Performance of 2IMO Differentially Transmit-Diversity Block Coded OFDM Systems in Doubly Selective Channels

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Abstract—By applying the differential space-time block coding in wireless multicarrier transmission, Diggavi et al. proposed the two-input-multiple-output (2IMO) differentially space-time-time block coded OFDM (TT-OFDM) system. In this paper, we recommend three novel differentially transmit-diversity block coded OFDM (DTDBC-OFDM) systems, namely, the FT-, FF-, and TF-OFDM systems. For instance, the TF-OFDM stands for the differentially space-time-frequency block coded OFDM. Moreover, the noncoherent maximum-likelihood sequence detector (NSD), and its three special cases, namely, the noncoherent one-shot detector, linearly predictive decision-feedback (DF) detector, and linearly predictive Viterbi receiver are incorporated to the 2IMO DTDBC-OFDM systems.

Keywords—Estimator-detector, noncoherent ML sequence detection, OFDM, transmit diversity, Viterbi algorithm.

I. INTRODUCTION

Based on the Alamouti space-time block code (STBC) [1], Tarokh and Jafarkhani (TJ) proposed a differential STBC (DSTBC) [2], which does not require the channel state information (CSI) for detection. The TJ DSTBC can be viewed as the differential space-time modulation (DSTM) [3] [4] with a nondiagonal constellation. The original detector, employed in [2]-[4], was designed by assuming the channel is constant over two space-time (ST) codeword durations and hence caused performance degradation in rapid fading channels. Accordingly, the noncoherent maximum-likelihood (ML) sequence detector [5]-[7] and its three special cases, namely, the noncoherent one-shot detector [5] [6], linearly predictive decision-feedback (DF) detector [5] [7], and linearly predictive Viterbi receiver [6] [7] were proposed to improve the original detector.

For the orthogonal frequency division multiplexing (OFDM) transmission [8], assuming the channel is constant over two ST codeword durations, Diggavi et al. applied the TJ DSTBC on each subcarrier and arrived at the differentially space-time-time block coded OFDM (TT-OFDM) system [9, III-A], which performs both the spatial and differential encodings in the time direction (see Fig. 1).

In this paper, in addition to the TT-OFDM, we propose three novel differentially transmit-diversity block coded OFDM (DTDBC-OFDM) systems, namely, the FT-, FF-, and TF-OFDM systems (see Fig. 1). For instance, the TF-OFDM stands for the differentially space-time-frequency block coded OFDM. Here, the first T or F denotes the spatial encoding direction and the next one indicates the differential encoding direction. Moreover, the noncoherent ML sequence detector (NSD) and its three special cases are incorporated to the 2IMO DTDBC-OFDM systems.

II. NONCOHERENT ML SEQUENCE DETECTOR

In this section, the NSD and its special cases for the single-carrier TJ DSTBC system [2] are reviewed hierarchically.

Suppose an $N$ -block sequence is transmitted and received over a 2IQO time-varying flat Rayleigh fading channel. After Gray mapping, the information symbol-matrix $V_n$ for the $n$th block interval is firstly transformed through

$$A_n = TV_n$$

where $A_n$ is differentially ST block encoded as

$$A_n = \begin{bmatrix} a_{2n+1} & a_{2n+2} \\ -a_{2n+2} & a_{2n+1} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \quad 1 \\ 1 \quad 1 \end{bmatrix} \begin{bmatrix} v_{2n+1} \quad v_{2n+2} \\ -v_{2n+2}^* \quad v_{2n+1}^* \end{bmatrix}^T,$$  \hspace{1cm} (1)

and $v_{2n+1} \in \Omega$ and $\Omega_n = \{(1/\sqrt{2}) \exp(j2\pi l/L), l = 0, 1, \ldots, L-1\}$. Then, $A_n$ is differentially ST block encoded as...
\[ X_s = A_s X_{s-1}, \quad X_s = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \]

where \( X_s = \begin{bmatrix} x_{2s+1} \\ x_{2s+2} \end{bmatrix} \).

The first row of the channel symbol-matrix \( X_s \) consists of the symbols, \( x_{2s+1} \) and \( x_{2s+2} \), being transmitted from the 0th and 1st transmit antennas, respectively, in the \((2n+1)\)th symbol duration. Similarly, the second row of \( X_s \) comprises the symbols being transmitted in the \((2n+2)\)th symbol duration. Let \( x_{2s+1} \in \Omega_s \) and \( x_{2s+2} \) be a \( L \)-ary constellation, then it is well known that \( L \geq L \) as a result of the constellation expansion [6].

Let \( h_{2s+1}^{(g)} \) be the path gain from the \( g \)th transmit antenna to the \( q \)th receive antenna. Here, we employ the quasi-static (QS) channel assumption that the channel is constant over a ST codeword duration. Then the piecewise-constant path gain for the \( n \)th ST codeword duration is defined as \( h_{2s+1}^{(g)} = h_{2s+2}^{(g)} = h_{2s}^{(g)} \) with mean zero and correlation function \( \phi_g \triangleq \mathbb{E}[h_{2s+1}^{(g)} h_{2s}^{(g)*}] = J_p(2\pi f_p, m \cdot 2T) \), assuming 2Q independent and identical Rayleigh fading channels with the classical Doppler spectrum. \( f_p \) is the maximum Doppler frequency, and \( T \) is the reciprocal of the system bandwidth. Also, let \( w_{2s+1}^{(q)} \) be the AWGN and \( w_{2s+2}^{(q)} \sim \mathcal{CN}(0, \sigma_q^2) \). Thus, the \( 2 \times Q \) received signal matrix is

\[ R_s = X_s H_s + W_s, \]

where \( R_s = \begin{bmatrix} r_{s}^{(0)} & r_{s}^{(1)} & \cdots & r_{s}^{(Q-1)} \end{bmatrix} \), \( H_s = \begin{bmatrix} h_{s}^{(0)} & h_{s}^{(1)} & \cdots & h_{s}^{(Q-1)} \end{bmatrix} \), and \( W_s = \begin{bmatrix} w_{s}^{(0)} & w_{s}^{(1)} & \cdots & w_{s}^{(Q-1)} \end{bmatrix} \). Also, \( r_{s}^{(g)} = \begin{bmatrix} r_{2s+1}^{(g)} & r_{2s+2}^{(g)} \end{bmatrix} \), \( h_{s}^{(g)} = \begin{bmatrix} h_{2s+1}^{(g)} & h_{2s+2}^{(g)} \end{bmatrix} \), and \( w_{s}^{(g)} = \begin{bmatrix} w_{2s+1}^{(g)} & w_{2s+2}^{(g)} \end{bmatrix} \).

Stacking up the \( N_t \) ST codewords gives a signal model for the noncoherent ML sequence detection as

\[ \mathbf{R} = \mathbf{A} \Sigma \mathbf{W}, \]

where \( \mathbf{R} = \begin{bmatrix} R_0' & R_1' & \cdots & R_{N_t-1}' \end{bmatrix} \), \( \mathbf{A} = \text{diag} \begin{bmatrix} X_{s}, X_{s+1}, \ldots, X_{s+1} \end{bmatrix} \), \( \Sigma = \begin{bmatrix} H_0' & H_1' & \cdots & H_{N_t-1}' \end{bmatrix} \), and \( \mathbf{W} = \begin{bmatrix} W_0 & W_1 & \cdots & W_{N_t-1} \end{bmatrix} \).

From (1) and (2), it is clear that there exists a one-to-one correspondence between the \( 2(N_t-1) \times 2 \) information super-symbol-matrix \( \mathbf{V} = \begin{bmatrix} V_0 & V_1 & \cdots & V_{N_t-1} \end{bmatrix} \) and the \( 2N_t \times 2 \) channel super-symbol-matrix \( \mathbf{X} = \begin{bmatrix} X_0 & X_1 & \cdots & X_{N_t-1} \end{bmatrix} \). Then the ML estimate of \( \mathbf{V} \) is obtained through [6, eq. (46)]

\[ \hat{\mathbf{V}} = \arg \min _{\mathbf{V}} \left\{ \text{tr} \left( \mathbf{R}^H \mathbf{A} \Sigma^{-1} \mathbf{A}^H \mathbf{R} \right) \right\}, \]

where \( \mathbf{A} \triangleq \mathbf{A}_1 \cup \sigma^2 \mathbf{I}_2 \) and \( \mathbf{A}^T \mathbf{A} = \mathbf{I}_{2N_t} \). Here, \( \otimes \) denotes the Kronecker product and \( \mathbf{C}_s \) represents the covariance matrix of \( \mathbf{h}^{(g)} = \begin{bmatrix} h_{2s}^{(g)} & h_{2s+1}^{(g)} & \cdots & h_{2s+1}^{(g)} \end{bmatrix} \). In the following, we introduce three special cases of this NSD, which are of lower complexity and can be carried out in practice.

### A. Linearly Predictive Viterbi Receiver

With the QS channel assumption, the NSD given in (5) brings a linear-prediction interpretation of its structure. The minimization problem (5) can be solved by using the Cholesky factorization approach [6] as follows. Given the Cholesky decomposition of the \( N_t \times N_t \) matrix \( \Phi = \mathbf{C}_s + \sigma^2 \mathbf{I}_{N_t} \) as

\[ \Phi = \mathbf{L} \Sigma \mathbf{L}^H, \]

where \( \mathbf{L} \) is lower triangular, \( \Sigma = \text{diag} \{ \sigma_1, \sigma_2, \ldots, \sigma_{N_t} \} \), and

\[ \mathbf{L}^T = \begin{bmatrix} -p_0^0 & 0 & 0 & \cdots & 0 \\ -p_0^1 & -p_1^0 & 0 & \cdots & 0 \\ -p_0^2 & -p_1^1 & -p_2^0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -p_0^{N_t-1} & -p_1^{N_t-2} & -p_2^{N_t-3} & \cdots & -p_{N_t-1}^{N_t-1} \end{bmatrix}, \]

Note that \( p_{n}^0 = -1 \) and \( p_{n}^g \) is the \( k \)th coefficient of the \( n \)th order linear predictor for each component of \( \{ \mathbf{h}^{(g)} + \mathbf{X}^q \mathbf{W}^{(q)} \} \) and \( \varepsilon_q \) is the prediction error. Substituting (6) and (7) into (5) results in a linearly predictive sequence detector as [6, eq. (51)]

\[ \hat{\mathbf{V}} = \arg \min _{\mathbf{V}} \sum _{s=0}^{N_t-1} \left\| \mathbf{Y}_s - \hat{\mathbf{Y}}_s \right\| / \varepsilon_q, \]

where \( \hat{\mathbf{Y}}_s = \sum _{s=0}^{N_t-1} p_{n}^g \mathbf{X}^q \mathbf{R}_s \) is the \( n \)th order linear prediction of the fading-plus-noise matrix \( \mathbf{Y}_s = \mathbf{X}^q \mathbf{R}_s = \mathbf{H}_s + \mathbf{X}^q \mathbf{W}_s \), and \( \left\| \cdot \right\| \) is the Frobenius norm of a matrix.

By truncating the memory of this estimator-detector, an approximate ML sequence detector containing a \( K \)th order linear predictor is derived as [6, eq. (52)]

\[ \hat{\mathbf{V}} = \arg \min _{\mathbf{V}} \sum _{s=0}^{N_t-1} \left\| \mathbf{X}^q \mathbf{R}_s - \sum _{s=0}^{N_t-1} p_{n}^g \mathbf{X}^q \mathbf{R}_s \right\| / \varepsilon_q. \]

Based on (9), a trellis structure with \( L_{2K} \) states is defined as: 1) each state of the trellis in the \( n \)th block interval is represented as \( \Gamma_s = \{ X_{s-1}, X_{s-2}, \ldots, X_{s-1} \} \); 2) there are \( L_{2} \) transitions emerging from each state and terminating in \( L_{2} \) different states \( \Gamma_{s+1} \); 3) the branch metric associated with each transition is defined as [6, eq. (53)]

\[ \Delta(\Gamma_s, V_s) = \sum _{s=0}^{N_t-1} \left\| \mathbf{X}^q \mathbf{R}_s - \sum _{s=0}^{N_t-1} p_{n}^g \mathbf{X}^q \mathbf{R}_s \right\|. \]
Then the Viterbi algorithm with a fixed decision delay $D$ can be used to solve the minimization problem in (9).

To further alleviate the implementation complexity, the reduced-state technique is incorporated: 1) a reduced state $\tilde{\Gamma}$ is defined with the most recent $U$ ($U < K$) channel symbol-matrices, i.e., $\tilde{\Gamma}_k \triangleq [X_{k-1}, X_{k-2}, \ldots, X_{k-U}]$; 2) in calculating the branch metric, one can extract the unavailable symbol-matrices from the survivor history according to the per-survivor processing (PSP) technique [10]; 3) the branch metric related to each transition is defined as

$$\Delta(\tilde{\Gamma}_s, \tilde{V}_s) = \sum_{i=0}^{U} \left\| X_i^s \tilde{a}_s^{(i)} - \sum_{i=1}^{U} p_i^s X_i^s \tilde{a}_s^{(i)} - \sum_{i=1}^{U} p_i^s \tilde{X}_s \tilde{a}_s^{(i)} \right\|_2,$$

where $\tilde{X}_s$, $\tilde{X}_{s-1}$, $\ldots$, $\tilde{X}_{s-U}$ are determined by the survivor entering the state $\tilde{\Gamma}_s$. Subsequently, this reduced-state trellis-based sequence detector [6] is named the Viterbi receiver (VR).

B. Linear Predictive Decision-Feedback Detector

Here, we consider an extreme case of $U = 0$ for the VR. In this case, only one path is allowed to survive and the sequence detector degenerates into a DF detector as

$$\tilde{V}_s = \arg \min_{\tilde{V}_s} \sum_{i=0}^{U} \left\| X_i^s \tilde{a}_s^{(i)} - \sum_{i=0}^{U} p_i^s X_i^s \tilde{a}_s^{(i)} \right\|_2,$$

which is exactly the linearly predictive DF receiver (DR) [5].

C. Noncoherent One-Shot Detector

By considering the DR with the prediction order $K = 1$ and employing the fact $p_1^s = 0$ [11], (12) is simplified to

$$\tilde{V}_s = \arg \max_{\tilde{V}_s} \sum_{i=0}^{U} \Re \left\{ \exp \left\{ i \tilde{a}_s^{(i)} \left( r_i^s \right)^H A^s \right\} \right\},$$

which is the noncoherent one-shot detector [5] [6] and is dubbed the conventional receiver (CR), subsequently.

D. Hierarchy of the NSD and its Special Cases

Now we interpret the hierarchy of the NSD (5) and its three special cases, namely, the CR (13), DR (12), and VR (11). Firstly, the VR is a suboptimum version of the NSD; secondly, the DR is the one-survivor version of the VR; finally, the CR is the one-order-prediction version of the DR. Moreover, since the NSD and its special cases are inherently of the estimator-detect structure, their performances are dominated by the qualities of their estimates regarding the fading-plus-noise processes. More specifically, the VR employs the PSP technique so that each state in the trellis has its own survivor. Then the estimates contained in the branch metrics are distinct from state to state and from path to path. Indeed, the VR benefits from the path diversity and thus performs better than its one-survivor version, namely, the DR. Furthermore, with a larger prediction order and hence better estimates, the DR performs better than its one-order-prediction version, i.e. the CR.

III. 2IMO DTDBC-OFDM SYSTEMS

In this section, the system and channel models, the differential encoding, and the receive design of the 2IMO DTDBC-OFDM systems are provided.

A. System and Channel Models

We consider the DFT-based OFDM transmission over the spatially independent and identical 2IMO WSSUS Rayleigh fading channels and assume sufficient cyclic prefix (CP) is inserted [8]. Let $V_d$ denote the L-PSK information symbol for the $d$th subcarrier in the $n$th OFDM block interval. The received signal from the $q$th receive antenna is [12]

$$R_{i,q}^{(n)} = \frac{1}{2} \sum_{d=0}^{D} H_{d,q}^{(n)} \sigma_{d,q}^{(n)} + W_{i,q}^{(n)} ,$$

where the channel symbols $\{ \sigma_{d,q}^{(n)} \}$ are uncorrelated with mean zero and variance half (i.e. $E_{d} = 1/2$) and the equivalent AWGN $W_{i,q}^{(n)} = \sum_{d=0}^{D} C_d^{(n)} + N_{d,q}^{(n)}$ is of mean zero and variance $\sigma_{d,q}^{(n)}$. $H_{d,q}^{(n)}$, $C_d^{(n)}$, and $N_{d,q}^{(n)}$ are the multiplicative distortion (MD), inter-carrier-interference (ICI), and AWGN, respectively. All of them are of mean zero, and each of which is of variance $\sigma_{d,q}^{(n)}$, $\sigma_{d,q}^{(n)}$, and $N_{d,q}$, respectively.

For the exponential power delay profile, we define the fading power of the $d$th tap as [12, eq. (12)]

$$\sigma_{d,q}^{(n)} = \frac{1-e^{-d/d}}{1-e^{-d/d}} e^{-d/d} ,$$

where the delay control $d$ determines the normalized root-mean-square (RMS) delay spread $\tau_{\text{rms}}/T$. Also, the correlation of the MD $\rho_{m}(\Delta t, \Delta k) = \rho_{m}(\Delta t, \Delta k)$ is as [12, eq. (13)]

$$\rho_{m}(\Delta t, \Delta k) = \frac{\sum_{k=0}^{K-1} \rho_{m} \left( \sum_{k=0}^{K-1} \sigma_{d,q}^{(n)} e^{j2\pi km/T} \right) \left( \sum_{k=0}^{K-1} \sigma_{d,q}^{(n)} e^{j2\pi km/T} \right) \left( \sum_{k=0}^{K-1} \sigma_{d,q}^{(n)} e^{j2\pi km/T} \right)}{N \sum_{k=0}^{K-1} \left( N-1 \right) \left( \sum_{k=0}^{K-1} \sigma_{d,q}^{(n)} e^{j2\pi km/T} \right) \left( \sum_{k=0}^{K-1} \sigma_{d,q}^{(n)} e^{j2\pi km/T} \right) \left( \sum_{k=0}^{K-1} \sigma_{d,q}^{(n)} e^{j2\pi km/T} \right)} \left( \sum_{k=0}^{K-1} \sigma_{d,q}^{(n)} e^{j2\pi km/T} \right) \left( \sum_{k=0}^{K-1} \sigma_{d,q}^{(n)} e^{j2\pi km/T} \right) \left( \sum_{k=0}^{K-1} \sigma_{d,q}^{(n)} e^{j2\pi km/T} \right).$$

Accordingly, we define the temporal correlations as $\rho_{m}(2,0) = 0.2$ and $\rho_{m}(1,0) = 0.1$. Also, the spectral correlations are defined as $\rho_{m}(0,2) = 0.1$ and $\rho_{m}(0,1) = 0.1$.

B. Differential Encoding and Receiver Design

According to the system structures (see Fig. 1), we introduce the mechanisms of DTDBC-OFDM systems. Subsequently, the transmission of $N_s$ OFDM blocks is assumed.

1) TT-OFDM

For the TT-OFDM [9], since the differential encoding and detection are independent and simultaneous on all $N_s$ subcarriers, only the transmission on the $k$th subcarrier is illustrated, subsequently. In light of (1) and (2), the $k$th ST codeword is transformed and time-domain (TD) differentially block encoded as follows.
\[ X_{i,k} = A_{i,k} X_{i-1,k} = T V_{i,k} X_{i-1,k} , \quad X_{i,0} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \]  
(17)

where \( X_{i,k} = \begin{bmatrix} X_{2i+1,2} \\ X_{2i+2,2} \end{bmatrix} \), \( A_{i,k} = \begin{bmatrix} A_{2i+1,2} \\ -A_{2i+2,2} \end{bmatrix} \), and \( V_{i,k} = \begin{bmatrix} V_{2i+1,2} \\ V_{2i+2,2} \end{bmatrix} \).

Thus, the received signals from all \( Q \) receive antennas are

\[ R_{i,k} = X_{i,k} H_{i,k} + W_{i,k}, \]  
(20)

where \( R_{i,k} = \begin{bmatrix} r_{i,k}^{(0)} \\ r_{i,k}^{(1)} \\ \vdots \\ r_{i,k}^{(Q-1)} \end{bmatrix} \). \( H_{i,k} \) and \( W_{i,k} \) are defined in the same form as \( R_{i,k} \). Then piling up the \( N_q/2 \) ST codewords yields the signal model for the NSD as

\[ R_{i} = A_{i} H_{i} + W_{i}, \]  
(21)

where \( R_{i} = \begin{bmatrix} R_{i,0}^{(0)} \\ R_{i,1}^{(0)} \\ \vdots \\ R_{i,Q-1,1}^{(0)} \end{bmatrix} \), \( A_{i} = \text{diag}\{X_{i,0} X_{i,1} \cdots X_{i,Q-1,1}\} \). \( H_{i} \) and \( W_{i} \) are defined in the same form as \( R_{i} \). Thus, from (5), the ML estimate of the \( 2(N_q/2-1) \times 2 \) super-symbol-matrix \( V_{i} = \begin{bmatrix} V_{i,0} \\ V_{i,1} \\ \vdots \\ V_{i,Q-1,1} \end{bmatrix} \) is

\[ \hat{V}_{i} = \arg \min_{\hat{V}_{i}} \left\{ \text{tr} \left[ H_{i}^{(0)} A_{i} \left( \Phi_{i}^{(T)} \right)^{-1} \hat{V}_{i} R_{i} \right] \right\}, \]  
(22)

where \( \Phi_{i}^{(T)} = \hat{\Phi}_{i}^{(T)} \otimes I_2 \triangleq (C_{i}^{(T)} + \sigma_{\text{d}} I_{N_q/2-1}) \otimes I_2 \) and \( C_{i}^{(T)} \),

the covariance matrix of

\[ h_{i}^{(e)} = \begin{bmatrix} H^{(e)} \quad H^{(e)} \quad \cdots \quad H^{(e)} \\ H^{(e)} \quad H^{(e)} \quad \cdots \quad H^{(e)} \quad H^{(e)} \quad \cdots \quad H^{(e)} \quad \cdots \quad H^{(e)} \quad \cdots \quad H^{(e)} \end{bmatrix}, \]  
(23)

is evaluated via (16) with \( \Delta k = 0 \).

In light of (13), since the fading correlation \( \rho_{\tau} \) is real-valued, the CR obtains the estimate for \( V_{i,k} \) according to

\[ \hat{V}_{i,k} = \arg \max_{\hat{V}_{i,k}} \sum_{j=0}^{K} \left\{ \text{tr} \left[ r_{i,k}^{(j)} (\hat{A}_{i,k}^H) r_{i,k}^{(j)} \right] \right\}. \]  
(24)

Based on (12), the DR performs detection through

\[ \hat{V}_{i,k} = \arg \min_{\hat{V}_{i,k}} \sum_{j=0}^{K} \left\{ X_{i,k}^{H} r_{i,k}^{(j)} - \sum_{j=0}^{K} P_{j}^{e} X_{i,k}^{H} r_{i,k}^{(j)} \right\}^2, \]  
(25)

where \( \{P_{j}^{e}\} \) are calculated via (6), (7), and (16).

According to (11), the VR with \( L'^{(e)} \) states is defined as follows. Each state of the trellis in the \( N \) th ST codeword duration is denoted as \( \tilde{\Gamma}_{i,k} = \begin{bmatrix} X_{i,k-1} \cdots X_{i,k-1} \cdots \cdots \cdots \cdots X_{i,k} \end{bmatrix} \), and the corresponding branch metric is expressed as

\[ \Delta(\tilde{\Gamma}_{i,k}, \hat{V}_{i,k}) = \sum_{j=0}^{K} \left| X_{i,k}^{H} r_{i,k}^{(j)} - \sum_{j=0}^{K} P_{j}^{e} X_{i,k}^{H} r_{i,k}^{(j)} \right|^2 \]  
(26)

where \( \{P_{j}^{e}\} \) are the same as the ones for the DR.

2) FT-OFDM

Here only the transmission on the \( k \)th pair of subcarriers is demonstrated. The \( N_q \) space-frequency (SF) codeword is transformed and TD differentially block encoded as

\[ X_{i,k} = A_{i,k} X_{i-1,k} = T V_{i,k} X_{i-1,k} , \quad X_{i,k} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \]  
(27)

where \( X_{i,k} = \begin{bmatrix} X_{2i+1,k} \quad X_{2i+2,k} \\ -X_{2i+2,k} \quad X_{2i+1,k} \end{bmatrix} \). \( A_{i,k} = \begin{bmatrix} A_{2i+1,k} \\ A_{2i+2,k} \end{bmatrix} \), and \( V_{i,k} = \begin{bmatrix} V_{2i+1,k} \\ V_{2i+2,k} \end{bmatrix} \).

Then the SF transmission proceeds as

\[ S_{i,k} = X_{i,k} \]  
(28)

Assume the MD is QS over a SF codeword duration (QSF), i.e., \( H_{i,k}^{(e)} \triangleq H_{i,k}^{(e)} = H_{i,k}^{(e)} \). Therefore, according to (14), the received signal from the \( q \)th receive antenna is

\[ r_{i,k}^{(q)} = X_{i,k} H_{i,k}^{(q)} + W_{i,k}^{(q)} \]  
(29)

and the received signals from all \( Q \) receive antennas are

\[ R_{i,k} = X_{i,k} H_{i,k} + W_{i,k}, \]  
(30)
where \( \mathbf{R}_{s,i} = \begin{bmatrix} \mathbf{r}_{s,i}^{(0)} & \mathbf{r}_{s,i}^{(1)} & \cdots & \mathbf{r}_{s,i}^{(S-1)} \end{bmatrix} \). \( \mathbf{H}_{s,i} \) and \( \mathbf{W}_{s,i} \) are defined in the same form as \( \mathbf{R}_{s,i} \). Stacking up the \( N_s \) SF codewords gives the signal model for the NSD as

\[
\mathbf{R}_{s,i} = \Lambda_{s,i} \mathbf{H}_{s,i} + \mathbf{W}_{s,i},
\]

where \( \mathbf{R}_{s,i} = \begin{bmatrix} \mathbf{R}_{s,i}^{(0)} & \mathbf{R}_{s,i}^{(1)} & \cdots & \mathbf{R}_{s,i}^{(N_s/2-1)} \end{bmatrix} \), \( \Lambda_{s,i} = \text{diag} \{ \mathbf{x}_{s,0}, \mathbf{x}_{s,1}, \cdots, \mathbf{x}_{s,N_s/2-1} \} \). \( \mathbf{H}_{s,i} \) and \( \mathbf{W}_{s,i} \) are defined in the same form as \( \mathbf{R}_{s,i} \).

From (5), the ML estimate of the \( 2(N_s-1) \times 2 \) information super-symbol-matrix \( \mathbf{Y}_{s,i} = \begin{bmatrix} \mathbf{Y}_{s,i}^{(0)} & \mathbf{Y}_{s,i}^{(1)} & \cdots & \mathbf{Y}_{s,i}^{(N_s/2-1)} \end{bmatrix} \) is

\[
\hat{\mathbf{Y}}_{s,i} = \arg \min_{\mathbf{Y}_{s,i}} \{ \text{tr} \left( \mathbf{R}_{s,i}^{(0)} \mathbf{Y}_{s,i} \mathbf{R}_{s,i}^{(0)^T} \right) \},
\]

where \( \mathbf{R}_{s,i}^{(0)} = \mathbf{R}_{s,i} \) and \( \mathbf{Y}_{s,i} = \begin{bmatrix} \mathbf{Y}_{s,i}^{(0)} & \mathbf{Y}_{s,i}^{(1)} & \cdots & \mathbf{Y}_{s,i}^{(N_s/2-1)} \end{bmatrix} \). In light of (10), the CR is given by

\[
\hat{\mathbf{V}}_{s,i} = \arg \max_{\mathbf{V}_{s,i}} \sum_{k=0}^{N_s/2-1} \text{tr} \left( \mathbf{R}_{s,i}^{(0)} \mathbf{V}_{s,i}^{(0)} \mathbf{R}_{s,i}^{(0)^T} \right) \}
\]

is computed via (16) with \( \Delta k = 0 \).

In the following, the operations of the CR, DR, and VR for the FT-OFDM are similar to that for the TT-OFDM.

3) FF-OFDM

Here only the transmission for the \( i \)-th block is described. From (1) and (2), the \( i \)-th SF codeword is transformed and frequency-domain (FD) differentially block encoded via

\[
\mathbf{X}_{s,i} = \mathbf{A}_{s,i} \mathbf{X}_{s,i-1} = \mathbf{T} \mathbf{Y}_{s,i-1} \mathbf{T}^{-1} \mathbf{X}_{s,i-1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}.
\]

The same as that for the FF-OFDM, the SF transmission proceeds through (27) and the received signals from all \( Q \) receive antennas after QSF. Then piling up the \( N/2 \) SF codewords produces the signal model for the NSD as

\[
\mathbf{R}_{s,i} = \Lambda_{s,i} \mathbf{H}_{s,i} + \mathbf{W}_{s,i},
\]

where \( \mathbf{R}_{s,i} = \begin{bmatrix} \mathbf{R}_{s,i}^{(0)} & \mathbf{R}_{s,i}^{(1)} & \cdots & \mathbf{R}_{s,i}^{(N_s/2-1)} \end{bmatrix} \), \( \Lambda_{s,i} = \text{diag} \{ \mathbf{x}_{s,0}, \mathbf{x}_{s,1}, \cdots, \mathbf{x}_{s,N_s/2-1} \} \), \( \mathbf{H}_{s,i} \), and \( \mathbf{W}_{s,i} \) are defined in the same form as \( \mathbf{R}_{s,i} \).

From (5), the ML estimate of the \( 2(N_s/2-1) \times 2 \) information super-symbol-matrix \( \mathbf{Y}_{s,i} = \begin{bmatrix} \mathbf{Y}_{s,i}^{(0)} & \mathbf{Y}_{s,i}^{(1)} & \cdots & \mathbf{Y}_{s,i}^{(N_s/2-1)} \end{bmatrix} \) is

\[\hat{\mathbf{V}}_{s,i} = \arg \min_{\mathbf{V}_{s,i}} \{ \text{tr} \left( \mathbf{R}_{s,i}^{(0)} \mathbf{V}_{s,i} \mathbf{R}_{s,i}^{(0)^T} \right) \},\]

where \( \mathbf{R}_{s,i}^{(0)} = \mathbf{R}_{s,i} \) and \( \mathbf{V}_{s,i} = \begin{bmatrix} \mathbf{V}_{s,i}^{(0)} & \mathbf{V}_{s,i}^{(1)} & \cdots & \mathbf{V}_{s,i}^{(N_s/2-1)} \end{bmatrix} \). The same as that for the FT-OFDM, the ST transmission proceeds according to (18) and the received signals from all \( Q \) receive antennas are expressions by (20), assuming the MD is QSF. Then piling up the \( N \) ST codewords produces the signal model for the NSD as

\[
\mathbf{R}_{s,i} = \Lambda_{s,i} \mathbf{H}_{s,i} + \mathbf{W}_{s,i},
\]

where \( \mathbf{R}_{s,i} = \begin{bmatrix} \mathbf{R}_{s,i}^{(0)} & \mathbf{R}_{s,i}^{(1)} & \cdots & \mathbf{R}_{s,i}^{(N_s/2-1)} \end{bmatrix} \), \( \Lambda_{s,i} = \text{diag} \{ \mathbf{x}_{s,0}, \mathbf{x}_{s,1}, \cdots, \mathbf{x}_{s,N_s/2-1} \} \), \( \mathbf{H}_{s,i} \), and \( \mathbf{W}_{s,i} \) are defined in the same form as \( \mathbf{R}_{s,i} \).

From (5), the ML estimate of \( \mathbf{V}_{s,i} = \begin{bmatrix} \mathbf{V}_{s,i}^{(0)} & \mathbf{V}_{s,i}^{(1)} & \cdots & \mathbf{V}_{s,i}^{(N_s/2-1)} \end{bmatrix} \) is

\[\hat{\mathbf{V}}_{s,i} = \arg \min_{\mathbf{V}_{s,i}} \{ \text{tr} \left( \mathbf{R}_{s,i}^{(0)} \mathbf{V}_{s,i} \mathbf{R}_{s,i}^{(0)^T} \right) \},\]

where \( \mathbf{R}_{s,i}^{(0)} = \mathbf{R}_{s,i} \) and \( \mathbf{V}_{s,i} = \begin{bmatrix} \mathbf{V}_{s,i}^{(0)} & \mathbf{V}_{s,i}^{(1)} & \cdots & \mathbf{V}_{s,i}^{(N_s/2-1)} \end{bmatrix} \). Here, \( \mathbf{h}^{(e)} \) is evaluated by (16) with \( \Delta t = 0 \). Here, \( \mathbf{h}^{(e)} \) is expressed as
Subsequently, the operations of the CR, DR, and VR for the TF-OFDM are similar to that for the FF-OFDM.

IV. NUMERICAL RESULTS

For the numerical results, the simulation parameters are: 1) the carrier frequency and system bandwidth are 1.8 GHz and 800 KHz, respectively, and thus the symbol duration is $T = 1.25 \mu s$; 2) the number of subcarriers and guard samples are $N = 128$ and $G = 32$, respectively, and hence the total OFDM block duration is $(N+G)T = 200 \mu s$; 3) the constellation is two-ary, i.e. $L = L = 2$; 4) the number of uncorrelated paths is $M = 12$; 5) the number of receive antennas is $Q = 1, 2$; 6) for both the DR and VR, the prediction order is $K = 5$; 7) for the VR, $U = 1$, i.e., the number of states is $L^U = 4$, and the decision delay is $D = 10$. For the exponential power delay profile, the parameters for the highly selective channel condition being considered are: 1) the delay control $d = 20$ resulting in the RMS delay spread $\tau_{25\%} = 4.27 \mu s$; 2) the maximum Doppler frequency $f_d = 200$ Hz corresponding to the vehicle speed $v = 120$ km/hr. These channel parameters are the same as that in [11] and [12].

As shown in Fig. 2, the bit-error-rates (BERs) of DTDBC-OFDM systems are compared in the highly selective channel condition. The matched filter bounds (MFBs) [11], the BERs of the single-carrier 2ISO and 2I2O systems employing the noncoherent one-shot detector in the QS Rayleigh fading channel, are also provided as the benchmarks. The reasons for the presence of the error floors are threefold: 1) the ICI induced when the channel is not constant over an OFDM block duration; 2) the failure of the estimator-detect algorithm occurred when the QS channel assumption (i.e. QST and QSF) does not hold (see Section III-B); 3) the worse estimates of the fading-plus-noise processes resulting from the larger channel variation along the differential detection direction (see Section II-D). Thereupon, the TF- and FT-OFDM perform better than the TT- and FF-OFDM. Also, both the VR and DR yield significant performance improvements over the CR. This is due to that the VR and DR detect signals with the aid of better estimates (see Section II-D). Moreover, it is revealed that the 2I2O systems outperform the 2ISO systems remarkably since they benefit simultaneously from the transmit diversity and receive diversity.

V. CONCLUSION

In this paper, based on the estimator-detect algorithm, we hierarchically interpret the NSD and its three special cases, namely, the CR, DR, and VR. Then they are applied to the proposed 2IMO DTDBC-OFDM systems, namely, the TT-, FT-, FF-, and TF-OFDM systems. Numerical results have revealed that the 2I2O DTDBC-OFDM systems employing the CR can obtain satisfactory performance. However, when only one receive antenna is available, the implementation of the DR or the VR is necessary for achieving better performance.

REFERENCES