The Split Delivery
Capacitated
Team Orienteering Problem

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In Europe a large percentage of trucks travel with no load (30%-40%)
Average load around 1/3 of the capacity

- Small size of the carriers
- Geographic dispersion of the customers
- Low level of cooperation among carriers
- Customers service level
Opportunities

Increase of the size of a carrier

Acquisition of other carriers

Major issue

Which carriers to acquire?

Which carriers ‘complement’ the fleet at best?
Opportunities

Cooperation among carriers

Exchange of customers (a customer may be inconvenient for a carrier but convenient for another one)

Major issue

How to cooperate?

Which customers to exchange?
Opportunities

Electronic interaction between shippers and carriers

Electronic database of spot demands by shippers

Electronic auctions

Major issue

How to choose the customers from a database?

On which customers to bid in an auction?
Opportunities

Dynamic pricing

Attract customers from low demand areas

Major issue

How to dynamically price the service?

How profitable are additional customers?
General issues

Given a set of potential customers, which are the most profitable ones?

How to price new customers?
Routing problems with profits
Routing problems with profits

New customers – additional profit
Orienteering problem (OP)
Max collected profit (constraint on time)

One uncapacitated vehicle - Feillet, Dejax, Gendreau (2005)
Team Orienteering Problem (TOP)

Potential customers – associated profit
Fleet of vehicles - maximum time available for each tour

Objective: Maximize the total profit

Capacitated TOP (CTOP) – vehicles are capacitated
# The literature

## Definition
- Butt, Cavalier (1994)
- Chao, Golden, Wasil (1996)

## Heuristic Algorithms
- Tsiligirides (1984): Heuristic algorithm for the OP
- Chao, Golden, Wasil (1996): Heuristic algorithm for the TOP
- Tang, Miller-Hooks (2005): Tabu search + Adaptive memory
- Archetti, Hertz, Speranza: (2007) Tabu search and VNS
- Ke, Archetti, Feng (2008): Ant colony

## Exact Algorithms

Motivation for Split Deliveries

Possible increase of profit in CTOP?
The state of the art

Before 2000

**SDVRP and variants**

The state of the art

SDVRP

C. Archetti, R. Mansini, M.G. Speranza, *Transportation Sci.*, 2005
K. Liu, PhD thesis, 2005
M. Jin, K. Liu, R.O. Bowden, IJPE, 2007
U. Derigs, B. Li, U. Vogel, *JORS*, 2010
E. Mota, V. Campos, A. Corberan, working paper

After 2000
The state of the art - Variants

Time windows
D. Feillet et al, working paper

Pick-up and delivery
M. Nowak, PhD Thesis, 2005
S. Mitra, *APJOR*, 2005

Profit maximization
J.E. Korsvik, K. Fagerholt, G. Laporte, working paper
The state of the art - Variants

**Inventory and production**
M.C. Bolduc et al, *EJOR*, 2010

**Minimum fraction served**

**Heterogenous fleet**

**Arc routing**
N. Labadi, C. Prins, M. Reghioui, *volume ‘Recent advances...’*, 2008
J.M. Belenguer et, *Transportation Sci.*, 2010
The state of the art - Variants

Real time

Applications
S. Song, K. Lee, G. Kim, Comp. & Ind. Engineering, 2002
P. Belfiore, H.T.Y. Yoshizaki, EJOR, 2010
**The k-split cycles**

*Definition:* Given any subset of k customers 1, 2, ..., k and k routes. Route 1 visits customers 1 and 2, route 2 visits customers 2 and 3, ..., route k−1 visits customers k−1 and k, and route k visits customers k and 1. The subset of customers 1, 2, ..., k is called a k-split cycle.

Some properties

**Properties:** If the cost matrix satisfies the triangle inequality, then there exists an optimal solution to the SDVRP where:

- there is no k-split cycle (for any k);
- no two routes have more than one customer with a split delivery in common;
- the number of splits is less than the number of routes.
CTOP and SDCTOP

\[ d_i \leq Q \]

Is it worthwhile to allow split deliveries?
How much can be gained by split deliveries?
Max. gain with split deliveries

\[
\frac{CTOP}{SDCTOP} \geq \frac{1}{2}
\]
and the bound is tight
Tightness of the bound

m vehicles, capacity $Q \geq 2m$

2m customers

demand $= \frac{Q}{2} + 1$

profit $= p$

Optimal solution CTOP: a direct trip to each customer. As m vehicles are available, profit $= mp$

Optimal solution SDCTOP: m-1 vehicles visit 2 customers each

$\frac{Q}{2} + 1, \frac{Q}{2} - 1$

and 1 vehicle delivers the 2 missing units to m-1 customers. profit $= 2(m-1)p$
Algorithms

- Branch-and-price
- Matheuristic
Branch-and-price

Adapted from Archetti, Bianchessi, Speranza (2010) for SDVRP

Column generation identifies many good columns/routes

- Identify good subsets of routes

- Use CPLEX to solve MILP exactly on subsets of routes

- Improved lower bound
A matheuristic

Heuristic or metaheuristics scheme

Mathematical programming models

Matheuristic
A matheuristic for CTOP

Initialize generates initial solution

A Tabu search is run

Whenever a new best solution is found

Optimize is run
Optimize

Goal: to modify the current solution by inserting new customers or removing currently served customers to increase as much as possible the profit

Binary variables:

insertion in a current route of a customer
removal of a customer from a current route

Continuous variables:

quantity delivered in each route to each served customer
Optimize

Optimal solution of a MILP model

Delicate issues:

- Complexity of the resulting model
- Good estimation of the value of the improvement

At most one customer can be removed
**Tested instances**

**Known sets of benchmark instances**, from 10 Christofides, Mingozzi, Toth (1979) instances taken from the VRP library with both capacity and time constraints

Number of vertices: from 51 to 200.

<table>
<thead>
<tr>
<th>Set</th>
<th>Number of instances</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
<td>10</td>
<td>original instances, large number of vehicles, not interesting as all customers are served</td>
</tr>
<tr>
<td>Set 2</td>
<td>90</td>
<td>a smaller number of vehicles (m=2,3,4) and various values of Q and Tmax</td>
</tr>
<tr>
<td>Set 3</td>
<td>30</td>
<td>changing the number of vehicles (m=2,3,4) with respect to the original values</td>
</tr>
</tbody>
</table>

**A new set of instances**

110 instances - 10 scenarios for each original instance
### Set 2

<table>
<thead>
<tr>
<th>Method</th>
<th># of optimal solutions</th>
<th>Average optimality gap (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch-and-price</td>
<td>50/90</td>
<td>0.37</td>
</tr>
<tr>
<td>Matheuristic</td>
<td>42/90</td>
<td>0.51</td>
</tr>
</tbody>
</table>
Set 2

Average profit improvement
(due to split deliveries)

- $Q = T_{MAX} = 50$
- $Q = T_{MAX} = 75$
- $Q = T_{MAX} = 100$
### Set 3

<table>
<thead>
<tr>
<th>Method</th>
<th># of optimal solutions</th>
<th>Average optimality gap (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch-and-price</td>
<td>2/30</td>
<td>5.61 (over 20 instances)</td>
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<tr>
<td>Matheuristic</td>
<td>20/30</td>
<td>0.18 (over 20 instances)</td>
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</tbody>
</table>

Almost no improvement with split deliveries
### Set 4

<table>
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<tr>
<th></th>
<th># of optimal solutions</th>
<th>Average optimality gap (%)</th>
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</thead>
<tbody>
<tr>
<td>Branch-and-price</td>
<td>14/55</td>
<td>1.84</td>
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<tr>
<td>Matheuristic</td>
<td>27/55</td>
<td>0.22</td>
</tr>
</tbody>
</table>
Set 4

Average profit improvement
(due to split deliveries)

B&P
Matheuristic
Conclusions

- Routing problems with profits are an interesting class to explore
- The branch-and-price can solve instances of reasonable size and provides optimality gaps
- The heuristic use of the columns and matheuristic are both excellent directions to find high quality heuristic solutions