

Physics Beyond the Standard Model (Theory): Introducing the Little Higgs

Martin Schmaltz^a *

^aPhysics Department, Boston University, Boston MA 02215, USA
email: schmaltz@bu.edu

Little Higgs theories are an exciting new possibility for physics at TeV energies. In the Standard Model the Higgs mass suffers from an instability under radiative corrections. This “hierarchy problem” motivates much of current physics beyond the Standard Model research. Little Higgs theories offer a new and very promising solution to this problem in which the Higgs is naturally light as a result of non-linearly realized symmetries. This article reviews some of the underlying ideas and gives a pedagogical introduction to the Little Higgs. The examples provided are taken from the paper “A Little Higgs from a Simple Group”, by D.E. Kaplan and M. Schmaltz [1].

1. Introduction

Given the Standard Model’s remarkable success in accurately describing Physics at length scales ranging from atomic scales all the way down to the shortest currently probed scales of about 10^{-18} m it may appear puzzling that so much theoretical and experimental work is devoted to discovering physics beyond the Standard Model (SM)

In the first part of this talk I review the basic argument - the hierarchy problem - which motivates much of Physics Beyond the SM (PBSM) research. I then argue that the hierarchy problem can be used to make specific predictions with regards to the quantum numbers of new particles we might expect to discover at future colliders, and I present those predictions. In the second part of my talk, I give a pedagogical introduction to a recently discovered new solution to the hierarchy problem, the “Little Higgs” mechanism.

2. The hierarchy problem of the Standard Model

The predictions of the Standard Model have been probed directly at LEP and at the Tevatron

up to energy scales of order of a few hundred GeV. Lower bounds on masses of new particles BSM are approximately [2]

100 GeV	new leptons
200 GeV	new quarks
300 GeV	leptoquarks
700 GeV	W' or Z'

In addition, precision measurements at LEP, SLC and the Tevatron have indirectly probed scales between 1 and 10 TeV and found no significant disagreement with SM predictions. Finally, bounds on new flavor and CP violating couplings indirectly probe scales as high as 100 to 1000 TeV for new flavor violating couplings or order one at those scales.

Altogether, the experimental evidence shows that the SM works extremely well, and one might be tempted to postulate a “minimal scenario”: we will discover the SM Higgs boson with a mass somewhere between the current lower bound of 114 GeV [2] and the triviality bound near 500 GeV, and there is no new PBSM up to very high scales. In particular, this would imply that the LHC will not see any PBSM. In the following I argue that this pessimistic option is not only disfavored by the data but is also extremely unlikely because it implies a very delicate and unnatural

*Work supported by the Department of Energy, Contract DE-FG02-91ER-40676

fine-tuning of parameters.

The bounds from precision data require that new physics at the TeV scale is approximately flavor preserving and sufficiently weakly coupled so as to not generate large radiative corrections. In particular, new particles with masses of order a TeV are not very constrained. Most radiative corrections to precision observables from weakly coupled TeV scale particles are suppressed by factors of $(M_W/TeV)^2$ relative to loops with SM particles. Experiments are sensitive at the level of SM loops but not at the level of small corrections to them. Well-known examples for such new physics which may be hiding at the TeV scale are supersymmetry, vector-like extra generations, new gauge bosons or extended Higgs sectors.

Nevertheless, one might argue that the simplest possibility is to assume that the SM remains valid even beyond scales at which it has been directly tested. What is wrong with that possibility, why are we convinced that new physics will be discovered at the TeV scale? The answer is provided by the hierarchy problem which I briefly review here; not because the hierarchy problem is unfamiliar, but because I would like to formulate the problem in a way which best motivates the Little Higgs solution. In addition, I want to connect the hierarchy problem to future experiments.

At the LHC the 1-10 TeV energy scale will be probed directly for the first time. Thus an important question to answer is whether it is natural for the SM to be valid up to these scales (or if we can expect to discover new physics at the LHC). To make the argument, let us assume that the SM is valid up to a cut-off scale of $\Lambda = 10$ TeV. At even higher energies new physics takes over, which implies that we do not know how to compute loop diagrams with momenta larger than Λ , thus we will cut such loops off at Λ . The hierarchy problem arises from the fact that there are quadratically divergent loop contributions to the Higgs mass which drive the Higgs mass to unacceptably large values unless the tree level mass parameter is finely tuned to cancel the large quantum corrections.

The most significant of these divergences come from three sources. They are - in order of decreasing magnitude - one-loop diagrams (Figure

1.) involving the top quark, the $SU(2) \times U(1)$ gauge bosons, and the Higgs itself.

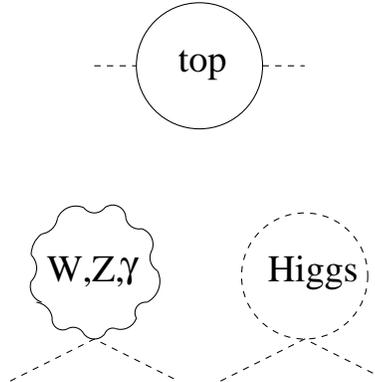


Figure 1. *The most significant quadratically divergent contributions to the Higgs mass in the Standard Model.*

All other quadratically divergent diagrams involve small coupling constants and do not significantly contribute for $\Lambda = 10$ TeV. The contributions from the three diagrams are

$$-\frac{3}{8\pi^2}\lambda_t^2\Lambda^2 \sim -(2TeV)^2 \quad (1)$$

from the top loop,

$$\frac{1}{16\pi^2}g^2\Lambda^2 \sim (700GeV)^2 \quad (2)$$

from the gauge loop, and

$$\frac{1}{16\pi^2}\lambda^2\Lambda^2 \sim (500GeV)^2 \quad (3)$$

from the Higgs loop. Thus the total Higgs mass is approximately

$$m_h^2 = m_{tree}^2 - [100 - 10 - 5] (200GeV)^2. \quad (4)$$

In order for this to add up to a Higgs mass of order a few hundred GeV as required in the SM fine tuning of one part in 100 (see Figure 2.) is required. This is the hierarchy problem.

Is the SM model already fine tuned given that we have experimentally probed to near 1 TeV and

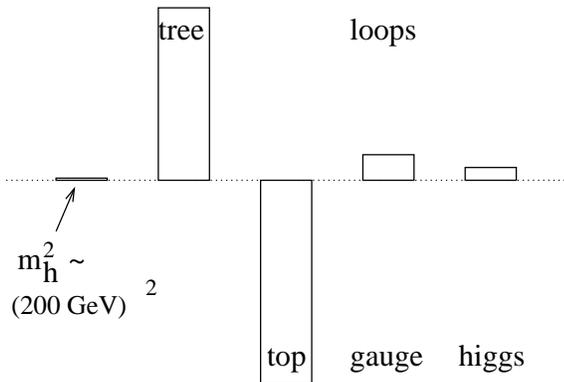


Figure 2. *The fine tuning required to obtain an acceptable Higgs mass in the Standard Model with cut-off 10 TeV.*

found no PSM? Setting $\Lambda = 1$ TeV in the above formulas we find that the most dangerous contribution from the top loop is only about $(200 \text{ GeV})^2$. Thus no fine tuning is required, the SM with no new physics up to 1 TeV is perfectly natural, and we should not be surprised that we have not yet seen deviations from it at colliders.

In the following, we will turn the argument around and use the hierarchy problem to predict what forms of new physics exist at what scale in order to solve the hierarchy problem. Consider for example the correction to the Higgs mass from the top loop. Limiting this contribution to be no larger than about 10 times the Higgs mass (limiting fine-tuning to less than 1 part in 10) we find a maximum cut-off for $\Lambda = 2$ TeV. In other words, we predict the existence of new particles with masses less than or equal to 2 TeV which cancel the quadratically divergent Higgs mass contribution from the top quark. In order for this cancellation to occur naturally, the new particles must be related to the top quark by a symmetry. In practice this means that the new particles have to have similar quantum numbers to top quarks. Thus the hierarchy problem predicts a new multiplet of particles with mass below 2 TeV which carry color and are easily produced at the LHC. In supersymmetric extensions these new particles

are of course the top squarks.

The contributions from gauge loops also need to be canceled by new particles which are related to the SM $SU(2) \times U(1)$ gauge bosons by a symmetry. The masses of these states are predicted to be at or below 5 TeV for the cancellation to be natural. Similarly, the Higgs loop requires new states related to the Higgs at 10 TeV. We see that the hierarchy problem can be used to obtain specific predictions.

SM loop	maximum mass of new particles
top	2 TeV
weak bosons	5 TeV
Higgs	10 TeV

2.1. Supersymmetry and the hierarchy

One successful approach to solving the hierarchy problem is based on supersymmetry (SUSY). Loosely speaking, in SUSY every quadratically divergent loop diagram in Figure 1. has a superpartner, a diagram with superpartners running in the loop (Figure 3.). The diagrams with superpartners exactly cancel the quadratic divergences of the SM diagrams. Generically, this happens because superpartner coupling constants are related to SM coupling constants by supersymmetry, but superpartner loops have the opposite sign from their SM partner because of opposite spin-statistics.

In the limit of unbroken supersymmetry the diagrams cancel completely. If weak scale SUSY occurs in nature superpartner masses softly break the supersymmetry. Then the cancellation only takes place above the scale of superpartner masses M_{SUSY} . Below M_{SUSY} only the SM particles exist, thus there the SM loop diagrams (Figure 1) are not canceled but the cut-off Λ is replaced by M_{SUSY} .

Experimental bounds on superpartner masses are somewhere in the few hundred GeV range, much lower than the upper bound of 2 TeV from fine tuning constraints top loops. Nevertheless, the MSSM is already somewhat fine tuned. The problem arises from the experimental lower bound on the Higgs mass. As is well known, the tree level Higgs mass in the MSSM is bounded

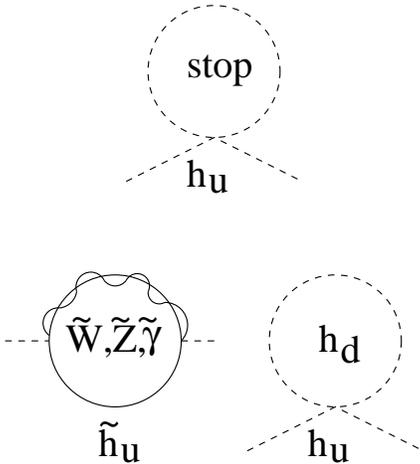


Figure 3. *Superpartner diagrams which cancel quadratic divergences of the Standard Model.*

from above by the Z mass. Radiative corrections from stop loops give a positive contribution and can lift the Higgs mass above the experimental bound of 114 GeV. However, a large enough correction requires heavy stops which in turn reintroduce fine tuning.

Current experimental bounds force a minimum fine tuning on the order of 10%.

2.2. SUSY, the only solution to the hierarchy problem?

Until recently it was widely believed that supersymmetry represents the only possible weakly coupled solution to the hierarchy problem. This belief was based on a lack of known alternatives, bolstered by a “folk theorem”. The “folk theorem” loosely states that supersymmetry is the only theory in which quadratic divergences cancel without tuning. The “folk proof” roughly goes as follows: *i.* boson and fermion loops have opposite signs due to a minus sign in the Feynman rules for fermion loops – *ii.* therefore cancellation of divergences only occurs between boson and fermion loops – *iii.* in order for the cancellation to be natural the boson and fermion loops need to be related by a symmetry: supersymmetry.

However, the folk theorem is wrong. Amus-

ingly, a counter example to step *ii.* in the above “proof” occurs in the MSSM itself. Because it is instructive let us briefly look at this example. The MSSM extends the Higgs sector of the SM to a two Higgs doublet model. The tree level quartic couplings for the Higgses arise from integrating out the D-auxiliary fields in the $SU(2) \times U(1)$ gauge vector multiplets. Looking for example only at the contribution from hypercharge, the relevant terms in the Lagrangian are

$$\mathcal{L} = \frac{1}{2}D^2 + \frac{g}{2}(h_u^* D h_u - h_d^* D h_d) \quad (5)$$

giving the usual quartic D-term in the Higgs potential

$$V = \frac{1}{2} \left(\frac{g}{2} \right)^2 (h_u^* h_u - h_d^* h_d)^2 \quad (6)$$

The cancellation of quadratic divergences is most easily understood by keeping the D-auxiliary fields in the theory. Then, the “quartic coupling” contained in eq. (5) leads to three distinct diagrams (Figure 4).

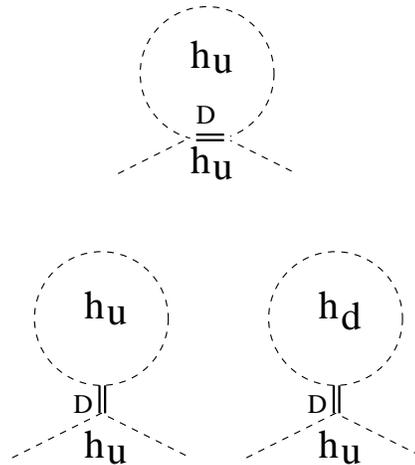


Figure 4. *Higgs loops in the MSSM.*

The first cancels against fermionic loops with gauginos as predicted by the folk theorem. However, the other two diagrams cancel between each

other. The cancellation occurs between diagrams with only bosons. It requires no fine tuning because the diagrams are proportional to the hypercharges of the Higgses in the loop which are opposite in sign and equal in magnitude (by gauge invariance and supersymmetry). Thus we see that even in the MSSM some of the quadratic divergences cancel between bosons, with the required difference in sign simply arising from a difference in signs between coupling constants.

3. Introducing the Little Higgs

In this part of the talk I give a brief pedagogical introduction to Little Higgs (LH) theories. The material covered is almost entirely contained in the recent literature which I briefly review here: Little Higgs theories are realizations of an old idea to stabilize the Higgs mass by making the Higgs a pseudo-Goldstone boson resulting from a spontaneously broken approximate symmetry. Early attempts at constructing such models [3,4] were not entirely successful and quadratic divergences to the Higgs mass remained. The first successful model which canceled all relevant quadratic divergences based on the pseudo-Goldstone idea was constructed by Arkani-Hamed, Cohen and Georgi in [5]. Subsequently, simpler and more elegant models were constructed. The models may be described by their global symmetry breaking patterns, $SU(5)/SO(5)$ [6], $SU(6)/SP(6)$ [7], the minimal moose $SU(3)^2/SU(3)$ [8], and general mooses $SU(3)^n/SU(3)^k$ [9]. Model building constraints on specific UV completions of LH have been discussed in [10,11], and preliminary studies of LH phenomenology appeared in [12]. More recently, a new variant of the LH mechanism was discovered which does not require duplication of gauge groups [1]. At this conference LH were already presented in two talks by Lane [13] and Wacker [14].

Let us begin by discussing LH theories from a phenomenological point of view. It should be clear from the previous section that they necessarily involve new particles related to the top quark, the $SU(2) \times U(1)$ gauge bosons and the Higgs. In LH theories the masses of these new states are given by a single scale f at which global symme-

tries are spontaneously broken. The spectrum of a generic theory is summarized in Figure 5. From

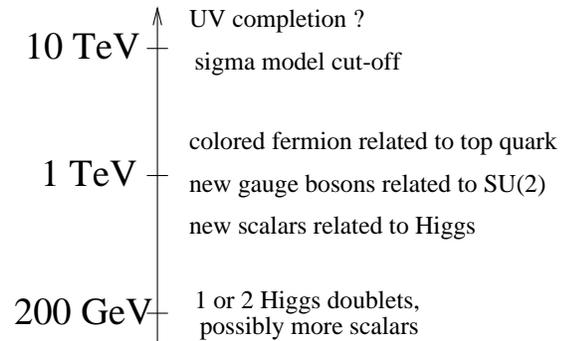


Figure 5. *Generic Little Higgs Spectrum.*

a purely mechanical point of view the cancellation of quadratic divergences in LH models is very simple. As in the case of supersymmetry there are diagrams with new particles in loops which precisely cancel the quadratic divergences. Consider for example the loop involving top quarks. LH Higgs theories contain a new vector like quark χ with mass of order f and couplings which are related to the top Yukawa coupling λ_t . The relevant interactions of this new fermion are

$$\lambda_t f \left(1 - \frac{1}{2} \frac{h^* h}{f^2} \right) \chi_R^\dagger \chi_L + h.c. \quad (7)$$

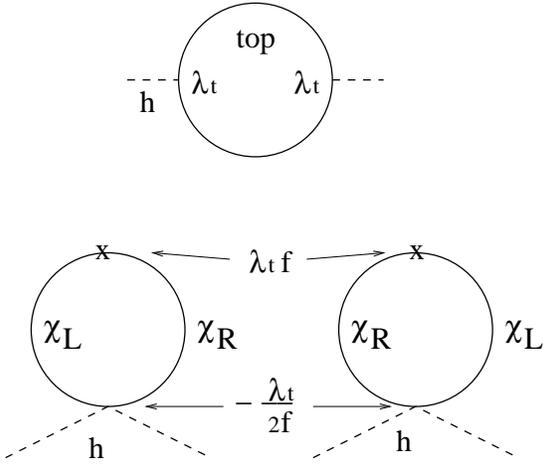
They allow a new quadratically divergent contribution to the Higgs mass from Figure 6 which involves the same integral as the top loop and is proportional to the coupling constants

$$\lambda_t^* f \left(-\frac{\lambda_t}{2f} \right) = -\frac{1}{2} |\lambda_t^2|. \quad (8)$$

There are two such diagrams which together exactly cancel the top loop divergence

As advertised, a fermion loop cancels a fermion loop. Similarly, the gauge and Higgs loops are canceled by diagrams with new bosons in loops.

Note that this alone is of course no solution to the hierarchy problem. Without a symmetry which enforces the relations between coupling

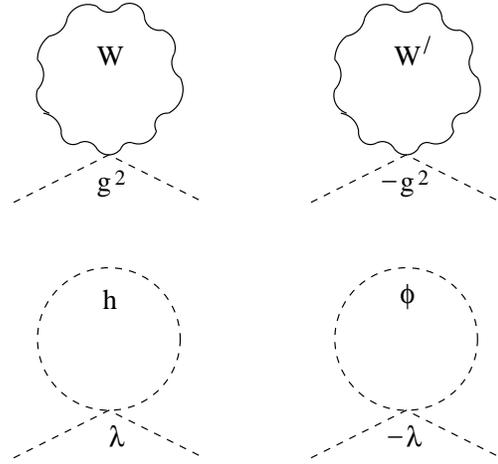
Figure 6. *Canceling the top quark loop.*

constants required for the cancellations one has only shifted the fine tuning from the Higgs mass to the coupling constants of the new fields. However, Little Higgs theories have such a symmetry built in. To discover it we will take a little detour.

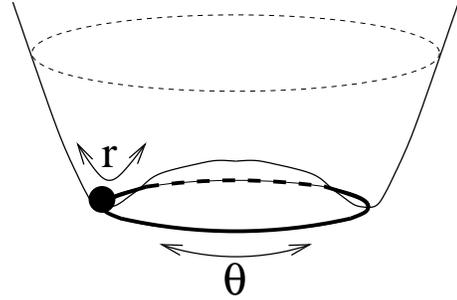
3.1. Higgs as a Pseudo-Goldstone boson

In this section we identify the symmetry which makes the identification of coupling constants of the previous discussion radiatively stable and therefore “technically natural”. To do so we first briefly review Goldstone bosons.

Massless Goldstone bosons always arise when global symmetries are spontaneously broken by the vacuum. Consider for example the theory of a complex scalar field Φ with potential $\mathcal{V} = \mathcal{V}(|\Phi|)$. This potential preserves a global $U(1)$ symmetry $\Phi \rightarrow e^{i\epsilon}\Phi$. If the potential induces a vacuum expectation value for $\langle \Phi \rangle = f$ then the $U(1)$ symmetry is spontaneously broken and a massless Goldstone boson arises. To see this explicitly, it is convenient to parameterize $\Phi = (v+r)e^{i\theta/f}$, where r and θ are real fields. Because of the global $U(1)$ symmetry θ can be removed from the potential by a space-time dependent $U(1)$ transformation with $\epsilon = -\theta/f$. The resulting Lagrangian does not contain θ except in derivative interactions. Therefore θ has no potential and in par-

Figure 7. *Canceling the gauge and Higgs loops.*

ticular also no mass. The “radial mode” r does obtain a mass from the potential and can be integrated out (Figure 7.). Note that this argu-

Figure 8. *The “Mexican hat” potential for Φ . The black dot represents the vacuum expectation value f , r is the radial mode and θ the Goldstone boson.*

ment is based only on the existence of the $U(1)$ symmetry and is therefore stable under radiative corrections.

To summarize, we now have a simple mechanism for generating massless scalar fields. Unfor-

unately, the symmetry needed forbids all non-derivative couplings. Thus θ is not a good toy example for a light Higgs, it has no quartic, gauge or Yukawa couplings.

Furthermore, θ also has the wrong quantum numbers to be the SM Higgs. This is easily fixed by generalizing to non-abelian symmetry breaking. Consider for example the breaking of a global $SU(3)$ symmetry to $SU(2)$ by an expectation value for a complex triplet field $\langle \Phi^T \rangle = \langle (\phi_1, \phi_2, \phi_3) \rangle = (0, 0, f)$. In this vacuum Φ is conveniently parameterized as

$$\Phi = e^{i\theta/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \quad (9)$$

where $\theta = \theta^a T^a$ is a hermitian 3×3 matrix containing the properly normalized five Goldstone bosons from the breaking of $SU(3) \rightarrow SU(2)$

$$\theta = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ h^\dagger & 0 & \end{pmatrix} + \frac{\eta}{4} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \quad (10)$$

Under the unbroken $SU(2)$ the four real fields in h transform as a complex doublet whereas η is a singlet. As in the abelian case, by performing a space-time dependent $SU(3)$ rotation it is possible to remove all non-derivative interactions of the Goldstone fields. This is still very different from the SM Higgs boson, our ‘‘Higgs’’ h is an $SU(2)$ doublet but it has no gauge, quartic, or Yukawa interactions and is exactly massless.

To add these couplings we need to explicitly break the global $SU(3)$ symmetry. Explicit symmetry breaking introduces non-derivative couplings of the Goldstone bosons, they become pseudo-Goldstone bosons. If the explicit symmetry breaking stems only from a small ‘‘spurion’’ parameter ϵ then all non-derivative couplings are proportional to the spurion. This remains true even including radiative corrections. Thus breaking of global symmetries by small spurions gives us control over quantum corrections. In the diagram of Figure 7. a small breaking of the symmetry corresponds to a small tilt of the ‘‘Mexican hat’’ proportional to ϵ .

This is all very nice and allows us to control the mass of pseudo-Goldstone bosons, however –

unfortunately – the SM Higgs couplings are not small. Thus even though radiative corrections are proportional to these couplings, this does not sufficiently suppress the contributions to the Higgs mass; i.e. simply adding the SM Higgs couplings and thereby explicitly breaking the $SU(3)$ symmetry leads to the radiative corrections of Figure 1. which are too large $\sim \lambda_i^2/16\pi^2 \Lambda^2$.

The new model building idea which led to the construction of Little Higgs models is collective breaking of symmetries. Instead of breaking the symmetry with a single coupling, one introduces two couplings in such a way that each coupling by itself preserves sufficient amount of symmetry to guarantee the masslessness of the pseudo-Goldstone boson. Schematically, we add two new sets of interactions \mathcal{L}_i to the $SU(3)$ preserving Lagrangian \mathcal{L}_0

$$\mathcal{L} = \mathcal{L}_0 + \epsilon_1 \mathcal{L}_1 + \epsilon_2 \mathcal{L}_2, \quad (11)$$

where each term is chosen such that by itself it preserves an $SU(3)$ symmetry but that together they break $SU(3)$ explicitly. Therefore radiative corrections to the Higgs mass are necessarily proportional to both spurions ϵ_1 and ϵ_2 . In the example we will study below, the fact that both spurions are required implies that quadratically divergent contributions arise only at two loops $\sim \epsilon_1^2/16\pi^2 \epsilon_2^2/16\pi^2 \Lambda^2$ which is sufficiently small for $\Lambda = 10$ TeV even for $\epsilon_i \sim 1$.

3.2. Toy Little Higgs theory

In order to study the LH mechanism without the notational complexity require by the many fields of the SM let us imagine a toy world without hypercharge in which the only fermions are top and bottom quarks with their normal Yukawa couplings and $SU(3) \times SU(2)$ gauge interactions. Particle theorists of this toy world have constructed a ‘‘Toy Standard Model’’ which suffers from the same hierarchy problem as the real SM. In order to protect the toy SM Higgs from quadratic divergences one can introduce an $SU(3)$ symmetry which is spontaneously broken to $SU(2)$ at the scale $f \sim 1$ TeV. The Higgs consists of four of the resulting Goldstone bosons as described before.

We generate Yukawa and gauge couplings with-

out reintroducing quadratic divergences by using collective symmetry breaking. This is achieved by introducing two $SU(3)$ groups which are each spontaneously broken to $SU(2)$ by expectation values for two scalar triplets Φ_1 and Φ_2 .

Concretely, $SU(2)$ -weak is enlarged into an $SU(3)$ gauge group. Two scalar fields Φ_i are both charged under this $SU(3)$ and both obtain expectation values. In order to understand the collective breaking, imagine turning off the gauge coupling to either one of the triplets. In this limit, there are two $SU(3)$ symmetries, one gauged, one global. Thus the model has two $SU(3)$ symmetries which are explicitly broken to the diagonal gauge group by the gauge couplings. But note that the gauge couplings to both Φ_1 and Φ_2 are required for the breaking. The $SU(3)$ gauge couplings of Φ_1 and Φ_2 play the role of the spurions ϵ_1 and ϵ_2 .

It is convenient to parameterize the light fields in the Φ_i as

$$\begin{aligned}\Phi_1 &= e^{i\theta/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \\ \Phi_2 &= e^{-i\theta/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}.\end{aligned}\quad (12)$$

Here θ are the pseudo-Goldstone bosons corresponding to breaking of the ‘‘axial’’ $SU(3)$. The eaten Goldstone bosons corresponding to the diagonal ‘‘vector’’ $SU(3)$ have been removed by a gauge transformation (to unitary gauge). The expectation values f of the Φ 's have been chosen to be equal for simplicity. Note that neither of the $SU(3)$'s here have anything to do with color.

Ignoring the singlet η , the pseudo-Goldstones are

$$\theta = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & h \\ 0 & 0 & 0 \\ h^\dagger & 0 & 0 \end{pmatrix}, \quad (13)$$

i.e. a complex Higgs doublet which is charged under the unbroken $SU(2)$ gauge group. Explicitly, the $SU(2)$ gauge interactions for h stem from the $SU(3)$ gauge couplings of the Φ 's

$$[(\partial_\mu + igA_\mu)\Phi_1]^\dagger (\partial_\mu + igA_\mu)\Phi_1$$

$$+ [(\partial_\mu + igA_\mu)\Phi_2]^\dagger (\partial_\mu + igA_\mu)\Phi_2. \quad (14)$$

In calculating the quadratic divergence from the gauge interactions it is most convenient to work in a manifestly $SU(3)$ invariant formalism. Then quadratic divergences simply come from a diagram with external Φ 's and an $SU(3)$ gauge boson loop (the first diagram in Figure 8.). The

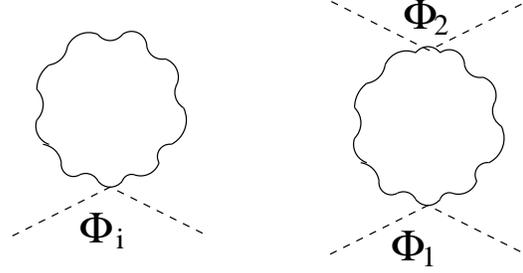


Figure 9. Quadratically and logarithmically divergent diagrams from the $SU(3)$ gauge interactions. The first diagram does not produce a Higgs mass.

quadratic divergence of this diagram produces the operators

$$\frac{g^2}{16\pi^2} \Lambda^2 (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2) \quad (15)$$

which respects both $SU(3)$ symmetries and therefore does not contain couplings of the pseudo-Goldstone bosons. This can also be seen explicitly by substituting eq. (12). The logarithmically divergent second diagram in Figure 8. gives rise to an operator which explicitly breaks $[SU(3)]^2 \rightarrow SU(3)$ and therefore contains a mass for h

$$\begin{aligned}\frac{g^4}{16\pi^2} |\Phi_1^\dagger \Phi_2|^2 \log(\Lambda^2/f^2) \rightarrow \\ \frac{g^4 f^2}{16\pi^2} \log(\Lambda^2/f^2) h^\dagger h,\end{aligned}\quad (16)$$

which is of order 100 GeV for $f \sim 1$ TeV.

The mechanism for adding a top Yukawa coupling without introducing a quadratic divergence

relies on the same principle. Since the weak interactions are embedded in an $SU(3)$ gauge group we need to introduce left-handed quarks in an $SU(3)$ triplet (t_L, b_L, χ_L) . The first two components are the usual quark doublet whereas the third component is a new quark. In addition to the triplet we also introduce three right handed quark singlets, two correspond to the right handed top and bottom quarks and one is the right handed component χ_R of the vector like heavy quark χ which is responsible for canceling the top loop divergence. To produce a mass for χ and a Yukawa coupling for the top quark we add

$$\frac{\lambda_t}{\sqrt{2}} (\chi_1 \Phi_1^\dagger + \chi_2 \Phi_2^\dagger) \begin{pmatrix} t_L \\ b_L \\ \chi_L \end{pmatrix} \quad (17)$$

where – again for simplicity – we have taken the two Yukawa couplings equal and factored out a $\sqrt{2}$ in order to simplify subsequent equations. Expanding the Φ_i to second order in the Higgs field and defining the two linear combinations

$$\chi_R = \frac{1}{\sqrt{2}} (\chi_1 + \chi_2), \quad t_R = \frac{1}{\sqrt{2}} (\chi_1 - \chi_2), \quad (18)$$

eq. (17) becomes

$$\lambda_t f \left(1 - \frac{h^\dagger h}{2f^2}\right) \chi_R^\dagger \chi_L + \lambda_t t_R^\dagger h^\dagger \begin{pmatrix} t_L \\ b_L \end{pmatrix}. \quad (19)$$

The first term yields a TeV scale mass $m_\chi = \lambda_t f$ and a higher-dimensional coupling to the Higgs for the vector-like quark. The second term is the top Yukawa coupling. Note that these couplings are exactly what we needed for the cancellation of the quadratic divergences in Figure 6. The cancellation of the divergence is again most transparent in an $SU(3)$ symmetric calculation (Figure 9.). The first diagram is quadratically divergent but preserves both $SU(3)$'s. Therefore the resulting operator

$$\frac{\lambda_t^2}{16\pi^2} \Lambda^2 (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2) \quad (20)$$

is independent of the pseudo-Goldstones. The second diagram is only logarithmically divergent and gives a contribution

$$-\frac{\lambda_t^4}{16\pi^2} |\Phi_1^\dagger \Phi_2|^2 \log(\Lambda^2/f^2). \quad (21)$$

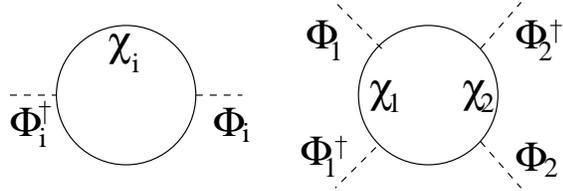


Figure 10. $SU(3)$ -symmetrized top loops.

This breaks the $SU(3)$ symmetry which protects the Higgs mass, but its coefficient is sufficiently small (compare to eq. (16)).

This concludes our discussion of the Little Higgs in the toy world. To promote the toy LH model to a fully realistic model requires three extra ingredients *i.* hypercharge, *ii.* the other quarks and leptons, *iii.* a quartic coupling for the Higgs. Adding hypercharge is very easy, one simply gauges one of the global $U(1)$ symmetries of the model. The mechanism for canceling quadratic divergences is unchanged by this. Adding the remaining fermions is also straightforward. Note that every SM fermion doublet has to be promoted to a triplet because of the $SU(3)$ gauge symmetry. Adding the Higgs quartic coupling is non-trivial and requires some additional structure, which the interested reader can find in [1].

4. Signatures and Conclusions

Little Higgs theories predict new states at the TeV scale which can be seen at the LHC.

Specifically, one generically finds a **vector-like quark** of charge $2/3$ (up-type) which is required to cancel the divergence from the top loop. The new quark can be pair-produced at Hadron colliders as long as its mass is within the kinematical reach (~ 2 TeV). χ 's are expected to decay predominantly to $h + t$, $W + b$ and $Z + t$. In addition we expect **new gauge bosons** which cancel the $SU(2) \times U(1)$ gauge loops. Their quantum numbers appear to be model dependent. Generically they carry weak charges and may be electrically charged. They couple with weak coupling

strength and can be singly produced at colliders. They have couplings to light fermions and can mix with the SM W and Z . These couplings typically lead to the strongest constraints on LH models from electroweak precision tests. Finally, one also expects **new scalars** with masses somewhere in the 100 GeV - 2 TeV range. The quantum numbers of the scalars are model dependent but additional Higgs doublets are often found. The new scalars couple most strongly to the Higgs, to the weak gauge bosons and to third generation fermions. In some models one of the neutral scalars is stable because of an unbroken discrete symmetry providing an excellent cold dark matter candidate.

In conclusion, I reiterate that despite the spectacular success of the SM in describing all particle physics experiments to date we have good reasons to believe that new physics will be discovered in the foreseeable future at the Tevatron or the LHC. This optimism is based on the belief that the hierarchy problem requires a TeV scale solution. Until recently, the only known such solution which is weakly coupled at the TeV scale and therefore does not suffer from fine tuning problems associated with precision data was supersymmetry. Thanks to recent developments supersymmetry has acquired a competitor, the Little Higgs. Many open questions remain: Are there even simpler Little Higgs models? Which signatures are generic? Are there compelling UV completions to Little Higgs models? What is the origin of flavor? What are the constraints from precision electro-weak fits? ... Obviously much more theoretical work is needed to answer these questions, but in the end experiments will have to tell us if the Little Higgs has its place in nature.

5. Acknowledgments

It is my pleasure to thank N. Arkani-Hamed, R.S. Chivukula, A.G. Cohen, H. Georgi, D.E. Kaplan, A.E. Nelson and W. Skiba for introducing me to the Little Higgs. I would also like to thank the organizers of ICHEP2002 for a perfectly organized conference and for inviting me to speak. This work is supported by the Department of En-

ergy with an OJI grant and under contract DE-FG02-91ER-40676.

REFERENCES

1. D. E. Kaplan and M. Schmaltz, "A Little Higgs from a Simple Group," to appear.
2. K. Hagiwara *et al.* [Particle Data Group Collaboration], Phys. Rev. D **66**, 010001 (2002).
3. H. Georgi and A. Pais, Phys. Rev. D **10**, 539 (1974); H. Georgi and A. Pais, Phys. Rev. D **12**, 508 (1975).
4. D. B. Kaplan and H. Georgi, Phys. Lett. B **136**, 183 (1984); D. B. Kaplan, H. Georgi and S. Dimopoulos, Phys. Lett. B **136**, 187 (1984); H. Georgi and D. B. Kaplan, Phys. Lett. B **145**, 216 (1984); H. Georgi, D. B. Kaplan and P. Galison, Phys. Lett. B **143**, 152 (1984); M. J. Dugan, H. Georgi and D. B. Kaplan, Nucl. Phys. B **254**, 299 (1985).
5. N. Arkani-Hamed, A. G. Cohen H. Georgi, Phys. Lett. B **513**, 232 (2001) [arXiv:hep-ph/0105239].
6. N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP **0207**, 034 (2002) [arXiv:hep-ph/0206021].
7. I. Low, W. Skiba and D. Smith, arXiv:hep-ph/0207243.
8. N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire, J. G. Wacker, [hep-ph/0206020] JHEP **0208**, 021 (2002).
9. T. Gregoire and J. G. Wacker, JHEP **0208**, 019 (2002) [arXiv:hep-ph/0206023].
10. K. Lane, Phys. Rev. D **65**, 115001 (2002) [arXiv:hep-ph/0202093].
11. R. S. Chivukula, N. Evans and E. H. Simmons, [arXiv:hep-ph/0204193] Phys. Rev. D **66**, 035008 (2002).
12. N. Arkani-Hamed, A. G. Cohen, T. Gregoire and J. G. Wacker, JHEP **0208**, 020 (2002) [arXiv:hep-ph/0202089].
13. K. Lane, arXiv:hep-ph/0210240.
14. J. G. Wacker, arXiv:hep-ph/0208235.