Spectrum Sensing of OFDMA Systems for Cognitive Radio Networks

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Abstract—To fully exploit wireless radio resource and, thus, increase spectrum efficiency, cognitive radios shall sense wireless environments and identify interference to allow opportunistic transmissions for secondary systems. Based on Chen et al. in their work about a terminal architecture for cognitive radio networks, by further obtaining transmission information with a rate–distance nature that is extended from an overlay concept, secondary systems can even leverage busy duration of primary systems to enhance the opportunity to use spectrum under derived tolerable interference to the primary orthogonal frequency-division multiple access (OFDMA) system. Our proposed novel multistate (i.e., existence, activity, and data transmission rate) spectrum sensing can thus effectively acquire a set of cognitive information for OFDMA mobile communication systems under a general system model. Through the received signal strength indicator (RSSI), fundamental symbol rate, cyclic property of the OFDMA signal, and the control and management signal, we can precisely determine the existence and activity of the primary (OFDMA) system within a subband via Neyman–Pearson (NP) criterion. With a priori knowledge of the frame structure of potential primary systems, communication parameters, including data transmission rate and resource allocation state, can be extracted by analyzing the frame header. Through appropriate parametric adjustments, our sensing procedure can be extended to state-of-the-art OFDMA systems.

Index Terms—Cognitive cycle, cognitive radio, orthogonal frequency-division multiple access (OFDMA), rate–distance relationship, spectrum sensing.

I. INTRODUCTION

TREMENDOUS mobile communication standards and technologies have been developed to cope with different applications, coverage, and transmission rates. Instead of developing a universal air interface, software-defined radios (SDRs) [1] for reconfiguring communication by adjusting system parameters are considered to be more feasible future mobile communication systems. In addition to low-cost highly flexible radios, spectrum efficiency, which is driven by the need for multimedia wireless applications in limited radio resources, is another important issue for state-of-the-art mobile communication technologies. According to recent measurements from the Federal Communications Commission (FCC) [4], [5], spectrum utilization at any time is roughly 10%. This instance highlights possible improvement of the current static spectrum allotment policy. Further generalizing the SDR concept, Mitola pioneered proposed cognitive radios [2], [3], which sense and cognize environments and then transmit data in idle/nonoccupied channels to increase spectrum efficiency. Therefore, spectrum sensing and cognition are among major functions in cognitive radios [7]. Conventionally, spectrum sensing is implemented by an energy detector, i.e., received signal strength indicator (RSSI) [10]–[14]. However, because of wireless fading channel and the existence of other noncollaborative interference, a simple energy detector is not sufficient to determine system existence and its operation state. Recently, Hsieh and Chen [31] have proposed a method to identify a frequency-hopping system and wideband systems by unique characteristics in frequency and time domains under maximum a posteriori criterion. However, this method is not enough for orthogonal frequency division multiple access (OFDMA) and multicast code-division multiple access (CDMA) [23] due to dynamic allocation of radio resources. Furthermore, the aforementioned methods only take account of spectrum holes [7]. To explore the extreme of radio resource utilization or medium access control (MAC) [44], [45] in cognitive radio “networks” so that the packets can be forwarded after using spectrum holes, we have to further identify the activity and transmission rate of primary systems. Consequently, more appropriate spectrum sensing for OFDMA is required, as proposed in this paper.

Let us recall a very fundamental situation in modern mobile communications. Based on the received power level, modern systems automatically adjust physical transmission rate accordingly and, thus, throughput via MAC [40]. We observe from a realistic operation of modern adaptive modulation and coding (AMC) [27] wireless systems such as the IEEE 802.11a/g that the physical layer (PHY) selects higher spectral efficient modulations for stronger received power, which are generally more sensitive to interference, noise, and fading, to yield high throughput. At lower received power, PHY selects lower spectral efficient modulations, which are more resistible to interference. We call such a nature as the rate–distance feature of wireless communications as an extension of overlay systems [8], [9]. Here, “distance” is considered as a measure of received signal power rather than just Euclidean or propagation distance. If we could guarantee “distance” between cognitive radios and primary systems in the channels that are occupied by
transmissions of the primary link, cognitive radios can reuse the channels and increase the opportunity to transmit. This feature has hardly been explored in traditional cognitive radio spectrum sensing research that considers only spectrum holes. Therefore, our spectrum sensing procedure also provides a methodology for sensing radio resource in the busy duration of primary systems.

In this paper, we focus on sensing OFDMA systems due to their potential applications for future mobile communications. The rest of this paper is organized as follows. In Section II, we present the system model, a set of cognitive information, and cognitive cycle. In Section III, we provide a procedure for identifying the existence and activity of the primary (OFDMA) system and design optimal detectors under a Neyman–Pearson (NP) criterion. Then, available radio resource to secondary systems in occupied channels is determined in Section IV. The spectrum sensing algorithm and block diagram are summarized in Section V. Section VI presents numerical results and comparisons. Concluding remarks are made in Section VII.

II. SYSTEM MODEL

A. Primary System

Consider a cell-structured OFDMA system. The base station (BS) lies at the center of cell with coverage radius \( R \) (see Fig. 1). Let \( W \) be the total bandwidth, which contains \( N_f \) subbands. One subband is allocated to one cell and is further divided into \( N \) channels. Similarly, the timing axis is segmented by the OFDMA symbol period. Then, the radio resource, which is composed of channels and OFDMA symbols, is allocated to active users in the cell. This information is usually specified in a frame header. Without loss of generality, we adopt the frame structure in IEEE 802.16 [6] as an example. The frame starts with a preamble, which is mainly used for synchronization and channel estimation, followed by a frame header, including DL_MAP and UL_MAP. In addition, the frame header also defines transmission parameters such as the forward error control (FEC) code rate and the modulation in DL and UL bursts, respectively. To simplify the model without loss of generality, assume that there are two data transmission rates, which is determined by AMC. Although the data transmission rate can adaptively be adjusted, the fundamental symbol rate is rather fixed in most systems.

B. Cognitive Radios

We categorize received signals at cognitive radios within a subband of the primary system into the following six classes:

- \( C_0 : w_{LP}(t) \) background noise;
- \( C_1 : s_1(t) \) primary system traffic signal;
- \( C_2 : s_c(t) \) primary system control signal;
- \( C_3 : i_s(t) \) interfering traffic signal with the same fundamental symbol rate as the primary system;
- \( C_4 : i(t) \) interfering traffic signal without the same fundamental symbol rate as the primary system;
- \( C_5 : i_c(t) \) interfering control signal.

Here, \( w_{LP}(t) \) denotes the equivalent baseband (BB) additive white Gaussian noise (AWGN) with zero mean, and the two-sided power spectrum density, i.e., \( N_0 \) and \( s_1(t) \), denotes the traffic signal of the primary system. In addition, \( i_s(t)/i(t) \) denotes the traffic signal from an interfering system with and without the same fundamental symbol rate as the primary system and is assumed white within a subband. By the central limit theorem [37], traffic signals are modeled as independent circular symmetric complex Gaussian processes. Finally, \( s_c(t) \) and \( i_c(t) \) denote control and management signals from the primary and interfering systems, respectively, and are periodically transmitted. In the following, we just denote “control and management signals” by “control signal” for simplicity.

By analyzing the received signals, cognitive radios determine the operation state of the primary system in the following five possible states:

- \( S_0 \): nonexistent, i.e., neither \( C_1 \) nor \( C_2 \) exists;
- \( S_1 \): existent and inactive, i.e., \( C_2 \) exists, but \( C_1 \) does not exist;
- \( S_2 \): existent, active, and high rate, i.e., \( C_1 \) exists, and traffic is transmitted at a high rate in a channel;
- \( S_3 \): existent, active, and low rate, i.e., \( C_1 \) exists, and traffic is transmitted at a low rate in a channel;
- \( S_4 \): existent, active, and idle, i.e., \( C_1 \) exists, and no traffic is transmitted in a channel.

In this paper, we further consider the state existent and inactive, in which primary users will enter the primary network, or more aggressively, cognitive radios could adjust system parameters and connect to the primary BS in cooperative communications. In addition, to find out potential radio resource of secondary systems within occupied subbands, we require information about data transmission rate and subband utilization, which is governed by the MAC protocol.

C. Cognitive Cycle

The operation of cognitive radios can be viewed as a finite-state machine and, thus, be considered by cognitive cycle [3], [7]. We facilitate our cognitive cycle, as shown in Fig. 2, with two sensing paths. Path A is responsible for detecting the existence and activity of the primary system, whereas Path B is used to decode the frame header and determine frequency-time...
utilization. For each OFDMA symbol, we establish two $N_F \times N$ tables—the channel-state table and the radio resource table—where the $(n, m)$th element records the operation state of the primary system and maximum transmission power at cognitive radios of the $m$th channel within the $n$th subband, respectively. The networking terminal operator in Fig. 2 collects sensing information and makes optimal decision of the operating mode, including configuration of software and hardware in MAC [30] and SDR, network routing and handoff, and maintenance of requirements from upper layer applications. Based on this system architecture, cognitive radio networks can practically be implemented in future mobile communications.

1) Cognitive Cycle—Path A: RSSI is a simple widely applied method in spectrum sensing; however, it is impossible to distinguish the signal of the primary system $C_1$ from interfering signals, i.e., $C_3$ and $C_4$. For example, in a 2.4-GHz ISM band, large RSSI may result from a microwave inside the band rather than the IEEE 802.11b network. In addition, noise uncertainty significantly degrades energy detector performance [15]. Therefore, we have to extract features of the traffic signal by transformation, which suppresses noise and interference signals. A fundamental symbol rate is invariant or belongs to a finite set (e.g., scalable WiMAX); thus, we model traffic signals as cyclostationary processes, which are induced by pulse trains, and detect such a feature by the spectral correlation function [24], [25]. Although preliminary spectrum sensing exists [16]–[18], we adopt a spectral-line generator to meet the need for quickness in cognitive radios.

However, in our system model, $C_1$ and $C_3$ have the same fundamental symbol rates. By means of a cyclic prefix (CP) in OFDMA systems, we can precisely detect the existence of the active primary system. Finally, BSs periodically broadcast a control signal (e.g., a beacon in IEEE 802.11) to maintain mobile network operation; thus, we could identify the existence of the inactive primary system by this property.

2) Cognitive Cycle—Path B: In Path A, we determined the state of the primary system: 1) nonexistent $S_0$; 2) existent and inactive $S_1$; and 3) existent and active $S_2, S_3, S_4$. We further consider the channels that are occupied by transmissions and apply rate–distance nature so that simultaneous communication is possible. The information about channel occupation and data transmission rate can be obtained by analyzing the frame header after cognitive radios achieve synchronization with the primary BS.

3) Cognitive Information: We summarize a set of cognitive information in spectrum sensing under OFDMA systems as follows:

- RF signal processing: includes (carrier) frequency, bandwidth, RSSI, signal-to-noise-and-interference ratio (SINR), and noise power;
- BB predetection signal processing: includes fundamental symbol rate, carrier and timing, pilot signal, and channel fading;
- BB post-detection signal processing: includes system identification, modulation parameters, and FEC type and rate;
- Network processing information: includes multiple access protocol and radio resource allocation.

Suppose that cognitive radios know a priori information about potential primary systems that include frequency planning, frame structure, subcarrier structure (e.g., fast Fourier transform (FFT) size and pilot positions), transmission parameters (e.g., fundamental symbol rate, CP length, and FEC type), and tolerant interference of primary systems, because spectrum usage is regulated, and system specifications are well defined.

Fig. 2. Cognitive cycle.
III. Discrimination of the State of the Primary System

In this section, we propose a simple design to identify the existence and activity of the primary (OFDMA) system within one subband by the RSSI, fundamental symbol rate, cyclic property, and control signal of the primary system, where RSSI is simply an energy detector. In general, we do not know the a priori probability of existence of the primary system. Hence, we adopt an NP criterion [36] to design the spectrum-sensing procedure.

A. Fundamental Symbol Rate

If there exists traffic signal from the primary system, the received signal at cognitive radios may be written as

\[
z(t) = \mathbb{R}\left\{ e^{j2\pi f_c t} \left( \sum_{n=-\infty}^{\infty} x(n) h(t - n T_s) + w_{LP}(t) \right) \right\}
\]

\[
= \mathbb{R}\left\{ e^{j2\pi f_c t} \left( s(t) + w_{LP}(t) \right) \right\} = \mathbb{R}\{y(t)\}.
\]

(1)

Here, \( f_c \) denotes the carrier frequency, \( T_s \) denotes the reciprocal of the fundamental symbol rate, \( h(t) \) denotes the pulse-shaping filter, and \( x(n) \) denotes the transmitted data in time domain. Alternatively, we have (2), shown at the bottom of the page, wherein \( X_m(k) \) denotes modulated data at the \( k \)th subcarrier in the \( m \)th OFDMA symbol, \( N_{FFT} \) is the FFT size, and \( N_{CP} \) is the CP length. Without loss of generality, \( h(t) \) is assumed the square-root raised cosine filter with a roll-off factor less than 100% [32], and (45) in Appendix A becomes

\[
E\left[ z^2(t) \right] \approx \frac{1}{2} \left( \frac{\sigma^2_x}{T_s} + \sigma^2_w \right) + \frac{\sigma^2_e}{T_s} \mathbb{R}\{ Z_1 e^{j2\pi \frac{\theta}{T_s}} \}
\]

(3)

where

\[
Z_m = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(j\omega) H^*(j(\omega - 2\pi m/T_s)) d\omega.
\]

(4)

Noise power \( \sigma^2_w \) is shown in (44) in the Appendix, \( Z_m = Z_m^* \), and \( Z_0 = 1 \). Note that the component at the fundamental symbol rate is independent of noise power, and thus, the challenge of noise uncertainty in an energy detector is relaxed. However, the received signal power in the second term of (3) is reduced by factor \( Z_1 \). Therefore, the performance of the detector depends on \( Z_1 \). Based on (4), we conclude that, if \( h(t) \) has larger bandwidth (i.e., a higher roll-off factor), the detector has better performance. A similar phenomenon happens in timing tracking systems [33], [34]. Furthermore, this algorithm can be implemented at the radio frequency (RF) part in front of an analog–digital converter (ADC); thus, it can be done quickly.

B. CP of OFDMA Signal

The initial setup for detecting the existence of CP is similar to [38]. Collect \( 2N_{FFT} + N_{CP} \) samples with sampling rate \( 1/T_s \) and assume that this region contains one complete OFDMA symbol. The detection problem becomes

\[
H_0: r(n) = w_0(n), \quad n = 0, 1, \ldots, 2N_{FFT} + N_{CP} - 1
\]

\[
H_1: r(n) = s(n) + w_1(n), \quad n = 0, 1, \ldots, 2N_{FFT} + N_{CP} - 1.
\]

Under \( H_1 \), there exist timing offset (TO) \( \theta \) and frequency offset \( \varepsilon \) due to the lack of synchronization. Then, the traffic signal of the primary system becomes

\[
s(n) = x(n - N_{CP} - \theta) e^{j2\pi n\varepsilon/N_{FFT}}.
\]

(5)

Let \( I \) and \( I' \) be two sampling intervals, which contain the CP and its replica, respectively, i.e.,

\[
I = \{\theta, \theta + 1, \ldots, \theta + N_{CP} - 1\}
\]

\[
I' = \{\theta + N_{FFT}, \theta + N_{FFT} + 1, \ldots, \theta + N_B - 1\}
\]

where \( N_B = N_{FFT} + N_{CP} \). If there exists the primary system, the samples in the CP and their copies are correlated [38], or

\[
E\left[ r(n)r^*(n + m) | H_1 \right] = \begin{cases} \sigma^2_x + \sigma^2_w, & m = 0 \\ \sigma^2_e e^{-j2\pi \varepsilon}, & m = N_{FFT}, n \in I \\ 0, & \text{otherwise}. \end{cases}
\]

(6)

We model \( w_1(n) \) as a white Gaussian process with zero mean and variance \( \sigma^2_w \), and \( E[|s(n)|^2] = E[|x(n)|^2] = \sigma^2_x \). On the other hand, \( w_0(n) \) denotes the superposition of interfering signals and background noise and is modeled as a white Gaussian process with zero mean and variance \( \sigma^2 = \sigma^2_x + \sigma^2_w \), i.e.,

\[
E\left[ r(n)r^*(n + m) | H_0 \right] = \begin{cases} \sigma^2, & m = 0 \\ 0, & \text{otherwise}. \end{cases}
\]

(7)

We assume that the total powers under both hypotheses are the same, which leads to the worst case, because the energy detector does not work. Then, the likelihood ratio test (LRT) becomes (see Appendix B)

\[
M(x) = |S(\theta)| \cos (2\pi \varepsilon + \angle S(\theta)) \geq \frac{\rho}{2} P(\theta) \geq \tau_{CP} \]

\[
H_1
\]

(8)

where

\[
S(\theta) = \sum_{n \in I} r(n)r^*(n + N_{FFT})
\]

(9)

\[
P(\theta) = \sum_{n \in I} \left[ |r(n)|^2 + |r^*(n + N_{FFT})|^2 \right]
\]

(10)

\[
\rho = \frac{\text{SINR}}{\text{SINR} + 1}
\]

(11)

Here, \( \rho \) is the SINR ratio.

\[
x(n + (N_{FFT} + N_{CP})m) = \begin{cases} \frac{1}{\sqrt{N_{FFT}}} \left( \sum_{k=0}^{N_{FFT}-1} X_m(k) e^{j2\pi \frac{k \varepsilon}{N_{FFT}}} \right), & \text{if } n \in \{0, 1, \ldots, N_{FFT} - 1\} \\ x(n + (N_{FFT} + N_{CP})m + N_{FFT}), & \text{if } n \in \{-N_{CP}, -N_{CP} + 1, \ldots, -1\} \end{cases}
\]

m \in \mathbb{Z}

(2)
and SINR = $\sigma^2_s/\sigma^2_w$. The decision metric $M(r)$ only depends on the samples in the CP and their copies whose energy $P(\theta)$ and correlation $S(\theta)$ are considered.

With the unknown parameters (i.e., $\theta$, $\varepsilon$, $\sigma^2$, and SINR) in (8), it becomes a composite detection problem, which is desirable for finding the uniformly most powerful (UMP) test [36]. However, the UMP test does not exist, because the decision region depends on $\theta$. In the absence of the UMP test, generalized LRT (GLRT) can be applied. We assume that the SINR is known and has (see Appendix B)

$$M(r) = \frac{\sum_n |r(n)|^2}{\sum_n |r(n)|^2 - \frac{2\rho}{\pi^2} \max \{|S(\theta)| - \frac{\theta}{2} P(\theta)| \} < \tau_{CP}}$$

(12)

Note that this detector is just the ratio of estimations of the total energy under two hypotheses, and the frequency offset does not affect (12). In addition, we evaluate the probability of a false alarm in Appendix C and have

$$P_F = \int_{\tau_{CP}}^{\infty} f(M(r)|H_0) dM \approx Q \left( \frac{-\mu_1 - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2}} \right)$$

(13)

where $f(\cdot)$ is a probability density function, $Q(x)$ denotes the right-tail probability of a Gaussian random variable with zero mean and unit variance [36], and (14)–(17), shown at the bottom of the page, holds. By taking the inverse of $Q(x)$, we can obtain the optimal threshold under the NP criterion. Next, we investigate a special case, i.e., $\rho = 0$, which results in the detection of a white random signal under AWGN with unknown noise power, and the decision metric in (12) becomes $M(r) = 1$. This case explains why a simple energy detector does not work with unknown noise power.

Finally, to deal with unknown SINR, note that (12) and (13) depend on SINR via $\rho$, and we design a robust CP detector at the target SINR SINR. The parameters $\rho$ and $\tau_{CP}$ can be calculated in advance by (11) and (13), with SINR = SINR. Furthermore, this detector explores the CP property of the target OFDMA system; thus, it can be used to recognize among OFDM-based systems with different system parameters $N_{FFT}$ and $N_{CP}$ (e.g., IEEE 802.11a and IEEE 802.16 in a 5-GHz unlicensed band).

C. Control and Management Signal

We do not specify control signals of potential primary systems but model them as white Gaussian processes. Therefore, this detector is robust to system variation. The control signal of the primary system is modeled as a zero-mean white Gaussian process with transmission-period $M_C$, samples and duration $N_C$ samples. To detect the control signal, we collect $M_C$ samples and assume that this interval contains one control signal period. The detection problem turns out to be

$$H_0 : r(n) = w(n), \quad n = 0, 1, \ldots, M_C - 1$$

$$H_1 : r(n) = \begin{cases} w(n), & n \notin J \\ s_r(n) + w(n), & n \in J \end{cases}$$

where $J$ denotes the sampling interval, which contains the control signal, or $J = \{\phi, \phi + 1, \ldots, \phi + N_C + 1\}$, where $\phi$ denotes the TO of the control signal. Note that the frequency offset is ignored, because it contributes phase shift, but only a magnitude of the received signal provides information in a likelihood function. In addition, the statistics of the received signal under hypotheses are shown as follows:

$$E[r(n)r^*(n+m)|H_1] = \begin{cases} \sigma^2_s + \sigma^2_w, & m = 0, n \notin J \\ \sigma^2_w, & m = 0, n \in J \\ 0, & \text{otherwise} \end{cases}$$

(18)

$$E[r(n)r^*(n+m)|H_0] = \begin{cases} \sigma^2_w, & m = 0 \\ 0, & \text{otherwise}. \end{cases}$$

(19)

The GLRT becomes

$$M(r) = \max_{\phi} \left\{ \sum_{n=\phi}^{\phi+N_C-1} |r(n)|^2 \right\} \frac{H_1 \geq \tau_{CP}}{H_0 \leq \tau_{CP}}$$

(20)

This detector finds the TO and measures the energy of the received signal in the interval of the control signal. Next, we release the assumption of knowing noise power, which occurs in detecting the superposition of the control signal of the primary system and noncollaborative interference or background noise with unknown power. In addition to noise power, signal power is another unknown parameter under $H_1$. Then, we have (see Appendix D)

$$M(r) = \max_{\phi} \left\{ M_C \ln \left( \sum_n |r(n)|^2 \right) - N_C \ln \left( \sum_{n \notin J} |r(n)|^2 \right) \right\} \frac{H_1 \geq \tau_{CP}}{H_0 \leq \tau_{CP}}$$

(21)

$$\mu_1 = \frac{(2N_{FFT} - N_{CP})(1 - \tau_{CP})}{1 - \rho^2}$$

(14)

$$\sigma^2_1 = \frac{(2N_{FFT} - N_{CP})(1 - \tau_{CP})^2}{1 - \rho^2}$$

(15)

$$\mu_2 = 2N_{CP} - \frac{2N_{CP} - \rho \sqrt{\pi N_{CP}}}{1 - \rho^2} \tau_{CP}$$

(16)

$$\sigma^2_2 = 2N_{CP} \left( 1 - 2 \frac{1 - \rho \sqrt{\pi} (\sqrt{N_{CP} + 1} - \sqrt{N_{CP}})}{1 - \rho^2} \frac{\tau_{CP}}{\sqrt{\pi} (\sqrt{N_{CP} + 1} - \sqrt{N_{CP}})} + \frac{1 + 2\rho^2(1 - \pi/4) - 2\rho \sqrt{\pi} (\sqrt{N_{CP} + 1} - \sqrt{N_{CP}})^2}{(1 - \rho^2)^2} \right)$$

(17)
Finally, consider a situation where a nontarget system exists and broadcasts control signal. If cognitive radios only measure the received energy, they fail to discriminate between this system and the primary system. Alternatively, it is reasonable to assume that periods of the control signal from different systems are distinct, because each standard is designed for specific purposes (e.g., coverage and data rate), which results in individual timing parameters. Then, cognitive radios can track the period of the control signal from the primary system to detect the control signal. We rewrite the control signal as follows:

\[ r(n) = s_c(n) \sum_{m=-\infty}^{\infty} g(n - mM_C) + w(n) \]

\[ n = 0, 1, \ldots, LM_C - 1 \]  \hspace{1cm} (22)

where

\[ g(n) = \begin{cases} 1, & n = 0, 1, \ldots, N_C - 1 \\ 0, & \text{otherwise} \end{cases} \]  \hspace{1cm} (23)

and \( L \) denotes the number of periods that we observe. To track the period of the control signal, we adopt an approach similar to fundamental-symbol-rate tracking, i.e., squaring the received signal and then extracting the magnitude at the fundamental frequency of the control signal. We rewrite the control signal as follows:

\[ E \left[ |r(n)|^2 \right] = \sigma_s^2 + \sum_{m=-\infty}^{\infty} g(n - mM_C) + \sigma_w^2 \]

\[ n = 0, 1, \ldots, LM_C - 1. \]  \hspace{1cm} (24)

We observe that (24) is a periodic function with period \( M_C \), or specifically, the magnitude at frequency \( 1/M_C \) is:

\[ L \frac{\sin \pi N_C/M_C}{\sin \pi/M_C} \sigma_s^2. \]  \hspace{1cm} (25)

To provide high throughput, the control signal is not frequently transmitted, and thus, \( M_C \) is large, which leads to long sensing duration. However, the primary BSs are reasonably assumed to be fixed; thus, this process is only initially taken.

Furthermore, this detector can also be used to detect OFDM-based signal with zero padding in the prefix, such as multi-band OFDM. However, in detecting the control signal by the received energy, we assumed that the observation interval contains one control signal period, which is true if \( M_C \gg N_C \). For OFDM-based signals, \( M_C = N_{\text{FFT}} + N_{\text{CP}}, \) \( N_C = N_{\text{FFT}}, \) and the aforementioned condition is not satisfied. To solve this problem, we could collect \( 2N_{\text{FFT}} + N_{\text{CP}} \) samples, as we did in the CP detector, and the optimal detector has similar structure as in (21). Alternatively, we could detect such OFDM-based signals by tracking the symbol period similar to the method in detecting the fundamental symbol rate.

IV. AVAILABLE RADIO RESOURCE TO SECONDARY SYSTEMS

To take advantage of the rate–distance nature, we reconsider the distance relation in Fig. 1 and the primary communication link to utilize some channel. Let \( \text{SINR}_{\text{min}} \) be the minimum SINR at which the primary receiver \( \text{Rx}^1 \) maintains the current link quality. Note that this value will depend on the modulation scheme and the FEC type and rate. Thus, we obtain an inequality, i.e.,

\[ \text{SINR}_1 = \frac{G_{11}P_1}{G_{21}P_2 + N_1} \geq \text{SINR}_{\text{min}} \]  \hspace{1cm} (26)

where \( P_1 \) denotes the transmission power at \( \text{Tx}^1 \), \( N_1 \) denotes the noise power at \( \text{Rx}^1 \), and \( G_{ij} \) denotes the power loss from \( \text{Tx}^i \) to \( \text{Rx}^j \). Then, we have

\[ P_{2,\text{max}} = \frac{1}{G_{21}} \left( \frac{G_{11}P_1}{\text{SINR}_{\text{min}}} - N_1 \right) = \frac{I_{\text{CR}}}{G_{21}} \]  \hspace{1cm} (27)

where \( I_{\text{CR}} \) denotes the maximum-tolerance interference of the primary system at the channel. \( I_{\text{CR}} \) depends on the signal quality of the primary communication link; thus, we can roughly infer it from the transmission rate and assume that it is known. In addition, “distance” is actually a measure of the received signal power; thus, we have to specify the relation between received power and propagation distance. Without loss of generality, we adopt large-scale path loss that includes log-distance path loss and log-normal shadowing [28], because spectrum sensing does not treat instantaneous signal reception. The power loss between two nodes \( G \) is given by

\[ G = K d^{-\alpha} 10^{\beta/10} \]  \hspace{1cm} (28)

where \( K \) is a normalization constant, \( d \) is the distance between these two nodes, \( \alpha \) is a path-loss exponent, and \( \beta \) is a shadowing parameter, which is modeled as a Gaussian random variable with zero mean and variance \( \sigma^2 \). In the following, we conditionally determine the radio resource on whether information about the distance between \( \text{Tx}^2 \) and \( \text{Rx}^1 \) is available or not, which corresponds to uplink and downlink environments.

A. Radio Resource: Uplink

In uplink, \( \text{Rx}^1 \) is the primary BS, and cognitive radios can trace its location by the received signal strength of a preamble. Then, we get the following inequality:

\[ KP_2 r^{-\alpha} 10^{\beta/10} \leq I_{\text{CR}}. \]  \hspace{1cm} (29)

Due to shadowing factor \( \beta \), the interference level is a random variable and, hence, can only be guaranteed in probability sense. The optimal criterion is maximum transmission power such that the probability of interfering with the primary system is less than \( \xi \), i.e.,

\[ P_{2,\text{max}} = \arg \max_{P_2} \left\{ \Pr \left( KP_2 r^{-\alpha} 10^{\beta/10} > I_{\text{CR}} \right) \leq \xi \right\}. \]
This criterion has the same concept as in the NP criterion, and we have
\[
P_{2,\text{max}}^{UL} = \frac{I_{CR}r^\alpha}{K} - 10^{\sigma_\beta Q^{-1}(\xi)/10}. \tag{30}
\]
Note that \(P_{2,\text{max}}^{UL}\) is proportional to \(I_{CR}R\).

**B. Radio Resource: Downlink**

In downlink, Rx\(^1\) becomes the primary mobile station (MS), and it is hard for cognitive radios to estimate the location of the MS. It is reasonable to assume that MSs are uniformly distributed around a coverage region; thus, by the Bayesian approach, we have
\[
\Pr\{P_2G_2 > I_{CR}\} = \int_0^\infty Q\left(\frac{1}{\sigma_\beta} 10\log_{10}\left(\frac{I_{CR}r^\alpha}{P_2 K}\right)\right) dF(r)
\]
where
\[
F(r) = \begin{cases} \frac{A_H(r,c,D)}{\pi D^2}, & \text{high-rate region} \\ \frac{A_L(r,c,D,R)}{\pi (D^2 - D)}, & \text{low-rate region.} \end{cases} \tag{31}
\]
In (32) (see Appendix E), we have
\[
A_H(r, c, D) = \begin{cases} \pi r^2 u(D-c), & 0 \leq r < |D-c| \\ \theta_1 r^2 + \theta_1 D^2 - cv, & |D-c| \leq r < D+c \\ \pi D^2, & D+c \leq r \end{cases}
\]
and
\[
A_L(r, c, D, R) = A_H(r, c, R) - A_H(r, c, D) \tag{33}
\]
where \(u(x)\) is the unit step function. In addition
\[
u = \sqrt{D^2 - u^2} \tag{36}
\]
and \(\theta_1, \theta_2 \in [0, \pi]\). Then
\[
P_{2,\text{max}}^{DL} = \arg_{P_2} \left\{ \int_0^\infty Q\left(\frac{1}{\sigma_\beta} 10\log_{10}\left(\frac{I_{CR}r^\alpha}{P_2 K}\right)\right) dF(r) = \xi \right\}
\]
and can numerically be obtained. By similar procedures, we can easily generalize to multilevel data transmission rates.

We now acquire a rough range of \(I_{CR}\) from \textit{a priori} information; however, it could dynamically be calculated according to the instantaneous link quality of the primary system. If cognitive radios can exchange information with the primary system, i.e., collaborative coexistence [29], after achieving synchronization with the primary BS, they can more efficiently squeeze the radio resource and further achieve channel capacity.

**V. Spectrum-Sensing Procedure**

The spectrum sensing algorithm serially senses the whole spectrum subband by subband. To quickly acquire spectrum utilization within a subband, we discriminate the traffic signal of the primary system from noise and interference by received energy and extracting signal feature, including the fundamental symbol rate and CP, and then assure the existence of the active primary (OFDMA) system. We then consider the existence of the inactive primary system by means of control signal under three conditions: 1) background noise \(C_0\); 2) interfering traffic signal \(C_3\) and \(C_4\); and 3) interfering control signal \(C_5\). Although there exists an active primary system, we acquire the utilization status of each channel along the time axis to further increase spectrum efficiency by decoding the frame header and then determining the radio resource to secondary systems. This general sensing algorithm, which can be applied to IEEE 802.16 and other OFDMA systems by adjusting the system parameters \(T_s, N_{FFT}, N_{CP}, M_C, N_C\), is summarized as follows.

1) Initially, set the channel-state table as \(S_0\) and reset the radio resource table.
2) For \(n = 1\) to \(N_F\) (subband), do the following substeps.
   Then, go to step 3.
   a) Measure the RSSI and distinguish the hypothesis test.
      \(H_0\): traffic signals do not exist.
      \(H_1\): traffic signals exist.
      If \(H_0\) is true, go to step 2.f; otherwise, go to step 2.b.
   b) Track the fundamental symbol rate of the primary system.
      \(H_0\): traffic signals with the fundamental symbol rate of the primary system do not exist.
      \(H_1\): traffic signals with the fundamental symbol rate of the primary system exist.
      If \(H_0\) is true, go to step 2.g; otherwise, go to step 2.c.
   c) By the CP of the OFDMA signal, separate the non-collaborative interference and traffic signal from the primary system.
      \(H_0\): traffic signals with the CP property of the primary system do not exist.
      \(H_1\): traffic signals with the CP property of the primary system exist.
      If \(H_0\) is true, go to step 2.g; otherwise, go to step 2.d.
   d) Synchronize with the primary BS, including carrier and timing synchronization, and channel estimation by the preamble [39]. Then, go to step 2.e.
   e) Decode the frame header (DL_MAP and UL_MAP in this case) and obtain the transmission parameters of the primary system, including the FEC rate, modulation scheme, and resource allocation. These parameters are used to set the \(n\)th row of the channel-state table from \(S_2\) to \(S_4\). Then, go to step 2.i.
   f) Measure the energy of the control signal and detect the hypothesis test.
      \(H_0\): control signals do not exist.
      \(H_1\): control signals exist.
If $H_0$ is true, go to step 2.i; otherwise, go to step 2.h.

g) Extract the control signal of the primary system from noncollaborative interference.

$H_0$: the control signal of the primary system does not exist.

$H_1$: the control signal of the primary system exists.

If $H_1$ is true, set the $r$th row of the channel-state table as $S_1$. Then, go to step 2.i.

h) Track the period of the control signal and discriminate between control signals from the primary system and other systems.

$H_0$: the control signal of the primary system does not exist.

$H_1$: the control signal of the primary system exists.

If $H_1$ is true, set the $r$th row of the channel-state table as $S_1$. Then, go to step 2.i.

i) (optional) Identify the system by the fundamental symbol rate, cyclic properties of OFDMA systems, and control signal. This step is critical information in cognitive and cooperative networking [26]. Then, go to step 2.

3) According to the channel-state table, set the radio resource table by (30) and (39). Then, end the sensing procedure.

In steps 2.4 and 2.5, if synchronization or packet decoding cannot be achieved due to the lack of frame information, this instance leads to a degenerative case, in which $\Pi_5$ cannot be achieved due to the lack of frame information, this step is critical information in cognitive and cooperative networking [26]. Then, go to step 2.

VI. NUMERICAL RESULTS

A. Detector of the State of the Primary System

To illustrate detector performance, we show the probability of detection $P_D$ versus the SINR under constraint on the probability of false alarm $P_F < 0.05$ in AWGN and frequency-selective fading channels. We adopt SUI-3 Channel [42], which is the channel model for IEEE 802.16. We develop the theory for spectrum sensing, but it is still effective for multipath channels by simulation. In addition, to protect the primary system, the probability of detection must be high enough, e.g., 0.95. Therefore, we also investigate the required sensing duration to achieve system requirement, $P_F < 0.05$, $P_D > 0.95$, at SINR = 0 dB. According to IEEE 802.16, we set the fundamental symbol rate to 20 MHz, FFT size $N_{\text{FFT}} = 1024$, and CP length $N_{\text{CP}} = N_{\text{FFT}}/32 = 32$.

1) Fundamental-Symbol-Rate Detector: We first show spectrum magnitude with a roll-off factor equal to 0.35 and 0.55, respectively, in Fig. 5(a). It is easy to observe the tone at the fundamental symbol rate. As our analytical result, noise does not affect magnitude at the fundamental symbol rate but only at the DC component, and the one with the larger roll-off factor has larger magnitude at the fundamental symbol rate. However, note that spectrum magnitude below 10 MHz is large to a certain degree, which comes from the Signal × Signal term due to the randomness of the signal and is called self noise [35]. Therefore, to design a multisymbol-rate detector, asymmetric mutual interference between decision metrics must be considered. This task is similar to [43], where the number of active users in CDMA systems is identified, and therefore, subspace signal processing and the multiple signal classification algorithm can be applied.
Fig. 5(b) shows the probability of detection versus SINR and sensing duration ($T_2$ in Fig. 4) in AWGN and SUI-3 fading channels. We observe that a larger roll-off factor outperforms. We also note that the fading channel does not much degrade detector performance, particularly in a large roll-off factor, because the periodicity of pulse trains is retained under the fading channel. Although the fading channel makes approximate white OFDMA signal to colored signal, if pulse shaping has a larger roll-off factor, the colored part can be ignored, like what we did in Appendix A. Therefore, the detector is robust to fading channels. In addition, note that the probability of detection in the fading channel approaches that in AWGN when the roll-off factor $=0$. This result reveals the quickness of this detector.

2) CP Detector: The probability of detection versus SINR and effective sensing duration $N_{CP}$ under AWGN and SUI-3 fading channels are depicted in Fig. 6. We compare the CP detector with a known total power, i.e., (8), and robust CP detector, i.e., (12), with SINR equal to 0 dB, and observe that they have the same performance in the AWGN channel. Therefore, the robust CP detector shows its robustness to noise and SINR uncertainty. However, in a fading channel, the robust CP detector experiences more degradation than the CP detector with
known total power, because the energy estimator in the robust CP detector cannot work well in fading channels. To achieve system requirement in fading channels, \( N_{CP} \) has to be larger than 104 samples, or we have to collect four complete OFDMA symbols. However, if there is constraint on sensing duration, exploring spatial diversity by multiple-input–multiple-output (MIMO) or cooperative spectrum sensing [19]–[22] is a feasible solution.

We also investigate the effect of the TO estimation error in the top figure in Fig. 6. Although detector performance degrades below SINR \( = 0 \) dB, the probabilities of detection of these two detectors approach one at SINR \( = 2 \) dB. Therefore, our CP detectors are robust to timing error.

3) Control Signal Detector: The period of control signal \( M_C \) is relatively large (on the order of frame period) in practice; thus, we only show the effectiveness of the detector and set \( M_C = 512 \) and \( N_C = 16 \). Similar to the CP detector, the probability of detection versus SINR and effective sensing duration

Fig. 6. Probability of detection versus SINR and sensing duration under constraint \( P_F < 0.05 \) for the CP detector with known total power and robust CP detector.

\( N_C \) under AWGN and SUI-3 fading channels are illustrated in Fig. 7. We compare the control signal detector with and without knowledge of noise power, i.e., (20) and (21), and there is observable degradation due to the lack of knowledge of noise power. We also show the performance of detecting the control signal by tracking the period of the control signal and the effect of observation interval (the number of periods \( L \)). Simulation results are presented in Fig. 7(b). The result with a longer observation interval has better performance but experiences more performance degradation in the fading channel.

B. Radio Resource to Secondary Systems

In this section, we normalize the cell coverage of the primary system \( R \) to unity and combine normalization factor and \( K \) to transmission power at cognitive radio. We set the radius of the high-rate region \( D = 0.4 \), the distance between the cognitive radio transmitter \( T_x^2 \) and \( T_x^1 = 0.5 \), path loss exponent \( \alpha = 4.725 \), and the variance of shadowing parameter \( \sigma_\beta^2 = 8.2 \). The parameters are set according to terrain type B (intermediate path loss) [42], with the height of the BS being equal to 20 m.

In the top row of Fig. 8(a), we present the relation between the scaled transmission power at \( T_x^2 \) \( P_2 \) and the probability of interfering with the primary system \( \xi \) under different tolerant levels \( I_{CR} \) in uplink. The normalized distance between the primary BS and \( T_x^2 \) is set to be 0.5. Simulation and analytical results are consistent, and \( \xi \) dramatically increases under tough constraint (i.e., a lower tolerance level). The same relation is illustrated in the downlink environment in the bottom row of Fig. 8(a), where high- and low-rate regions are shown for comparison. It is interesting to observe that \( \xi \) more gradually increases when the primary MS lies in the low-rate region. In addition, Fig. 8(b) illustrates the maximum transmission power \( P_{2,\text{max}} \) at \( T_x^2 \) if \( \xi \) is constrained to 0.1. The results show that cognitive radios deliver more radio resource in uplink where cognitive radios know the distance of the primary receiver \( R_x^1 \) and \( T_x^2 \), particularly at a high-tolerance level. In addition, note

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that as our derivation that radio resource to secondary systems is linearly proportional to a tolerable interference level. In downlink, cognitive radios can utilize more radio resources when the primary MS lies in a low-rate region, because there is a higher possibility for the primary MS in a low-rate region to be apart from the cognitive radio transmitter. These results suggest that the radio resource to secondary systems heavily depends on the information available to cognitive radios; therefore, in downlink, utilization in the spatial domain should be exploited by MIMO or cooperative spectrum sensing.

VII. CONCLUSION

To achieve effective spectrum sensing for cognitive radio networks (beyond traditional cognitive radio links) by taking advantage of the rate–distance nature, we have proposed to acquire a set of cognitive information for OFDMA systems. To avoid interfering with the overlay primary system, cognitive radios could determine the operation state of the primary system, including the frequency band, existence, activity, resource allocation, and data-transmission rate. Spectrum sensing has therefore been generalized to multistate discrimination and is illustrated in a sensing tree. Elaborating on existing energy detection and cyclostationary feature detection, we have designed the optimal detectors to recognize the operation state of the primary system in the generalized system model. Simulation results showed that these detectors satisfactorily achieve system requirements (i.e., $P_F < 0.05, P_D > 0.95, \text{SINR} = 0 \text{ dB}$) in AWGN and frequency-selective fading channels by increasing sensing duration.

After identifying the operation state of the primary system, we have generalized the conventional cognitive radio and pioneered sensed the radio resource in the busy duration of the primary system by considering the rate–distance nature to improve the opportunity to use spectrum. We have shown that radio resource to secondary systems critically depends on the information (e.g., the link quality of the primary system and distance information) available to cognitive radios and is linearly proportional to the tolerable interference level of the primary system.
Furthermore, these methodologies can be applied to system identification, which is critical information in cognitive and cooperative networking. By adjusting the sensing parameters of the cognitive radio, our proposed sensing procedure can be extended to state-of-the-art OFDMA systems and even to most mobile communication systems, along with MIMO processing technologies.

**APPENDIX A**
**DERIVATION OF (3)**

By the circular symmetric property, we have

\[
E[z^2(t)] = 1/2E[|y(t)|^2] = 1/2E[|s(t)|^2 + |w_{LP}(t)|^2].
\]  
(40)

The second equality comes from the independence between \(s(t)\) and \(w_{LP}(t)\). The randomness of \(s(t)\) comes from data \(x(n)\); thus, we first find out the statistical characteristics of \(x(n)\). Let

\[
R_{xx}(n_1, n_2, m_1, m_2) = E[x(n_1 + N_B m_1)x^*(n_2 + N_B m_2)]
\]  
(41)

where \(N_B = N_{FFT} + N_{CP}\). By the existence of the CP, we have

Case 1: \(n_1, n_2 \in [0, N_{FFT} - 1]\)

\[
R_{xx}(n_1, n_2, m_1, m_2) = \sigma_X^2 \delta_{m_1, m_2} \delta_{n_1, n_2}
\]

Case 2: \(n_1, n_2 \in [-N_{CP}, -1]\)

\[
R_{xx}(n_1, n_2, m_1, m_2) = \sigma_X^2 \delta_{m_1, m_2} \delta_{n_1, n_2}
\]

Case 3: \(n_1 \in [-N_{CP}, -1], n_2 \in [0, N_{FFT} - 1]\)

\[
R_{xx}(n_1, n_2, m_1, m_2) = \sigma_X^2 \delta_{m_1, m_2} \delta_{n_1 + N_{FFT}, n_2}
\]

Case 4: \(n_1 \in [0, N_{FFT} - 1], n_2 \in [-N_{CP}, -1]\)

\[
R_{xx}(n_1, n_2, m_1, m_2) = \sigma_X^2 \delta_{m_1, m_2} \delta_{n_1 - N_{FFT}, n_2}
\]

where \(\sigma_X^2 = E[|X(k)|^2]\). The autocorrelation function of \(x(n)\) depends on the absolute value of the time index; thus, the output signal of the OFDMA system is not a stationary process [37]. However, the nonstationary property can be ignored by considering (42), shown at the bottom of the page. Note that the second term comes from the nonstationary property of the OFDMA signal. In the case where the pulse-shaping filter is a raised-cosine filter, \(h(t)\) has fast decay outside \(T_s\) in the time domain. As a result, its contribution can be ignored. Then, using Poisson’s sum formula [32], we have

\[
\sum_{m=-\infty}^{\infty} |h(t - mT_s)|^2 = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} Z_n e^{j2\pi nT_s/T_s}
\]  
(43)

where \(Z_n\) is shown in (4). On the other hand, it is obvious that

\[
E[|w_{LP}(t)|^2] = \sigma_w^2 = N_0 W / N_F.
\]  
(44)

By inserting (42)–(44) into (40), we can get

\[
E[z^2(t)] = \frac{\sigma_X^2}{2T_s} \sum_{m=-\infty}^{\infty} Z_m e^{j2\pi mT_s} + \frac{\sigma_w^2}{2}.
\]  
(45)

**APPENDIX B**
**LRT AND GLRT OF THE CP DETECTOR**

The likelihood function under two hypotheses can be written as

\[
f(r|H_0) = \prod_{n} f(r(n)|H_0)
\]  
(46)

\[
f(r|H_1) = \prod_{n \in \{1\}} f(r(n)|H_1)
\]  
(47)

where

\[
f(r(n)|H_i) = \frac{\exp \left( \frac{-(|r(n)|^2)}{\sigma_x^2 + \sigma_w^2} \right)}{\pi (\sigma_x^2 + \sigma_w^2)}, \quad i = 0, 1
\]  
(48)

and

\[
\frac{\exp \left( \frac{-(|r(n)|^2 + |r(n + N_{FFT})|^2)}{(1-\rho^2)(\sigma_x^2 + \sigma_w^2)} \right)}{\pi^2 (1-\rho^2)(\sigma_x^2 + \sigma_w^2)}
\]  
(49)

As a result, the LRT becomes

\[
K_1 \exp \left[ K_2 \left( S(\theta) \cos (2\pi \varepsilon + \angle S(\theta)) - \frac{\theta}{2} P(\theta) \right) \right]_{H_1} \overset{\sim}{\underset{H_0}{\geq}} \tau
\]  
(50)

where \(K_1\) and \(K_2\) are constants, which are independent of the received signal. Finally, by discarding constants, we can get (8).

If there are unknown parameters, we have to derive the GLRT. Under \(H_0\), the maximum likelihood estimation (MLE) of the total power can easily be derived as

\[
\sigma_0^2 = \frac{1}{2N_{FFT} + N_{CP}} \sum_{n} |r(n)|^2.
\]  
(51)
Under $H_1$, the MLE of $\theta$ and $\varepsilon$ have been derived in [38], or
\[
\hat{\theta}_{ML} = \arg \max_{\theta} \left\{ |S(\theta)| - \frac{P}{2} \right\}
\]
\[
\hat{\varepsilon}_{ML} = -\frac{1}{2\pi} \int S(\hat{\theta}_{ML}) d\theta.
\]
With some algebraic manipulations, we can get the MLE of the total power under $H_1$. We have
\[
\hat{\sigma}_1^2 = \sum_{n \notin I} |r(n)|^2 + \frac{P|\hat{\theta}_{ML}| - 2|S(\hat{\theta}_{ML})|}{2N_{FFT} + N_{CP}}.
\]  
\[ (52) \]
Note that the second term in the numerator is an energy estimator of the correlated signal. Finally, the GLRT becomes
\[
f(r|\hat{\sigma}_1^2, \hat{\theta}_{ML}, \hat{\varepsilon}_{ML}, H_1) = \frac{1}{1 - \rho^2} \frac{2N_{CP} + N_{CP}}{\hat{\sigma}_1^2} \geq \tau, \quad \frac{1}{1 - \rho^2} \frac{2N_{CP} + N_{CP}}{\hat{\sigma}_1^2} \\
\quad \leq \frac{1}{1 - \rho^2} \frac{2N_{CP} + N_{CP}}{\hat{\sigma}_1^2} \quad \text{for small } \rho.
\]  
\[ (53) \]
With further simplification, we can obtain (12).

**APPENDIX C**

**PROBABILITY OF A FALSE ALARM IN (13)**

To derive the probability of a false alarm in (13), we first derive the statistics of $a|S(\theta)| + bP(\theta)$, where $a$ and $b$ are constants, whereas $S(\theta)$ and $P(\theta)$ are shown in (9) and (10).

1) **Mean:** We have
\[ E[a|S(\theta)| + bP(\theta)] = aE[|S(\theta)|] + bE[P(\theta)]. \]  
\[ (54) \]
Thus, we separately derive the mean as follows. Under $H_0$, \{\{r(n), n = 1, 2, \ldots, 2N_{FFT} + N_{CP}\} are independent identically distributed (i.i.d.) Gaussian random variables with zero mean and variance $\sigma^2$. In the first term, $S(\theta)$ is the sum of $N_{CP}$ i.i.d. random variables, and we can approximate it as a Gaussian random variable by the central limit theorem. The mean and variance of $S(\theta)$ are
\[ E[S(\theta)] = \sum_{n \in I} E[r(n)r^*(n + N_{FFT})] = 0 \]  
\[ (55) \]
\[ \text{Var}(S(\theta)) = \sum_{n \in I} \text{Var}(r(n)r^*(n + N_{FFT})) = N_{CP}\sigma^4. \]  
\[ (56) \]
Therefore, $|S(\theta)|$ is a Rayleigh random variable, and
\[ E[|S(\theta)|] = \frac{\sqrt{\pi N_{CP}}}{2}\sigma^2. \]  
\[ (57) \]
In terms of $P(\theta)$, the observation spaces are $2N_{FFT} + N_{CP}$ i.i.d. Gaussian random variables with zero mean and variance $\sigma^2$; thus, \{\{\{r(n)|^2, n = 1, 2, \ldots, 2N_{FFT} + N_{CP}\} are i.i.d. exponential random variables [37] with parameter $1/\sigma^2$. Then, $P(\theta)$ is a Gamma random variable with parameter $(2N_{CP}, \sigma^2)$, and
\[ E[P(\theta)] = 2N_{CP}\sigma^2. \]  
\[ (58) \]
By inserting (57) and (58) into (54), we have
\[ E[a|S(\theta)| + bP(\theta)] = \left( a\frac{\sqrt{\pi N_{CP}}}{2} + 2bN_{CP} \right)\sigma^2. \]  
\[ (59) \]
2) **Variance:** Similarly, we have
\[ \text{Var}(a|S(\theta)| + bP(\theta)) = a^2\text{Var}(|S(\theta)|) + b^2\text{Var}(P(\theta)) + 2ab\text{Cov}(|S(\theta)|, P(\theta)). \]  
\[ (60) \]
Thus, we discuss these three terms, respectively. For the first two terms, from the last part, we have
\[ \text{Var}(|S(\theta)|) = N_{CP} \left( 1 - \frac{\pi}{4} \right)\sigma^2 \]  
\[ (61) \]
\[ \text{Var}(P(\theta)) = 2N_{CP}\sigma^4. \]  
\[ (62) \]
For the last term, by definition [37], we have
\[ \text{Cov}(|S(\theta)|, P(\theta)) = E[|S(\theta)|P(\theta)] - E[|S(\theta)|]E[P(\theta)]. \]  
\[ (63) \]
In (63), the second term is the product of (57) and (58), and we derive the first term in the following equation:
\[ E[|S(\theta)|P(\theta)] = \sum_{n \in I} E\left[ \sum_{m \in I} r(n)r^*(n + N_{FFT})r^2(m) \right] \]  
\[ + \sum_{n \in I} r(n)r^*(n + N_{FFT})r^2(m + N_{FFT}) \]  
\[ (64) \]
Similarly, we can get
\[ E\left[ \sum_{n \in I} r(n)r^*(n + N_{FFT})r^2(m) \right] = 0 \]  
\[ (65) \]
\[ \text{Var}\left( \sum_{n \in I} r(n)r^*(n + N_{FFT})r^2(m) \right) = 2(N_{CP} + 2)\sigma^8. \]  
\[ (66) \]
However, it is hard to specify the joint PDF of this variable, which is a product of the correlated Gaussian and exponential random variable. By adjusting the coefficients, we can approximate
\[ E[|S(\theta)|P(\theta)] \approx N_{CP}\sqrt{\pi(N_{CP} + 1)}\sigma^4. \]  
\[ (67) \]
Finally, by inserting (57), (58), (61), (62), and (67) into (60), we have
\[ \text{Var}(a|S(\theta)| + bP(\theta)) = N_{CP}\sigma^4\left[ a^2\left(1 - \frac{\pi}{4}\right) + 2b^2 \right. \]  
\[ \left. + 2ab\sqrt{\pi(N_{CP} + 1 - \sqrt{N_{CP}})} \right]. \]  
\[ (68) \]
Based on (12) and (13), we have
\[ P_F = \Pr\left( \frac{\sum_{n \notin I, i} |r(n)|^2 + P(\theta)}{\sum_{n \notin I, i} |r(n)|^2 + P(\theta) + \frac{\theta}{1 - \rho^2} - \frac{2P|\hat{\theta}_{ML}|}{1 - \rho^2}} > \tau_{CP}|H_0 \right) \]  
\[ = \Pr(X + Y > 0|H_0) \]  
\[ (69) \]
where

\[ X = (1 - \tau_{CP}) \sum_{n \in J, J} |r(n)|^2 + (1 - \tau_{CP}) \sum_{n \in I, J} |r(n)|^2 \quad \text{(70)} \]

\[ Y = \frac{2\rho \tau_{CP}}{(1 - \rho^2)\sigma^2} |S(\theta)| + \left(1 - \frac{\tau_{CP}}{1 - \rho^2}\right) \frac{P(\theta)}{\sigma^2} \quad \text{(71)} \]

The summation intervals of \(X\) and \(Y\) are nonoverlapping; thus, \(X\) and \(Y\) are independent random variables. Based on (69), \(X\) is the sum of i.i.d. random variables; thus, its distribution can be approximated by Gaussian. In terms of \(Y\), although it is the weighted sum of a Rayleigh random variable \(S(\theta)\) and a Gamma random variable \(P(\theta)\), this term is dominated by \(P(\theta)\). \(P(\theta)\) is also the sum of i.i.d. random variables; thus, the random variable \(Y\) can also be approximated by Gaussian. Then, we have

\[ X|H_0 \sim \mathcal{N} \left( \mu_1, \sigma_1^2 \right) ; \quad Y|H_0 \sim \mathcal{N} \left( \mu_2, \sigma_2^2 \right) \]

where \(\mu_1, \mu_2, \sigma_1^2, \text{and } \sigma_2^2\) are shown in (14)–(17). Finally, we have (13).

**APPENDIX D**

**GLRT OF THE CONTROL SIGNAL DETECTOR**

The likelihood functions can be written as

\[ f(r|H_0) = \prod_n \exp \left( -\frac{|r(n)|^2}{\sigma_w^2} \right) \quad \text{(72)} \]

\[ f(r|\theta, H_1) = \prod_{n \in I, J} \exp \left( -\frac{|r(n)|^2}{\sigma_s^2 + \sigma_w^2} \right) \prod_{n \notin I, J} \exp \left( -\frac{|r(n)|^2}{\pi \sigma_w^2} \right) \quad \text{(73)} \]

Under \(H_0\), the MLE of the noise power has been shown in (51) with \(2N_{\text{FFT}} + N_{\text{CP}} = M_C\). Under \(H_1\), we can get the MLEs of the noise power and signal power as follows:

\[ \hat{\sigma}_1^2 = \frac{1}{M_C - N_C} \sum_{n \notin I, J} |r(n)|^2 \quad \text{(74)} \]

\[ \hat{\sigma}_s^2 = \frac{1}{N_C} \sum_{n \in I, J} |r(n)|^2 - \hat{\sigma}_1^2 \quad \text{(75)} \]

Then, by inserting (51), (74) and (75) into a likelihood ratio function, we can get the GLRT as

\[ \frac{f(r|\hat{\sigma}_1^2, \hat{\sigma}_s^2, \hat{\phi}_{\text{ML}}, H_1)}{f(r|\hat{\sigma}_0^2, H_0)} = \max_{\phi} \left( \frac{(\hat{\sigma}_s^2)^{M_C}}{\sigma_s^2(\hat{\sigma}_1^2 + \hat{\sigma}_s^2)^{M_C - N_C}} \right) \quad \text{(76)} \]

In the implementation, we avoid power functions due to large \(M_C\) and \(N_C\) in practice and take logarithm on both sides. With some algebraic manipulations, it yields (21).

Fig. 9. Intersection area of a high-rate region and a circle with the center at cognitive radio with radius \(r\).

**APPENDIX E**

**DERIVATION OF (32)**

Assume that the coordinates of the primary BS and cognitive radio transmitter are \((0, 0)\) and \((0, c)\), respectively. Without loss of generality, we assume \(c > 0\). We first derive the \(F(r)\) when the primary MS lies in a high-rate region and then apply the results to a low-rate region. The derivation mainly refers to [41]. We assume that MSSs are uniformly distributed; thus, \(F(r)\) in a high-rate region is the ratio of an intersection area to an area of a high-rate region \(\pi D^2\). We compute the intersection area, depending on the location of the cognitive radio, which is shown in Fig. 9. We have

Case 1: \(0 < c < D\)

Case (i): \(0 \leq r < D - c\)

\[ A_H(r, c, D) = \frac{\pi r^2}{2} \]

Case (ii): \(D - c \leq r < D + c\)

\[ A_H(r, c, D) = \theta_2 r^2 + \theta_1 D^2 - cv \]

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where \((u, v)\) is cross point of these two circles by solving

\[
\begin{align*}
\begin{cases}
u^2 + v^2 - D^2 = (u - c)^2 + v^2 - r^2 \\
u^2 + v^2 = D^2
\end{cases}
\end{align*}
\]

(77)

and is shown in (35)–(38). Combining these two cases, we get (33). On the other hand, if MS lies in a low-rate region, the intersection area is the difference between the intersection with an outer circle and the intersection with an inner circle, i.e.,

\[
A_L(r, c, D, R) = A_H(r, c, R) - A_H(r, c, D).
\]

(78)

Then, \(F(r)\) in a low-rate region is obtained by dividing the intersection area by the area of a low-rate region and is shown in (32).

**ACKNOWLEDGMENT**

The authors would like to thank the anonymous reviewers for their constructive comments that helped improve this paper.

**REFERENCES**


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