Performance Analysis of Dual-Hop Systems with Decode-and-Forward Relays over Generalized Fading Channels

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Abstract—Dual-hop transmission systems employing decode-and-forward relay are studied in this paper. The examination has been done for independent, but not necessarily identical Extended Generalized-K fading channels. New, exact, closed-form expressions are derived for the outage probability, the nth moments of the end-to-end signal-to-noise ratio (SNR), and the ergodic capacity. Moreover, the average symbol error probability (ASEP) for coherent and non-coherent modulation schemes is derived. Analytical results along with simulation results are obtained with excellent agreement.

Index Terms—decode-and-forward relays, Extended Generalized-K fading channels, outage probability, end-to-end SNR, ergodic capacity, average symbol error probability

I. INTRODUCTION

The emergence of a demand for high data rates, increasing connectivity, and capacity in current and future wireless communication systems, has made multi-hop transmission attract the attention of researchers [1]-[4]. In cooperative transmission, a destination terminal staying or moving in the radio range of the source terminal may receive signals due to the existence of two distinct paths: i) a direct transmission path originating from the source terminal; and ii) an indirect path, arriving through the intermediate relaying (neighboring) terminal. However, if the line-of-sight (i.e., direct) path is completely blocked, the resulting received signal will be the received signal of the relaying path only. In a wireless relay system, the complexity depends on the number of relay terminals between the source and the destination, i.e. when a single relay terminal exists, multi hop transmission can be simplified to it is special case of which the dual-hop transmission, this type of transmission was encountered originally in bent-pipe satellites [5]-[7].

With three general classifications of cooperative strategies, i.e., amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF), it is uncertain which is more practically better [8]. All are deployed in practical systems to facilitate secure communication, but each class has its own advantages and disadvantages, compared to each other. The advantage of using DF relaying is that the relay node forwards a clean copy of the decoded source message to the destination node, by contrast to AF relaying, in which the relay node scales and forwards the received signal to the destination node. The noise amplification while repeating the received signal in AF relaying makes the amplification of both the actual desired signal and the noise happens simultaneously. On the other hand, there are several advantages of using AF relaying starting with the low implementation complexity and simplicity. The other advantages are the high utilization of the available resources and the increased performance gains, by contrast to DF relaying in which the relay node facilitates data transmission only if it can reliably decode the source message. In addition to the aforementioned advantages, AF ends up with much less delay along with much less power consumption as no decoding or quantizing operation is performed at the relay side.

Recently, an extensive amount of research has been accomplished on the performance of dual hop transmission over fading channels. In the work presented in [9], Hasna et al. (2004) have analyzed the end-to-end outage probability of DF relaying system over Rayleigh fading channels. In [10], Suraweera et al. have studied the outage probability of DF relaying system over Nakagami-m fading channels. Following the work investigated in [11], Wei-Guang Li and Ming Chen have presented useful performance, approximated by finite series, for the outage capacity of dual-hop wireless communication systems with DF relays operating over \(\eta-\mu\) and \(\kappa-\mu\) fading channels.

More recently, the Extended Generalized-K (EGK) fading model has been proposed [12]. EGK can represent a wide range of realistic fading conditions and can also be used to perfectly fit experimental data, simplifying the way how to model the source-relay and relay-destination channels. However, concerning DF relaying systems, to the best of the authors’ knowledge, there are no analytical results for the outage probability, the nth moment of end-to-end SNR and the ergodic capacity in the technical literature, even for the upper bound. Thus, the aim of this paper is to analyze the performance of dual-hop DF relaying system in terms of outage probability, ergodic capacity, and ASEP for different coherent and non-coherent modulations. The received signals from the two hops are subjected to independent, not necessarily identical EGK fading. New, closed-form expressions for
the outage probability, the ergodic capacity, and the ASEP in terms of H-Fox and bivariate H-Fox functions [13] are derived. These derived results are applicable for both integer-order as well as non-integer order fading. Our results depict the effects of the power imbalance between the two hops. We showed and discussed how these effects can be positive or negative on the overall system performance.

The remainder of this paper is structured as follows. Section II introduces the considered system and channel models. Section III evaluates the end-end performance of wireless communication systems with DF relays. These results are applied and discussed in Section IV. Finally, Section V concludes the paper.

II. SYSTEM AND CHANNEL MODELS

Consider a dual-hop DF relaying system. In the system, a single relay node, which is equipped with one antenna, decodes and forwards the received signal to the destination node. Employing an efficient relay communication, we assume that the source node communicates with the destination node via the relay node where the relay node is employed in half-duplex mode to reduce the relay self-interference. For the considered system, the end-to-end SNR can be approximated by its upper bound $\gamma_{eq}$ [14] as

$$\gamma_{eq} = \min(\gamma_1, \gamma_2)$$

As mentioned above, $\gamma_i$ is an EGK random variable with PDF given by (2). With the reasonable assumption that $\gamma_1$ and $\gamma_2$ are statically independent due to large spacing between the source, relay and destination nodes, then their joint PDF is $f_{\gamma_1, \gamma_2}(\gamma_1, \gamma_2) = f_{\gamma_1}(\gamma_1)f_{\gamma_2}(\gamma_2)$. Accordingly, the CDF of $\gamma_{eq}$ can be derived as

$$F_{\gamma_{eq}}(\gamma) = P\{\gamma_{eq} \leq \gamma\} = 1 - P\{\gamma_1 > \gamma\} P\{\gamma_2 > \gamma\}$$

III. PERFORMANCE ANALYSIS

A. Cumulative Distribution Function (CDF)

As mentioned above, $\gamma_i$ is an EGK random variable with PDF given by (2). With the reasonable assumption that $\gamma_1$ and $\gamma_2$ are statically independent due to large spacing between the source, relay and destination nodes, then their joint PDF is $f_{\gamma_1, \gamma_2}(\gamma_1, \gamma_2) = f_{\gamma_1}(\gamma_1)f_{\gamma_2}(\gamma_2)$. Accordingly, the CDF of $\gamma_{eq}$ can be derived as

$$F_{\gamma_{eq}}(\gamma) = P\{\gamma_{eq} \leq \gamma\} = 1 - P\{\gamma_1 > \gamma\} P\{\gamma_2 > \gamma\}$$

B. Outage Probability Analysis

The outage probability (OP), in regenerative systems, is defined as the probability that $\gamma_{eq}$, described in (1), goes below a predefined threshold, $\gamma_{th}$. Significantly, knowing the outage probability provides an important indication of the QoS level.

For the prescribed system model, the outage probability can be found as

$$P_{out}(\gamma_{th}) = 1 - F_{\gamma_{eq}}(\gamma_{th})$$

A closed-form expression for the outage probability can be obtained after performing some mathematical manipulations as (8).
It can be observed that the outage probability can be accurately and efficiently expressed by using the H-Fox function. To the best of our knowledge, (8) is novel. Furthermore, it should be noted that the H-Fox function is not a standard built-in function in any of the mathematical software packages such as MATLAB, MAPLE or MATHEMATICA. However, in this paper we used a similar approach as in [16] for its numerical evaluation.

C. Moments of End-to-End SNR
Assuming the overall end-to-end SNR at the destination in a dual-hop transmission model with DF relay as in (1). The \( n \)th moments of the end-end SNR can be calculated from \( \mu_n = E[\gamma_{eq}^n] \), where \( E[\cdot] \) denotes the expectation operator.

In this paper, in order to obtain a closed expression of \( \mu_n \), the PDF of \( \gamma_{eq} \) is derived by taking the derivative of (8) with respect to \( \gamma_{th} \) as

\[
f_{\gamma_{eq}}(\gamma) = \frac{dP_{out}(\gamma_{th})}{d\gamma_{th}} \bigg|_{\gamma_{th} = \gamma}
\]

Having derived a closed form expression of \( f_{\gamma_{eq}}(\gamma) \) as (10), finally we can get \( \mu_n \) as

\[
\mu_n = \int_0^{\infty} \gamma^n f_{\gamma_{eq}}(\gamma) d\gamma
\]

After performing integrals involving H-Fox functions we can obtain (12). A special case of \( \mu_n \), when \( n = 1 \), is the average end-to-end SNR, \( \bar{\gamma}_{eq} \), expressed as (13).
communication over fading channels. For a dual-hop relay, the ergodic capacity can be obtained as [17]
\[
C = \frac{\eta W}{2} E[\log_2(1 + \gamma_{\text{overal}})] \tag{14}
\]

After some manipulations involving Mellin Barnes integral we can obtain (15). It can be observed that the ergodic capacity can be expressed by using the bivariate H-Fox function. To the best of our knowledge, (15) is novel. Furthermore, it should be noted that the bivariate H-Fox function is not a standard built-in function in any of the mathematical software packages such as MATLAB, MAPLE or MATHEMATICA. However, in this paper we used a similar approach as in [16] for its numerical evaluation.

\[
C = \frac{\eta W}{2} \int \frac{1}{\Gamma(m_2)\Gamma(m_{s_2})} H^4_{0.5,4} \left[ \left( \frac{b_{s_1}b_{s_2}}{\sqrt{Y_2}} \right)^{-1} \left( \frac{b_{s_1}b_{s_2}}{\sqrt{Y_1}} \right)^{-1} \left( 1,1,1,1 \right), \left( 1 - m_{s_2} \frac{2}{\beta_{s_2}} \right), \left( 1 - m_{s_1} \frac{2}{\beta_{s_1}} \right), \left( 1,1,0,1 \right) \right] + \frac{1}{\Gamma(m_1)\Gamma(m_{s_1})} \times \nabla \nabla
\]

E. Error Performance Analysis

The average symbol error probability, \( P_{se} \), is obtained in this work for both coherent and non-coherent frequency shift keying (FSK) and phase shift keying (PSK) by averaging the conditional symbol error rate for optimum detection of nonfading binary signals in Gaussian noise, \( P_{se}(\gamma) \), over the distribution of the end-to-end SNR \( \gamma_{eq} \) as [18]
\[
P_{se} = \int_{-\infty}^{\infty} P_{se}(\gamma)f_{\gamma_{eq}}(\gamma) \tag{16}
\]

In (16) the symbol error rate, \( P_{se}(\gamma) \), for coherent and non-coherent detection of both PSK and FSK with optimum matched filter receiver is given by [19]
\[
P_{se}(\gamma) = a \cdot e^{-by}
\]

\[
P_{se} = \frac{\sqrt{\pi m_2} \Gamma(m_{s_2})}{\sqrt{\pi m_1} \Gamma(m_{s_1})} H^5_{0.5,4} \left[ \frac{b_{s_1}b_{s_2}}{\sqrt{Y_2}} \right]^{-1} \left( 1 - m_{s_2} \frac{2}{\beta_{s_2}} \right), \left( 1 - m_{s_1} \frac{2}{\beta_{s_1}} \right), \left( 1,1,0,1 \right), \left( 0,1,0,1 \right) \right] +
\]

\[
\frac{\sqrt{\pi m_1} \Gamma(m_{s_1})}{\sqrt{\pi m_2} \Gamma(m_{s_2})} H^5_{0.5,4} \left[ \left( \frac{b_{s_1}b_{s_2}}{\sqrt{Y_1}} \right)^{-1} \left( 1 - m_{s_1} \frac{2}{\beta_{s_1}} \right), \left( 1 - m_{s_2} \frac{2}{\beta_{s_2}} \right), \left( 1,1,0,1 \right), \left( 0,1,0,1 \right) \right] -
\]

\[
\frac{\sqrt{\pi m_1} \Gamma(m_{s_1})}{\sqrt{\pi m_2} \Gamma(m_{s_2})} H^5_{0.5,4} \left[ \left( \frac{b_{s_1}b_{s_2}}{\sqrt{Y_2}} \right)^{-1} \left( 1 - m_{s_1} \frac{2}{\beta_{s_1}} \right), \left( 1 - m_{s_2} \frac{2}{\beta_{s_2}} \right), \left( 1,1,0,1 \right), \left( 0,1,0,1 \right) \right] \times
\]

\[
\left\{ H^2_{0.3,2,2,1,1,1,1} \left[ \left( \frac{b_{s_1}b_{s_2}}{\sqrt{Y_1}} \right)^{-1} \left( 0,1,1 \right), \left( 1,1,0,1 \right), \left( 0,1,0,1 \right) \right] \right\} \tag{19}
\]

\[
Evaluating the integral in (16) for non-coherent FSK and PSK, then \( P_{se} \) can be expressed as (19). Similarly, the closed form of, \( P_{se} \), for coherent FSK and PSK, is obtained as (20). To the best of our knowledge, the derived error performance results reported in this paper are new.

\[
P_{se}(\gamma) = a \cdot \text{erfc}(\sqrt{2by})
\]

\[
a = 0.5, b = 0.5 \quad \text{for NCFSK}
\]

\[
a = 0.5, b = 1.0 \quad \text{for DCFSK}
\]

where \( \text{erfc}(\cdot) \) is the complementary error function [15]. Evaluating the integral in (16) for non-coherent FSK and PSK, then \( P_{se} \) can be expressed as (19). Similarly, the closed form of, \( P_{se} \), for coherent FSK and PSK, is obtained as (20). To the best of our knowledge, the derived error performance results reported in this paper are new.
\[ F_{se}^{-} = \frac{A}{\Gamma(m_2)\Gamma(m_{s2})} H_{4,4}^{1,4} \left[ B \left( \frac{b_{s2}b_{2}}{\gamma_2} \right)^{-1} \left( 1 - m_{s2}^2 \frac{2}{\beta_2} \right) \right] \left( 1 - m_{s2}^2 \frac{2}{\beta_{s2}} \right) \left( 1,1,0,1 \right) + \]

\[ \frac{A}{\Gamma(m_1)\Gamma(m_{s1})} H_{4,4}^{1,4} \left[ B \left( \frac{b_{s1}b_{1}}{\gamma_1} \right)^{-1} \left( 1 - m_{s1}^2 \frac{2}{\beta_1} \right) \right] \left( 1 - m_{s1}^2 \frac{2}{\beta_{s1}} \right) \left( 1,1,0,1 \right) \]

\[ \frac{A}{\Gamma(m_1)\Gamma(m_{s1})\Gamma(m_2)\Gamma(m_{s2})} \times \left\{ H_{0,1,2,2;1}^{1,0,4,2;2,2} \left( \frac{b_{s1}b_{1}}{\gamma_1} \right)^{-1} \left( 1; -1; -1; 0,0,1,1,0,0,1,0,1,1,1 \right) \right\} \]

\[ H_{0,1,2,2;1}^{1,0,4,2;2,2} \left( \frac{b_{s2}b_{2}}{\gamma_2} \right)^{-1} \left( 1; -1; -1; 0,0,1,1,0,0,1,0,1,1,1 \right) \left( m_{s2}^2 \frac{2}{\beta_{s2}} ; m_{s1}^2 \frac{2}{\beta_{s1}} ; m_{s2}^2 \frac{2}{\beta_2} ; m_{s1}^2 \frac{2}{\beta_1} \right) \]

(20)

IV. DISCUSSION OF RESULTS

In this section, we present the numerical results of outage probability, moments of end-to-end SNR, ergodic capacity and ASEP for dual-hop DF relaying system. In Fig. 1, the outage probability is plotted as a function of the average SNR per hop \( \gamma_1 \), assuming \( \gamma_2 = 1 \) dB. As it can be observed, OP improves as the fading parameter \( m \) increases from \( (m_1 = m_2 = 2) \) to \( (m_1 = m_2 = 8) \). Also, by considering the same value for \( \gamma_i \), it can be seen that the power imbalance may be either advantageous or harmful for the overall system performance. Indeed, for \( \gamma_2 > \gamma_1 \) it is beneficial, otherwise, it is detrimental.

In Fig. 2, the average SNR is plotted versus the average SNR per hop \( \gamma_1 \). The average SNR increases with increasing \( \gamma_1 \) and also with increasing the fading parameter \( m \). Also, it can be seen that, by considering the same value for \( \gamma_1 \), the power imbalance may be either advantageous or harmful for the overall system performance. Indeed, for \( \gamma_2 > \gamma_1 \) it is beneficial, otherwise, it is detrimental.

The results for the average SNR from (13) over EGK fading channels with different values of the fading parameter \( m \) are shown in Fig. 2. As expected, the average SNR increases with increasing \( \gamma_1 \) and also with increasing the fading parameter \( m \). In the same figure, it can be seen that, by considering the same value for \( \gamma_1 \), the overall system performance improves for \( \gamma_2 > \gamma_1 \) and degrades for \( \gamma_2 < \gamma_1 \).
Finally, the closed form mathematical results for the ASEP given in (19) and (20) are presented in Fig. 4 for BPSK and DBPSK as a function of the first hop average SNR, $\gamma_1$, for $\gamma_2 = 2\gamma_1$, different values of shadowing severity $m_1 = m_2 = m$, and different values of shadowing shaping factor $\beta_1 = \beta_2 = \beta$. Depending on Fig. 4, the ASEP decreases with the increase of $m$, and $\beta$ because increasing $m$, decreases the fading severity and increasing $\beta$, skews the PDF of the fading around the average power. Moreover, in Fig. 5, the performance of the aforementioned signaling constellations is studied for both balanced and imbalanced links. This figure shows that the error performance improves as $y_2$ exceeds $y_1$. Similar performance results are derived for BFSK and CBFSK using Fig. 6 and Fig. 7. All these figures include both analytical and Monte Carlo simulation results with excellent agreement.

V. CONCLUSION

In this paper, we investigated the end-to-end performance of dual-hop wireless communication systems with DF relays over EGK fading channels. The outage probability and the moments of the end-to-end SNR were extracted in closed form in terms of the H-Fox function. The results were used to study important performance criteria, such as the average end-to-end SNR. Furthermore, a closed form expressions of the ergodic capacity and ASEP were derived in terms of the bivariate H-Fox function. The results showed that the impact of the power/fading imbalance between the two hops may have gainful or harmful effects on the overall system performance.

REFERENCES


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