homogenization method, evolutionary structural optimization, stochastic optimization, biological-inspired method etc. The boundary-based shape optimization [6] is a direct method to change the structure outline for finding an optimal shape design. However, it is not capable of handling topology changes of structural optimization. The homogenization method [7-10], first proposed by Bensøe and Kikuchi [3], consists in dealing with continuous density distribution of material used for structure. In order to make the optimal solution practical for real-world application, there are different approaches were applied to force the final density distribution toward value of 0 or 1. Especially, the solid isotropic material with penalization (SIMP) method has been widely used in different fields for its implementation simplicity. The mathematical programming methods were frequently used in homogenization or SIMP methods for searching optimal solution. But there is a high probability to find the local optimum solution by using mathematical programming algorithms.

In order to find global optimum solution many researches focused on the population-based optimization algorithms. A simple method for the shape and layout optimization, evolutionary structural optimization (ESO), has been proposed by Xie and Steven [11-13]. Its original concept is to remove the redundant elements carrying low stress gradually and update the criterion for element elimination. In addition, numerous stochastic or biological-inspired method imitating natural phenomena and physical processes have been developed to find optimal structures such as simulated annealing [14-15], genetic algorithms [16-19] and immune algorithm [20]. In these methods, the genetic algorithms(GAs) is used in dealing with the optimal design of discretized trusses for sizing [21-22], shape [23-24], and topology [17-19]. The GAs, based on the natural mechanism, is applied in searching global optimum for many real-world engineering problems. GAs begins from randomized strings to represent the individuals of population and proceed repeatedly from generation to generation through three basic genetic operators: reproduction, crossover, and mutation. In the evolution procedure, strong individuals will be found and kept, but weak ones will be eliminated. The purpose of these mechanisms is mainly to increase the probability of finding the optimal solution and avoid premature convergence of the search. The artificial immune algorithm has capability of performing several tasks including adaptive learning, memory acquisition, generation of diversity, and distributed detection. Those characteristics are also the system features of optimization algorithm and it is useful to
transform the biological immune system into searching algorithm for design optimization. In this paper, the ACO algorithm, a stochastic optimization method, was adopted and modified to solve topology optimization problems from the framework of original ACO algorithm.

The first ant colony optimization (ACO) algorithm, called Ant System (AS) [26-30], was proposed by Dorigo in 1992 to solve the Traveling Salesman Problem (TSP). Ant algorithms [29], population-based methodology, are generally applied to NP-hard combinatorial optimization problems such as TSP and Vehicle Routing Problem (VRP). The objective of these problems is to find the shortest route in all visited spots once. But topology structure optimization is a combinatorial optimization problem that cannot be solved by using original ACO algorithm. The idea of using ACO to solve the TSP is modified and adopted to solve the topology optimization of structure. In the design domain, a topology of structure will be composed of elements that have been visited by an ant’s food foraging tour. The map of tour is transformed into structure and the finite element method is applied for solving displacement and stress. Then the deposited pheromone of all elements visited by the ant is calculated using objective function for the problem. The ant could follow pheromone distribution information for optimum searching in the next step.

In order to increase the diversity for optimization search, some mechanisms are used to prevent premature convergence. Search stagnation described in [26] is the situation that all ants follow the same path and construct the same solution over and over again, such that the better solutions can not be found anymore. Hence, some simple mechanisms such as MAX-MIN Ant System (MMAS) [31-33] and Pheromone Trail Centralization (PTC) [34] are integrated in the algorithm developed in this study to improve the performance of optimum search. Although the definitions of maximum and minimum limits are different in those mechanisms, they all modulate the strength range of pheromone trails to avoid the premature convergence of original ACO.

The ACO algorithm is modified and adopted to solve topology optimization problems in this study. In the design domain, a topology of structure will be composed of elements visited by an ant that finishes its tour of food forging. Through finite element analysis of structure and calculation of objective function, the pheromone value deposited on each element was updated and used for searching optimal topology for structure.

2. Ant Colony Optimization

Ant System, the first ACO algorithm, was proposed by Dorigo [29] to iteratively calculate and construct candidate solutions to numerous NP-hard combinatorial optimization problems. These ants have been inspired by the spontaneous behavior of real ant colonies, especially by the foraging behavior. The indirect communication among real ants is based on depositing some chemical substance, called pheromone, on the trails. The pheromone trails are a kind of distributed numeric information. Hence, by means of accumulating information foraging by ants, an ant algorithm could be applied to solve some particular engineering problems. For the TSP problem, the ant must play the role of traveling salesman to look a route which covers all cities with minimal total distance. This problem could be expressed as a graph \( G \), with nodes, also called cities and edges connecting all of nodes. The algorithm will start with small amount of initial pheromone on all edges. In the solving process, each ant begins its trip from a randomly selected starting city and must only have visited every city once. All the trails finished by all ants are then used to update the pheromone intensity on all edges. The path-taking probability equation for the \( k \)th ant to decide to go the next city \( s \) at city \( r \) is showed as follows.

\[
P_k(r,s) = \begin{cases} 
\frac{[\tau(r,s)]^\beta [\eta(r,s)]^\alpha}{\sum_{u \in J_k(r)} [\tau(r,u)]^\beta [\eta(r,u)]^\alpha}, & \text{if } s \in J_k(r) \\
0, & \text{otherwise} 
\end{cases}
\]

(1)

Where \( \tau(r,s) \) is the pheromone intensity of the path between cities \( r \) and \( s \); \( \eta(r,s) \) is the visibility which is the reciprocal of the distance between cities \( r \) and \( s \). \( J_k(r) \) is the set of unvisited cities for the \( k \)th ant. \( \beta \) is the relative weighting factor to control the influence of \( \tau \) and \( \eta \). The trails accumulated in Eq.(1) is then used to update the original pheromone intensity by the following rule.

\[
\tau(r,s) = (1-\rho) \cdot \tau(r,s) + \sum_{s \in \mathcal{R}^k} \Delta \tau_k(r,s)
\]

(2)

\[
\Delta \tau_k(r,s) = \begin{cases} 
\frac{1}{L_k}, & \text{if } (r,s) \in \text{route made by ant } k \\
0, & \text{otherwise} 
\end{cases}
\]

(3)

Where \( \rho \) (with \( 0 \leq \rho < 1 \)) is the parameter of pheromone evaporation; \( m \) is the number of ants; \( L_k \) is the length of the route between cities \( r \) and \( s \) made by the \( k \)th ant; \( \Delta \tau_k(r,s) \) is the iterative increment of pheromone on edge \( (r,s) \) by the \( k \)th ant. Each ant follows the transition rules defined in Eq.(1) to construct a path step by step until a tour is finished. Then the pheromone intensity is calculated and deposited on the path. The overall pheromone intensities will be modified according to the new deposited pheromone and evaporation rate. The updated pheromone intensity of every path is used to guide the path-taking decision in the next system cycle. Through the iterative search, the objective of finding optimum tour with minimum traveling distance for visiting every city of \( G \) once will be finally achieved. But solving the problem of topology optimization of structure is impossible by using the original framework of ACO, so the basic concept of ACO will be modified and adopted for topology optimization problem.

3. Topology Optimization Using ACO

A two-dimensional design domain is discretized into elements and each element represents either material (with code value of 1) or void (with code value of 0). A topology model is mapped by the distribution of material and void in the design domain. If an ant search food in this domain, its trail will form a map. The elements with material are treated as the cities to be visited in TSP problem. Then this map will be translated into a string. The code “1” in the string means a solid element of the structure in the finite element model and the code “0” represents the void element. The string can be converted into a two-dimensional structure as shown in Fig.1.

The following steps were used to illustrate the procedures for the integration of new ACO algorithm with the finite element method.
to solve the topology optimization problem and the flowchart is showed in Fig.2.

![Flowchart of new ACO algorithm](image)

**Fig. 2** Flowchart of new ACO algorithm

(1) Random initialization of ant population

In the initial process, the design domain for topology optimization is divided into elements for finite element model and it is set as the food-seeking space for ants. In order to avoid trapping into local search, the initial pheromone laid on all elements is defined by a small fixed value. The binary code of initial ant population is randomly generated uniformly. A travel of ant search connects all supports and loadings of structure is used to imitate the condition that real ant completed a tour for food forging. The elements located on the tour path will form a topology map inside design domain and it will be transformed into a structure for finite element analysis. When the structure transformed from food forging tour is a discontinuous structure a stochastic procedure is used to modify for continuity for finite element analysis. The objective function value obtained by the results of finite element analysis will be used to define the pheromone deposited on the trails. The accumulation of pheromone after every ant of population finishing it travel will form an original pheromone map, called A map. The pheromone distribution of A map will be used in transition rule for guiding ants to select path for food forging. It means that the topology of structure will be improved for design goal.

(2) Node transition rule

The movement of an ant search should be one of the eight neighboring elements in the design domain. But there are only seven possible seeking directions of the ant as shown in Fig. 3. The element that the ant has just visited will be excluded from candidate list used in path selection movement and it is illustrated with element with code 1 in the Fig. 3.

Two methods, as shown in Fig.4, are used to create the topology of structure and promote the diversity for modified ACO algorithm.

(a) The locations of support and loading are treated as food for food forging as shown in Fig.4 (a). These locations must be visited by every ant of population and every ant will start food forging from the nest randomly placed on the element of design domain. The roulette wheel method was used for selecting the element for next movement. The ant starts the search from the nest and the travel is completed only when all food has been visited once.

(b) The location of support was treated as the entrance or exit of nest and the location of loadings were defined as food as shown in Fig.4 (b). Each of ants will start from any the exit to find the food. The roulette wheel method is also used in selecting the element for next movement. The process is repeated until the ant found the food and returned to nest. If ant cannot find the way back to nest, the search will be stop and starts the search again from another entrance.

The city in TSP problem can only be visited by ant only once. If the element in modified ACO algorithm can be visited by ant once also, the ant may be trapped in corner or in an area surrounded by visited elements.

There is no way out for ant to search other area, then the food searching process may be ended without accomplished a tour of food forging. Element in modified ACO algorithm is allowed to be visited by ant more than once to avoid the ant trapped in a closed area. The trail finished by ants using the developed methods will become a topology map of structural in the design domain. The topology of structures must be checked with the requirement of continuity before starting the finite element analysis. If the structure is discontinuous, it will go through some trials of stochastic modification for continuity. If the structure is
still a discontinuity structure after several trials of discontinuity modification, every element of structure will get a minimal pheromone value. The continuous structure will be further analyzed using finite element method to obtain the value of pheromone.

In order to improve the performance of modified ACO algorithm for topology optimization, operators of global search and local enhancement will be adopted in transition rule. The accumulated and updated pheromone map from all ants, $\mathcal{J}$ map, is used as the fundamental map for global search. Two global search mechanisms, pheromone trail centralization and max-min ant system, and one local enhancement mechanism, elitist policy, are applied in this study. Three corresponding probability thresholds are set for those three mechanisms separately. A random number is generated and one of those mechanisms is selected randomly to generate its own pheromone map according equation used for corresponding mechanism respectively. Then the roulette wheel selection or tournament selection will be used in companion with individual pheromone map for searching tour of ant. The fundamental pheromone map is updated after the ant finished the tour of food foraging.

The transition probability for selecting next direction are defined in Eq.(3) and Eq.(4).

$$P_k = \begin{cases} \frac{[\tau_i]^\alpha \cdot [\eta_i]^\beta}{\sum_{j=1}^{N_i} ([\tau_j]^\alpha \cdot [\eta_j]^\beta) }, & \text{if } R < -0.5 \\ \frac{\tau_i^+}{\sum_{j=1}^{N_i} (\tau_j^+) }, & \text{otherwise} \end{cases}$$  

Where $\tau_i$ is the pheromone value of the $i$th element for food foraging search for $k$th ant. $P_k$ is the probability for $k$th ant to select next path. $N_i$ is the number of selected trials and $\eta_i$ is the stress value of the best ant in a population on the $i$th element. $\alpha$ and $\beta$ are weighted values. $R$ is the probability of selecting operator. $\tau_i^+$ and $\tau_i^-$ are the pheromone value transformed from the operator of pheromone trail centralization or max-min ant system on the $i$th or $j$th trial.

Eq.(3) is the local enhancement operator utilizing the stress value of the best ant in the elitist population for improving the best solution further. But optimum search using Eq.(3) excessively will result in the premature of convergence and the local optimum will be found. In order to increase the diversity of ants’ search, Eq.(4) is applied for the global search operator. The equation included mechanisms of PTC and MMAS are showed in the followings.

(a) Pheromone trail centralization (PTC)

The phenomenon of premature convergence often appears in the process of ACO iteration and affects the performance of ACO. It will lead to the situation that all ants follow the same path and construct the same solution repeatedly. The current best solution can’t be improved anymore. It means that the algorithm has ceased to explore new possibilities of finding better solution. In order to improve this situation, the method of Pheromone Trail Centralization [32] is integrated in ACO to promote its performance and avoid falling into the local optima. The method is shown in the following equations.

$$\tau_{i}^+ = \tau_{i} + \lambda (\tau_{cen} - \tau_{i})$$  

$$\tau_{cen} = 0.7 \cdot \tau_{max}$$  

Where $\tau_{i}$ and $\tau_{i}^+$ are the pheromone values before and after the centralization on the element of topology. $\tau_{cen}$ is the central pheromone value. $\tau_{max}$ is the maximum pheromone value in the every iteration of ACO. $\lambda$, a proportion value, will be set to 0.5 according to the experimental data [32].

(b) Max-min ant system (MMAS)

This max-min ant system [31] is used to limit the strengths of the pheromone trails effectively and avoid premature convergence of the search. The equations used in this study are shown in the followings.

$$\tau_{max} = \frac{1}{(1 - \rho) \cdot f(S^*)}$$  

$$\tau_{min} = \frac{\tau_{max}}{2n}$$  

Where $f(S^*)$ is the current best object value in the iteration and $\rho$ is evaporation rate. $\tau_{max}$ is the upper and $\tau_{min}$ is the lower pheromone limit. $n$ is the number of selected paths/elements. A normalized value of pheromone, $\tau_i^+$, will be got by this method.

(3) Connectivity analysis

For any two elements in a topology of structure to be considered as connected they must share at least one common edge while element sharing only one corner is considered as disconnected as shown in Fig.5.

Fig. 5 (a) The continuous structure (b) discontinuous structure

A topology contained disconnected elements will undergo a structure modification procedure. In the procedure, the removal of disconnected elements or the adding of elements to neighbor of disconnected element will be done randomly until the discontinuous structure is compensated as shown in Fig.6. Fig.6 (b) and (c) shows the procedure of filling element and Fig.6 (d) is the removal procedure. The continuous topology will be further analyzed via the finite element computation to obtain the required displacements and stresses.

Fig. 6 Filling or removal operator in the structure modification
If the structure is still a discontinuity structure after several trials of discontinuity modification, every element of structure will get a minimal pheromone value. The continuous structure will be further analyzed using finite element method to obtain the value of pheromone of every elements using Eq.(9) in the followings.

\[
Phv^k_{obj} = \frac{UY_{ref}}{UY^k} \frac{V_{ref}}{V^k} \frac{1}{\rho^k}
\]  

(9)

Where \(Phv^k_{obj}\) denotes the objective value of pheromone intensity on every element visited by the kth ant. \(UY_{ref}\) and \(V_{ref}\) denote the reference maximum displacement of full structure at the point of loading application and the maximum volume of full structure. \(UY^k\) is the maximum displacement of topology transformed by the kth ant’s trace. \(V^k\) is the volume of the kth ant’s topology. \(n^k\) is the number of elements over the allowable stress of the kth ant’s topology. The goal of this pheromone function is to find the design with uniform stress distribution and minimum volume simultaneous without violating allowable stress.

(4)Pheromone update

After \(n^k\) ants completed the food forging tour and deposited the pheromone on the elements they visited, pheromone intensity on the trails were updated for evaporation. This is done by decreasing the pheromone intensity of trails by a constant factor. In this study, the update rule is showed as follows:

\[
\Delta \tau^k_i = phv^{k}_{obj}
\]  

(10)

\[
\tau^*_i = (1-\rho) \cdot \tau^k_i + \sum_{k=1}^{n^k} \Delta \tau^k_i
\]  

(11)

Where the parameter \(\rho\) (with \(0 \leq \rho < 1\)) is the trail persistence and the term \((1-\rho)\) models the evaporation process. \(\Delta \tau^k_i\) is the increment of pheromone which ant \(k\) deposits on the \(i\)th element it has visited in its route. \(\tau^*_i\) is the final pheromone value remained on the \(i\)th element. This mechanism can avoid unlimited accumulation of the pheromone on trails. While an element is not chosen by the ants, its pheromone on trail is decreased exponentially. This prevents the ants to run in the same route and avoids the occurrence of the stagnation.

(5) Elitist policy

The mechanism presented by Stützle and Linke [35] is utilized in this study to raise the speed of ants’ food searching and increase the convergence of finding the optimal solution efficiently. The equation used in the mechanism is showed as follows:

\[
\tau^\text{elite}_i = \tau^*_i + \tau^\text{elite}_i
\]  

(12)

Where \(\tau^\text{elite}_i\) is the accumulated value formed by many ants in the elitist model on the \(i\)th element.

(6) Stop criterion

In the search process, it is difficult to judge whether the ants already found the optimal topology structure or not. So the maximum iteration number is defined as the stop criterion.

4. Simulation results and discussions

In this paper, two examples with different objective functions will be used to demonstrate the performance of the modified ACO algorithm for topology optimization. Constraints, material properties, and simulation results of the topology models will be discussed respectively as follows.

4.1 Case one

This design domain is showed in Fig.7. This is the optimization of a cantilever beam which is subjected to a vertical load of 3 kN applied at the finite element node on the middle of its right hand side. The dimensions of this beam are set as \(L_x = 0.16m, L_y = 0.1m\), and the thickness \(t = 0.001m\). The Young’s modulus is \(E = 200 \text{ GPa}\) and the Poisson ratio is \(\nu = 0.3\). The allowable stress \(\sigma = 285 \text{ MPa}\) is assumed. In addition, this design domain is divided into 32x20 elements. The Eq.(9) is the pheromone objective function for this case.

Fig. 7 The design model of case one

Fig.8 is the topology optimum solution obtained through 200,000 objective function evaluations (2000 generations by 100 ants). It took about 2 hours on computer with the Intel Core2 Duo CPU. Due to the reason of seven possible seeking directions of the ant without undergoing the structural modification procedure, there were many small voids existing in the final optimal structure. It easily generated many discontinuous topology structures and wasted a lot of computation time by using this method. We will try to find a new method to improve its performance in the future without interfering with the trend of searching the optimal solution.

Fig. 8 The result of topology optimum solution

4.2 Case two

The example proposed by Chapman [17-19] minimizing a cantilevered plate’s compliance is used for further test the performance of modified ACO algorithm developed in this study. Due to constraints, loads and material properties are not clearly illustrated in those papers, but the same problem is found in another reference paper [36] and the parameters used in the paper are also used in this study. Fig.9 is the structure model for test. The left side of the beam is fixed and a vertical load of 3KN is applied at the middle of the right side. The dimensions of the model are \(L_x = 0.16m, L_y = 0.10m\) and the thickness \(t = 0.001m\). The Young’s modulus is \(E = 207 \text{ GPa}\) and the Poisson’s ratio is \(\nu = 0.3\). The design domain is divided into 32x20 quadrilateral elements for finite element analysis. According to the definition of fitness functions from papers, the pheromone value is voted as the
topology’s compliance which is equal to the inner product of the
topology’s maximum displacement ($\delta_{\text{max}}$) at the point of load application with the applied load $F_{\text{APPLIED}}$ as shown in Eq.(13). The objective is to minimize the compliance of plate subjected to maximum volume constraint of 25%. And the equations (14), (15), and (16) are selected from the literature [17]. If the structure’s volume ($V$) is greater than the maximum volume constraint ($V_{\text{max}}$), the structure is assigned to value of pheromone using Eq.(15) and (16). If volume if structure is not greater the maximum volume constraint, the structure is assigned to the value of pheromone using Eq.(14). But if the execution generation is less than 175, Eq.(15) is used as the objective function. In order to understand the performance of modified algorithm developed in this study, the solutions from Chapman (1994) and Rodrigues (1993) will be re-calculate using commercial software Ansys to get the compliance value with the same constraints and material properties. Fig.10 is the recalculated results of the reference papers. After computation of 500 generations, the best topology structure will be used for comparison of the compliance of different researches. From the result of comparison, it can be seen that the optimal solution, shown in Fig.11, is very similar to the solutions in Fig.10. Table 1 shows comparison of results for different optimum solutions. They are very similar in the aspect of volumes, but the compliance of solution of modified ACO algorithm shows 22% smaller in compared with GA-based solution and 8% less than the solution of homogenization method. The modified ACO algorithm can actually solve the topology optimization of structure effectively.

Compliance = $\delta_{\text{max}} \cdot F_{\text{APPLIED}}$ \hspace{1cm} (13)

$$P_{h_{ij}}^{k} = \frac{1}{\ln(\text{compliance})}$$ \hspace{1cm} (14)

$$P_{h_{ij}}^{k} = \begin{cases} \frac{1}{175} \cdot 0.6 \cdot \frac{V - V_{\text{max}}}{V_{\text{max}}} & \text{if } V > V_{\text{max}} \\ \frac{1}{\ln(\text{compliance})} & \text{otherwise} \end{cases}$$ \hspace{1cm} (15)

$$P_{h_{ij}}^{k} = \begin{cases} 1 - 0.6 \cdot \frac{V - V_{\text{max}}}{V_{\text{max}}} & \text{if } V < V_{\text{max}} \\ \frac{1}{\ln(\text{compliance})} & \text{otherwise} \end{cases}$$ \hspace{1cm} (16)

5. Conclusions

In this study, there are some conclusions can be drawn in the following.

(1) A novel concept, a modified ACO algorithm imitating the behavior of real ant colonies, is presented for topology optimization of structure. From the results of two examples with different pheromone definitions, the purposed algorithm can solve the topology optimization of structure successfully and effectively.

(2) The results obtained in this study are better than results of previous literatures without violating volume and displacement constraints. The compliance of optimal structure in the case two is smaller than results of previous studies [17-19].

(3) The global search and local enhancement in transition rule lead to many voids existed in the final optimal topology structure. In the future, there should be a better mechanism to solve this problem.

(4) Because the algorithm developed in this study is applied to single objective function only and it is easy to be trapped into local optimum. Some advanced mechanisms such as multimodal optimization, multi-objective optimization, and parallel processing will be developed further to improve the efficiency and promote the ability of modified ACO algorithm in solving other complex real life engineering optimization problems.

6. References

