Adaptive Load Management: Multi-Layered And Multi-Temporal Optimization Of The Demand Side In Electric Energy Systems

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Adaptive Load Management: Multi-Layered And Multi-Temporal Optimization Of The Demand Side In Electric Energy Systems

Submitted in partial fulfillment of the requirements for
the degree of
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in
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Abstract

Well-designed demand response is expected to play a vital role in operating power systems by reducing economic and environmental costs. However, the current system is operated without much information on the benefits of end-users, especially the small ones, who use electricity. This thesis proposes a framework of operating power systems with demand models including the diversity of end-users’ benefits, namely adaptive load management (ALM). Since there are a large number of end-users having different preferences and conditions in energy consumption, the information on the end-users’ benefits needs to be aggregated at the system level. This leads us to model the system in a multi-layered way, including end-users, load serving entities, and a system operator. On the other hand, the information of the end-users’ benefits can be uncertain even to the end-users themselves ahead of time. This information is discovered incrementally as the actual consumption approaches and occurs. For this reason ALM requires a multi-temporal model of a system operation and end-users’ benefits within. Due to the different levels of uncertainty along the decision-making time horizons, the risks from the uncertainty of information on both the system and the end-users need to be managed. The methodology of ALM is based on Lagrange dual decomposition that utilizes interactive communication between the system, load serving entities, and end-users. We show that under certain conditions, a power system with a large number of end-users can balance at its optimum efficiently over the horizon of a day ahead of operation to near real time. Numerical examples include designing ALM for the right types of loads over different time horizons, and balancing a system with
a large number of different loads on a congested network. We conclude that with the right information exchange by each entity in the system over different time horizons, a power system can reach its optimum including a variety of end-users’ preferences and their values of consuming electricity.
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Part I

Introduction
Chapter 1

Introduction

1.1 Background

This thesis starts with a simple motivation: current power system operation lacks a model of demand that includes a variety of the end-users’ objectives of using the system. System operators take the most part of the demand as an exogenous value that is irresponsible to the rest of the system. Historically, demand has been regarded as a parameter that has to be met with generation resources by the system operator.

Recently, this has been changing as the cost of meeting the ever-rising capacity of demand increases and the technology to connect and control some types of loads develops. Nonetheless, policies to promote demand response in a system largely view demand as a resource that can replace or substitute generation resources when needed. Markets accept bids from load serving entities so that they can curtail demand in place of using generation resources when demand curtailment is seen as less expensive. However, this does not fully incorporate, or even distorts, what end-users intended to achieve from the system. The issues with such demand response policies have been well discussed in the literature, e.g., [5].

The fundamental problem with seeing demand of the system as an alternative/substitute of generation resources is simple in the sense that the objectives of demand and supply in the system are not interchangeable. Loads exist in the power system to achieve
end-users’ objectives, whether it is lighting a building to be able to work, heating a house to keep warm and comfortable, or running a motor to operate a business. However, when the cost of using electric energy outweighs the benefit of consuming energy, at least in theory, end-users will cease or reduce their consumption. On the other hand, when the benefit is greater than the cost, users will continue or even increase their consumption.

In reality the equilibrium of demand and supply of energy is not found this simply for various reasons. First, to most end-users the price they pay for electricity does not reflect the system condition; whether the system supply is scarce or not, they pay a uniform price. Second, even if end-users pay a time-varying price that reflect the system condition, the opportunity cost occurring from inconvenience of adjusting consumption with respect to the price can outweigh the benefit of reducing energy cost. This inconvenience is worse without automated and reliable infrastructure for end-users to communicate with the system and to manage consumption. Third, even if a system and end-users are equipped with communication and control infrastructure, we do not know whether the different end-users and suppliers can be coordinated to keep the system in balance and/or at the optimum, or how to achieve such an optimum.

Assuming the first two said problems are resolved, this thesis attempts to address the last question of whether and how a system can reach its optimum including a large number of end-users’ objectives. We propose a framework where the cost of providing energy service to end-users with diverse consumption preferences and benefits is communicated between the system operator and the end-users. The true cost of providing energy comes from the suppliers, and the benefit of consuming it is specific to an end-user. Since the system operator oversees the production and consumption of electricity in the whole system, the cost of supply and the benefit of demand by the end-users are coordinated by the system operator.

However, it is not easy, if not impossible, to account for every single individual end-user’s benefits into system operation. Even if it is achievable, it is hard to justify the reason
why the system operator should know every end-user’s profile of energy consumption. Due to this privacy concern, the information on the end-users’ energy consumption and their preferences should propagate as little as possible. Also because a system operator cannot oversee every end-user’s energy consumption model, this information needs to be aggregated at a certain level to the system, while keeping the particularity of each end-user’s benefit model. This role is fulfilled by load serving entities, who provide electric energy service to their customers and purchase energy on behalf of them in the market.

Since the price in the market is uncertain, load serving entities face the risk from this uncertainty when purchasing electricity. Most end-users make contracts with their load serving entities (LSEs) at a predetermined rate in order to avoid this risk. As a result, LSEs effectively hedge the risk from the uncertain market price for the end-users. In addition to hedging the risk of the uncertain market price for the end-users, since load serving entities interact directly with the end-users, they can differentiate the service according to the end-users’ needs and choices. They can offer various tariffs to the end-users on one side, and purchase electric energy in a variety of ways in the market on the other side.

For the above reasons, we model the power system with a system operator who oversees the overall supply and demand, various end-users who have different objectives in consuming electric energy, load serving entities who aggregate the demand of their customers in the system/market, and power producers on the supply side. While in the real system there may be entities who involve in purely financial transactions without producing or consuming energy, we do not consider these entities in this work.

By modeling end-users, load serving entities, and a system operator in a power system comprehensively, we attempt to show a proof of concept in this thesis how this model, namely adaptive load management (ALM), can theoretically work to coordinate the objectives of the different entities in the system when the objectives of the end-users are fully incorporated in system operation over multiple time steps. We especially focus on the fact that end-users have their own local systems of using electric energy in order to maximize
their benefits. They have different preferences on how much energy they are willing to consume, and how much they are willing to pay for it. Load serving entities can also have different objectives and risk preferences in the market.

ALM provides a consumer-centric, rather than grid-centric, view of operating a power system. By communicating the preferences with respect to the signal from the system level, the end-users can influence the market price and act as a price maker rather than a mere price taker. In communicating their optimal demand with respect to the signal from the system, the ALM framework enables end-users to optimize their objectives with respect to their own preferences on risks and consumption costs, and physical dynamics and limits of their local system of demand.

While our focus in modeling the system is on incorporating different physical dynamics and economic preferences of end-users, our concept of exchanging information between entities with different purposes can be extended to include other entities and components in the system. Examples can range from physical components such as transmission control devices that communicate with the system to optimize its performance, to economic entities such as electric vehicle chargers and their aggregators. We provide a generic information exchange framework that captures the local objectives and preferences that is readily extensible to include these components and entities in system operation.

ALM also relates the objectives over different time horizons, as shown in Figure 1.1. We recognize the relationship between the long- and short-term objectives. Long-term in ALM is defined as any time horizon longer than day-ahead scheduling; long-term decisions include multiyear capacity/energy decisions, monthly energy contract decisions. Short-term decisions in ALM range from day-ahead scheduling to near-real-time\(^1\) adjustment of the amounts settled a day ahead of time. The long-term decisions of any entity cannot be made without projection of information on cumulative short-term conditions of the system.

\(^1\)Real-time in this work refers to hour-ahead or shorter ahead of actual operation and consumption, which is different from the conventional use of the term in the market. For example, real-time market price is determined \textit{ex post}, i.e., \textit{after} the actual consumption and operation occurs.
Figure 1.1: The timeline of adaptive load management (Note: the lengths not to scale)

system. The information of the end-users’ benefits can be uncertain even to the end-users themselves ahead of time, and the external factors and conditions that suppliers or the system operator assumed for the future can be highly uncertain as well. The information used for decision making is discovered incrementally as the actual consumption approaches and occurs, and the uncertainty of the information gradually decreases. On the other hand, the amount of time available to make such decisions and actions also diminishes [6].

For example, in near real-time dispatch, the uncertainty of the system conditions and the demand is very low while decisions on the actions of the entities need to be made within a very short time frame. ALM recognizes this time-varying uncertainty of information and the different time horizons for making decisions based on available information.

1.2 Problem statement

This section describes the problems that we tackle in this thesis. The ultimate goal is to find an information exchange framework among the entities, including end-users, that reach the system optimum over different time horizons. The settings and assumptions change with respect to the perspective, i.e., who is making the decision, and the time horizon over which the entity is making decisions. Specific mathematical models are presented in Parts II.

First, we model the benchmark problem of the power system that encompasses the
objectives of all entities, including end-users, over the longest time horizon. This includes planning for capacity, long-term energy contracts, and short-term energy scheduling. We reason why this problem cannot be solved in its intact form, and divide the problem into more workable forms. Since the problems are formed over multiple time horizons, a lot of parameters and conditions are uncertain at the time the entities make decisions. We specify problems of a load serving entity hedging the risk from these uncertainties.

Second, we propose a specific framework where end-users’ benefits can be effectively scheduled in the short-term economic dispatch, and show the proof of concept by numerical examples. The method is based on dual decomposition of the system-level global problem. We propose two different approaches for different time horizons within the economic dispatch timeline. In doing so, we examine the relationship between the optimum of the global system and the optima of the local entities, i.e., suppliers and end-users, and how they should be coordinated efficiently. We especially model price-responsive loads with linear intertemporal\textsuperscript{2} dynamics by vectorizing the state and control variables of end-users’ demand systems. This makes the system-level global problem convex and decomposable and converge to the primal optimum under mild conditions that we specify.

Third, we identify a methodology to design ALM in the real-world power network. While we model the ALM framework in a generic way, deploying demand resources for system operation is specific to the system due to distinctive system conditions and a composition of loads and generation resources. We explore 1) what loads can used for ALM and 2) how they can be used, based on the specific examples and data analysis of the actual system and loads.

1.3 Related work

The idea of finding the optimum of the power system by decomposing the problem has been around for a long time. [7] introduced the idea of trading electric energy as a commodity and the concept of price as a signal that entities can react to in order to balance the

\textsuperscript{2}Intertemporal in this work implies dynamics over multiple discretized time steps.
system. Since then, the US and a lot of other countries have developed electricity markets. However, the current market does not necessarily fully communicate the market price in order to balance the system, but rather have the participants bid in the price based on what they think the market price will be. In literature, many technical methods have been proposed to solve a complex system balance problem in a decomposed way, if not with demand. For example, [8] compared different methods to decompose an augmented Lagrangian function of an optimal power flow problem. Decoupling a power flow by control areas has been a popular way of partitioning the global problem, e.g., [9, 10, 11]. Theories behind decomposing optimal power flow with power network equations is studied in [12], and [13] applies it to including the benefits of demand in the system. More recent literature including benefits of demand in power balance problems appear in [14, 15, 16, 17]. The limitations with the recent literature on demand response include 1) setting the end-users as a price taker and solving a local problem, 2) obscure models of end-users’ benefits based on assumptions not based on the physical models of the loads, and 3) ambiguity of the settings and subjects of the problems such as who is a coordinator of the problem, who defines the objective of the problem, etc.

The concept of multi-temporal electricity markets has been proposed in various forms by several authors [5, 18]. Although markets that trade the long-term reliability or capacity have been designed and implemented in some regional systems [19], their model of demand is set rather arbitrarily by the system operator, without regard to any information from the actual system users. There has been work that attempted to formulate this dynamic system of the market and the demand responding to the system by price signal [20, 21]. On the other hand, there have been practical efforts in testing out and analyzing the response of end-users to the time-varying system condition or price, especially with increasing rollout of smart meters [22, 23, 24, 25]. Much of the previous work relating the load aggregator’s problem focuses either on a single time stage [26, 27], or a single or total load that a load aggregator needs to serve [28, 29]. As a result, different risk sources and possibly different
risk criteria at various time stages have not been considered. Risk managing strategies of a load aggregator has little been considered in relation with the demand resources that are adjustable [30, 31].

1.4 Contributions

By solving the problems in Section 1.2, we have the following results from this thesis.

1. We propose a framework that comprehensively includes the objectives of end-users in a power system over the longest time horizon ranging from years to hours. Especially,
   (a) we specify the end-users’ objectives of the system based on their various economic preferences and physical intertemporal dynamics and limitations, and
   (b) define a load serving entity’s problem where she hedges the risk from the uncertainty of demand and market prices.

2. We propose a specific consumer-centric short-term information exchange framework among the entities that
   (a) effectively captures the variety of end-users’ needs without revealing their preferences,
   (b) while efficiently communicating with the system operator through load serving entities within reasonable timeframe and communication limits,
   (c) does not interrupt with the nature of the market where competitive participants do not share information with each other, if the system is balanced through a market,
   (d) coordinates the entities’ objectives and intertemporal dynamics, including the end-users, to manage congestion of a power network, and
   (e) can furthermore be extended to incorporating other components such as storage into system operation.

3. We design ALM based on the real-world data of the loads and power system in the
Azores Islands where we
(a) identify the loads that can be deployed that are specific to the system,
(b) specify feasible methods by the types of loads,
(c) analyze the potential benefits of utilizing the different types of loads, and
(d) show the proof of concept how ALM works to achieve the system optimum with the selected loads.
Part II

Formulation and Methodology
In this part of the thesis, we formulate the benchmark problem for electric energy systems over multiple time horizons, including the long-term procurement of resources and the short-term scheduling of the available resources. Then we examine this big problem into smaller subproblems with respect to different time horizons and different optimization entities. In Chapter 3, we propose a specific method to coordinate short-term decisions made by entities in the system.

The objective of the system looking over the entire electric energy system of interest, considering linearized network constraints, is to maximize the long-run social welfare subject to cumulative short-run energy balance and long-run reliability criteria, and the network flow limitations. It can be formulated as

\[ \begin{align*}
\text{maximize} & \quad P_{G_i} P_{D_j} \sum_{i=1}^{N_G} \sum_{t=1}^{T} -c_i(P_{G_i}(t)) + \sum_{j=1}^{N_D} b_j(P_{D_j}(t)) \\
& \quad - \sum_{y=1}^{Y} \sum_{i=1}^{N_G} C_i^G(K_{G_i}[y]) + \sum_{j=1}^{N_D} \{-C_j^D(K_{D_j}[y]) + B_j^D(P_{D_j}^{\text{max}}[y])\} \\
\text{subject to} & \quad \sum_{i=1}^{N_G} P_{G_i}(t) = \sum_{j=1}^{N_D} P_{D_j}(t) \quad \forall t \\
& \quad |H(C_y P_G(t) - C_y P_D(t))| \leq F(t) \quad \forall t \\
& \quad \dot{P}_{G_i}(t) = f_i(P_{G_i}(t), \Theta_i(t)) \quad \forall i, t \\
& \quad \dot{P}_{D_j}(t) = f_j(P_{D_j}(t), \Theta_j(t)) \quad \forall j, t \\
& \quad R_y \sum_{i=1}^{N_D} P_{D_j}^{\text{max}}[y] \leq \sum_{i=1}^{N_G} \{P_{G_i}^{\text{max}}[y] + K_{G_i}[y]\} + \sum_{j=1}^{N_D} K_{D_j}[y] \quad \forall y.
\end{align*} \]

where \( t \) denotes the time step in energy market, typically an interval of an hour, that ranges from 1 to \( T \) over the horizon, and \( y \) denotes the time step of a year for which the capacity market clears. Supply entities in the market are indexed with \( i = 1, \cdots, N_G \), and demand entities with \( j = 1, \cdots, N_D \). The energy produced by \( i \)-th entity at time step \( t \) is \( P_{G_i}(t) \) while the energy consumed by \( j \)-th entity is named \( P_{D_j}(t) \). \( c_i(\cdot) \) and \( b_j(\cdot) \) denote the short-run cost and benefit functions of the \( i \)-th supply entity and \( j \)-th demand entity, respectively.
\( C^G_i (\cdot) \) and \( B^D_j (\cdot) \) denote the long-run cost and benefit function for providing or consuming additionally provided capacity of \( K_i[y] \) at year \( y \), respectively. Note that a demand entity can also provide additional capacity \( K_D_j[y] \) by investing in energy efficiency. This means that investing in energy efficiency by end-users can reduce the peak demand and has the same effect of adding in additional capacity to the system. \( P_{D_j}^{\text{max}}[y] \) and \( P_{G_i}^{\text{max}}[y] \) denote the peak demand and the maximum existing capacity at year \( y \), and \( R_y \) is the pre-determined rate of reserve for reliability. \( H \) is the power transfer distribution factor (PTDF) matrix of the network, and \( F(t) \) is the transmission limits at time step \( t \). \( C_g \) (\( C_d \)) is a supply (demand) connection matrix with binary elements 0 and 1 in the dimension of (number of buses except the slack bus)-by-\( N_G \) (\( N_D \)) that relates each supply (demand) entity to the bus that it is connected to.

Equations (1.1c)-(3.2b) are related with the short-term decision making, and Equation (1.1g) defines the constraint for the long-term decision making. More specifically, Equation (1.1c) dictates that the total generation and total demand need to match at any given time period \( t \). Equation (1.1d) limits the transmission line flows at each time period \( t \). Even though the real network dynamics also include reactive power and bus voltages as its variables, since our interest is only in active power, we use a linearized network model. Equations (1.1e) and (3.2b) define the physical short-term dynamics of each supply/demand entity’s power output/consumption with exogenous parameters denoted as \( \Theta \). Equation (1.1g) ensures an adequate amount of capacity resources in the long run.

There are a few reasons why we cannot solve this benchmark problem in the form above. First, the short-term external parameters and functions that decide the optimum of the problem are highly uncertain at the time when the long-term decisions are made. Therefore, the long-term decisions are based on forecast of these parameters and functions. Second, there are two many different objectives and constraints attached to the problem. Typically there are numerous end-users and their loads that have their own objectives and intertemporal dynamics. In other words, \( N_D_j \) is too large and not even known to
the system level, and will entail as many equations such as (3.2b). The last reason also has to do with the complexity of this problem, but in a slightly different perspective than the large dimension of the problem. If we relieve the linear approximation of the network dynamics (1.1d) and have complex nonlinear dynamic equations for (1.1e) and (3.2b), the problem is structurally difficult to solve, regardless of its dimension. Tackling the nonconvexity/nonlinearity of an optimization problem is out of the scope of this work, and for this reason we assume linear network constraints and local dynamics in this problem. The detailed models are presented in the following chapters.

For these reasons, we separate the problem in two interrelated ways in the next chapter: by entities that have an objective and by time horizons. In the first section of Chapter 2, we discuss the multi-layered aspect of the system. We describe the entities that compose the system in more detail and how they are related in terms of their objectives. The physical and economic interconnections of the entities are discussed. Section 2.2 presents the multi-temporal aspect of the system optimization. We especially focus on the relation of the optima in different time scales. We point out how disconnected the decisions at different time scales are in the real-world system, and suggest a direction to change the disconnection between the long-term procurement and the short-term scheduling.
Chapter 2

Multi-layered and multi-temporal decision making

2.1 Multi-layered aspect: the entities in the system

We view the power system as three layers based on their economic and physical objectives; a system operator, load serving entities and power producers who make transactions in the market, and the end-users. Figure 2.1 shows the system and the entities that we consider in ALM.

2.1.1 System Operator

A system operator looks over the physical power system at a high-voltage transmission level. The objective of the system operator is to meet the expected demand in the most cost-efficient and reliable way. In the most comprehensive form, the system operator’s problem is the same as Problem (1.1). Note that the system operator does not observe the individual loads in making decisions on operation. Small end-users’ demand is aggregated and represented in the system by load serving entities. The system operator, however, still needs to forecast system demand and schedule resources to meet the expected time-varying demand.
2.1.2 Load Serving Entities

Load serving entities (LSEs) purchase energy on behalf of end-users in markets or directly from the power producers and provide energy to their end-use customers. The objective of a load serving entity is to maximize profit by minimizing the cost of energy purchase and maximizing the revenue from their end-users. In this work we assume that there is no gaming between different load serving entities and the revenue from the end-users are fixed. This simplifies the objective of the load serving entities into minimizing the cost of energy that their end-users consume.

Since the system operator does not observe individual small loads of end-users, load serving entities play an important role in ALM. First, for end-users who like to avoid the risk of uncertainty of the market price, load serving entities usually provide energy at a uniform or “insulated” price. Load serving entities are said to usually procure up to about 80% of their customers’ energy through long-term contracts with energy traders and power producers ahead of the scheduling markets. This is one way for a load serving entity and a power producer to hedge against the risk of uncertainty in market prices. More on the risk management of a load serving entity will be discussed in Section 2.2.3. Second, in order to balance the system with the information of demand, the system operator needs an
aggregator of this information so that it does not communicate with numerous individual end-users. LSEs serve as an information exchange medium for end-users in specific clearing methods of ALM, which is described in Chapter 3.

### 2.1.3 End-users

An end-user’s objective is to maximize her benefit from consuming energy and minimize the incurring cost. The price of electricity depends on the contract that each user makes with her own load serving entity. Most of the small residential users pay a uniform rate of electricity to their load serving entities. However, in our formulation, we assume the ideal connection between the true price of electricity in the system and the end-user’s benefit. In reality, end-users may have various contracts with their load serving entities. The purpose of formulating the end-users’ problems in terms of the true cost of electricity provides a control scheme to adjust the flexible loads of the end-users. In other words, the true cost of electricity acts as the control signal that affects the end-users’ systems of electricity consumption. This way, the end-users are able to respond to the system condition by reacting to the price.

While enabling the end-users to react to the system condition with the price is important, the end-users’ benefit should not be compromised as long as they are willing to pay the price. We model the end-users’ benefit in a quantitative way and include it in their objective function and the constraints. We include in our formulation the fact that each end-user can have a desirable state that she tries to achieve by consuming energy. An end-user’s energy consumption is seen as a dynamic system where the possibly time-varying desirable state is to be achieved with the input of energy consumption.

### 2.2 Multi-temporal aspect: the time horizons and risk management

In order for the system operator to achieve cost efficiency and reliability of the system, operational decisions need to be made over different time horizons. First, in order to
guarantee that the highest point of demand is met, the system operator needs to project the long-term change in demand and supply, and procure additional capacity if needed. Since obtaining additional capacity can take as long as several years, it is important to forecast the trend of long-term demand and plan ahead on procuring enough capacity. Once the capacity of the system is set, the system operator needs to schedule the energy resources in a weekly, daily, and hourly bases to meet the expected demand. System operator should take into account contingencies, physical network constraints and physics, and unexpected events in demand and/or supply.

What we particularly emphasize in the multi-temporal formulation is the connection between the decisions in the long- and short-term horizons. The long-term decisions should be made based on the projection of the short-term decisions forthcoming, and once the decisions are made for the long-term decisions, they become the constraints for the short-term decisions. Especially since the long-term decision making involves a great amount of uncertainty in short-term decisions in the future, accurate forecast is important. On the other hand, however, in making long-term decisions, the detailed information on how the short-term decisions are to be made is not needed. In the following subsections, we detail the decision making problems on long- and short-term horizons.

2.2.1 Long-term decision making

The objective of a power system in the long term is to secure an adequate amount of capacity to meet the projected system demand in the future at the least cost. Therefore, the most crucial information needed to make a decision in the long run is to project the demand in the most accurate way. Electricity demand has a pattern depending on the day of the week, season, and in the longer run, a trend dependent on the economic and sociopolitical factors. For decisions in the long term, which we define to be longer than three years, the most important information on demand is its long-run trend. Also the projected load factor can be of importance; if the peak demand is expected to grow faster than the rate of the average demand, fast-ramping resources may need to be planned prior
to others.

In our framework of ALM, we formulate the long-term decision making of the power system including information from end-users. While it may be more difficult for end-users to project their individual demand in the long run, they do make decisions to save energy in the long run. For example, some end-users purchase energy-efficient products that are more expensive over cheaper and less efficient ones. Many end-users also improve insulation of their buildings so that they use less energy for space conditioning. We suggest that these decisions of the end-users should be included in the long-term decisions of the system. Moreover, there should be appropriate incentives for end-users to improve energy efficiency of their energy consumption systems, especially to overcome the inertia and indifference. While policy making for such objectives is out of scope of this thesis, we formulate the system’s decision making with the end-users’ long-run decisions.

Taking only the long-term variables from Problem (1.1), the long-term decision making problem of a system operator can be rearranged as follows:

\[
\text{maximize} \quad - \sum_{y=1}^{Y} \sum_{i=1}^{N_G} C_i^G(K_{G_i}[y]) + \sum_{j=1}^{N_D} \{ -C_j^D(K_{D_j}[y]) + B_j^D(P_{D_j}^{\text{max}}[y]) \} \]
\[
\text{subject to} \quad R_y \sum_{i=1}^{N_D} P_{D_i}^{\text{max}}[y] = \sum_{i=1}^{N_G} \{ P_{G_i}^{\text{max}}[y] + K_{G_i}[y] \} + \sum_{j=1}^{N_D} K_{D_j}[y] \quad \forall y. \]

In this formulation, individual constraints are not seen by the system operator. In order to incorporate the decisions made by the end-users in the long-run efficiency, how much capacity can be saved by each end-user should be known to the system. We assume that this information can be expressed as a monetary benefit with respect to the capacity that an end-user can provide, which is analogous to generation capacity bids.

An interactive long-term decision making of power producers and the system operator has been studied to an extent, for example, [38]. Prca proposes that the short-term electricity price should be the signal for each of the power producers and the load serving entities to optimize their own sub-objectives, and the bids calculated with their optimum
should be coordinated with the system operator. In this work the benefit of demand entities is also modeled into the system objective, but the procedure on how to get the bids from the demand remains abstract and it is modeled as an aggregate load.

We propose that the benefit of the demand side should be given by the end-users, and be represented in the market by their load serving entities. The individual end-user (or her agent, i.e., load serving entity) should calculate their optimal additional capacity with respect to electricity price input. This price should include both the capacity price and the energy price. With the expected price given by the load serving entity, the end-user calculates their expected energy consumption and the expected capacity, i.e., the peak demand, over the predetermined time horizon of interest. There can be different ways in reaching the optimal capacity and energy quantities between the end-users and their load serving entity. In any case, the expected system condition should be transferred to the end-users through the load serving entity as the price to the end-users, and the end-users should send their optimal consumption in response to the system condition. The methodology of reaching the optimum in this case will be discussed in more detail in the next chapters in terms of short-term decision making procedures. The same methods can be applied to the long-term decision making, which we leave as one item of future work, due to the difficulty in modeling policies and various social behaviors of end-users.

There are programs that help the end-users calculate the potential energy cost savings based on the appliances they own and their specifications, the characteristics of the building, heating and cooling system specifications, etc. One example is shown in Figure 2.2. With information shown in the figure, end-users decide on whether and which energy efficiency measures to invest in, along with the energy rate that they pay.

We can think of a more general way to obtain information of various energy efficiency measures that the end-user chooses to invest in, and the tradeoff between the investment cost and the energy cost savings. For example, each end-user can calculate, based on the expected rate of charge for energy and capacity in the coming years that is given from
his/her LSE, the expected peak demand assuming that they install energy efficiency measures such as improving insulation of his/her house. If the investment in energy efficiency measures is expected to be paid off by the energy cost savings within the time horizon that the end-user is satisfied with, then s/he can give the information to the LSE in order to bid into the capacity market in expectation of getting rewards from the capacity market. Recently, creative business models that connect energy efficiency of end-users’ buildings and capacity market earnings have also emerged [39].

How the rewards should be shared between the LSE and the end-user will depend on the contract between the two parties. This procedure should be able to reduce the needs for additional generation capacity if the end-users are willing to pay for the investment in energy efficiency measures; if not, it means that the end-users have agreed to pay for the additional capacity cost. Since the aggregation of investment in energy efficiency measures is likely to be less expensive than installing a new peak generator, the end-users should be compensated for their investment cost in the capacity market. How to incent the end-users...
with this reward from the capacity market, including defining the time horizon so that
the market gives enough time for demand to see the payoff of their investment, is an open
policy question.

2.2.2 Short-term decision making

Most of the energy resources in power systems need to be scheduled ahead of operation
because of their physical operation constraints such as ramp rates, startup and shutdown
times and costs, etc. The resources are usually scheduled a day ahead of operation by
the system operator as a unit commitment problem. This problem is solved with binary
variables including the on/off decisions of the resources, and can be solved efficiently as
a mixed-integer programming. Once the resources to be online are determined by unit
commitment, in order to make sure the resources are scheduled in the most economic way, or
to recalculate the optimum with slightly different settings than when units are committed,
the system operator can schedule the online resources more efficiently without the binary
variables. This problem is called economic dispatch, and we focus on this problem with
demand resources. The decisions made in the long-term procurement problem will affect
the capacity of the available resources when making the short-term scheduling decisions.

\[
\begin{align*}
\text{maximize} & \quad \sum_{t=1}^{T} \left[ \sum_{i=1}^{N_G} -c_i(P_{G_i}(t)) + \sum_{j=1}^{N_D} b_j(P_{D_j}(t)) \right] \\
\text{subject to} & \quad \sum_{i=1}^{N_G} P_{G_i}(t) = \sum_{j=1}^{N_D} P_{D_j}(t) \quad \forall t \\
& \quad |H(C_gP_G(t) - C_dP_D(t))| \leq F(t) \quad \forall t \\
& \quad \dot{P}_{G_i}(t) = f_i(P_{G_i}(t), \Theta_i(t)) \quad \forall i, t \\
& \quad \dot{P}_{D_j}(t) = f_j(P_{D_j}(t), \Theta_j(t)) \quad \forall j, t \\
& \quad P_{G_i}^{\text{min}}(t) \leq P_{G_i}(t) \leq P_{G_i}^{\text{max}}(t) \quad \forall i, t \\
& \quad P_{D_j}^{\text{min}}(t) \leq P_{D_j}(t) \leq P_{D_j}^{\text{max}}(t) \quad \forall j, t
\end{align*}
\]
The long-term decisions that are already made in Problem (2.1) determine $P_{G_i}^{\text{max}}$ and $P_{D_j}^{\text{max}}$ in Equations (2.2f) and (2.2g). In this work we model all the constraints in linear forms, and assume all the variables to be continuous, which makes the problem convex. Even so, this is a large problem depending on how many supply and demand entities you have in your system. Also, within the short-term horizon, conditions of the system and each entity can change after the scheduling has been settled, for example, a day ahead of operation. The next part of the thesis discusses the methodology we use to solve this problem.

2.2.3 Risk management

Since the benchmark problem cannot be solved deterministically without any uncertainty in the future, decisions made by each entity involve risks from the uncertainty. Note that in all our formulations for risk management, if we exclude the risk term in the objective functions, then it is equivalent to solving the benchmark problem in a decomposed way by each entity over a time horizon of interest. The bottom line of our approach to risk management is that the end-users should be able to have enough information to make decisions on savings and costs of investing in long-term energy efficiency so that the choices of end-users are reflected into the system operation. While enabling this involves a great deal of policy and business incentive issues, we focus on modeling end-users’ choices into decision makings of a load serving entity. We point out the importance of enabling the end-users’ choices into both long- and short-term energy purchase.

Long-term energy procurement of a load serving entity

As been pointed out in this section, the risks in long-term decision making of the power system come from different sources such as sociopolitical changes, climate changes, and uncertainty of demand as their result. To account for the uncertainty of demand in a long-term decision making, scenarios of system demand change with different probabilities can be set up.

However, incorporating the long-term decisions from demand resources into the system
optimization has not been considered widely. One of the reasons is because it is more difficult to measure the capacity savings from energy-efficient actions on the end-users’ side. Some Regional Transmission Organizations (RTOs) such as PJM offer a demand response program in the capacity market. PJM evaluates energy efficiency that a load serving entity likes to offer in the market and the capacity savings from it according to their predetermined rules [32]. However, it still remains questionable how many demand entities a system operator can evaluate and how reliable the evaluation is.

Another reason why it is difficult to include long-term decisions from demand is the reliability of the resources. For example, when a demand resource is measured to be able to potentially save certain megawatts by improving energy efficiency, can the system operator rely on the number on a particular day when the system demand hits the peak? In order to go around this problem, PJM mandates the demand resources that bid into the capacity market to be able to respond when they are needed. However, this mandate discourages the demand entities to participate in the capacity market because it is highly uncertain when their resources are to be deployed throughout the contract period.

Lastly, it is not entirely obvious what incentivizes end-users to choose long-term energy efficiency despite the investment cost and inertia of keeping their energy consumption system as is. While the monetary incentive seems to matter, there are other situational and behavioral factors that make a particular end-user choose energy efficient products and actions. In other words, there lies a great deal of uncertainty in the end-users’ behaviors and modeling of them, which remain as open problems.

In this subsection we formulate the problem of a load serving entity where she procures energy that her customers will consume in the future [33]. We divide the time horizons into yearly and monthly contracts. We especially focus on the multiple portfolio that an LSE can choose such as bilateral contracts with suppliers in addition to the energy markets. In this setting, an LSE is interested in hedging risks from the uncertainty of market prices, while purchasing adequate amount of energy for her customers. The end-use customers
are assumed to be divided into $S$ groups by contracts they already settled with the LSE. The rate charged to each group of end-users per unit of energy is denoted as $r$, a vector of length $S$. The anticipated total energy usage by each group of end-users during a month or a year is denoted as $\hat{P}$, also a vector of length $S$. The sum of all the elements of $\hat{P}$ will be equal to the estimate of $P_{D_j}$ in our benchmark problem (1.1), where the LSE is seen as demand entity $j$. The LSE’s long-term energy procurement by annual and monthly bilateral contracts are denoted as $\phi_a$ and $\phi_m$, respectively, while the spot market purchase amount for hour $t$ is denoted as $\phi_{sp}(t)$. End-users are charged a stratified rate for electricity hours, months, and years, denoted by $r[y]$, $r[m]$, and $r(t)$, respectively.

**Decision making on energy years** An LSE is given the long-term bilateral contract offer for the years to come; a price per MWh on certain blocks of time during the year, e.g. peak hours on weekdays from March to July. The LSE also has an estimate of how its load would evolve for the period based on historic data, and the estimate of the spot market price along with the monthly contract offers. With this price information of the system and the information of the energy consumption of the end-users that it serves, the LSE decides on how much energy to procure from the yearly contract; we call this amount of energy *energy year*\(^1\). The information on the demand of the end-users is formed as a function of the yearly charge of energy to the users. In a long-term decision making, the demand is also a function of investment in long-term energy efficiency measures such as insulating a building or replacing an old refrigerator with a more energy-efficient one.

Then the LSE’s optimization problem can be formulated as minimizing the energy cost and the risk from the uncertainty of demand and price minus the revenue from the

\(^1\)The concept of *energy minutes* was coined by Professor Daniel Siewiorek at Carnegie Mellon University, in the context of credit of energy bought by end-users that can be exchanged among each other.
end-users:

$$\min_{\phi, r, \zeta} \sum_{t=1}^{T} \left\{ p_{lt}^y \phi_a(t) + p_{mt}^y \phi_m(t) + \hat{p}_{sp} \phi_{sp}(t) \right\} + \beta F_{\alpha}(\phi_a, \zeta) - \sum_{y=1}^{Y} r^T[y] \hat{P}_y(r[y], C_{inv,y})$$

(2.3a)

subject to

$$E\{d_t\} = \phi_a(t) + \phi_m(t) + \phi_{sp} \quad \forall t$$

(2.3b)

$$\phi_{a,{\text{min}}} \leq \phi_a(t) \leq \phi_{a,{\text{max}}} \quad \forall t$$

(2.3c)

$$\phi_{m,{\text{min}}} \leq \phi_m(t) \leq \phi_{m,{\text{max}}} \quad \forall t$$

(2.3d)

$$E\{\sum_{t=t_{y,{\text{start}}}^{t_{y,{\text{end}}}}} d_t\} = \hat{P}_y(r[y], C_{inv,y}) \quad \forall y$$

(2.3e)

$$\hat{P}_{y,{\text{min}}} \leq \hat{P}_y \leq \hat{P}_{y,{\text{max}}} \quad \forall y$$

(2.3f)

where

$$F_{\alpha}(\phi_a, \zeta) = \zeta + \frac{1}{1-\alpha} E\{[p_{lt}^y \phi_a(t) + p_{mt}^y \phi_m(t) + \hat{p}_{sp} \phi_{sp}(t) - \zeta]^+\}$$

is a term for risk from the uncertainty of demand $d_t$ and price $\hat{p}_m$ and $\hat{p}_{sp}$.

We assume a linear demand function throughout this paper, which is:

$$\hat{P}(r[\cdot]) = Ar[\cdot] + b$$

where $A$ is an $S$-dimensional square matrix and $b$ is a column vector of length $S$. The diagonal elements of $A$ are usually negative, in other words, the higher the price the lower the demand. If we assume that the behavior of each group of end-users does not have any influence on each other, then $A$ is diagonal. In this yearly time scale including the investment in energy efficiency, the demand function is

$$\hat{P}_y(r[y], C_{inv}) = A'_y r[y] + b'_y.$$

If we define the demand function with no investment as $\hat{P}_y(r[y], 0) = A_y r[y] + b_y$, then the
elements of $A'_y$ are likely to be greater than those of $A_y$. This is because if the end-user’s premise is highly energy-efficient, then their energy usage is likely to be low without regard to the price, and thus less sensitive to the price change. Also, $b'_y$ is likely to be smaller than $b_y$ because of the reduction in energy consumption overall. This is depicted in Figure 2.3.

Consumer surplus from investing in energy efficiency is the difference of the energy cost from the case where they do not invest. If they do invest, it will incur them a certain capital cost, but the energy cost is likely to reduce. If they decide not to, then they have no initial capital cost, but the energy cost will be higher than the case where they improve their energy efficiency. Consumer surplus with $r[y]$ as a variable now can be defined as

$$CS(r[y]) = r^T[y] \hat{P}_y(r[y], 0) - r^T[y] \hat{P}_y(r[y], C_{inv}).$$

It only makes sense for an end-user to invest in energy efficiency when this $CS$ is greater than zero over the course of the optimization time horizon. Note that this optimization is done by the end-users with $r[y]$ offered by the LSE, which reflects the long-term price of electricity in the system. The resulting optimal demand with respect to the given $r[y]$ will be sent back to the LSE in order to decide on the energy year she needs to purchase. This procedure can be repeated until both the LSE and the end-user agree on the level of price.
and energy amount.

**Decision making on energy months**  Now after settling on the yearly contract for the next years, LSE likes to decide on, over a course of one year, how much energy to procure for each month on a monthly contract, and how much to charge within each month to each group of end-users. Monthly long-term bilateral contract energy prices are given to the LSE by a supplier. We also assume that LSEs have forecast of the anticipated monthly energy usage of the end-users as a function of the rate charged for each month, in a similar way from the yearly optimization. It also has information on anticipated hourly spot market price. As opposed to Equation 2.3a, the monthly contract price offer $p_{lt}^m$ is given deterministically instead of as an expected value $\hat{p}_{lt}^m$. Also, now you have better estimate on the demand $d_t$ than in the previous problem.

\[
\begin{align*}
\text{minimize} & \quad \sum_{t=1}^{T} \{ p_{lt}^m \phi_m(t) + \hat{p}_{sp,h} \phi_{sp}(t) \} + \beta F_\alpha(\phi_m, \zeta) - \sum_{m=1}^{12} r^{T}[m] \hat{u}_m(r[m]) \\
\text{subject to} & \quad E\{d_t\} - \phi^*_a(t) = \phi_m(t) + \phi_{sp}(t) \quad \forall t \\
& \quad \phi_{m,min} \leq \phi_m(t) \leq \phi_{m,max} \quad \forall t \\
& \quad E\{\sum_{h=\text{h}_{m,\text{start}}}^{	ext{h}_{m,end}} P_h\} = \hat{P}_m(r[m]) \text{ for } m = 1, \ldots, 12 \\
& \quad \hat{d}_{m,min} \leq \hat{d}_m \leq \hat{d}_{m,max} \text{ for } m = 1, \ldots, 12
\end{align*}
\]  

where

\[
F_\alpha(\phi_m, \zeta) = \zeta + \frac{1}{1 - \alpha} E\{[p_{lt}^m \phi_m(t) + \hat{p}_{sp,h} \phi_{sp}(t) - \zeta]^+\},
\]

is a term for risk from the uncertainty of demand $d_t$ and price $\hat{p}_{sp}$.

Note that solving for the hourly purchase amounts $\phi_m(t)$ and $\phi_{sp}(t)$ is a linear programming problem and can be solved independently of $r[m]$, if there were not the coupling constraint 2.4d. This means that without the coupling relationship between the expected
demand level and the monthly end-user rate, if $\hat{p}_{sp,h} - p_{lt}^{m} > 0$ then the optimal $\phi_{sp}(t)$ is its lower bound, and if $\hat{p}_{sp,h} - p_{lt}^{m} < 0$ then $\phi_{sp}(t)$ is the upper bound. This makes sense since when the spot market price is higher than the long-term contract price, then it is most profitable for the LSE to purchase all its energy from the contract, and vice versa. In reality, however, since the LSE does not have perfect knowledge on the anticipated price of the spot market in the future, the optimum will depend on the tradeoff between the expected cost that the LSE pays and the CVaR, the risk measure of the uncertainty.

Based on the optimal spot market purchase obtained from this formulation, we calculate the long-term monthly contract purchase amount and the monthly end-user rate, which is apart from the yearly end-user rate and energy amount locked in from the energy year optimization. In other words, the end-users will have a different rate and the amount limits on the energy that is purchased on the monthly contract, on top of the yearly contract that they made with the LSE.

**Implications of multi-temporal decision making of demand** This proposed framework calls for a fundamental change in the current market structure on the demand side and the demand response programs. There needs a pricing structure between the LSEs and the end-users to communicate and choose for the proper rate that the both parties can agree on. The information needed from the end-users does not necessarily have to be calculated by the end-users themselves. With the communication and computation infrastructure rolled out and used more widely, the terminal devices (smart meters) can do the job for the end-users when needed. On the end-users’s side, the interface should be intuitive and simple so that the complicated and intelligent computation is conducted by the computing terminal. LSEs can also think of a way to interpolate the end-users’ demand and price information from the historic data, if available. This assumption will make the implementation of this framework much more feasible even to small end-users. In any case, the information of the demand and their desired level of end-user rate should

---

\(^2\)We use the term spot market to include both day-ahead and real-time markets.
be communicated to the LSEs, and ultimately to the system.

This extended demand subscription framework [34] can change the demand market structure where the LSEs bear all the risk from the uncertainty of the demand and price, while the end-users in return pay for the high premium of avoiding this risk with a high flat rate. Since the end-users can opt to participate in procuring energy in advance with a possibly lower rate of bilateral contracts, the LSEs can relieve some of the risk from 1) the uncertainty from the demand by communicating the end-users’ needs, and 2) the uncertainty from the market price by procuring the more desired level of energy.

In minimizing the risk, the LSEs and the end-users will have a diversity in how much risk they are willing to take. This can also open up more choices for the end-users to subscribe to different energy services with various risk-reliability profiles. For example, given the same expected demand profile for a year, an LSE that is financially risk-prone (i.e. willing to take higher risks in price uncertainty) may choose to procure less amount of energy from the bilateral contracts than its counterpart who is financially risk-averse. This concept is depicted in Figure 2.4, where the vertical arrows denote the level of energy procurement with long-term bilateral contracts and the rest of the load is purchased from the spot market.

The long-term contracts can take different forms, and it can affect the risk from the price uncertainty. For example, with the same risk aversion, the optimal portfolio will differ if the LSE is allowed to sell back the energy that they procured from the previous longer-term contracts. If they are not allowed to do so, then it would limit the LSE’s
transactions on long-term contracts and result in more inflexible and conservative (i.e. risk-averse) portfolios. Therefore, in designing the markets in various time scales, it should be considered which markets should bind the physical transactions and which should allow for financial sellbacks.

**Short-term energy scheduling of a load serving entity**

In this subsection, we propose a method where a load serving entity can minimize the risk of uncertainty in day-ahead and real-time market prices [3]. The risk from the uncertainty of demand can be hedged by running different scenarios of the end-users’ behaviors in consuming energy, which we do not cover in this thesis.

**Financial risks from the day-ahead and real-time markets** We first start from managing the financial risks from the spot market, without regard to any physical model of the loads. In this setting, the LSE tries to minimize the risk from the spot market taking into account the correlations between the hourly market prices in both the day-ahead and real-time markets. Since the load does not have any dynamics, LSE only allows a minimum and maximum constraints for the load to deviate and optimizes their purchase with respect to the market price forecast.

A classical Markowitz portfolio optimization problem solves for the optimal mix of purchase of assets with different risk levels, and can be formulated as a quadratic programming [35]. Assuming that each of the assets has a normal distribution, the risk level is quantified as the variance of the distribution. We apply this modern portfolio theory, also called Markowitz mean-variance optimization, to minimizing the risk of energy purchase in the spot market.

\[
\begin{align*}
\text{minimize} & \quad w_r u^T \Sigma u + w_c (\bar{\lambda} - \mathbf{1} r)^T u \\
\text{subject to} & \quad u_{\min} \leq u \leq u_{\max}
\end{align*}
\]
For simplicity, we denote $P_D$, in Problem (1.1) as $u$. It is the amount of energy purchase by an LSE at each hour from day-ahead (DA) and real-time (RT) markets, where $u = [u_{DA}^T, u_{RT}^T]^T = [u_{DA,1}, u_{DA,2}, \cdots, u_{DA,24}, u_{RT,1}, \cdots, u_{RT,24}]^T$. $u_{\min}$ and $u_{\max}$ are the minimum and maximum hourly energy usage, respectively. $w_r$, $w_c$, $w_T$ are weights on risk, cost and temperature deviations in the objective function, respectively, which are all scalars. $I$ is a 48-by-24 matrix with binary elements defined as

$$I = \begin{cases} 
I_{ii} = 1 & \text{if } i = 1, \cdots, 24 \\
I_{(i+24),i} = 1 & \text{if } i = 25, \cdots, 48 \\
I_{ij} = 0 & \text{otherwise.}
\end{cases}$$

$\lambda$ is the price of electricity in day-ahead and real-time markets at all hours, where $\lambda = [\lambda_{DA}^T, \lambda_{RT}^T]^T = [\lambda_{DA,1}, \lambda_{DA,2}, \cdots, \lambda_{DA,24}, \lambda_{RT,1}, \cdots, \lambda_{RT,24}]^T$. Note that for the price in the future, this is a random vector with mean $\bar{\lambda}$ and covariance $\Sigma$ known. Otherwise, it is a fixed-value vector.

Each of the hourly day-ahead and real-time market price is assumed to have the Gaussian distribution, with mean and covariance calculated from the historic price data. The covariance matrix shows how correlated the hourly market prices are, and the variance of each of the hourly market prices itself. The covariance matrix can be divided into four different submatrices as follows:
$$\Sigma = \begin{bmatrix} \Sigma_{DA} & \Sigma_{DA-RT} \\ \Sigma_{DA-RT} & \Sigma_{RT} \end{bmatrix} = \begin{bmatrix} \sigma^2_{DA,1-DA,1} & \cdots & \sigma^2_{DA,1-DA,24} \\ \vdots & \ddots & \vdots \\ \sigma^2_{DA,1-RT,1} & \cdots & \sigma^2_{DA,1-RT,24} \\ \sigma^2_{DA,1-RT,24} & \cdots & \sigma^2_{DA,24-DA,24} \\ \vdots & \ddots & \vdots \\ \sigma^2_{RT,1-RT,1} & \cdots & \sigma^2_{RT,1-RT,24} \\ \vdots & \ddots & \vdots \\ \sigma^2_{RT,1-RT,24} & \cdots & \sigma^2_{RT,24-RT,24} \end{bmatrix}$$

(2.6)

Each component in this matrix is calculated as

$$\sigma^2_{ij} = E[(\lambda_i - E[\lambda_i])(\lambda_j - E[\lambda_j])]$$

(2.7)

where a sample of an hourly price is denoted as $\lambda_i$ or $\lambda_j$.

Covariance matrices are semi-positive definite by definition. If the covariance matrix in this problem is positive definite, then it has a closed-form solution

$$u(t) = \begin{cases} 
    u_{\text{min}} & \text{if } u^*(t) \leq u_{\text{min}} \\
    u_{\text{max}} & \text{if } u^*(t) \geq u_{\text{max}} \\
    u^*(t) & \text{otherwise}
\end{cases}$$

(2.8)

for all hour $t$

$$u^* = \frac{w_c}{2w_r} \Sigma^{-1}(\text{Ir} - \overline{\lambda})^T.$$ 

(2.9)

Relating the financial risk and physical constraints of the load  Now if we assume that the LSE can adjust some part of the loads and bid their price-responsiveness in the form of demand functions into the day-ahead market, then the formulation should include
the physical intertemporal dynamics of these loads. For the day-ahead market bidding, we choose air-conditioning load as controllable. This is because air-conditioning loads have a flexibility over multiple hours; in other words, the amount of energy consumed for air-conditioning within an hour, which is the interval of the day-ahead market, can be varied depending on the LSE’s optimization problem. On the other hand, the reason why loads with storage with a shorter time constant such as a residential refrigerator are not suitable for hourly bidding is because the state (or temperature) of a refrigerator varies with a much shorter time constant than an hour. This makes the amount of energy that a refrigerator can adjust very limited.

The modified Markowitz optimization including the physical dynamics of the loads as a constraint now includes an additional term in its objective function: temperature discomfort of the end-users. In this problem, it is defined as the squared error from the temperature setpoint determined in an hourly interval by the end-users. The detailed formulation of the first-order state space model of the air-conditioning load used in this formulation can be found in [36].

\[
\begin{align*}
\text{minimize} \quad & w_r (u^T \Sigma u) + w_c (\bar{\lambda} - \mathbf{1} \mathbf{r})^T u + w_T (x - x_{\text{set}})^T (x - x_{\text{set}}) \\
\text{subject to} \quad & x(t + 1) = \varepsilon x(t) + (1 - \varepsilon) \{x(t)^{\text{out}} + \gamma (u(t) + u(t + 24))\}, \\
& u_{\text{min}} \leq u(t) \leq u_{\text{max}}, \\
& x_{\text{min}} \leq x(t) \leq x_{\text{max}} \quad \text{for all } t = 1, 2, \cdots, 24.
\end{align*}
\]
Chapter 3

Methodology for short-term decision making

In this chapter, we discuss specific methodology to solve the short-term scheduling problems posed in the previous part. While we discussed both long- and short-term decision making of the problem, we put more emphasis on the short-term problem and propose a new concept to schedule resources in economic dispatch and in near real time.

Since the short-term benchmark problem cannot be solved in one problem as is, we decompose it with respect to different entities that optimize their sub-objectives and coordinate them by a system operator. There are a couple of reasons why we propose a decomposed optimization of this problem. First, the problem is of too high a dimension. If we like to consider all the individual loads that can react to the system condition, the dimension of the problem can reach millions\(^1\).

Secondly, related with the first argument, the system operator cannot have information on the dynamics of each supply and demand entity that participates in the system. In other words, the system operator can only know the key information that is needed to operate and plan for the system in the most reliable and cost-efficient way. In the current U.S. markets, RTOs usually take bids (i.e., quantity and price) from the market participants and clear the optimum within the global constraints such as transmission limits. However, the current market mechanism does not have a clear way to reconcile the global (system/market) objectives and constraints with the local units (supply and demand entities), especially one

\(^1\)The total number of end-use customers of electricity in the United States in 2011 was 144,509,146 [37].
that takes into account the intertemporal dynamics of the units. After clearing the market, if any physical constraints of the local units were violated, then the system operator simply re-clears the market with some modification. In our proposed framework, we overcome this problem by iterative communication between the units and the system operator, which we describe in Section 3.1.

While the iterative method for a day-ahead scheduling is effective, it may not be suitable for the near-real-time dispatch of supply and demand units. This is because the communication between the entities can take many iterations. Therefore, we suggest a different approach for near-real-time dispatch. We also note that the real-time dispatch functions as the correction of the result of day-ahead scheduling. This procedure is presented in Section 3.2.

3.1 Short-term decision making I: day-ahead iterative clearing

We restate the short-term decision making problem formulated in Chapter 2:

\[
\begin{align*}
\text{maximize} & \quad \sum_{i=1}^{N_G} \left[ \sum_{t=1}^{T} -c_i(P_{G_i}(t)) + \sum_{j=1}^{N_D} b_j(P_{D_j}(t)) \right] \\
\text{subject to} & \quad \sum_{i=1}^{N_G} P_{G_i}(t) = \sum_{j=1}^{N_D} P_{D_j}(t) \quad \forall t \\
& \quad |H(C_gP_{G_i}(t) - C_dP_{D_j}(t))| \leq F(t) \quad \forall t \\
& \quad \dot{P}_{G_i}(t) = f_i(P_{G_i}(t), \Theta_i(t)) \quad \forall i, t \\
& \quad \dot{P}_{D_j}(t) = f_j(P_{D_j}(t), \Theta_j(t)) \quad \forall j, t \\
& \quad P_{G_i}^{\min}(t) \leq P_{G_i}(t) \leq P_{G_i}^{\max}(t) \quad \forall i, t \\
& \quad P_{D_j}^{\min}(t) \leq P_{D_j}(t) \leq P_{D_j}^{\max}(t) \quad \forall j, t.
\end{align*}
\]

The objective function (3.1a) is the sum of objectives of the individual entities, and the constraints (3.1d)-(3.1g) can be separated with respect to each entity’s problem. The complicating constraints that include variables from multiple entities are (3.1b) and (3.1c).
It is well known that the Lagrange multipliers associated with these network constraints are related with the price of electricity. The Lagrange multiplier associated with constraint (3.1b) gives the uniform price throughout the system, and the one with constraint (3.1c) is the congestion cost at each bus. If there is no congestion, the system has a uniform price for all the buses with congestion costs. Moreover, the uniform price without congestion is the same as the price in a setting where you do not consider the network topology and simply match the demand and supply. This argument is proven in Appendix A.

We assume that variables $P_{G_i}$ and $P_{D_j}$ are continuous, and the cost/benefit functions are smooth convex/concave functions. The dynamics of the supply and demand entities in Equations (3.1d) and (3.1e) are linear, but we discretize the time intervals in accordance with the system operation rules. We assume hourly intervals in this work. Taking into these assumptions, we slightly modify the constraints (3.1d) and (3.1e) into

$$|P_{G_i}(t + 1) - P_{G_i}(t)| \leq R_i \quad \forall i, t = 0, \cdots, T - 1 \quad (3.2a)$$

$$P_{D_j}(t + 1) = a_j P_{D_j}(t) + \Theta_j(t) \quad \forall j, t = 0, \cdots, T - 1. \quad (3.2b)$$

We assume that only the loads that respond to the system price can be modeled as 3.2b. Therefore, we differentiate the demand entities that are controllable and not, and treat the uncontrollable load (or inelastic demand) as an exogenous parameter. Moreover, an end-user’s system dynamics is governed not only by the electric energy that she consumes. An end-user’s system has a possibly time-varying desired state, which is driven by her electricity consumption. This way an end-user’s benefit can be more clearly stated as the proximity to the desired state. Incorporating these remarks, Problem (3.1) can be rewritten as

$$\max_{P_{G_i}, P_{D_j}, x_j} \sum_{t=1}^{T} \left[ \sum_{i=1}^{N_G} -c_i(P_{G_i}(t)) + \sum_{j=1}^{N_D} b_j(P_{D_j}(t), x_j(t)) \right] \quad (3.3a)$$
subject to \( \sum_{i=1}^{N_G} P_{G_i}(t) = \sum_{j=1}^{N_D} P_{D_j}(t) + P_{D_{in}}(t) \) \( \forall t \) (3.3b)

\[ |H\{C_s P_G(t) - C_d(P_D(t) - P_{D_{in}}(t))\}| \leq F(t) \] \( \forall t \) (3.3c)

\[ |P_{G_i}(t + 1) - P_{G_i}(t)| \leq R_i \] \( \forall i, t = 0, \cdots, T - 1 \) (3.3d)

\[ x_{D_j}(t + 1) = a_j x_j(t) + b_j P_{D_j}(t) + \Theta_j(t) \] \( \forall j, t = 0, \cdots, T - 1 \) (3.3e)

\[ P_{G_i}^{\min}(t) \leq P_{G_i}(t) \leq P_{G_i}^{\max}(t) \] \( \forall i, t \) (3.3f)

\[ P_{D_j}^{\min}(t) \leq P_{D_j}(t) \leq P_{D_j}^{\max}(t) \] \( \forall j, t. \) (3.3g)

### 3.1.1 Decomposing the problem

There are many ways to decompose a large optimization problem such as our short-term decision making problem. We focus on the practical implication in decomposing this problem while minding the technical and mathematical perspectives. It means that the methodology that we propose for ALM framework has the following criteria: 1) it is feasible from the technical and policy perspectives, and 2) it guarantees the optimal global solution at least in our approximated problem settings. The first criterion considers the practical barriers in communication and control infrastructure of the system; the communication time should be short. The policy perspective is important in the sense that the information of various entities’ objectives cannot or will not be shared if they compete for limited revenue, customers, etc. The last criterion comes from the mathematical analysis of the problem.

We choose a dual decomposition method to solve this problem due to its direct economic interpretation and confidentiality between the market participants. Primal decomposition methods are not appropriate in a practical sense because it involves subproblems’ solutions being exchanged with each other to reach the global optimum\(^2\). 

\(^2\)For comparison of primal and dual decomposition methods, see for example, [40]).
Lagrange-relaxed dual decomposition

The Lagrange dual function of the global Problem 3.3 is defined as

\[ g(\lambda, \overline{\nu}, \mu) = \sup_{P_G, P_D, x} \mathcal{L}(P_G, P_D, x, \lambda, \overline{\nu}, \mu) \tag{3.4} \]

where

\[ \mathcal{L}(P_G, P_D, x, \lambda, \overline{\nu}, \mu) = \sum_{t=1}^{T} \left[ \sum_{i=1}^{N_G} -c_i(P_G_i(t)) + \sum_{j=1}^{N_D} b_j(P_D_j(t), x_j(t)) \right] \]

\[ + \sum_{t=1}^{T} \lambda(t) \{ \sum_{i=1}^{N_G} P_G_i(t) - \sum_{j=1}^{N_D} P_D_j(t) - P_{D_{in}}(t) \} \]

\[ - \sum_{t=1}^{T} \overline{\nu}_i^T(t) \{ H_i \{ C_g P_G_i(t) - C_d(P_D(t) - P_{D_{in}}(t)) \} - F(t) \} \]

\[ - \sum_{t=1}^{T} \mu_j^T(t) \{ -H_j \{ C_g P_G_j(t) - C_d(P_D(t) - P_{D_{in}}(t)) \} - F(t) \}. \tag{3.5} \]

With certain assumed values for the dual variables \( \lambda, \overline{\nu}, \) and \( \mu \), each supply entity \( i \) can solve an individual subproblem

\[ \text{maximize} \sum_{i=1}^{T} \left[ -c_i(P_G_i(t)) + \lambda(t) P_G_i(t) - \overline{\nu}_i^T(t) H_i P_G_i(t) + \mu_i^T(t) H_i P_G_i(t) \right] \tag{3.6a} \]

subject to \(|P_G_i(t + 1) - P_G_i(t)| \leq R_i \) for \( t = 0, \cdots, T - 1 \) \hspace{1cm} (3.6b)

\[ P_{G_{i_{\min}}}(t) \leq P_G(t) \leq P_{G_{i_{\max}}}(t) \quad \forall t, \tag{3.6c} \]

and each load serving entity (demand entity) \( j \) solves

\[ \text{maximize} \sum_{i=1}^{T} \left[ b_j(P_D_j(t), x_j(t)) - \lambda(t) P_D_j(t) + \overline{\nu}_j^T(t) H_j P_D_j(t) - \mu_j^T H_j P_D_j(t) \right] \tag{3.7a} \]

subject to \( x_j(t + 1) = a_j x_j(t) + b_j P_D_j(t) + \Theta_j(t) \) for \( t = 0, \cdots, T - 1 \) \hspace{1cm} (3.7b)
\[ P_{D_j}^{\text{min}}(t) \leq P_{D_j}(t) \leq P_{D_j}^{\text{max}}(t) \quad \forall t \]  

where \( \mu_{i(j)} \geq 0 \) is the congestion prices at the buses where supply(demand) unit \( i(j) \) is located\(^3\), and \( H_{i(j)} \) is the column of \( H \) that corresponds to the supply(demand) entity \( i(j) \). \( \mu \) is a vector that has the length equal to the number of the buses that the entity is connected to minus the slack bus since the power transfer distribution matrix \( H \) is defined to be of size, (the number of lines)-by-(the number of buses without slack). \( H_{i(j)} \) is also a vector with the same length as \( \mu_{i(j)} \) that includes only the elements related to the entity \( i(j) \) and the lines connected to it.

When the dual variables are known, each entity can calculate its optimum with respect to them. However, since the dual variables cannot be known to either the system operator or the entities, they need to be solved in an iterative way with the primal solutions of the individual entities without information of their objectives and constraints. In practical terms, the prices of electricity should be determined by communicating an estimate of the prices and the optima of each entity with respect to the price iteratively. In the next subsection, we discuss the way to coordinate the optima between the local primal optima (i.e., supply/demand quantities) of the entities and the global dual optimum (i.e., price).

Observing Problems (3.6) and (3.7), we can deduce the locational marginal price from the economic interpretation of the objective functions. Each problem can be viewed as a supply entity’s maximization of its profit, and a demand entity’s maximization of its benefit. (3.6) maximizes negative cost of production, the first term, plus revenue from the market, while (3.7) maximizes benefit of consumption minus cost of purchase. Therefore, the locational marginal price at bus \( i \) \( \pi_i \) can be defined as

\[ \pi_i = \lambda - H^T_i \bar{p}_i + H^T_i \mu_i \]  

(3.8)

\(^3\)When we refer to both \( \bar{p} \) and \( \mu \), we simply denote them as \( \mu \). Also when we use the variable in a general sense without regard to particular supply or demand entity \( i \) or \( j \), we simply drop the subscript \( i \) and \( j \).
Coordinating decomposed objectives: subgradient method

The dual of the original global problem (3.3) is

\[
\minimize_{\lambda, \mu} g(\lambda, \mu) = \sup_{P_G, P_D} \mathcal{L}(P_G, P_D, x, \lambda, \mu) \tag{3.9a}
\]

\[
= \sum_{t=1}^{T} \sum_{i=1}^{N_G} -c_i(P^*_G(t)) + \sum_{j=1}^{N_D} b_j(P^*_D(t), x^*_j(t))
\]

\[
+ \sum_{t=1}^{T} \lambda(t) \{ \sum_{i=1}^{N_G} P^*_G(t) - \sum_{j=1}^{N_D} P^*_D(t) - P_{D_{in}}(t) \}
\]

\[
- \sum_{t=1}^{T} \mu^T(t) [H\{C_gP^*_G(t) - C_d(P^*_D(t) - P_{D_{in}}(t))\} - F(t)]
\]

\[
- \sum_{t=1}^{T} \mu^T(t) [-H\{C_gP^*_G(t) - C_d(P^*_D(t) - P_{D_{in}}(t))\} - F(t)] \tag{3.9b}
\]

subject to \( \mu \geq 0, \mu \geq 0 \) \tag{3.9c}

where \( * \) denotes the optimum value of the variable. There are many ways to find the optimum of this dual problem in an iterative way. Since this problem is convex and continuous, a gradient-based method is appropriate.

There are a number of other ways to update the dual variables of a decomposed convex problem. We choose the subgradient method because it requires the least amount of information from the local entities to solve the global system problem. Cutting plane method, bundle method, trust region method all require that the central coordinator know at least part of the objective functions of the local problems, since the dual functions need to be updated at every iteration [41]. Meanwhile, the subgradient method can be slow to converge since it only requires the subgradient of the dual problem to update the dual variables. However, exactly because of this reason, it is more desirable in the market environment where the participants like to reveal the least amount of their information to the system and the competitors. Faster algorithms such as the augmented Lagrange relaxation method also compromise the level of local information exposure to the global
With the subgradient method, the dual variables can be updated at each iteration with respect to the subgradient of the dual function. The subgradients are obtained from the local primal optima calculated with the dual variables evaluated at the previous iteration. Therefore, omitting \((t)\) for simplicity since each equation applies to all \(t\)’s, we have

\[
\lambda^{(\nu+1)} = \lambda^{(\nu)} + \alpha^{(\nu)} \left\{ \sum_{i=1}^{N_G} P^*_G(\lambda^{(\nu)}, \overline{\mu}^{(\nu)}, \mu^{(\nu)}) - \sum_{j=1}^{N_D} P^*_D(\lambda^{(\nu)}, \overline{\mu}^{(\nu)}, \mu^{(\nu)}) - P_{Di} \right\} 
\]

\[
\overline{\mu}^{(\nu+1)} = \left[ \overline{\mu}^{(\nu)} + \alpha^{(\nu)} \left[ H\{C_g P^*_G(\lambda^{(\nu)}, \overline{\mu}^{(\nu)}, \mu^{(\nu)}) - C_d(\overline{P}_D(\lambda^{(\nu)}, \overline{\mu}^{(\nu)}, \mu^{(\nu)}) - P_{Di})\} \big] + F \right]^{+}
\]

\[
\mu^{(\nu+1)} = \left[ \mu^{(\nu)} + \alpha^{(\nu)} \left[ -H\{C_g P^*_G(\lambda^{(\nu)}, \overline{\mu}^{(\nu)}, \mu^{(\nu)}) - C_d(\overline{P}_D(\lambda^{(\nu)}, \overline{\mu}^{(\nu)}, \mu^{(\nu)}) - P_{Di})\} \big] + F \right]^{+}
\]

where \(\nu\) denotes the iteration step, and \([·]^{+}\) denotes the projection onto the nonnegative orthant.

**Convergence of dual problem to the primal optimum**

If the cost and benefit functions are differentiable regardless of convexity of the problem, the necessary Karush-Kuhn-Tucker (KKT) optimality conditions of the Lagrangen function in (3.9a) are

\[
\partial_{P_G} \mathcal{L} = \sum_{i=1}^{N_G} \left\{ \frac{-dc_i(P_G_i)}{dP_G_i} + [\mathbf{I}_b^T : - \mathbf{I}_b^T] \nu_i \right\} + \lambda - H^T_G(\overline{\mu} - \underline{\mu}) = 0 
\]

\[
\partial_{P_D} \mathcal{L} = \sum_{j=1}^{N_D} \left\{ \frac{\partial b_j(P_{D_j}, x_j)}{\partial P_{D_j}} + \frac{\partial F_j(P_{D_j}, x_j, \Theta_j)}{\partial P_{D_j}} \eta_j \right\} - \lambda + H^T_D(\overline{\mu} - \underline{\mu}) = 0 
\]

\[
\sum_{i=1}^{N_G} P_G_i - \sum_{j=1}^{N_D} P_{D_j} = 0 
\]

\[
F_j(P_{D_j}, x_j, \Theta_j) = 0 \quad \forall j = 1, \cdots, N_D 
\]
\[
\eta_i^T \begin{bmatrix} I_b P_{G_i} - R_i \\ -I_b P_{G_i} - R_i \end{bmatrix} = 0, \quad \eta_i \succeq 0, \quad \text{and} \quad \begin{bmatrix} I_b P_{G_i} - R_i \\ -I_b P_{G_i} - R_i \end{bmatrix} \succeq 0 \quad \text{for } i = 1, \cdots, N_G \quad (3.15)
\]

Note that in this condition, we include the local constraints in addition to the global objectives and constraints, and all the variables are vectorized, e.g., \( P_{G_i} = [P_{G_i(1)}, \cdots, P_{G_i(T)}]^T \).

\( \eta_i \) and \( \eta_j \) are the local Lagrange multipliers associated with the local constraints (3.6b) and (3.7b), which are both rearranged in an implicit form, i.e., they are

\[
\begin{bmatrix} I_b P_{G_i} - R_i \\ -I_b P_{G_i} - R_i \end{bmatrix} = 0 \quad (3.16)
\]

and

\[
F_j(P_{D_j}, x_j, \Theta_j) = 0, \quad (3.17)
\]

respectively where \( I_b \) is a \((T - 1)\)-by-\( T \) bidiagonal matrix defined as

\[
I_b = \begin{bmatrix}
1 & -1 & 0 & \cdots & 0 \\
0 & 1 & -1 & \cdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & 0 \\
0 & \cdots & \cdots & 1 & -1
\end{bmatrix}
\]

Assuming that the local problems are always feasible and the global problem is convex, the KKT conditions are necessary and sufficient for the strong duality to hold. In this case the marginal price of the system is

\[
\lambda = \sum_{i=1}^{N_G} \left\{ \frac{dc_i(P_{G_i})}{dP_{G_i}} - \begin{bmatrix} I_b^T & -I_b^T \end{bmatrix} \eta_i \right\} + H_i^T (\overline{p} - \mu) \quad (3.18)
\]

\[
= \sum_{j=1}^{N_D} \left\{ \frac{\partial b_j(P_{D_j}, x_j)}{\partial P_{D_j}} + \frac{\partial F_j(P_{D_j}, x_j, \Theta_j)}{\partial P_{D_j}} \eta_j \right\} + H_j^T (\overline{p} - \mu) \quad (3.19)
\]

We observe that the system price, or the locational marginal price that includes the
congestion cost for the matter, is not exactly equal to the marginal cost/benefit of the system supply/demand. The added second terms in both (3.18) and (3.19) are the multipliers associated with the local dynamics of a supply and demand unit. The difference between these two multipliers is that \( \eta_i \) is for the inequality constraint while \( \eta_j \) is for an equality constraint. Therefore, all the elements of \( \eta_i \) will be zero except when the inequality constraint is binding.

More precisely, the inequality constraint is related with the ramp rate constraints of a supply unit binding its minimum and maximum output at each time step. Therefore, only at a time step \( t \) when generator \( i \) is running at its minimum or maximum ramp rate constraint will \( \eta_i(t) \) be nonzero and affect the market price. On the other hand, since the constraint related with \( \eta_j \) is an equality constraint, it will be nonzero in order for the solution to be feasible. This can be interpreted as the marginal benefit adjusted by the equality constraint, i.e., the physical dynamic model of the load.

If the strong duality holds for the original problem (3.3), the duality gap is zero and the dual optimum obtained by (3.10) is equivalent to the primal solution of (3.3). The most well-known condition for a convex problem to hold the strong duality is Slater’s condition [42]. In addition to the global primal problem being convex, the local problems should be strictly convex and finite for the local optima to converge to the global dual optimum. Since we assume that the individual cost and benefit functions \( c_i(P_{Gi}) \) and \( b_j(P_{Dj},x_j) \) are strictly convex/concave [40, 43, 44], this condition is also satisfied. The inequality constraint of a generator (3.6b) is linear, and the demand dynamics (3.7b) is also modeled as a linear system. While the total system load usually cannot be model as a linear system, some loads with energy storage such as space conditioning, battery charging, heating/cooling devices can be represented as a linear state space model with electric energy usage as its input. In order for the local problems to be finite, there must always exist a solution within the feasible regions of the problems. The iteration step sizes in (3.10) can be chosen to satisfy \( \lim_{\nu \to \infty} \alpha^{(\nu)} = 0, \sum_{\nu=0}^{\infty} \alpha^{(\nu)} = \infty \) in order for the global
dual problem (3.3) to converge [45].

Regarding the current market mechanism and generation cost models, we see that the non-convexity of the cost functions comes not from the nature of the cost of production, but rather from the modeling of the production cost and the rules of electricity markets. Most markets take the cost of each generator as a single value with respect to its supply quantity, and construct a stepwise increasing marginal cost supply curve for the whole system. The steps (i.e., costs) of the generators here are already an approximation with respect to the supply quantity. By setting the rule of cost functions to be convex in the market, the costs of generators can actually be more accurately modeled with respect to the quantity than with the single-valued costs.

Aggregation of end-users by load serving entities

If we assume that a load serving entity’s revenue is fixed from predetermined contracts with its end-users and the LSE is simply trying to minimize the cost of purchasing energy from the market, the load serving entity’s problem can be further decomposed into individual end-users’ problems. The load serving entity’s objective function will be the aggregate of the end-users’, and the end-users’ systems will have individual dynamic equations similar to Equation (3.7b). In this case a load serving entity represents the aggregate load of its customers in the market and simply passes on the system price information to the end-users and the optimal quantity information from the end-users to the system operator.

However, the load serving entity’s problem in practice can be much more complicated. If the revenue of the load serving entity depends heavily on the flexible energy usage, the revenue should also be a function of $P_{Dj}$. There can be a variety of contracts with end-users, and finding the optimal contract with each end-user is a nontrivial problem by itself. This also involves modeling other competing load serving entities for customers, and can be approached as a game theory problem, which we leave as future work.
3.1.2 The algorithm of day-ahead iterative clearing

The procedure of day-ahead iterative clearing is depicted in Figure 3.1. Each plot has the supply and demand curves of the whole system. These can be thought as the aggregate of different cost and benefit functions of the supply and demand entities. Note that these curves are not known to the system operator. The system operator simply broadcasts an initial guess of the global dual variables, which is the locational marginal price. The y-axes of the plots denote this price that is sent to each local entity. The price can be different at different time steps, as shown in the figure. After the local entities calculate their optimal quantities and send them to the system operator, the system operator updates the global dual variables (or the price) by (3.10) and send the new price to the local entities. This procedure is repeated until the predetermined stopping criterion is satisfied. Note that this procedure is done over a multi-step horizon, and the local entities calculate their optimum over the horizon within their physical limits and subject to their local dynamics.

The following are the steps of the algorithm.

1. Set $\nu = 0$.
2. System Operator (SO) broadcasts a set of $\pi^{(\nu)}_i$ for all buses, which consists of $\lambda^{(\nu)}, \bar{\mu}^{(\nu)}, \underline{\mu}^{(\nu)}$, to all the market participants, supply $i = 1, \cdots, N_G$ and demand $j = 1, \cdots, N_D$ entities, for the whole time horizon of $T$ time steps. Note that $\bar{\mu}^{(\nu)}$ and $\underline{\mu}^{(\nu)}$ are particular to the bus, and each entity receives different values for the bus where it is located.

Figure 3.1: The procedure of day-ahead iterative clearing
located. The initial $\lambda^{(0)}$ can be calculated based on the forecast of system load and the correlation between the price and load.

3. Each market participant calculates its optimal energy production/consumption with respect to $\pi^{(\nu)}$ by solving either (3.6) or (3.7). The results are $P_{G_i}^{*^{(\nu)}}$ for $i = 1, \cdots, N_G$ and $P_{D_j}^{*^{(\nu)}}$ for $j = 1, \cdots, N_D$.

(a) Each load serving entity broadcasts the locational marginal price $\pi^{(\nu)}$ that she received from SO to its end-users.

(b) The end-users calculate their optimal energy consumptions with respect to the locational marginal price and send them to the load serving entity.

(c) Load serving entities send the sum of their end-users’ consumption as her optimal energy consumption to the SO.

4. The results from the previous step, which are vectors of length $T$, are sent to SO.

5. If the convergence criterion (e.g., the mismatch between the total supply and total demand at each time step being less than a preset tolerance) is satisfied, it concludes the algorithm. If the convergence criterion is not satisfied, SO updates the prices $\lambda^{(\nu)}, \pi^{(\nu)}, \bar{\mu}^{(\nu)}$ as in (3.10), set $\nu = \nu + 1$ and go back to 2).

### 3.2 Short-term decision making II: real-time functional clearing

#### 3.2.1 Background

For scheduling resources hours to a day ahead of operation, iterative clearing is effective since it finds the global system optimum that also satisfies the local constraints and dynamics without having the local entities expose their information. However, it may take a number of iterations to find the system optimum. For this reason, if the communication time for scheduling is constrained, a different method should be considered. Moreover, after the resources are scheduled with iterative clearing as discussed in the previous chapter, the conditions of either the system network or the local entities can change. For this
reason, there needs an algorithm that can modify the day-ahead scheduling in a faster time scale.

We adopt the functional clearing method of Dynamic Monitoring and Decision Making System (DYMONDS) [46] for the near-real-time adjustment. Each entity calculates its own optimal quantity with its price sensitivity (i.e., price-quantity bid) and submit this information to the system operator. The system operator clears the bids and each entity operates the quantity dispatched from the system operator. The price sensitivity can be obtained by calculating optimal quantities with respect to different prices, where the ratio of the change in price to the change in quantity is defined as the price sensitivity of demand/supply. Since the system balances at discrete time steps, this can be calculated at each time step. However, the dynamics of the local units are intertemporal. Therefore, at each time step, with the price sensitivity of supply/demand, the minimum and maximum quantity limits should also be specified from the local units so that the system operator dispatches within the feasible limits.

The exact quantity limits of supply and demand bids are difficult to set for further into the future, since they are intertemporal and dependent on the initial quantity. For example, if a power plant whose ramp rate is 3MW/hour is running at 40MW, in 10 hours this plant can be operating anywhere between 10MW and 70MW or the plant’s capacity, whichever is smaller. This is a pretty wide range, and if the plant ends up operating at 42MW in 9 hours, this range is not valid any more. For this reason of intertemporality of the local units, we perform real-time functional clearing in a receding horizon.

3.2.2 Obtaining the price sensitivity of demand

In order to show the procedure to obtain the price sensitivity of supply/demand bids, in this section we show the procedure for a demand entity. The same algorithm can be applied to a supply entity.

The price sensitivity of demand $\rho_j$ given the reference price $\pi_0$ and demand $P_{D_j,0}$ is
defined as
\[ \rho_j = \frac{\Delta \pi}{\Delta P_{D_j}} = \frac{\pi'-\pi_0}{P_{D_j}'-P_{D_j,0}}. \] (3.20)

Note that the price sensitivity of demand is specific to the reference price and demand quantity. Prices \( \pi_0 \) and \( \pi' \) are given to the demand entity, and come from the system condition as described in Problem (3.7). In fact, \( \pi \) can be considered as \( \pi_j = \lambda + \mu_j^T H_j - \mu_j^T H_j \), i.e., the locational marginal price of demand entity \( j \) that includes the congestion costs.

Assume that the real-time clearance is done in an hourly interval, and the quantities of supply and demand are settled from the day-ahead market. Thus we know how much energy is purchased from the day-ahead market, and the price at which we pay the day-ahead energy. Now we want to adjust the near-real-time energy consumption from the day-ahead settlement. We reiterate Problem 3.7 but with slightly different notations to account for the amount \( P_{DA}^{D_j} \) already purchased and settled from the day-ahead market.

\[ \maximize_{P_{RT}^{D_j}} \sum_{t=1}^{T} \{b_j(P_{D_j}(t), x_j(t)) - \pi_{RT0}^{RT}(t)P_{RT}^{D_j}(t)\} \] (3.21a)

subject to \( x_{D_j}(t+1) = a_j x_j(t) + b_j \{P_{DA}^{D_j}(t) + P_{RT}^{D_j}(t)\} + \Theta_j(t) \) for \( t = 0, \ldots, T-1 \) (3.21b)

\[ P_{min}^{D_j}(t) \leq P_{DA}^{D_j}(t) + P_{RT}^{D_j}(t) \leq P_{max}^{D_j}(t) \quad \forall t \] (3.21c)

Solving this problem gives the optimum \( P_{RT}^{RT*}(t) \) for \( t = 1, \ldots, T \), with a given set of price \( \pi_{0}^{RT} = [\pi_{0}^{RT}(1), \ldots, \pi_{0}^{RT}(T)]^T \). By replacing \( \pi_{0}^{RT} \) in the problem with

\[ \pi' = [\pi^{RT}(1), \pi^{RT}(2), \ldots, \pi^{RT}(T)]^T \]

and solving the problem, we obtain \( P_{RT}^{RT*}(t) \) for \( t = 1, \ldots, T \). Note that the price at only the next time step \( t = 1 \) is changed and the price at the other time steps are intact. Since we solve real-time functional clearing in a receding horizon, we calculate the price sensitivity
of demand only for the next time step \( t = 1 \). Therefore, we get the price sensitivity of demand at the next time step \( t = 1 \) by taking only the first elements of the solutions \( P_{D_j,0}^\star(1) \) and \( P_{D_j,1}^\star(1) \) and plugging them in (3.20). In our simulations, to account for the price sensitivity of demand for both price increase and decrease, we calculated three sets of different demand quantities with respect to the price, e.g., \( P_{D_j,0}^\star(1) \), \( P_{D_j,1}^\star(1) \), \( P_{D_j,-1}^\star(1) \) with respect to \( \pi_{0,RT} \), \( \pi_{1,RT} \), \( \pi_{-1,RT} \), and extrapolated the sensitivity by least-square estimation. This procedure is shown in Figure 3.2.

![Figure 3.2: Procedure of calculating price sensitivity of demand](image)

There can be different ways to construct demand functions as long as it captures the sensitivity of demand with respect to the price. For example, assuming different sets and values of \( \pi_{0,RT} \), \( \pi_{1,RT} \) can yield different demand functions. The price sensitivity of demand can also be calculated based on the historical data of price and demand and by learning the correlation between the two. We leave improvement of calculation of demand functions as future work.

**Aggregation of end-users by load serving entities**

In day-ahead iterative clearing discussed in Section 3.1, the optimal quantities and the price can be passed on through the load serving entities by the end-users and the system.
operator. However, since the real-time functional clearing requires the price sensitivity of *aggregate* demand to be sent to the system operator, there should be a more careful coordination by the load aggregator. After a set of different prices for the next coming time step, say \( \pi_{0}^{RT}, \pi_{1}^{RT}, \pi_{-1}^{RT} \) are sent to the end-users by the load aggregator, the end-users calculate their optimal real-time energy usage \( P_{Dj,0}^{RT}, P_{Dj,1}^{RT}, P_{Dj,-1}^{RT} \) with respect to the set of prices. The end-users send the maximum and minimum energy usage limits along with these energy quantities. The load serving entity calculates the price sensitivity of the aggregate demand with the sum of the individual energy usages, along with the aggregate maximum and minimum limits to bid into the system operator.

After the system operator clears the time step with the aggregate demand bids from the load serving entities within the limits of each entity, the load serving entity needs to realize the dispatch. Since the dispatch that a load serving entity received from the system operator will lie within the limits that she bid in, she can calculate the portion that she should consume, as an aggregate. For example, if the load serving entity bid in between \( P_{\text{min}} \) and \( P_{\text{max}} \) MWh and received a dispatch of \( d \) MWh, then the load serving entity dispatches the portion, which is \( p = \frac{d - P_{\text{min}}}{P_{\text{max}} - P_{\text{min}}} \) in this case, to the end-users. Each end-user uses \( p \) of her energy bandwidth (the maximum minus the minimum limit that she calculated and sent to the load serving entity earlier) and updates her initial state to calculate her demand bids for the next time step.

### 3.2.3 The algorithm of real-time functional clearing

The procedure of real-time functional clearing is shown in Figure 3.3. Note that the procedure is conducted on a moving horizon with updated initial states of the end-users. The figure shows the demand side of the procedure, but the same goes with the supply side as well. The following lists the steps of the procedure.

1. Let the current time step \( t \). The real-time price up to time step \( t-1 \) is known, since the price is determined ex post.

2. System Operator (SO) broadcasts a set of \( \lambda = [\lambda(t), \ldots, \lambda(T)]^T, \lambda^{(\nu)} = [\lambda(t), \ldots, \lambda(T)]^T, \)
Figure 3.3: Procedure of real-time functional clearing

\[ \mu = [\mu(t), \ldots, \mu(T)]^T \] to all the market participants, supply \( i = 1, \ldots, N_G \) and demand \( j = 1, \ldots, N_D \) entities, for the whole time horizon of \( T - t + 1 \) time steps.

3. Each market participant calculates its price sensitivity of optimal energy production/-consumption with respect to \( \lambda \) and \( \mu \) by solving (3.21) for the first forthcoming time step. This price sensitivity is interpreted as supply/demand function of a supply/demand unit, which is a marginal cost/benefit with respect to a given supply/demand quantity. Note that each entity calculates its quantity limits of supply/demand for the next time step given the current state (e.g., indoor temperature, generation output) and its constraints (e.g., temperature dynamics, ramp rates).

4. SO clears the price \( \lambda(t) \). The optimal \( P_{G_i}^*(\lambda(t))'s \) and \( P_{D_j}^*(\lambda(t))'s \) are dispatched to each supply/demand unit. This is equivalent to Step 2.

5. Each unit realizes the dispatch from SO and calculates the state at the next time step. Given this state as the initial state, each unit can repeat the procedure in Step 3.
3.3 Summary of the methodology and implications

In this chapter we proposed two different approaches for different time horizons in the short-term decision making of the benchmark problem. For scheduling a day or shorter in advance of operation, iterative clearing is effective for many reasons. It captures the intertemporal dynamics and constraints of the local entities while settling to the global system optimum, when modeled as a decomposable convex problem. The subgradient method is used to coordinate the local optima and the global objective of the system. This method allows the supply and demand entities, (and the end-uses) to keep their objectives covert to the system operator (and their load serving entities).

After the day-ahead scheduling is settled, a near-real-time adjustment may be needed. Since the iterative method used for day-ahead scheduling can take a large number of iterations to reach the system optimum, we use the functional clearing method in a moving horizon. The supply and demand entities bid in their price sensitivity of demand/supply and their operational limits for the next coming time step only. The load serving entities calculate this from the aggregate of their individual loads. After the system dispatches the adjustment amount from the day-ahead settlement to each entity, a supply/demand entity realizes the dispatch. A load serving entity does so by dispatching again the amount to its individual end-users. With the current dispatch amount from the system operator, each market participant and end-user updates her initial condition and repeats the procedure of obtaining the price sensitivity of demand/supply for the next coming time step. This functional clearing allows a timely update between the end-users, market participants, and the system operator since it requires only one-time communication among the entities for each time step.

The information exchange framework we proposed has implications on policy regarding system operation when demand resources are included. We observe that day-ahead scheduling should be done in a way that the intertemporal dynamics of demand systems are included. On the other hand, since real-time adjustment is done for one time step
in a moving horizon, different information, i.e., the price sensitivity and quantity limits, is required from the day-ahead scheduling. We also emphasize that information exchange between load serving entities and end-users shape the retail energy service products offered from LSEs to end-users. For example, LSEs can offer a subscription service [34] that fixes the day-ahead scheduled amount with the end-users and charges real-time market price (or an approximated fixed time-varying rate) for real-time functional clearing. LSEs can also make contracts with end-users to charge them a flat rate in exchange of having some of their loads controlled within predetermined range. The variations can be numerous and the optimal policy between LSEs and end-users remains as an open problem.
Part III

Numerical Examples
Chapter 4

The Azores Islands, Portugal

In this chapter we apply the short-term scheduling methods of Adaptive Load Management (ALM) on the Azores Islands, Portugal. The contents are excerpted and edited from [47]; the studies in the book were done for the year 2008. We put emphasis on modeling the demand resources and scheduling them with the rest of the generation resources over various time horizons, based on scenarios including a large amount of wind power in two of the islands, Flores and São Miguel. We identify the types of loads that can be used for different time horizons, and calculate the potential benefits of adjusting some of the loads. For short-term scheduling, we apply the functional clearing method a day ahead of operation and point out the limitations of the method. This shows how the functional clearing method described in Chapter 3.2 should be used over a moving horizon with constant updates of the initial conditions of the local end-user systems, in order to guarantee the feasibility of the local system. In order to focus on the clearing methods, we consider the system without network constraints. The case of using both iterative and functional clearing over the short-term scheduling horizon will be shown in the following Chapter 5.

4.1 Overview of the power systems in Flores and São Miguel

The Archipelago of the Azores ( Açores) is composed of nine volcanic islands situated in the North Atlantic Ocean, and is located about 1,500 km west of Lisbon, Portugal(Figure 4.1) [48]. We study the systems of two of the islands, Flores and São Miguel. The main
industries of the archipelago are agriculture, dairy farming, and tourism. The climate is mild throughout the year, with daily maximum temperature between 15 to 25°C and minimum between 11 to 18°C. Winter has a higher precipitation of 136 mm on average in December while summer is dry where July has about 32 mm of precipitation. Flores has a population of about 3907 inhabitants in an area of 143 km², whereas São Miguel is the biggest and most populous island in the archipelago with an area of 759 km² and about 140,000 inhabitants.

4.1.1 Loads

Flores

For the year 2008, total electric energy produced in Flores was 11.6 GWh. Energy consumption can be allocated by consumer type as shown in Figure 4.2 [49]. Statistics by Electricidade dos Açores (EDA), the system operating utility for the Azores Islands, show that residential customers used roughly 4.5 GWh of energy. The load duration curve of
Figure 4.2: Energy Consumption by Consumer Type for the Island of Flores, 2008

Figure 4.3: Flores Island System Load Duration Curve

our data for Flores Island in 2008 is plotted below in Figure 4.3. Figure 4.3 shows that the system load is between 1800 kW and 1000 kW for the vast majority of the time. The system reaches a maximum of 1978 kW and a minimum of 701 kW. Figure 4.4 shows the annual averaged system load pattern, which is stratified into weekdays, Saturdays, and Sundays.

São Miguel

In the same year, total electric energy produced in São Miguel was 441 GWh. Energy consumption can be allocated by consumer type as shown in Figure 4.5 [49]. EDA statistics
show that residential customers used roughly 132 GWh of energy. The load duration curve of our data for São Miguel Island in 2008 is plotted below in Figure 4.6. The vast majority of the hours have loads between 30 and 70 MW. The system reaches a maximum load of 73.9 MW and a minimum of 25.4 MW. The annual average system load pattern for São Miguel Island is shown in Figure 4.7.

4.1.2 Generation

Flores

Flores Island is powered by a fleet of diesel, hydropower, and wind generators. In 2008, 52% of energy produced was by diesel, 31% by hydropower, and 17% by wind power.
Figure 4.6: São Miguel Island System Load Duration Curve for 2008

Figure 4.7: Averaged Annual System Load Profiles for São Miguel Island
The energy available from hydropower and wind power changes significantly by season. Monthly averages of the daily profile of hydropower output on Flores are plotted below in Figure 4.8. Duration curves of wind and hydropower are also shown in Figures 4.9 and 4.10 for the different seasons.

Figures 4.8 and 4.9 show the seasonal variation in availability of hydropower on Flores Island. November through March have the most hydropower availability while the summer months have lower availability. Hydro output in the summer is below 400 kW for the majority of the hours, while staying between 300 and 800 kW for the vast majority of the winter.

Figure 4.10 below shows the variation in seasonal availability of wind power. Summer clearly has the lowest wind resource. During the summer there are only 6 hours at maximum output, and the majority of hours have output below 100 kW. The other 3 seasons achieve maximum output for roughly 200 hours. Winter and autumn appear to have the best wind resource availability. Table 4.1 shows the power capacity, minimum output, and fuel type of individual generators installed on Flores Island.
Figure 4.9: Seasonal Duration Curves for Hydro Power on Flores Island

Figure 4.10: Seasonal Duration Curves for Wind Power on Flores Island
Table 4.1: Data for Installed Energy Generation Equipment on the Island of Flores as of 2008

<table>
<thead>
<tr>
<th>Power plant</th>
<th>Type</th>
<th>$P_{\text{min}}$ (MW)</th>
<th>$P_{\text{max}}$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Além-Fazenda</td>
<td>Diesel</td>
<td>0.18</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.28</td>
<td>0.7</td>
</tr>
<tr>
<td>Boca da Vereda</td>
<td>Wind</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>Além-Fazenda</td>
<td>Hydro</td>
<td>0.371</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.371</td>
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<tr>
<td></td>
<td></td>
<td>0.371</td>
<td>0.76</td>
</tr>
</tbody>
</table>

São Miguel

São Miguel currently has no wind power installed, and gets only 4.5% of its energy from hydropower. However, São Miguel obtains nearly 40% of its energy from geothermal power plants. The remainder of São Miguel’s energy is generated from heavy fuel oil [49].

Seasonal duration curves of geothermal power output are shown in Figure 4.11. This plot shows the sum of the output from São Miguel’s two geothermal power plants. During winter, spring, and summer, the power plants consistently produce over 18 MW. Autumn has the weakest resource availability, yet output is greater than 20 MW half of the time. The generation equipment installed on São Miguel is shown in Table 4.2.

4.2 Designing adaptive load management in the Islands

Even though the power systems in the Azores islands are not operated by a market mechanism and vertically integrated, one of the operational criteria is the cost of producing energy. Also, while there are not any load aggregators or mediators that represent the end-users’ value of electricity on the market, the end-users do respond to the tariffs that their bills are based on. This implies that even when the market is not explicitly run in the Azores’ systems, we can capture the costs of producing energy, and the value of consuming
Figure 4.11: Seasonal Power Duration Curves of Geothermal Power on São Miguel Island

Table 4.2: Data for Installed Energy Generation Equipment on the Island of São Miguel as of 2008

<table>
<thead>
<tr>
<th>Power plant</th>
<th>Type</th>
<th>$P_{\text{min}}$ (MW)</th>
<th>$P_{\text{max}}$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Caldeirão</td>
<td>Fuel oil</td>
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</tr>
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<td></td>
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<td>3.848</td>
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<td></td>
</tr>
<tr>
<td>Canário</td>
<td></td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>Ribeira Quente</td>
<td></td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Ribeira da Praia</td>
<td></td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Faiã Redonda</td>
<td></td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>Pico Vemelho</td>
<td>Geothermal</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>Ribeira Grande</td>
<td></td>
<td>14.8</td>
<td></td>
</tr>
</tbody>
</table>

69
energy for the end-users, in order to optimize system operations.

To design the right demand response program for the islands, we explore various ways of adjusting the demand to reduce the costs of installing or producing electric energy in the systems in both the long and short terms. Some types of demand can be adjusted with respect to the cost of producing electricity, while others may not be very flexible. The design process to utilize demand most efficiently on the islands can be summarized in three steps.

The first step is to use more energy when electricity is abundant and available, and to suppress consumption when it is not, in both the long and short terms. For the long term, we analyze the correlation between wind power and the loads and explore the benefit of shifting the load according to the availability of wind. For short-run operations, we give an hourly expected operational cost to responsive loads as a price signal to help them adapt to the availability of power.

The second is to relate the uncertainty of supply with the rate of response of demand. In order to operate volatile generation resources more efficiently in the short run such as wind power, it is important for the suitable loads to obtain the signal of supply availability (i.e., price) and respond to it within the right period of time. For the loads that are less uncertain and can be shifted, such as loads that operate particular machinery in a factory, generation resources with less volatility can be scheduled to supply their needs. Intermittent resources may be more suitable for more flexible and uncertain loads that can respond quickly within a certain range.

This brings us to the third step: to relate the physical characteristics of the loads to the time interval of the system dynamics. The storage effect time constant, or the period of time that the load can withhold its consumption without violating its physical constraints, is crucial when designing the right demand response program for a particular type of load. This also leads us to categorize loads with respect to their own suitable time scales.

Considering all these steps, we try to find the right program and signal for various
types of demand. It is also important to note that within the ALM framework, control of demand should minimize discomfort of the end-users. We also show this point in the demand response schemes that we suggest and in the numerical examples.

4.3 Characterizing different loads

4.3.1 Candidate loads for adaptive load management

In order to see a significant impact in terms of the magnitude of the demand reduction or shift, we first investigate if the largest consumers in the system are flexible at all and which of their loads can be managed. Another way to have effective demand control is to aggregate small flexible loads. We explain in more detail later what types of loads should be used, and at which points they should be scheduled over the course of the dispatch in the long and short terms.

We look at the types of loads on Flores and São Miguel, and generalize the idea of utilizing different types of loads on different dispatch time scales, or shifting the loads in the longer term.

Flores

Since the climate is very mild throughout the islands, and there are not many large business and commercial end-users on Flores, we explore the possibility of aggregating small residential loads. A 2004 analysis of energy consumption in the Azores attributes roughly 42% of residential consumption to household refrigerators [50]. Residential energy consumption is a large part of the total load on all the islands, making refrigerators a significant energy consumer in this system. We use ALM to model refrigerators on Flores as price-responsive loads.

São Miguel

The largest end-users in São Miguel are mostly commercial or industrial: a large shopping mall, and a cement company with an electricity bill of around 50,000 Euros a month, are among them. Shopping malls generally consume most of their electricity on lighting and
air conditioning. While lighting is not so flexible in terms of load adjustment since lights need to be consistently on during business hours, air conditioning can be more flexible; the air conditioning load can be adjusted as long as the people in the mall feel comfortable with the temperature. There are also a few other industrial end-users that have potential demand resources: a dairy farm that runs boilers, a pig farm that runs biofuel plants using animal waste and sells the surplus electricity to EDA.

Smaller end-users include small businesses and residential homes. Potential demand resources of these users are air conditioning, lighting and laundry loads in hotels, and refrigeration loads in large grocery stores. An average three-star hotel with 200 rooms and a grocery store both pay about 4,000 Euros a month. Residential users have small appliances such as dehumidifiers and refrigerators. Note that since the climate in the Azores is very moderate, the air conditioning load, especially from the residential sector, is not significant; in fact, only 2.4% of residential houses have air conditioning. However, due to humid winters, about 30% of residential households use dehumidifiers at that time of year [51].

4.3.2 Identifying types of loads over different time horizons

We note that there is no one-fits-all solution that applies to every type of load to be utilized as a demand resource. First, the different physical and economic characteristics of the loads define the suitable time frame for the optimal demand control frame. The physical characteristics of the loads to consider include storage availability of any type (whether the load has thermal/electric storage that can shift its consumption) and the storage time constant (how long the load is capable of physically withholding consumption).

The economic characteristics of the loads have more to do with the end-users’ use of the loads, e.g., temperature/humidity setpoints of the air conditioners/dehumidifiers, or the maximum or minimum energy consumption limits that an end-user would allow or set for a certain appliance. This information can be included in a demand function as the price sensitivity of demand by calculating the optimal energy usage with respect to
different prices. For example, air conditioning loads with different temperature deviation bands show different price sensitivities of demand [52].

There are also various factors, such as regulations on emission or noise, business hours, labor laws, etc., that can affect the controllability of the loads. All these different factors determine which types of technology are suitable, and when and how to schedule them. In this chapter, we attempt to utilize as various loads as possible, based on the characteristics of the loads.

The bottom line of our approach is to apply different frameworks for demand response according to the time intervals and time scales of different types of loads; in other words, loads that are more deterministic and can be pre-programmed (e.g., factory operation schedules, A/C, dehumidifiers) may be scheduled ahead of time whenever the necessary information is available. This information includes not only available loads and their characteristics, but also system conditions such as wind power forecast. On the other hand, more unpredictable and volatile loads (e.g., refrigerators, dehumidifiers) can be adjusted in real time if there exists an adequate two-way communication infrastructure between the loads and the system. However, we also understand that the only current framework in the Islands to induce more demand response is the tariff that charges different prices by season for different time blocks within a day, which can be categorized as time-of-use (TOU) pricing.

4.3.3 End-users’ response to current tariff and energy policies, and the alternatives

End-users, especially large industrial users are already responding to the current TOU by 1) shifting their loads to cheaper time blocks, (e.g., a dairy company running its boilers at night, even when it has to pay the labor force overtime) and 2) installing more energy-efficient equipment (e.g., a big grocery store replacing light bulbs in the refrigerators with LED lights). Also, in the Azores, intensive energy consumers are required by law to reduce their energy consumption by 6% in six years. In response to this longer time-scale energy
Figure 4.12: MT tariff over a day

savings plan, large industrial end-users are trying to curtail their energy bills. However, there are some end-users that are not able to shift their loads, due to problems such as regulatory issues and/or the characteristics of their loads. For example, a pig farm that also runs a cookie manufacturing business runs its cookie mills only at night because electricity is cheaper, but it cannot run its fodder production machines at night because of noise regulations.

The current TOU system is shown in Figure 4.12 for MT (medium voltage level). It is not clear how the rate for each time block is calculated. The system operators for the Azores simply take the end-user rates that the regulatory body responsible for the whole Portuguese electric power system imposes. From observing the rates shown in Figure 4.12, the tariff is designed to suppress demand during peak hours by imposing a higher rate, and the time blocks also change by season, reflecting the general seasonal operation conditions. However, it is not clear whether this tariff is effective even in terms of the operational cost. The tariff is applied across all the Islands, and the system conditions and/or the generation resource mix are quite different depending on the island. Therefore, it is not cost-efficient to impose a one-size-fits-all tariff on the Azores as a whole.

We propose that this tariff can improve by deciding on the rates for every season, or even month, based on the cost of the generation resources unique to each island and on the information available on the system condition and the changing demand. This information can include the availability of the generation resources, especially that of wind and hy-
The accuracy or credibility of wind power forecasts can differ significantly in different time frames, especially if the system aims to operate on a large portion of wind power that can be dispatched. Therefore, modifications in system operations planning are needed ahead of time according to the different predictions of how much wind power is available. This can be done on a seasonal, monthly, weekly, and/or daily basis. The right signal, that depends on available information about system conditions along each of these different time lines, should be sent to the flexible loads. The signals can be the expected value of the marginal cost of system generation that incorporate the availability of renewable resources, etc. The system operator should, in designing the signal, also consider the general price-responsiveness of the loads so that the responsive loads can most efficiently respond to system conditions over different time scales.

The right signal for responsive loads depends on the regulatory constraints (market existence, price tariffs to the end-users, etc.), system operation conditions and priorities (deploying more wind, reducing gas emissions, generation resource mix), and the technology of the loads to be deployed (the response/communication rate of the loads). In the case of the Azores, where they do not have a real-time market, a communication system/infrastructure between the loads and the wind availability can be constructed to exploit more wind in real time and make the loads respond to it more promptly.

In order for ALM to work, the system operations and scheduling, and the tariff designs, should consider: 1) the value of energy to the users, 2) the forecast accuracy of each time line and the dependency of the different time lines on the forecast, and 3) the physical characteristics of loads.

Demand functions capture the information on how the users value their energy usage [53]. A demand function characterizes the relationship between price and demand and tells what the marginal demand is for a given price, or how much the optimal willingness-
to-pay is for an additional unit of a given level of demand. This is an important piece of information that can be incorporated into system operations in order to reflect the economic value of energy as seen by an end-user.

For forecast accuracy and the dependency of time lines on it, the current goals of the Azores to include more renewable generation resources should heavily integrate this information into system operations and optimization. For example, how much wind should be scheduled before a season, a day ahead or an hour ahead, etc. will determine how much of the generation and demand resources available can be scheduled and dispatched when.

Finally, the physical characteristics of the loads should be determined in accordance with the time scales and intervals of the scheduling of the resources. The questions to consider include: how long the storage time constant of each load is, how fast it can respond and communicate with the system or the price signal, and whether it can be scheduled a day ahead or it can be adjusted flexibly in real time.

These three factors in combination determine the optimal framework of ALM to incorporate demand and renewable resources as much and as efficiently as possible.

4.4 Types of adaptive load management (ALM) frameworks for different loads

Scheduling of adaptive loads for ALM should be done in a way such that loads with higher uncertainty, and loads that cannot be well-predicted and pre-scheduled, pay for the corresponding cost of the risk to the system operation. Loads that can be planned ahead with higher certainty should be scheduled in advance so they can be met with lower-cost base generation. On the other hand, the more volatile and uncertain loads are adjusted or simply met with by generation resources that are more expensive and fast ramping.

In accordance with this idea, we categorize the loads into several different groups with respect to their physical characteristics, and to the time lines. We give examples of loads that can be scheduled in each of long and short time scales.
4.4.1 Better Time-Of-Use (long-term scheduling)

This section analyzes the potential for reduction in fossil fuel-supplied electric energy by simply scheduling energy consumption over long time horizons. In a hypothetical scenario where 33 MW of wind power have been installed on São Miguel, we quantify the possible benefits of some energy consumption being moved from weekdays to weekends. The benefits of such a shift are analyzed probabilistically because of the random nature of wind power. The factors that influence the size of these benefits are also analyzed. It is proposed that energy consumers and producers can negotiate an agreement on how to share the benefits and risks of such a shift.

Motivating and investigating load shifting

To reduce the amount of fossil fuels burned, energy consumption should be shifted to times when there is an excess of renewable energy from times when it does not meet the total load. As shown in Section 4.1, wind power shows steady daily and seasonal patterns on average, but is highly variable over the course of any particular day. Without the use of communication and control systems, it is difficult for energy consumers to react to real-time wind power conditions and shift consumption times. Because of this difficulty, we will investigate the use of long-term scheduling to reduce the amount of wind power that goes unused or is “spilled” on average.

Section 4.1 shows that electricity demand is generally higher on the weekdays than on the weekends, yet the day of the week does not affect wind power. This leads to a situation where the load exceeds the output of clean energy sources more often on weekdays than on weekends. More importantly, essentially-free wind power is more likely to go unused on the weekends. Our method for estimating the average wind power spilled on weekdays and weekends is described next.

Using historical generation dispatch data from São Miguel in 2008 and normalized wind data scaled to a proposed capacity, we can calculate the amount of wind power that would likely be spilled in each half-hour of the year. The wind power is scaled to represent a
Table 4.3: The mean and standard deviation of the weekly savings for each month, 2008, on São Miguel

<table>
<thead>
<tr>
<th>Month</th>
<th>Mean fossil fuel energy savings</th>
<th>Standard deviation of fuel savings (MWh/week)</th>
<th>Mean hourly benefit per firm ($/hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>7.91</td>
<td>6.87</td>
<td>36.58</td>
</tr>
<tr>
<td>February</td>
<td>8.53</td>
<td>12.26</td>
<td>39.46</td>
</tr>
<tr>
<td>March</td>
<td>14.39</td>
<td>14.61</td>
<td>66.54</td>
</tr>
<tr>
<td>April</td>
<td>16.06</td>
<td>14.16</td>
<td>74.26</td>
</tr>
<tr>
<td>May</td>
<td>8.59</td>
<td>12.86</td>
<td>39.72</td>
</tr>
<tr>
<td>June</td>
<td>2.56</td>
<td>4.90</td>
<td>11.86</td>
</tr>
<tr>
<td>July</td>
<td>1.55</td>
<td>3.87</td>
<td>7.16</td>
</tr>
<tr>
<td>August</td>
<td>-0.25</td>
<td>1.37</td>
<td>-1.18</td>
</tr>
<tr>
<td>September</td>
<td>3.91</td>
<td>5.93</td>
<td>18.1</td>
</tr>
<tr>
<td>October</td>
<td>6.68</td>
<td>11.53</td>
<td>30.91</td>
</tr>
<tr>
<td>November</td>
<td>5.87</td>
<td>8.47</td>
<td>27.13</td>
</tr>
<tr>
<td>December</td>
<td>0.23</td>
<td>7.56</td>
<td>1.05</td>
</tr>
</tbody>
</table>

power output with an installed capacity of 33MW. If hydropower and geothermal power are assumed to be uncontrollable, then we only need to compare wind power with fossil fuel power. The wind that would be spilled at each time step of the year was calculated as the wind power output subtracted by the oil power output, or zero, whichever is greater.

Based on this method, we find that an average of 42.75 MWh of wind power is spilled per weekday while an average of 53.82 MWh of wind power is spilled on each weekend day. This indicates an opportunity to reduce the amount of fuel burned and wind spilled by shifting consumption to the weekends. Our calculations show that the average amount of money per hour that the electricity supplier would be able to pay to each of the load shifting firms is 185 $/MWh, if we assume five firms participate in this load shifting, with Table 4.3 showing the statistics of the savings for each month [54]. This may or may not be an appealing offer, depending on the preferences of the firms management and employees.

4.4.2 Scheduling loads over different time horizons

In order to schedule generation resources with the loads a day to several hours ahead of operation, the loads can submit one of two different types of information to the system. First,
some loads can inform the system of their price-responsiveness. For more deterministic or pre-programmable loads such as automated machinery operations in a factory, or loads that have storage with a longer time constant such as air conditioning or water heating, can be grouped in this category. With respect to the anticipated price that is either given by the system operator or calculated by the end-user, the load aggregator, or the electricity distributor, the demand functions can be calculated based on this price information [52]. The minimum and maximum energy consumption constraints should also be included in addition to the price sensitivity information of the load sent to the system.

Another form of information that the loads can exchange with the system operator for day-ahead scheduling is the energy minutes/hours. This is to notify the system how many kilo or megawatts of energy the end-user plans to use each hour on the following day. The loads that have a pre-determined amount of energy usage within a time interval are more suitable to give this information to the system. The end-users with this type of load can notify the system operator the minimum amount of energy that they must consume, which can be considered as an inelastic demand for the hour.

After day-ahead scheduling is settled, with the more certain and predictable loads with the cheapest and generation resources, the demand can still be met with the next least expensive supply of reserve in real time. The day-ahead scheduled loads may or may not have contracts with hard constraints (e.g., a high penalty if the goal is not met). Regardless, there is always some degree of uncertainty surrounding the predicted or pre-scheduled demand.

Real-time adjustment of flexible loads should therefore require fast information exchange between the status of the loads and the system condition. More unpredictable loads, or loads that have storage with a shorter time constant than the time step of day-ahead scheduling (e.g., refrigerators), are suitable for real-time adjustment scheme. Price sensitivity with respect to the real-time signal, such as wind availability, that reflects the status of the system, should be sent to the system operator from the loads. The minimum
and maximum energy constraints calculated from the current status of the load (e.g., the current temperature inside a refrigerator or the current motor speed of a dryer) should also be communicated to the system operator, so that the system dispatch of this adjustable load is within the physical limits and is met with the end-users’ preferences of particular loads.

**Data preparation for dispatch with price-responsive demand**

**Flores**

: *Calculating the demand functions of refrigerators*  For the year 2008, the total electrical energy produced on Flores was 11.6 GWh. The statistics of the system operator show that residential customers used roughly 4.5 GWh of energy. Knowing the percentage of residential consumption that is used for refrigeration, we can calculate the annual energy consumption of household refrigerators. Because refrigerators run constantly, we can divide the annual consumption by the number of time steps to get the energy per unit of time. On Flores, the aggregate energy consumption of refrigerators is 35.7 kWh per 10 minutes, or a constant load of 214.2 kW. Because the duty cycle of refrigerators is 50% [55], 35.7 kWh represents the consumption when half of all the refrigerator compressors on the island are running. Therefore, ALM assumes that the maximum amount of energy that can be consumed in a ten-minute period by price responsive refrigerators is double the value, or 71.4 kWh per 10 minutes.

For ALM-enabled refrigerators to participate in energy markets, a physical model must be used to derive the demand functions for energy. First, we model the temperature dynamics of an individual refrigerator. We assume a linear temperature increase/decrease according to the on/off state of the compressor, within the maximum and minimum temperature bounds [56]. We assume both the cooling and warming time constants to be 20 minutes [55]. The minimum temperature bound is 3°C, and the maximum 8°C.
Based on the uncontrolled dynamics of the refrigerator’s temperature in Equation 4.1 from [57], we derive the temperature dynamics of the refrigerator with control allowed. This yields

\[
x(t) = x_i + at \quad \text{with} \quad \begin{cases} 
a = a_{\text{cooling}} = \frac{(x_{\text{min}} - x)_{\text{max}}}{\tau_{\text{cooling}}} \\
a = a_{\text{warming}} = \frac{(x_{\text{max}} - x)_{\text{min}}}{\tau_{\text{warming}}}
\end{cases}
\]

(4.1)

where \( t \) and \( x(t) \) are the final and initial time points, \( x(\cdot) \) is the temperature in the refrigerator at a given time step, \( P_r \) is the power rating [kW] of the refrigerator, and \( a_d \) and \( a_u \) are the heat transfer rates [°C/minute] for the cooling and warming periods, respectively. \( P_e \) is the electric energy input [kWh] within the time period.

Second, giving this temperature dynamic equation as a constraint and \( u \) in the equation above as the control variable, we solve an optimization problem of minimizing the total energy cost. The 10-minute interval electricity prices of a day are given as input, and the price is denoted as \( \lambda \) in the following problem formulation:

\[
\text{minimize} \quad \sum_{t=1}^{144} \lambda(t)P_e(t) \\
\text{subject to} \quad x(t + 1) = x(t) + 10\left\{ \frac{6}{P_r}P_e \ast a_d + (1 - \frac{6}{P})P_e \ast a_u \right\} \\
\quad x_{\text{min}} \leq x(t) \leq x_{\text{max}} \quad \forall t \\
\quad P_{e,\text{min}} \leq P_e(t) \leq P_{e,\text{max}} \quad \forall t
\]

(4.2)

This optimization problem can be transformed into simple linear programming with an equality constraint and minimum and maximum bounds. By solving this optimization with respect to the given set of 144-by-1 vector \( \lambda \), we obtain an optimal energy usage for
the whole time horizon of the day.

Third, in order to obtain the price sensitivity of this individual refrigerator load, we repeat the same optimization with respect to different price settings. We obtained the different values of optimal energy usage at each time step by perturbing the expected price given by $\pm 10\%$ and $\pm 20\%$. This way, we have five different pairs of price and demand at each time step. We interpolate, for each time step, these five points of price and demand quantity to obtain a demand function, which is the relation between the demand quantity and the price that the demand is willing to pay. We assume a linear (first-order polynomial) demand function. The details for calculating a demand function and the overall idea of ALM can be found in [52]. Note that this procedure is the same as Chapter 3.2 except that the price sensitivity of demand is calculated for all hours at once a day ahead of time. The quantities are also cleared at the system level for all 24 hours at once, instead of solving for one time step and moving the horizon to repeat the procedure at the next time step. For this reason, we call this procedure day-ahead static scheduling in order to differentiate with day-ahead iterative clearing in Chapter 3.1.

The price sensitivity of demand calculated this way corresponds to only a portion of the whole system demand. Therefore, in order to include this in the economic dispatch of the system, the demand sensitivities for an individual refrigerator were scaled to coincide with the value of the total refrigeration load size, 214.2 kW by our calculation.

The price sensitivities of a refrigeration load for a day were calculated with the expected price at each time step of the day.

The resulting price sensitivities of demand on April 16, assuming two wind turbines installed (with a total wind capacity of 0.66 MW), are shown in Figure 4.13. Demand function slopes indicate the level of the demand’s sensitivity with respect to the price. Note that a higher value (or a value closer to zero) of the demand function slope indicates a higher price sensitivity of demand, i.e., a more elastic demand with respect to the price.

The overall tendency in this study is that a higher price induces the demand to be more
Figure 4.13: Expected market price and the corresponding demand function slopes on April 16, 2008

inelastic to price. Also, an interesting point to note is that at the time points where there is an abrupt change in price level, such as at the first time step (0:00 a.m. in Figure 4.13) and around hour 18 (5:50 p.m. in Figure 4.13), the demand was inelastic with respect to the price. At Point 1, the optimal demand was fixed to be at its maximum level, while at Point 2 the inelastic optimal demand was the minimum bound. This shows that look-ahead optimization works with respect to the price and adjust the demand based on the price forthcoming. Demand functions at some representative time points are plotted in Figure 4.14.

São Miguel

: Calculating the demand functions of an air-conditioning load in a shopping mall We noted that a large shopping mall is operating business in São Miguel. Since the climate of the Azores is moderate in summer, as described in Section 4.1, we find that there is little air-conditioning usage there, except for perhaps big commercial buildings and offices. Therefore, based on estimations of the physical parameters of the shopping mall building, we simulated the air conditioning usage of the shopping mall and attempted to prove how ALM can help move forward the efficient and clean use of electric energy.
Figure 4.14: Demand functions at 0 a.m., 0:30 a.m., 12 p.m., 5:50 p.m. and 9 p.m. on April 16, 2008
The detailed procedure is as follows: we first obtain the market price data from the economic dispatch for the given day. Then we calculate the optimal energy usage for 24 hours with respect to the price. Note that we optimize the energy usage based on the whole 24-hour horizon instead of one interval at a time; we call this look-ahead optimization. Besides calculating the optimal hourly energy usage, we also calculate the price sensitivity of demand by obtaining the optimal energy usage with respect to a slightly perturbed value from the expected price.

For the representative summer day of July 16th, 2008, we first obtain the operational cost of energy in 10-minute intervals from the system. We take this as the hourly price input of the optimization problem for controlling the air conditioning system inside the mall. We assume that the mall is open from 11 a.m. to 10 p.m., so the thermostat is set to be 21°C during those hours. We also assume that for one hour both before and after business hours, the mall shop owners and staff will prepare for opening or closing, so we set the temperature setpoints at 22°C for those hours. We assume that the initial temperature is 22°C, and we set the last temperature setpoint to go back to this initial state, too.

Since the weather temperature is close to the setpoints throughout the day, and the inertia factor of the indoor temperature is large because of the vast area of the mall, the largest heat sources are the lighting and people. Therefore we attempt to estimate the indoor temperature change by the heat sources first. The recommended illumination for supermarkets is 750 lux or lumen/m² and this intensity of light will emit approximately 25 W of heat per meter squared, according to the following equation\(^1\).

\[
P_e = \frac{b}{(\eta_e \eta_r I_s)} \quad (4.4)
\]

where

\(P_e\): installed electric power (W/m\(^2\) floor area)

\(^1\)All the equations and parameters regarding the heat sources and the temperature from them were taken from The Engineering Toolbox (http://www.engineeringtoolbox.com).
The total land area of the shopping mall is estimated to be about 25,000 m². Since the mall has two stories, the total floor area is 50,000 m², and the total emitted heat is 1.25 MW.

The heat emitted from the people in the stores is estimated at 220 btu/hour per person, which is equivalent to 4.795 joules/hour per person. Assuming there are about 300 persons in the mall at all business hours, the total heat that people emit will be 4.795 \times 300 \text{joules/3600 sec} = 0.4 \text{ W}. This is negligible compared to the heat emitted from the lighting; therefore we only consider the heat from the lighting.

Since we have a dynamic equation of the temperature inside a building with a cooling system, we are interested in by how much this heat will raise the indoor temperature. The amount of heat needed to increase the temperature of a subject is expressed as:

\[ Q = c_p m \, dT \]  

where
\[ Q: \text{ amount of heat (kJ)} \]
\[ c_p: \text{ specific heat (kJ/kg⋅K)} \]
\[ m: \text{ mass (kg)} \]
\[ dT: \text{ temperature difference between hot and cold sides (K)}. \]

In one hour, the heat from the lighting will emit 4,500 MJ. The volume of the air in the shopping mall, assuming the height of the whole building (two-story) is 30 meters, 25,000
m² × 30 m = 750,000 m³. The air density at 20°C is 1.204 kg/m³, so the mass of the air in the mall is 903,000 kg. Applying these values to the equation above, we have 4,500,000 kJ = 1 kJ/kg·K × 903,000 kg × dT, and dT = (4,500/903) K = 4.98 K = 4.98°C. Therefore, when the lights are on in the shopping mall, the indoor temperature will rise by 4.98°C in an hour, or 0.83°C in 10 minutes, without any temperature control.

Based on all these estimations and the temperature dynamics of the air conditioning system [36], the resulting indoor temperature dynamic equation becomes

\[ x(t + 1) = εx(t) + (1 - ε)(T_{\text{out}}(t) + γP_e(t)) \]

(4.6)

where \( x(t) \): the indoor temperature at hour \( t \)

\( T_{\text{out}}(t) \): the outdoor temperature at hour \( t \)

\( P_e(t) \): the electric energy usage of the air conditioning system at hour \( t \)

\( ε \): air inertia factor calculated to be \( e^{-\tau/TC} \) where \( τ \) is the time interval and \( TC \) is the time constant (equal to the total thermal mass divided by the thermal conductivity)

\( γ \): steady-state temperature gain (– for cooling, + for heating)

The optimization of the whole time horizon of 24 hours can be formulated as

\[
\text{minimize} \sum_{t \in \text{open}} \{αλ(t)P_e(t) + (1 - α)(x(t) - x_{\text{set}}(t))^2\} + \sum_{t \in \text{closed}} αλ(t)P_e(t) \]

(4.7)

subject to \( x(t + 1) = εx(t) + (1 - ε)(T_{\text{out}}(t) + γP_e(t)) + 4.98 \) for \( t \in \text{open hours} \)

\( x(t + 1) = εx(t) + (1 - ε)(T_{\text{out}}(t) + γP_e(t)) \) for \( t \in \text{closed hours} \)

\( P_{e,\text{min}} \leq P_e(t) \leq P_{e,\text{max}} \quad \forall t. \)
Figure 4.15: Optimal energy usage with different optimization methods

Note that the objective functions are different depending on the hours when the desired temperature is set (open hours) or not (closed hours). Figure 4.15 shows the difference in the calculated optimal energy usage between the look-ahead approach and static optimization. Static optimization is defined here as adjusting the electric energy usage according to the expected temperature only at the very next time step. Static optimization is a more myopic temperature control than look-ahead optimization, and does not include price information in its optimization. In this Figure 4.15, one can see that look-ahead optimization has a lower energy usage the during peak hours than the static approach, and the energy usage during the peak hours is shifted to the off-peak hours. This is more obvious in Figure 4.16. With look-ahead optimization, we can observe that they pre-cool the air before business hours when the electricity price is cheaper (Figure 4.17). For this simulated day alone, the look-ahead approach cost 127 Euros less than the static approach.

4.4.3 Direct load control

Direct load control is also an option to utilize flexible demand. Direct load control in this context refers to a demand control scheme where a one-way signal from the system is sent to the end-user to respond to. The response of the end-users’ loads in direct load control, as opposed to other price-responsive demand schemes that we used in this thesis, is not
Figure 4.16: Controlled indoor temperature with different optimization methods

Figure 4.17: Hourly price input for look-ahead optimization
taken as a signal/input for the system or other suppliers to respond to. Loads that can be interrupted on a short notice and for a short period of time are good candidates for this. On São Miguel, dehumidifiers fit this description. End-users should notify the system operator about how much of their loads can be curtailed, and the maximum disconnection time allowed for the loads. Depending on the contract, they may also want to specify how long in advance they would like to be notified before any upcoming curtailment.

According to a report on the energy use of the residential users in São Miguel, 14.2% of residential households on São Miguel have their dehumidifiers kept on most of the time during the winter [51]. This means that the number of dehumidifiers running in residential homes would be about 7,570. Since one dehumidifier consumes about 0.5 kW, the amount of power consumed by the dehumidifiers at a random moment would be 3,785 kW. This is a substantial amount of power considering that the peak capacity in winter is about 60 to 70 MW. Assuming that turning off the dehumidifiers for about 10 minutes will not discomfort end-users much, the system operator can consider shaving small spikes of oil dispatch (shown in Figure 4.18). Turning off oil generators for 10 minutes fives times each day can save about 700 Euros a season.
4.5 Formulation of dispatch with price-responsive demand

In this section, we take the distributed look-ahead dispatch of generators in [58] and modify it to fit elastic or price-responsive demand. This look-ahead dispatch is similar as the functional clearing method described in Chapter 3.2, but instead of clearing deviations from day-ahead scheduled quantities, it directly clears for actual operation quantities. In this framework, price-responsive demand takes the anticipated price of electricity as the input for its optimization over a time horizon. As we discussed in the previous sections, the time horizon that a certain load or end-user oversees varies according to the physical characteristics of the load and the needs and preferences for the use of electric energy. This section discusses economic dispatch with price-responsive demand over a course of a day.

The sensitivity of demand to price is formulated as a demand function as shown in the previous section. The demand functions of different loads are calculated with respect to their unique physical dynamics and attributes as discussed in Section 4.4.2. Given these demand functions, we can construct quadratic benefit functions that are analogous to the quadratic cost functions of supply, by integrating the demand functions [59]. The following notations are used for the formulation.

- $G$ set of all available generators
- $G_r$ set of intermittent energy generators
- $Z$ set of load zones
- $\hat{P}_z(t)$ expected demand at load zone $z$ time step $t$
- $c_i(P_{G_i})$ cost function of generator $i$
- $b_z(P_z(t))$ benefit function of load $z$ consuming $P_z(t)$
- $P_{G_i}^{\text{min}}, P_{G_i}^{\text{max}}$ minimum and maximum output of generator $i$
- $\hat{P}_{G_w}^{\text{min}}(t), \hat{P}_{G_w}^{\text{max}}(t)$ expected minimum and maximum wind generation output
at time step $t$, $w \in G_r$

$g_w(\hat{P}_{G_w})$ forecast of available output for generator $w$

$R_i$ ramping rate of generator $i$, $i \in G$

$T$ number of time steps in the optimization period

$F, F^{\text{max}}$ vector of line flows and their limits.

\[
\begin{align*}
\text{minimize} & \quad \sum_{t=1}^{T} \left( \sum_{i \in G} (c_i(P_G(t))) - \sum_{z \in Z} (b_z(P_z(t))) \right) \\
\text{subject to} & \quad \sum_{i \in G} P_G(t) = \sum_{z \in Z} P_z(t) \\
& \quad \hat{P}_{G_w}^{\text{max}}(t) = g_w(\hat{P}_{G_w}^{\text{max}}(t-1)), \quad w \in G_r \\
& \quad \hat{P}_{G_w}^{\text{min}}(t) = h_j(\hat{P}_{G_w}^{\text{min}}(t-1)), \quad w \in G_r \\
& \quad \hat{P}_{G_w}^{\text{min}}(t) \leq P_G(t) \leq \hat{P}_{G_w}^{\text{max}}(t), \quad w \in G_r \\
& \quad 0 \leq P_z(t) \leq P_z^{\text{max}}, \quad z \in Z \\
& \quad P_{G_i}^{\text{min}}(t) \leq P_{G_i}(t) \leq P_{G_i}^{\text{max}}(t), \quad i \in G \setminus G_r \\
& \quad |P_{G_i}(t+1) - P_{G_i}(t)| \leq R_i, \quad i \in G \\
& \quad |F(t, P_G, P_z)| \leq F^{\text{max}} \quad \forall k
\end{align*}
\]
market is cleared with all the supply and demand bids. They calculate the demand/supply bids for the next day at every time step where the interval of each time step is set by the system/market operator. The system operator clears quantity based on the bids at every time step.

*Real-time adjustment* is a demand response scheme that we propose should be used in real-time energy balance of supply and demand, in a moving time horizon. Assuming an adequate communication infrastructure and the control of small devices such as refrigerators on the end-users’ premises, the end-users’ appliances and the system operator communicate every time step in the real-time market (e.g., 5 or 10 minutes) to exchange real-time price signals and the price sensitivity of demand based on the current physical status of the appliance. The procedure of real-time adjustment is shown in Figure 4.19.

### 4.6 Simulation of scheduling dispatch with ALM

We discuss the simulation results of the dispatch with elastic loads for both Flores and São Miguel. Both the day-ahead static scheduling and real-time adjustment methods are simulated and presented in this section. The simulations were conducted for each island, with different candidate loads for ALM that were determined in the previous sections. The
time interval for all the simulations is 10 minutes, and optimization is done for a day or 24 hours.

4.6.1 Dispatch with refrigerators on Flores

Calculating the demand functions as shown in Section 4.4.2, we calculate the optimal dispatch for four seasonally representative days in 2008. As in the generation dispatch of the Flores system, described in [58], the power supply sources on Flores consist of diesel, hydro and wind power generators. The same marginal costs were used for these simulations as well.

Day-ahead static scheduling

The algorithm of day-ahead static scheduling is identical to what is shown in [58], except that now we have an additional unit “elastic demand” also bidding into the system. The procedure of getting the demand bids was explained in Section 4.4.2, and the system dispatch formulation is shown in Section 4.5. In the simulations for the Flores system with the refrigerator loads, we assume that the aggregate refrigeration load acts as one large refrigerator. In other words, we do not include an algorithm that aggregates multiple refrigerators with different temperature statuses. The results of the system dispatch with this algorithm are shown for the four seasonally representative days in Figures 4.20-4.23.

Issues with day-ahead static scheduling

The dispatch results for day-ahead static scheduling do not keep track of the physical state of the elastic load at each time step, and thus the cleared dispatch can be physically infeasible. The bids that are submitted by supply and demand entities are based on expected price, but cannot be forecast by the amount of energy to be consumed. The bid curves are highly dependent on the current state of a bidder. If the state of a bidder deviates from the state calculated ahead of the actual consumption, then the current and future bid functions are not guaranteed to be feasible or representative of the current price sensitivity. The first instance where the system clears at something other than the expected price will cause this deviation.
Figure 4.20: Day-ahead static scheduling with control on the refrigerators load on January 16, 2008 for Flores

Figure 4.21: Day-ahead static scheduling with control on the refrigerators load on April 16, 2008 for Flores
Figure 4.22: Day-ahead static scheduling with control on the refrigerators load on July 16, 2008 for Flores

Figure 4.23: Day-ahead static scheduling with control on the refrigerators load on October 15, 2008 for Flores
Using the day-ahead static scheduling described above and the model of ALM-enabled refrigerators described in Section 4.4.2 results in the violation of temperature bounds for the price-responsive refrigerator. Figures 4.20 - 4.23 show the generation dispatch resulting from using this day-ahead static scheduling algorithm. The price-responsive load consumes less than in the inelastic case at nearly all the time steps, as shown by the gap between the stacked generation output and the baseline load. Figure 4.24 shows how the modeled temperature state of ALM-enabled refrigerators would evolve if operated according to the dispatch results. The refrigerator temperature model should only be considered valid within a reasonable proximity to the minimum and maximum temperature bounds, so one should disregard the resulting temperature evolution after the maximum temperature constraint has been violated. Still the results clearly show that day-ahead static scheduling dispatch results in an inadequate amount of energy consumption to satisfy the temperature constraints. This pushes one toward the use of the real-time adjustment algorithm where a new bid curve is formulated at each time step using the current state of a bidder in a moving horizon.
Real-time adjustment

In real-time adjustment dispatch, we overcome the problems of day-ahead static scheduling by making a closed loop of information between the elastic load dispatch and the physical dynamics of the elastic load, i.e., the temperature of the refrigerator. At each time step, once the system operator clears the market, ALM uses the energy dispatched from the system to calculate the temperature of the refrigerator at the next time step. Now the demand bid function of the next time step will be calculated in the same way as the system dispatch with day-ahead static scheduling, but with a specific initial temperature calculated from the systems dispatch to the price-responsive load. This process is iterated at every time step so that the dispatched energy amount follows the temperature dynamics of the refrigerator across the time horizon. Figures 4.25 - 4.28 show that the total amount of energy consumed over a day is close to the daily consumption of the baseline load. Figure 4.29 shows the evolution of the refrigerator temperature when using this real-time adjustment algorithm. We note that the temperature is kept within the bounds of 3 to 8°C in this real-time adjustment case.
Figure 4.26: Real-time adjustment with control on the refrigerators load on April 16, 2008 for Flores

Figure 4.27: Real-time adjustment with control on the refrigerators load on July 16, 2008 for Flores
Figure 4.28: Real-time adjustment with control on the refrigerators load on October 15, 2008 for Flores

Figure 4.29: Temperature inside the refrigerator with load dispatch under real-time adjustment on October 15, 2008 for Flores
4.6.2 Dispatch with an air-conditioning load on São Miguel

We calculated the air conditioning load in the shopping mall described in Section 4.4.2, optimizing the load with the anticipated operational cost for July 16th, 2008. Compared to the total load, the air conditioning load was insignificant in terms of the magnitude. However, as shown in Section 4.4.2, if the price signal given to the end-user reflected the true cost of the system operations, then the savings from shifting the load during peak hours to off-peak was considerable at least from the end-user’s perspective.

São Miguel has four different sources of generation: oil, hydro, wind, and geothermal, as described in Section 4.1.

Day-ahead static scheduling

The generation and demand dispatch results of day-ahead static scheduling are shown in Figure 4.30. As can be noted, the elastic demand is very small. The air conditioning load is separately plotted in Figure 4.31.

As pointed out in the previous simulations for Flores, day-ahead static scheduling...
dispatch results can be infeasible for the load. Therefore, the resulting temperature change in the mall is plotted in Figure 4.32 assuming that the air conditioning system follows the day-ahead static scheduling dispatch. As with the results from the day-ahead static scheduling on Flores, the results on São Miguel also turn out to be infeasible. This is more obvious in a much warmer weather temperature setting, as shown in Figure 4.33.

**Real-time adjustment**

The algorithm for real-time adjustment on Flores was applied to the system on São Miguel as well for the air conditioning load in the mall. The resulting generation and demand dispatch, the air-conditioning load, and the temperature change inside the mall are shown in Figure 4.34, 4.35 and 4.36, respectively. We note that with this algorithm, the temperature inside the mall is kept close to the desired temperature setpoints.

**4.7 Discussions and Summary**

In this section, we attempted to select the right types of loads for demand response on the islands of Flores and São Miguel. We recognize that there are many different types of loads that are suitable for a certain framework of demand response with the system dispatch or
Figure 4.32: Temperature inside the mall assuming air-conditioning load dispatch under day-ahead static scheduling on July 16, 2008 for São Miguel

Figure 4.33: Air-conditioning load dispatch under day-ahead static scheduling assuming a warmer weather condition
Figure 4.34: Real-time adjustment with control on the air-conditioning load on July 16, 2008 for São Miguel

Figure 4.35: Air-conditioning load dispatch under real-time adjustment on July 16, 2008 for São Miguel
longer-term scheduling. Table 4.4 summarizes the overall view of the possible tariffs or dispatch frameworks and the corresponding loads that are suitable for each of them.

Each of the demand response technologies has different costs and savings associated with it. Real-time adjustment demand dispatch requires near real-time communication and control devices on both the end-users’ premises and the system operator, while the longer-term demand scheduling by Better Time-Of-Use may not require any investment in sophisticated infrastructure. Therefore, in order to fully evaluate the potentials of the demand response programs suggested, further research on the tradeoff between the investment costs and the benefit of each scheme should follow this work.

To look at how flexible loads can be scheduled a day ahead, demand and generation dispatch on the islands of Flores and São Miguel were presented. Two distinct algorithms for this dispatch, day-ahead static scheduling and real-time adjustment, were analyzed. By comparing the results of these two methods, we conclude that timely information exchange between the demand unit and the system operator is crucial for two reasons. First, the demand can be adjusted within tolerable bounds due to accurate energy consumption limits based on the current physical status of the load. Second, the system can guarantee the commitment of the participating load by obtaining accurate energy consumption limits.
Table 4.4: Comparison of different tariffs

<table>
<thead>
<tr>
<th>Tariff</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time-Of-Use</td>
<td>Rates fixed within a season</td>
</tr>
<tr>
<td>Better Time-Of-Use</td>
<td>Better representation of seasonal or monthly changes of generation and demand resources</td>
</tr>
<tr>
<td></td>
<td>Transitional tariff between the current one and the more advanced and detailed day-ahead static scheduling and real-time adjustment</td>
</tr>
<tr>
<td></td>
<td>Induce large loads to schedule to shift to lower-cost periods</td>
</tr>
<tr>
<td>Day-ahead static scheduling</td>
<td>Loads that can be scheduled a day ahead by quantity (physical commitment)</td>
</tr>
<tr>
<td></td>
<td>or that can give information about price sensitivity (financial contract)</td>
</tr>
<tr>
<td>Day-ahead static scheduling + real-time adjustment</td>
<td>Real-time two-way communication with the appliance and the system operation</td>
</tr>
<tr>
<td></td>
<td>Loads that can respond promptly within a time step of the real-time operation</td>
</tr>
</tbody>
</table>

and load flexibility.

This implies that a successful demand response program for “greening” a system requires far more than simply getting more end-users or loads enrolled. An adequate communication and control infrastructure is crucial for both the end-users’ and the system operator’s objectives. The time interval and the duration of the communication between the loads and the system must be well designed depending on the types of loads.
Chapter 5

IEEE 30-bus test system

In this chapter we apply the methods proposed in Chapters 3.1 and 3.2 for short-term scheduling of generation resources and demand. Day-ahead iterative clearing schedules supply and demand entities with their intertemporal dynamics and constraints. Real-time functional clearing adjusts the scheduled amounts in a moving horizon based on the bids that are submitted by supply and demand entities.

5.1 System configuration

The system studied here is based on the IEEE 30-bus test system [60]. Transmission line limits were added as in [61, 62], but in order to make one line congested at the peak hour, the transmission limit of the line connecting bus 1 and 2 were reduced to 23 MW from the original value of 130 MW (Figure 5.1).

Since the test system does not include the specifics of the loads, we configure the flexible loads based on statistics and inference. We choose air conditioners in residential premises as the flexible loads to be scheduled and adjusted with the generators in the system. In order to estimate the number of air conditioners for each load bus, first we obtained the number of residential air conditioning systems in Northwestern Power Coordinating Council (NPCC) region [63], including part of the PJM and ISO New England areas, Ontario, and Maritimes, and the hourly load in the same region on August 8th 2007, the hottest day of the year. The numbers of residential air conditioners were obtained from
Figure 5.1: Modified IEEE 30-bus test system (adjusted from [4])
and the load data was obtained from Professor Daniel Shawhan at Rensselaer Polytechnic Institute who had processed the data from the websites of the RTOs in the region for his own research purpose.

The test system only provides load data for a single time step. We calculated the ratio of each hourly load to the maximum load of the day in the NPCC system. We then multiplied each load value of the 30-bus test system by the 24 hourly ratios obtained from the NPCC system, which gave us a 24-hour load profile at each bus in the test system. The hourly system load calculated this way is depicted in 5.2.

The number of residential air conditioners at each bus of the test system was calculated to be the same ratio of the number of air conditioners in NPCC system region to the NPCC load values. The numbers were separately calculated for window- and central-unit air conditioners. We assume that 10% of the central units and 1% of the window units are participating in our framework, to make the case more realistic. Table 5.1 shows the numbers of total air conditioners calculated for each load bus.

There are six generators in this 30-bus system. The profiles of these generators are shown in Table 5.2.
Table 5.1: Number of air conditioners at each bus of the system

<table>
<thead>
<tr>
<th>Bus</th>
<th>Central units</th>
<th>Window units</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1,532</td>
<td>114,592</td>
</tr>
<tr>
<td>3</td>
<td>169</td>
<td>12,641</td>
</tr>
<tr>
<td>4</td>
<td>536</td>
<td>40,092</td>
</tr>
<tr>
<td>7</td>
<td>1,609</td>
<td>120,351</td>
</tr>
<tr>
<td>8</td>
<td>2,118</td>
<td>158,424</td>
</tr>
<tr>
<td>10</td>
<td>409</td>
<td>30,593</td>
</tr>
<tr>
<td>12</td>
<td>791</td>
<td>59,166</td>
</tr>
<tr>
<td>14</td>
<td>438</td>
<td>32,762</td>
</tr>
<tr>
<td>15</td>
<td>579</td>
<td>43,309</td>
</tr>
<tr>
<td>16</td>
<td>247</td>
<td>18,475</td>
</tr>
<tr>
<td>17</td>
<td>635</td>
<td>47,497</td>
</tr>
<tr>
<td>18</td>
<td>226</td>
<td>16,905</td>
</tr>
<tr>
<td>19</td>
<td>671</td>
<td>50,190</td>
</tr>
<tr>
<td>20</td>
<td>155</td>
<td>11,594</td>
</tr>
<tr>
<td>21</td>
<td>1,235</td>
<td>92,377</td>
</tr>
<tr>
<td>23</td>
<td>226</td>
<td>16,905</td>
</tr>
<tr>
<td>24</td>
<td>614</td>
<td>45,927</td>
</tr>
<tr>
<td>26</td>
<td>247</td>
<td>18,475</td>
</tr>
<tr>
<td>29</td>
<td>169</td>
<td>12,641</td>
</tr>
<tr>
<td>30</td>
<td>748</td>
<td>55,950</td>
</tr>
</tbody>
</table>

Table 5.2: Profiles of generators in the system

<table>
<thead>
<tr>
<th>Bus</th>
<th>Capacity (MW)</th>
<th>Ramp rates (%/min)</th>
<th>Cost($) coefficients 2nd order</th>
<th>Cost($) coefficients 1st order</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150</td>
<td>3</td>
<td>0.02</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>5</td>
<td>0.0175</td>
<td>1.75</td>
</tr>
<tr>
<td>22</td>
<td>62.5</td>
<td>10</td>
<td>0.0625</td>
<td>1</td>
</tr>
<tr>
<td>27</td>
<td>48.7</td>
<td>40</td>
<td>0.00834</td>
<td>3.25</td>
</tr>
<tr>
<td>23</td>
<td>40</td>
<td>30</td>
<td>0.025</td>
<td>3</td>
</tr>
<tr>
<td>13</td>
<td>44.7</td>
<td>40</td>
<td>0.025</td>
<td>3</td>
</tr>
</tbody>
</table>

110
5.2 Modeling flexible loads

An end-user’s air conditioning system is modeled the same way as described in Chapter 4 for the air conditioning system in a shopping mall in São Miguel. The dynamic equation for the air conditioning system is rewritten here.

\[ x_e(t + 1) = \varepsilon_e x_e(t) + (1 - \varepsilon_e)(T_{out}(t) + \gamma_e P_e(t)) \]  
\[ \text{subject to } x_{e,\text{min}} \leq x_e(t) \leq x_{e,\text{max}} \quad \forall t, \forall e \]  
\[ P_{e,\text{min}} \leq P_e(t) \leq P_{e,\text{max}} \quad \forall t, \forall e \]  

where \( x_e(t) \): the indoor temperature at hour \( t \) of end-user \( e \)  
\( T_{out}(t) \): the outdoor temperature at hour \( t \)  
\( P_e(t) \): the electric energy usage of the air conditioning system at hour \( t \) of end-user \( e \)  
\( \varepsilon_e \): air inertia factor calculated to be \( e^{-\tau /TC_e} \) of end-user \( e \)  
\( \gamma_e \): steady-state temperature gain of end-user \( e \)

The difference from the system in the Azores is that in this work we have a total of 11,329 air conditioners to control along with 6 generators on a network with 30 buses and 42 lines. Therefore, we assign different parameters \( \varepsilon_e \) and \( \gamma_e \) for each end-user \( e \), while we assume that the weather temperature \( T_{out} \), shown in Figure 5.3, is the same in the whole area for this network.

The power ratings of the most popular window and central air conditioners in the market range from 7 to 10.6 kW, and from 2.3 to 3.5 kW, respectively. Since we look at a time step of an hour, we assume that an air conditioner’s energy output within an hour can be controlled anywhere between zero and its power rating. We generated uniformly distributed random values between the ranges of the power ratings to decide \( P_{e,\text{max}} \) for every end-user. \( \varepsilon_e \) and \( \gamma_e \) are randomly generated in a similar way to create different
parameters for the end-users’ cooling systems. To account for different energy consumption preferences of end-users, we randomized and differentiated the temperature set points (the desired state) of all the end-users over 24 hours. We also assumed that about 70% of the residential users are away between 8 a.m. and 7 p.m., 20% of users occupant all day, and 10% of users away only during 11 a.m. to 4 p.m.

5.3 Short-term scheduling

Apart from the experiments in the Azores Islands in Chapter 4, we consider the power network model in short-term scheduling. We apply day-ahead iterative clearing in Chapter 3.1 in order to schedule generators and loads a day ahead of consumption, and conduct real-time functional clearing in Chapter 3.2 to account for any changes in the real-time operation. In order to subject our framework to a congested network, we set the limit of Line 1 connecting bus 1 and 2 to 23 MW, so that the line is congested at the peak hour, which is hour 17 in Figure 5.2.
5.3.1 Linearized network constraints with congestion

The network equation is linearly approximated as described in Chapter ??, with the power transfer distribution factor and power injection at each bus. Lagrange multipliers associated with this equation are the marginal costs of congestion on each line; therefore it is nonzero only when the line is congested. The locational marginal price at a bus is defined as Equation (3.8).

Loads located at each bus receive this locational marginal price. In iterative clearing, end-users calculate their optimal hourly consumption while in functional clearing, they calculate the price sensitivity of demand along with their feasible consumption band (maximum minus minimum limit of consumption). Since the network is only congested at one line, we use [12] to further simplify calculations of locational marginal prices. [12] proves that if only one line is congested in a network, the locational marginal prices at all buses can be calculated with respect to the price at one-end bus of the congested line and the congestion cost.

5.3.2 Day-ahead iterative clearing

The initial locational marginal prices (LMPs) for all buses to start iteration were calculated based on the demand without any price-responsiveness, and without any network constraints. This is a good initial value for iterative clearing. The resulting initial price over 24 hours is shown in Figure 5.4. Since we don’t take the network equation in this calculation, all buses have the same price. An end-user $j$’s benefit function is defined as $b_j = (x_j - x_{j,\text{set}})^T (x_j - x_{j,\text{set}})$ where $x_{j,\text{set}}$ denotes the hourly temperature set points of end-user $j$.

After only 100 iterations, the system demand and supply match very closely; the mismatch of supply and demand is less than $1e-5$ at all hours except $-0.0286$ MWh at hour 17, which is the peak hour when a line is congested. Figure 5.5 shows the system demand and supply over 24 hours after 10 and 100 iterations, respectively. Note that this iteration is done among a system operator and six generators and over 11,000 end-users, over 24
hours including the intertemporal dynamics of the generators and the end-users’ cooling systems.

We compared the energy consumption of the end-users with and without iterative clearing with the system price. Assuming that the end-users only satisfy their temperature comfort without regard to the hourly price, we obtain the case of the demand irresponsible to the price. Figure 5.6 shows the compared results. Over the 24-hour horizon, the loads scheduled with iterative clearing reduced the system load by 18.6 MWh, about 0.4% of the total energy demand 3,770.8 MWh. Since we set the end-users’ temperature constraints as hard constraints, there is little energy savings during the peak hours. This shows that it is important to give the right price signal to the end-users if the system operator intends to adjust demand. However, this should be done in a way that end-users are given a choice of consuming energy at a high price or reducing consumption. The system operator is still expected to benefit, by scheduling demand a day of operation with iterative clearing, from having better information on how demand will behave. The resulting system price over 24 hours is shown in Figure 5.7. The congestion cost at hour 17 was $0.1294$/MWh on bus 1 and 2.
Figure 5.5: System supply and demand over 24 hours after 10 and 100 iterations
(a) Air-conditioning loads with and without iterative clearing  
(b) System load with and without iterative clearing

Figure 5.6: Comparison of demand with and without iterative clearing

Figure 5.7: System price from iterative clearing
5.3.3 Real-time functional clearing

After the end-users and generators are scheduled a day ahead with iterative clearing, the conditions of the system or a generator/end-user may change from what was assumed during day-ahead scheduling. In order to account for this unexpected change from the day-ahead scheduling, we apply real-time functional clearing described in Chapter 3.2, with the outdoor temperature higher than assumed for day-ahead scheduling in Figure 5.3. Specifically, the temperature from 10 a.m. to 2 p.m. was higher by 1°F, and from 2 p.m. to midnight by 2°F. We applied the moving horizon method for functional clearing described in Chapter 3.2, with locational marginal prices as the input signal for suppliers and end-users to calculate their price sensitivity of supply/demand. The end-users’ information on the price sensitivity of demand and consumption limits were aggregated by a load serving entity, so that the system worked with six suppliers and one demand representing all the end-users.

Figure 5.8 shows the difference between the hourly system demand from day-ahead and real-time clearing. As a result of the higher temperature, the demand increased during hotter hours. We can also observe that there is a small peak at hour 5. This is suspected to be a result of look-ahead optimization; since the price at hour 5 was the lowest, a lot of end-users consumed more energy and precondition their buildings at this hour. This implies that in order to avoid unexpected system peaks as a result of price responsive demand adjustment, a more sophisticated price signal system may be needed that can disperse concentration of energy consumption at hours with a low price.

Meanwhile, the resulting transmission at all lines are successfully limited within the bounds. Figure 5.9 shows the absolute values of the transmission flows and limits for all lines.

5.3.4 Summary and discussion

We applied day-ahead iterative clearing and real-time functional clearing methods for short-term scheduling of supply and demand entities on IEEE 30-bus test system over a 24-hour
(a) Adjusted system demand from day-ahead clearing

(b) Comparison of system demand

Figure 5.8: Comparison of demand with day-ahead and real-time clearing

Figure 5.9: Transmission flows as a result of real-time functional clearing
time horizon. The system network parameters were slightly adjusted to account for one congested line at a peak hour. Air conditioners were chosen to respond to a locational marginal price at an end-users’ premise. Iterative clearing could efficiently schedule the generators and the end-users’ systems including their vectorized intertemporal dynamics and constraints. The generators’ dynamics were linearly approximated with their ramp rates, and the end-users’ cooling systems of their buildings were modeled as a linear system with the indoor temperature as the state and the hourly energy consumption as the input. Real-time functional clearing was conducted for near-real-time adjustment of both demand and supply after the hourly supply and demand of each entity was cleared with day-ahead iterative clearing. We assumed that the weather temperature rose higher than forecast at the time when day-ahead scheduling was done. The system demand increased as a result in order to satisfy the end-users’ benefit, which we set as a hard constraint, the system could adjust from the day-ahead scheduling efficiently over moving horizons on a congested network. For future work we can think of a way to eliminate secondary peak loads that occur in real-time adjustment with look-ahead optimization of end-users. One of the ways to tackle this issue can be to devise a cooperative coordination among end-users within a load serving entity’s area.
Part IV

Conclusion
Motivated by the efforts of including more demand resources into power systems yet failing to include the end-users’ benefit in the current system operation, this thesis provides a framework of operating a power system with end-users’ benefits, namely adaptive load management (ALM). In order to represent a large number of end-users in the system where the system supply and demand are scheduled to be balanced at its optimum, we consider load serving entities to play a critical role of aggregating end-users’ demand in ALM. Coordinating the objectives of a large number of different end-users and power producers in the system subject to the system network requires a careful design of information exchange scheme among the entities. We note that the information on the condition and external factors of the system, end-users, and generators varies along the timeline of operating and planning the system. For this reason, the information exchange framework needs to be designed differently over various time horizons, and needs to be determined according to the risks of uncertainty of this information. ALM provides a multi-layered (from end-users to load serving entities to the system operator), multi-temporal (ranging from a long-term capacity and energy decision making to a short-term scheduling including day-ahead clearing and near-real-time adjustment) information exchange framework that relates the decisions made by each entity over different time horizons. The decisions of each entity and the information on the system condition were modeled based on Lagrange dual decomposition of the system-level problem. The thesis provides a numerical example where we design an ALM framework that is specific to the characteristics of the loads and generators of the system. Another example shows that ALM can efficiently schedule a large number of end-users with generators a day ahead of operation, and adjust the scheduled amounts in near real time even when the system condition has changed from what was expected. The biggest contribution of this thesis is in proposing and showing the proof of concept of a system operation framework, which enables the choices of end-users that have different energy consumption preferences and loads with physical dynamics. We show the conditions under which the system optimum can be achieved with various objectives of the
entities coordinated by the system operator.

Our ALM framework suggests changes in policy regarding system operation. For day-ahead scheduling, the intertemporal dynamics and constraints of local supply and demand units, including end-users, should be well incorporated in optimizing the system's objective. We showed that information to be exchanged in different time horizons, e.g., day-ahead and real-time clearing, should be designed differently. The types of loads and their physical characteristics should also be of concern when scheduling these resources with the rest of traditional supply units, and especially with more uncertain and volatile renewable generation resources. The information exchanged between load serving entities and end-users has implications on the service products of LSEs, such as demand subscriptions suggested in [34]. Moreover, due to the generality of the ALM framework, the information exchange protocol can be applied to other components in a power system as well.

There is much future work ahead in order to relax many assumptions we made to show the proof of concept. The objectives of load serving entities were approximated since we assumed perfect competition and no gaming among them. In reality they need to make decisions on the rates of electricity to offer end-users so that they keep customers and maximize their revenue. Physics of a power system network was approximated to a linear model with only active power supply and demand. However, our examples of air conditioning loads are inductive and can have an impact on reactive power of the system. For extension of the short-term scheduling, a novel methodology extending day-ahead and real-time clearing methods may be needed to solve a nonconvex problem, including the uplift costs of generators and nonlinear load dynamics.
Appendices
Appendix A

Proof regarding DC OPF without network congestions
In this appendix, we prove that the optimal solution and the price of electricity defined by the Lagrange multiplier of a Direct Current Optimal Power Flow (DC OPF) problem is the same with the solution of an OPF problem without network constraints, when 1) the network does not contain any shunt bus elements, and 2) not any transmission flows are binding, i.e., at their limits.

Consider a power system network without any shunt bus elements and enough transmission capacity without congestions. The solution of DC OPF of this network, modeled with the linearized network equation including the bus voltage angles, is the same with the solution of economic dispatch without considering the network equation. Economic dispatch, in this context, simply matches total supply and demand. Moreover, the locational marginal prices (LMPs) at all buses are the same, and are equal to the universal price from the economic dispatch.

Assume the network has a total of \( n \) buses where bus 1 is assumed to be a slack bus, and \( n_l \) lines. \( P_i \) denotes the net power injection at bus \( i \), and \( P = [P_1, \cdots, P_n]^T \). The admittance matrix is linearized to a \( n \)-by-\((n-1)\) susceptance matrix with the first column (slack bus column) omitted to avoid singularity, which is defined as

\[
B_{\text{bus}} = \begin{bmatrix}
B_{12} & B_{13} & \cdots & B_{1n} \\
\vdots & \ddots & \ddots & \vdots \\
B_{n2} & \cdots & \cdots & B_{nn}
\end{bmatrix}
\]

where \( B_{ij} \) denotes the susceptance on the line connecting bus \( i \) and \( j \). The bus voltage angles are \( \theta_i \), and since bus 1 is a slack bus and \( \theta_1 = 0 \), the bus voltage vector is defined as \( \Theta = [\theta_2, \cdots, \theta_n]^T \). The convex cost function of injecting power \( P_i \) at bus \( i \) is defined by \( f_i \), which can be generalized as a negative benefit function of consuming power \( P_i \), if \( P_i < 0 \). The \( n_l \)-by-\((n-1)\) linear mapping of the bus voltage angles to the line flows is \( B_{\text{line}} \) and \( F_{\text{max}} \) a vector of length \( n_l \) that denotes the lines’ active power flow limits.
Then the problem of DC OPF on this network is

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{n} f_i(P_i) \\
\text{subject to} & \quad B_{bus} \Theta - P = 0 \quad \text{(A.1b)} \\
& \quad |B_{line} \Theta| \leq F_{\text{max}} \quad \text{(A.1c)} \\
& \quad P_{i,\text{min}} \leq P_i \leq P_{i,\text{max}} \quad \forall i. \quad \text{(A.1d)}
\end{align*}
\]

The Lagrange relaxation of this problem is

\[
\mathcal{L}(P, \Theta, \lambda, \mu, \mu) = \sum_{i=1}^{n} f_i(P_i) + \lambda^T (B_{bus} \Theta - P) + \pi^T (B_{line} \Theta - F_{\text{max}}) + \mu^T (-B_{line} \Theta - F_{\text{max}}),
\]

where \(\lambda, \pi, \mu\) are Lagrange multipliers associated with (A.1b), and (A.1c). Since this is a convex problem, the necessary and sufficient Karush-Kuhn-Tucker (KKT) conditions for the optimum are

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial P_i} &= \frac{df_i(P_i)}{dP_i} - \lambda_i = 0 \quad \forall i \quad \text{(A.3a)} \\
\frac{\partial \mathcal{L}}{\partial \Theta} &= B_{\text{bus}}^T \lambda + B_{\text{line}}^T (\pi - \mu) = 0 \quad \text{(A.3b)} \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= B_{\text{bus}} \Theta - P = 0 \quad \text{(A.3c)} \\
\pi^T (B_{\text{line}} \Theta - F_{\text{max}}) &= 0, \quad \pi \geq 0, \quad B_{\text{line}} \Theta - F_{\text{max}} \leq 0 \quad \text{(A.3d)} \\
\mu^T (-B_{\text{line}} \Theta - F_{\text{max}}) &= 0, \quad \mu \geq 0, \quad -B_{\text{line}} \Theta - F_{\text{max}} \leq 0 \quad \text{(A.3e)}
\end{align*}
\]

where \(\lambda_i\) denotes the locational marginal price at bus \(i\). (A.3d) and (A.3e) imply that if the inequality constraints (A.1c) are not binding, i.e., the lines are not congested, then

\[
\pi = \mu = 0,
\]
and as a result,

\[ B_{\text{bus}}^T \lambda = 0 \]

from (A.3b), or

\[
\begin{bmatrix}
B_{12} & \cdots & B_{n2} \\
\vdots & \ddots & \vdots \\
B_{1n} & \cdots & B_{nn}
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\
\vdots \\
\lambda_n
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
\vdots \\
0
\end{bmatrix}.
\]

Since this is linearly dependent, by removing the first row, we can rearrange it as

\[
B_{\text{red}} 
\begin{bmatrix}
\lambda_2 \\
\vdots \\
\lambda_n
\end{bmatrix}
= 
-B_{12} 
\begin{bmatrix}
\lambda_1
\end{bmatrix}
\]  \hspace{1cm} (A.4)

where \( B_{\text{red}} \) is a full-ranked reduced susceptibility matrix of dimension \((n - 1)\) defined as

\[
B_{\text{red}} = 
\begin{bmatrix}
B_{22} & \cdots & B_{n2} \\
\vdots & \ddots & \vdots \\
B_{2n} & \cdots & B_{nn}
\end{bmatrix}
\]

Meanwhile, observing \( B_{\text{bus}} \), since there are no shunt elements in the lines,

\[
B_{ii} = -\sum_{j=1, j\neq i}^{n} B_{ij} = -\sum_{j=1, j\neq i}^{n} B_{ji} \hspace{0.5cm} \forall i.
\]

Therefore,

\[
\begin{bmatrix}
B_{12} \\
\vdots \\
B_{1n}
\end{bmatrix}
= 
\begin{bmatrix}
-B_{22} - B_{32} - B_{42} - \cdots - B_{n2} \\
\vdots \\
-B_{2n} - B_{3n} - B_{4n} - \cdots - B_{nn}
\end{bmatrix}
= -B_{\text{red}} 
\begin{bmatrix}
1 \\
\vdots \\
1
\end{bmatrix},
\]
and (A.4) becomes

\[
B_{\text{red}} \begin{bmatrix}
\lambda_2 \\
\vdots \\
\lambda_n
\end{bmatrix} = B_{\text{red}} \begin{bmatrix} 1 \\
\vdots \\
1
\end{bmatrix} \lambda_1,
\]

therefore

\[
\lambda_1 = \lambda_i \quad i = 2, \cdots, n.
\]

This means that the locational marginal price at every bus is the same.

Also, according to (A.3a),

\[
\lambda_i = \frac{\partial f_i(P_i)}{\partial P_i}.
\]  

This means that a globally uniform system price \( \lambda \) determines the optimal power injection \( P_i \) at every bus \( i \).

Now we compare this with the solution of an OPF problem without network constraints. An OPF problem without a network is formulated as

\[
\text{minimize} \quad \sum_{i=1}^{n} f_i(P_i) \tag{A.6a}
\]

subject to \( \sum_{i=1}^{n} P_i = 0 \),  

\[
\text{subject to} \quad \sum_{i=1}^{n} P_i = 0, \tag{A.6b}
\]

and the Lagrangian relaxation is

\[
\mathcal{L}(p, \lambda) = \sum_{i=1}^{n} f_i(P_i) - \lambda \sum_{i=1}^{n} P_i. \tag{A.7}
\]

The KKT condition for the optimum is

\[
\frac{\partial \mathcal{L}}{\partial P_i} = \frac{\partial f_i}{\partial P_i} - \lambda = 0.
\]
So the system price

$$\lambda = \frac{\partial f_i}{\partial P_i}$$

which is exactly the same as (A.5).

This proof implies that if a network modeled without any shunt elements have no congestions in any lines, it can be solved as a much simpler economic dispatch problem where only supply and demand are matched without regard to the network.
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