Labour-market Volatility in a Matching Model with Worker Heterogeneity and Endogenous Separations*

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Abstract

This paper shows that introducing worker heterogeneity into a standard search and matching model can help increase the volatility of unemployment without violating the tight negative correlation between vacancies and unemployment, i.e., the Beveridge curve. In the model, periods of high job destruction and unemployment correspond with periods of more severe mismatch between the demands of firms and the qualifications of job seekers. A more severe mismatch translates into fewer successful employment matches conditional on the number of contacts per firm and, as a result, into a higher expected recruitment cost per worker hired, with adverse effects on incentives to open vacancies.

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INTRODUCTION

Consider a negative shock to labour productivity in the standard search and matching model where job separations are exogenous. On impact, firm profitability worsens and fewer vacancies are opened. Unemployment increases through fewer matches. But the decline in the number of new vacancies and the rise in unemployment make the labour market less tight, with a lower number of vacancies per unemployed worker. This creates a positive externality on firms and ultimately puts a break on the decline in vacancies and consequent increase in unemployment. This externality is the main reason why, as shown in Shimer (2005), a reasonably calibrated version of the textbook matching model grossly fails to account for the observed volatility of unemployment. The model can achieve more unemployment volatility only if some mechanism is introduced to offset the positive externality on vacancy creation.

A natural alternative mechanism is replacing the assumption of exogenous separations with endogenous job destruction caused by the productivity shock. There is ample evidence that transitions in and out of unemployment both contribute to the cyclical volatility of unemployment, with the inflow rate contributing about one third to one half of the volatility of unemployment.\(^1\) With a negative productivity shock, job destruction rises and the entry into unemployment is increased. The rise in unemployment is reinforced, adding to the volatility due to the lower matching rate. But the rise in job destruction also reinforces the positive externality on vacancy creation, because the further increase in unemployment lowers the vacancy to unemployment ratio even more. As Mortensen and Nagypál (2007b) demonstrate, in the most widely used model of endogenous job destruction, due to Mortensen and Pissarides (1994), the externality is strong enough to make firms increase vacancies, leading to a positively sloped Beveridge curve.

This paper shows that the positive externality on vacancy creation from the fall in tightness can be mitigated when there is heterogeneity in the labour force. In a recession, firms become more selective in terms of the profitability of the employment relationships they choose to commence and their threshold for hiring a worker becomes higher than usual. Firms also become more selective with respect to what workers they retain, thus job destruction rises and unemployment entry is increased. But the employment rela-

\(^1\)See, e.g., Shimer (2007), Fujita and Ramey (2009) and Barnichon (2012). Evidence reported in Rogerson and Shimer (2011) suggests that spikes in job destruction drive part of the initial decline in unemployment during most downturns. Moreover, evidence from Barsky et al (1994), Bowles et al (2002) and Liu (2003) that the average labour quality increases in economic downturns, suggests that at least some part of job separations is driven by endogenous decisions in response to aggregate productivity shocks.
tionships that are endogenously terminated in a recession are those with workers whose qualifications are not strong enough to secure positive rents to firms, i.e., workers that are less likely to be employable during bad times. Consequently, at times of low aggregate productivity and high job destruction there are a lot more unemployed workers looking for jobs, but at the same time, there is a larger distance between the demands of firms and the range of qualifications offered by unemployed workers. Because a firm searches for a good match among a heterogeneous group of workers, a larger distance translates into fewer successful matches, conditional on the vacancy to unemployment ratio, i.e., into a deterioration in matching efficiency, with adverse effects on incentives to post vacancies. Thus, in a recession outflows from unemployment are achieved with greater difficulty than in a boom. This idea is formalized in a relatively standard search and matching model extended to allow for worker heterogeneity in terms of ability. I show quantitatively that the model with worker heterogeneity is capable of increasing the volatility of unemployment through endogenous separations without violating the tight negative correlation between vacancies and unemployment. A downward-sloping Beveridge curve continues to hold.

Recent empirical literature suggests that matching efficiency can, indeed, decline substantially in a recession. For instance, Elsby et al (2010) report that the outflow rate from unemployment, conditional on the vacancy-unemployment ratio, has been very low during the 2008-2009 recession. Likewise, Davis et al (2010) show a dramatic decline in the vacancy yield during the same period. Evidence of a strong cyclical component in matching efficiency over a longer time period can be found in Barnichon and Figura (2011a,b) and Sahin et al (2012). Further, a large empirical literature studying the matching function (see Petrongolo and Pissarides, 2001, for a survey) links changes in matching efficiency to aggregation issues often disguised under the term mismatch: the disparity between the characteristics of job seekers and the requirements of firms. In line with the mismatch hypothesis, the model developed in this paper captures an endogenous mechanism that generates mismatch-driven cyclical changes in matching efficiency.

2In addition, Daly et al (2011) show that in the recent recession vacancy yields have been below expectations across all industries, suggesting a broad-based change in the firms hiring decisions: the depressed aggregate conditions may have caused firms that create vacancies to become more selective about filling them, in line with the assumption used in this paper.

3The model proposed relies on unobservable heterogeneity and emphasizes a type of mismatch that is more likely to occur within small segments of the labour market and between observationally similar workers and vacancies, due to firms becoming more selective with respect to what workers to hire and retain. Admittedly, because the degree of disaggregation that can be achieved by available data is limited, the empirical studies that examine the mismatch hypothesis rely on broad disaggregations such as differences in the distribution of industrial sectors or occupations between unemployed workers and vacancies.
The model also accounts for the coexistence of a large number of short unemployment spells with a small number of workers who stay unemployed for much longer. Shimer (2008) documents a strong negative duration dependence in re-employment probabilities and interprets his finding as evidence for the view that workers who fail to quickly find jobs need to wait for new vacancies to come into the market. Likewise, Coles and Petrongolo (2003) find that the re-employment rate of some of the newly unemployed workers depends statistically on the inflow of new vacancies and not on the vacancy stock. They interpret this finding as evidence that some newly unemployed workers are on the long-side of their market. Consistent with these findings, in the model presented here, some of the newly unemployed workers - particularly those with lower ability - will have better chances of finding a job only when economic conditions improve and the hiring margin becomes looser again, in the sense that workers with lower ability are hired.

A few other papers also explore the role of heterogeneity in generating more cyclical volatility in matching models. The papers most related to this one are Pries (2008) and Bils et al (2010). Pries (2008), incorporates worker heterogeneity in terms of productivity into a relatively standard matching model to demonstrate that the changing composition of unemployment can increase labour market volatility. In particular, Pries argues that if during downturns the unemployment pool consists of a larger than usual share of low-productivity workers, who generate lower surplus to employers, then firms have less incentive to open vacancies. The model in this paper is complementary to Pries’s model in that it allows for endogenous separations, while Pries allows for only exogenous separations. But my analysis emphasizes a different channel through which worker heterogeneity can generate more volatility in job creation. In the model developed here, the driving force behind the enhanced responsiveness of vacancies to productivity shocks are not the compositional changes in the unemployment pool, but the adjustments in the match acceptance/continuation threshold that cause procyclical changes in matching efficiency. Pries’s model precludes such effects due to the assumption that all matches are

\[4\] Another related paper is Guerrieri (2007). She also pursues the idea that cyclical adjustments in the hiring margin can be a potential source of volatility in job creation, but her model differs from the present model in several aspects. It is a competitive search model with homogeneous workers and only exogenous separations. Her main finding is that in such a model the adjustments in the hiring margin do not contribute to more volatility in job creation. Fugita and Ramey (2012) also explore the quantitative ability of a Mortensen-Pissarides model with match quality shocks and endogenous separations, but homogeneous labour force. They show that allowing for on-the-job search enables to model to replicate the Beveridge curve. Some other studies, such as Krause and Lubik (2007), Nagypál (2007) and Tasci (2007), also explore the interaction between worker heterogeneity and labour market volatility, but focus on the heterogeneity introduced by on-the-job search. Finally, Reicher (2010), adopts an alternative approach. He abstracts from search costs and frictional unemployment and shows that an indivisible-labour real business cycle model with sector-level heterogeneity can generate a realistic Beveridge curve.
acceptable at all times.\footnote{In Pries's model, the shift in the unemployment pool towards low-productivity workers is imposed exogenously by considering a larger increase in the exogenous separation rate for low-productivity workers. It is not clear cut, however, that a model with endogenous separations can predict such a shift. In such a model, just as in the present model, the workers laid off in a recession are more productive than those laid off in a boom, meaning that in a recession the share of low-productivity workers in the unemployment pool is smaller, not larger, than usual. Evidence suggest that the skill composition of the unemployed does not change much over the business cycle (see, e.g., Elsby et al 2010 and Barnichon and Figura 2010), while evidence based on unobservable heterogeneity point to a reversed impact than the one assumed by Pries. Using CPS data Mueller (2011) finds that the average residual wage of the unemployed is strongly countercyclical.}

Bils et al (2010) consider a variant of Mortensen and Pissarides (1994) where workers are risk averse and therefore heterogeneous in terms of their willingness to trade search for work (i.e., in terms of their reservation wage). The question they ask is whether their model can produce both realistic fluctuations in unemployment and a realistic dispersion in wage growth within matches. For this reason, they abstract from other sources of heterogeneity and consider only match-quality shocks, which are necessary to generate wage changes within matches.\footnote{In two related papers (Bils et al 2007 and 2012) the same authors consider versions of the model where workers are also heterogeneous in terms of labour market ability, but assume that markets are segmented by ability.} In their model, differences in worker’s reservation wages reflect differences in their wealth that, in turn, reflect differences in their histories of match qualities and unemployment spells. Having both worker heterogeneity and a match acceptance decision, their model captures a channel for volatility in vacancies that is similar to the one emphasized in this paper. As in the present model, endogenous separations may depress vacancy creation if they are concentrated on workers with high reservation wages who are less likely to be employable during bad times. However, in their calibrated model the dispersion in reservation wages across workers is relatively small, meaning that differences in rents across matches reflect mainly match-quality shocks. The Beveridge curve correlation in their calibrated model is therefore weak, because the matches that endogenously break up in their model are mainly low-quality matches, as opposed to matches with workers whose reservation wage is high. Freeing up workers by destructing such low-quality matches facilitates the creation of new more productive jobs.

There are also a few other important contributions on how to generate higher cyclical volatility in search and matching models. Examples are Hall (2005) and Gertler and Trigari (2009) that emphasize the role of wage rigidity and Pissarides (2009) that emphasizes the role of fixed matching costs. However, these studies focus on the volatility in the unemployment outflow rate, i.e. the job finding rate, and due to the positive externality from job destruction on job creation, mentioned above, they are better able to
explain the negative correlation between unemployment and vacancies if they restrict the unemployment inflow rate (i.e. the separation rate) to be constant. This paper develops a model in which sharp and short-lived movements in the separation rate contribute to unemployment fluctuations, in line with the evidence.\textsuperscript{7} In addition, in the model developed here, movements in the separation rate amplify the volatility of job creation. Hence, this paper contributes to this literature by showing a mechanism that can help generate the right share of variation in the unemployment rate due to its two margins: the inflow and the outflow rate.

The paper is organized as follows. Section I lays out the set up of the model under study and characterizes the steady-state equilibrium. Section II presents steady-state comparisons that characterize the response of key labour-market variables to aggregate productivity shocks. Section III presents some quantitative results. Section IV briefly discusses some of the model’s assumptions and Section V concludes.

\textbf{I \hspace{0.5em} The Model}

The model is in discrete time. The economy is populated by ex-ante heterogeneous risk-neutral workers of measure one and firms of a large measure. Workers differ in terms of their ability, which is measured by $x$. Ability is distributed according to the cumulative distribution function $F(\cdot)$ with support $X \equiv [x, \bar{x}]$ and associated density function $f(\cdot)$. In any period, a worker may be either employed or unemployed, while a firm may be either matched with a worker and producing or posting a vacancy. A type-$x$ worker produces $y_t p(x)$ units of output, where $y_t$ is a stochastic aggregate productivity component and $p(x)$ is a constant worker-specific productivity component that increases with the worker’s ability: $p'(x) > 0$. Unemployed workers receive a constant flow benefit $b$ per period. Firms that post a vacancy pay a constant cost $c$ per period. The number of vacancies is determined by free entry. Hence, firms open vacancies until the expected value of doing so becomes zero.

Ability is assumed to be observable to the firm, but only when the firm actually meets with the worker. Firms cannot learn about the workers’ abilities prior to meeting with them. For this reason, they cannot direct their search to workers of a particular ability level. There is therefore a single matching market with a meeting function determining the number of contacts.matches.\textsuperscript{8} More precisely, let $u_t(x)$ and $v_t$ denote the number of

\textsuperscript{7}See, e.g., Rogerson and Shimer (2011) and Barnichon (2010).
\textsuperscript{8}The idea is that observationally equivalent workers (i.e, workers with similar education and experi-
unemployed workers of type $x$ and posted vacancies, respectively, in period $t$. The total number of matches between searching workers and firms in period $t$ is determined by a matching function $M(v_t, u_t) = v_t^{1-\alpha} u_t^\alpha$, where $u_t = \int_0^x u_t(x) dx$ gives the total number of unemployed workers in period $t$.$^9$ The probability that a worker matches with a firm can be written as $m(\theta_t)$, where $\theta_t = \frac{v_t}{u_t}$ measures the tightness of the labour market. Likewise, a vacancy matches with a worker (of any type) with probability $q(\theta_t)$ and with a worker of type $x$ with probability $q(\theta_t) \frac{u_t(x)}{u_t}$.

Each period, before production takes place, matched workers and firms (including those in ongoing employment relationships) negotiate on a contract that divides the surplus of the match according to the Nash Bargaining solution. The worker’s bargaining weight is $\beta$ and the disagreement point is separation. Let $S_t(x)$ denote the surplus of a match between a firm and a worker of type $x$ in period $t$. A worker and a firm will choose to continue or begin an employment relationship only if $S_t(x) > 0$, and will agree to separate if $S_t(x) = 0$, in which case separation is jointly optimal. Since the surplus of an employment relationship is increasing in the productivity and therefore the ability of the worker, there will be a reservation productivity $p(R_t)$ and a reservation ability $R_t$, such that $S(R_t) = 0$. Hence, the worker and the firm will choose to continue or commence any employment relationship with $x > R_t$. Aside from the jointly optimal separations, known as endogenous separations, employment matches also face a risk of separating for exogenous reasons with a probability $s$.

The timing of events and decisions within a period is as follows. At the beginning of each period matches between unemployed workers and vacancies are realized. At the same time, a randomly selected fraction $s$ of ongoing employment relationships is destroyed for exogenous reasons. Subsequently, aggregate productivity, $y_t$, is realized. Upon observing $y_t$, workers and firms in surviving relationships bargain a new wage if there is still a surplus to share, i.e., if $x > R_t$. In the opposite case, they optimally separate. Likewise,$^9$With the term “match” I refer to a meeting between a searching worker and a firm with a vacancy. As I explain below, a meeting may or may not lead to the beginning of a new employment relationship, because an agreement may not be reached. I use the terms “employment relationship” and “employment match” to refer to the cases where an agreement has been reached and the pair has decided to start producing.
the newly matched workers decide whether or not to begin an employment relationship with the wage reflecting worker-firm bargaining. If, given the realization of aggregate productivity, the worker’s ability is sufficiently large, i.e., if \( x > R_t \), so that the surplus of the employment relationship is positive, then a new employment relationship begins. Otherwise, the firm and the worker continue searching. Finally, production takes place and unemployed workers and vacancies engage in search.

**Value Functions**

The unemployment value, \( U_t(x) \), and the value of a match, \( W_t(x) \), to a worker of ability \( x \) satisfy:

\[
U_t(x) = b + \gamma E_t [m(\theta_t)W_{t+1}(x) + (1 - m(\theta_t))U_{t+1}(x)]
\]

\[
W_t(x) = \max \{w_t(x) + \gamma E_t [sU_{t+1}(x) + (1 - s)W_{t+1}(x)], U_t(x)\}
\]

where \( E_t \) is the expectation operator, \( w_t(x) \) the wage rate and \( \gamma = \frac{1}{1+r} \) the discount factor.

The value of a vacancy is given by

\[
V_t = -c + \gamma E_t \left[ q(\theta_t) \int_x \frac{u_t(x)}{u_t(x)} J_{t+1}(x)dx + (1 - q(\theta_t))V_{t+1} \right]
\]

and the value of a match with a type-\( x \) worker to a firm is given by

\[
J_t(x) = \max \{y_t p(x) - w_t(x) + \gamma E_t [sV_{t+1} + (1 - s)J_{t+1}(x)], V_t\}
\]

In (1) the payoff in the current period for an unemployed worker is \( b \); with probability \( m(\theta_t) \) the worker matches with a vacancy yielding a value \( W_{t+1}(x) \) and in the opposite case the worker remains unmatched yielding a value \( U_{t+1} \). The first term in the bracket of equation (2) is the value of an employment relationship to a type-\( x \) worker. An employed type-\( x \) worker earns the wage \( w_t(x) \) and faces the risk of an exogenous separation that occurs with probability \( s \). If the relationship exogenously breaks up, the worker becomes unemployed yielding a value \( U_{t+1} \), but if the relationship survives, the continuation value is \( W_{t+1}(x) \). A worker will choose to stay in (or commence) an employment relationship only if the value of being in the employment relationship is greater than the value of being unemployed. Accordingly, the value of a match to the worker is the maximum between the two. Likewise, in (4), the value of a match with a type-\( x \) worker to the firm is the maximum between the value of being in an employment relationship with that worker and
the value of being vacant. If the firm chooses to commence (or stay in) an employment relationship with the worker, it produces output \( y_t p(x) \), pays the wage \( w_t(x) \) and faces the risk of an exogenous separation that occurs with probability \( s \). In (3), a firm with a vacancy incurs a cost \( c \) and matches with a type-\( x \) worker with probability \( q(\theta_t) \frac{m(x)}{u_t} \), yielding a value \( J_{t+1}(x) \). With probability \( 1 - q(\theta_t) \) the firm fails to match with a worker yielding a value \( V_{t+1} \).

The wage rate, \( w_t(x) \), satisfies the Nash conditions,
\[
W_t(x) - U_t(x) = \beta S_t(x)
\]
\[
J_t(x) - V_t = (1 - \beta)S_t(x)
\]
Moreover, in a free-entry equilibrium \( V_t = 0 \) holds for all \( t \). Using these conditions we can write the surplus of a match, when the worker’s ability is \( x \) as
\[
S_t(x) = \max \{ y_t p(x) - b + \gamma E_t S_{t+1}(x) [1 - s - \beta m(\theta_t)] , 0 \} \tag{5}
\]
and the value in (3) as
\[
\frac{c}{q(\theta_t)} = \gamma (1 - \beta) E_t \int_x^x u_t(x) S_{t+1}(x) dx \tag{6}
\]
The law of motion for the unemployment of a type-\( x \) worker is given by
\[
u_{t+1}(x) = u_t(x) + s [f(x) - u_t(x)] - u_t(x)m(\theta_t)I_t(x) + (1 - s) [f(x) - u_t(x)] [1 - I_t(x)] \tag{7}
\]
where \( I_t(x) \) is an indicator function, which takes the value of 1 if the worker’s ability is equal or above the reservation ability and 0 otherwise. The last term in the above expression captures discrete jumps from employment to unemployment due to endogenous separations.

Equations (5) to (7) determine the free-entry equilibrium path of \( \theta_t \) for given realizations of the aggregate productivity process.

**The steady-state equilibrium**

Here I characterize the properties of the non-stochastic steady state, where the the aggregate state, \( y \) and the distribution of unemployment across different types of workers are constant.

The steady-state surplus of a match when the worker’s type is \( x \) is given by
\[
S(x) = \max \{ \frac{y p(x) - b}{\gamma (r + s + \beta m(\theta))} , 0 \} \tag{8}
\]
It is evident from (8) that the steady-state reservation productivity satisfies:

\[ p(R) = \frac{b}{y} \quad (9) \]

The steady-state unemployment is given by

\[ u(x) = \begin{cases} \frac{sf(x)}{s+m(\theta)}, & \text{if } x > R \\ f(x), & \text{otherwise} \end{cases} \quad (10) \]

The overall unemployment rate, \( u = \int_{x}^{R} f(x)dx + \int_{x}^{\bar{x}} \frac{sf(x)}{s+m(\theta)}dx \), can be written as

\[ u = \frac{s + F(R)m(\theta)}{s + m(\theta)} \quad (11) \]

Employed workers with \( x > R \) face only the risk of an exogenous separation that occurs with probability \( s \). However, workers with \( x \leq R \) separate at rate 1, because even if they manage to survive the exogenous separation shock, they will separate endogenously. The overall separation rate, denoted by \( \tilde{s} \), is therefore given by

\[ \tilde{s} = \int_{x}^{R} f(x)dx + \int_{x}^{\bar{x}} sf(x)dx \]

and can be written as

\[ \tilde{s} = F(R)(1 - s) + s \quad (12) \]

It is clear from (9) that \( R \) is decreasing in \( y \), meaning \( F(R) \) is also decreasing in \( y \). The model therefore features countercyclical fluctuations in the separation rate: a reduction in \( y \) raises the reservation productivity leading to more endogenous separations.

The average job finding and job filling rates differ from the matching rates \( m(\theta) \) and \( q(\theta) \), respectively, because only matches with workers whose productivity is above the reservation productivity will continue as employment matches. In particular, the average job finding and filling rates can be calculated as

\[ \tilde{m} = \int_{R}^{\bar{x}} \frac{u(x)}{u} m(\theta) \]

\[ \tilde{q} = \int_{R}^{\bar{x}} \frac{u(x)}{u} q(\theta) \]

which give:

\[ \tilde{m} = \phi(R, \theta)(1 - F(R))m(\theta) \]

\[ \tilde{q} = \phi(R, \theta)(1 - F(R))q(\theta) \quad (13) \]

where

\[ \phi(R, \theta) = \frac{s}{s + F(R)m(\theta)} \quad (14) \]
The term \( \phi(R, \theta) \) measures the probability that an unemployed worker has ability \( x > R \).

The free-entry condition that determines the steady-state value of \( \theta \) is given by,

\[
\frac{c}{q(\theta)} = \gamma (1 - \beta) \int_R^\infty \frac{u(x)}{u} S(x) dx
\]  

(15)

With (8) and (10) substituted in, the free-entry condition can be written as

\[
\frac{c}{q(\theta) \phi(R, \theta)} = \frac{(1 - \beta) \int_R^\infty (yp(x) - b) f(x) dx}{\gamma (r + s + \beta m(\theta))}
\]  

(16)

The free-entry condition is such that the expected surplus from filling a vacancy equals the expected recruitment cost. If the expected surplus is higher than the expected recruitment cost (i.e., if the right-hand side of (16) is higher than its left-hand-side), firms open more vacancies per job seeker until all rents are exhausted.

The main difference between this model and other models that allow for endogenous separations is the presence of the term \( \phi(R, \theta) \) in the free-entry condition. A larger \( \phi(R, \theta) \) means that the number of employment relationships that are expected to be formed, conditional on the number of contacts per firm, is larger. In other words, a larger \( \phi(R, \theta) \) implies an improvement in matching efficiency, and therefore, a decline in the recruitment cost a firm expects to pay on average in order to fill a vacancy. Notice from (14) that \( \phi(R, \theta) \) is decreasing in \( R \), meaning that a rise in the reservation productivity (and thus ability) deteriorates matching efficiency and causes the expected recruitment cost to rise, with adverse effects on incentives to open vacancies. Intuitively, when firm profitability is lower and thus firms are more selective with the workers they are willing to hire, they are also more reluctant to open vacancies, because they anticipate that they will have more difficulty finding suitable workers to fill them. Hence, the model captures a new source of cyclical fluctuations in vacancies, that comes from the impact of changes in the reservation productivity on the expected recruitment cost. In this model, the rise in unemployment that occurs in a recession due to the rise job destruction, i.e., due to the rise in \( R \), lowers the vacancy to unemployment ratio without reinforcing the positive externality on vacancy creation. In contrast, the further increase in unemployment acts to further depress job creation, because the workers that enter unemployment due to endogenous separations are those whose productivity falls below the firms’ acceptance threshold. Such workers congest the market during downturns, making it more difficult for firms to locate workers whose productivity is above the acceptance threshold. These
workers will make it easier for firms to fill their vacancies only when economic conditions improve and the acceptance threshold falls again, as captured by the negative relation between $R$ and $\phi(R, \theta)$.\footnote{It is perhaps useful to clarify that this feature of the model is due to firms becoming more selective about the profitability of the jobs they choose to create during bad times; it is not due to endogenous separations being concentrated on low-ability workers. Even if one assumed that profits per worker are non-monotonic in ability, or even falling monotonically with ability, so that the workers laid off are not necessarily the least able, this feature would still be present. For instance, one could assume that higher ability workers generate lower profits to employers, say because they have a much better outside option and so need to paid much more. If this was the case, the workers laid off in a recession would be the most able. Nevertheless, the congestion effects mentioned here would still be present, because the most able workers in this case would be those that generate smaller profits, making employers reluctant to hire them during downturns.}

For the results below, it is also useful to characterize the replacement ratio. The replacement ratio in the model is $\tilde{b} = \frac{b}{yy}$, where $\tilde{p} = \frac{\int_R^\infty p(x) dF(x)}{1-F(R)}$ is the average worker-specific productivity among the employed.

\section{Steady-State Comparisons}

Next, I derive results that describe how the key labour-market variables in the model respond to changes in aggregate productivity.\footnote{I follow a common practice and use comparative static results to characterize the cyclical response of the model. Examples of studies that follow the same approach are Mortensen and Nagypály (2007a,b) and Pissarides (2009). The steady-state comparisons help make the mechanism that this paper emphasizes more transparent to the reader. Appendix A aims to address the question of how good an approximation to the dynamic responses are the steady-state comparisons by reporting results on the dynamic responses in a simplified version of the dynamic stochastic model outlines in Section I. For a discussion of how good an approximation to the dynamic responses are the comparative static results, see also Mortensen and Nagypály (2007b).}

By taking logs of (11) and differentiating the result with respect to $\ln y$ we obtain the following expression for the elasticity of the unemployment rate with respect to aggregate productivity:

$$\frac{\partial \ln u}{\partial \ln y} = -\frac{(1-\alpha)m(\theta)}{s + m(\theta)} \left[ \frac{s(1-F(R))}{s + F(R)m(\theta)} \frac{\partial \ln \theta}{\partial \ln y} + \frac{m(\theta)f(R)R}{s + F(R)m(\theta)} \frac{\partial \ln R}{\partial \ln y} \right]$$

(17)

where it may be recalled that $\alpha$ denotes the elasticity of the matching function with respect to the unemployment rate. The first term captures the effect of changes in aggregate productivity on the unemployment rate through the impact of such changes on market tightness. The second term captures the impact of changes in the reservation ability. Clearly, the negative response of the reservation ability amplifies the negative response of
unemployment to aggregate productivity shocks. Thus, this model can generate a larger volatility of unemployment than the model with a constant separation rate (canonical model, henceforth), analyzed in Shimer (2005). However, if the model fails to also generate sufficiently larger volatility in tightness than the canonical model, then it will fail to generate a realistic Beverage curve, because

\[
\frac{\partial \ln v}{\partial \ln y} = \frac{\partial \ln \theta}{\partial \ln y} + \frac{\partial \ln u}{\partial \ln y} \quad (18)
\]

If the positive elasticity of \(\theta\) with respect to aggregate productivity is not much larger, while the negative elasticity of unemployment is much larger than in the canonical model, then the resulting elasticity of vacancies will be small or even negative; equivalently, if the elasticity of \(\theta\) with respect to aggregate productivity is not sufficiently larger than that of the canonical model, the model will generate a very small or even positive covariance between unemployment and vacancies.

Substituting (17) into (18) yields

\[
\frac{\partial \ln v}{\partial \ln y} = \left[ 1 - \frac{(1 - \alpha)m(\theta)}{s + m(\theta)} \frac{s(1 - F(R))}{s + F(R)m(\theta)} \right] \frac{\partial \ln \theta}{\partial \ln y} + \frac{m(\theta)f(R)R}{s + F(R)m(\theta)} \frac{\partial \ln R}{\partial \ln y} \quad (19)
\]

Because the term in the bracket is positive, a larger response in market tightness implies a larger (positive) response in the vacancy rate. But the negative response of the reservation ability dampens the response of the vacancy rate, as captured by the second term in the above expression. This means that the model can explain jointly the cyclical behavior of unemployment and vacancies, only if in addition to the larger volatility in unemployment, it generates a sufficiently larger volatility in market tightness than the canonical model.

The most widely used model of endogenous separations, due to Mortensen and Pissarides (1994), predicts a counter-cyclical vacancy rate, because it delivers significantly larger volatility in unemployment, but no more volatility in market tightness than the canonical model. The elasticity of tightness in the canonical model with only a constant separation rate \(s\) and homogeneous workers that produce \(p\) is given by

\[
\epsilon_{\theta,y} = \left[ r + s + \beta m(\theta) \right] \left[ \frac{1}{\alpha(r + s) + \beta m(\theta)} \right] \quad (20)
\]

where \(\tilde{b} = \frac{b}{yp}\) gives the replacement ratio. As shown in Mortensen and Nagypál (2007b), the elasticity of market tightness in the Mortensen and Pissarides model (the MP model,
henceforth) is observationally equivalent to that in the canonical model, given in (20). Thus, when both models are calibrated in the same way, i.e., given equal replacement ratios, average job finding and separation rates, and parameter values for $\alpha, r$ and $\beta$, they yield identical elasticities of market tightness. As mentioned in the introduction, the reason behind this result is the endogenous response of vacancies to the fall in tightness, caused by the rise in unemployment.

The current model has the potential to deliver more volatility in market tightness, because, as explained above, it captures a new mechanism that can help mitigate the positive externality on vacancy creation from the fall in tightness. By taking logs of the free-entry condition in (16) and differentiating the result with respect to $\ln y$ we obtain the following expression for the elasticity of market tightness in the current model:

$$\frac{\partial \ln \theta}{\partial \ln y} = \left[ \frac{r + s + \beta m(\theta)}{\alpha(r + s) + \beta m(\theta)} \right] \left[ \frac{\partial \ln \phi(R, \theta)}{\partial \ln y} + \frac{1}{1 - \tilde{b}} \right]$$

(21)

Notice that the term $\frac{\partial \ln \phi(R, \theta)}{\partial \ln y}$ that reflects the impact of changes in aggregate productivity on matching efficiency enters with a positive sign in (21) but is absent from (20). Consequently, if this term is positive, the elasticity of tightness with respect to aggregate productivity in the present model can be higher than that in the MP model.

By taking logs of (14) and differentiating with respect to $\ln y$ we obtain:

$$\frac{\partial \ln \phi(R, \theta)}{\partial \ln y} = -\frac{(1 - \alpha)F(R)m(\theta)}{s + F(R)m(\theta)} \frac{\partial \ln \theta}{\partial \ln y} - \frac{m(\theta)f(R)R}{s + F(R)m(\theta)} \frac{\partial \ln R}{\partial \ln y}$$

(22)

The first term reflects the impact of changes in market tightness: an increase in $\theta$, and as a consequence, an increase in the workers’ matching rate, $m(\theta)$, deteriorates matching efficiency, because it lowers the share of workers with $x > R$ in the unemployment pool. The second term captures the impact of changes in the reservation ability: an increase in $R$ implies a lower share of workers with $x > R$ in the unemployment pool and therefore a deterioration of matching efficiency. While the effect of changes in market tightness is negative on the elasticity of $\phi(R, \theta)$, the effect of changes in the reservation ability is positive, because the response of the reservation ability is countercyclical. As shown in Section III, for realistic parameter values the effect of changes in reservation ability dominates that of changes in market tightness so that $\frac{\partial \phi(R, \theta)}{\partial y} > 0$.\(^{12}\)

\(^{12}\)By combining equations (21) and (22) and using the expression for the average job separation rate (equation (12)), the expression for the average job filling rate (equation (13)) and the expression for the elasticity of the separation with respect to aggregate productivity (equation (23), below), we can write
The elasticity of the job finding rate, $\hat{m}$, with respect to aggregate productivity can be expressed as:

$$\frac{\partial \ln \hat{m}}{\partial \ln y} = \left[ \frac{s(1 - \alpha)}{s + F(R)m(\theta)} \right] \frac{\partial \ln \theta}{\partial \ln y} - \frac{f(R)R}{1 - F(R)} \left[ \frac{s + m(\theta)}{s + F(R)m(\theta)} \right] \frac{\partial \ln R}{\partial \ln y} \tag{23}$$

The job finding rate responds to changes in aggregate productivity due to the impact of such changes on both the market tightness (first term) and the reservation ability (second term).

The separation rate responds to aggregate productivity shocks due to the impact of such changes on the reservation ability. Specifically,

$$\frac{\partial \ln \hat{s}}{\partial \ln y} = \frac{f(R)R(1 - s)}{\hat{s}} \frac{\partial \ln R}{\partial \ln y} \tag{24}$$

Finally, by taking logs of (9) and differentiating with respect to $\ln y$, we can write the elasticity of the reservation ability with respect to aggregate productivity as:

$$\frac{\partial \ln R}{\partial \ln y} = -\frac{1}{\epsilon_p(R)} \tag{25}$$

where $\epsilon_p(R)$ denotes the elasticity of the productivity function $p(x)$ with respect to $x$, evaluated at $x = R$.

It is also worth mentioning here that an additional channel through which the model can generate a larger change in the vacancy rate relative to that in labour productivity is the divergence between aggregate and labour productivity, which is a common feature of models that allow for endogenous separations. Because the reservation productivity moves countercyclically, the average worker-specific productivity among the employed workers, $\tilde{p}$ is lower at higher $y$. For this reason, a percentage increase in the aggregate component of productivity, $y\tilde{p}$, translates into a smaller percentage increase in average labour productivity, $y\tilde{p}$. Specifically,

$$\frac{\partial \ln y\tilde{p}}{\partial \ln y} = 1 + \frac{f(R)R}{1 - F(R)} \frac{\partial \ln R}{\partial \ln y} \tag{26}$$

the elasticity of matching efficiency as:

$$\frac{\partial \ln \phi(R, \theta)}{\partial \ln y} = \frac{-(1 - \alpha)(\hat{s} - s)\hat{m}\epsilon_{\theta,y} - \hat{m}\hat{s} \frac{\partial \ln \hat{s}}{\partial \ln y}}{(\hat{s} - s)(1 - s + (1 - \alpha)\hat{m}\epsilon_{\theta,y})}$$

It follows from this expression that $\frac{\partial \ln \phi(R, \theta)}{\partial \ln y} > 0$ as long as $-\frac{\partial \ln \hat{s}}{\partial \ln y} > (1 - \alpha)(1 - \frac{\epsilon_{\theta,y}}{2})\hat{m}\epsilon_{\theta,y}$. 

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Apparently, $\frac{\partial \ln y^p}{\partial \ln y}$ is less than one, because $\frac{\partial \ln R}{\partial \ln y}$ is negative. When confronting the model with the data, the appropriate measure of the changes in a variable, say $z$, relative to the changes in labour productivity is given by

$$\frac{\Delta_y \ln z}{\Delta_y \ln y^p} \equiv \frac{\partial \ln z / \partial \ln y}{\partial \ln y^p / \partial \ln y}$$

(27)

This means that the change in variable $z$ relative to that in labour productivity is larger in this model than that in the canonical model. Moreover, since both models are calibrated to match the empirical volatility of the average productivity of labour, $y$-shocks are larger in this model than in the canonical model. In turn, larger $y$-shocks generate larger fluctuations in the key labour market variables.

## III Quantitative Results

Next, I present some quantitative results of the model. In my baseline calculations I use the same parameter values and targets used by Shimer (2005), who reports the results of the canonical model. Hence, aggregate productivity is normalized to $y = 1$ and the quarterly discount rate is $r = 0.012$. I set the elasticity parameter to $\alpha = 0.72$, let worker’s bargaining power take the same value, $\beta = 0.72$ and set the replacement ratio to 0.40. Finally, I target an average separation rate of 0.10 and an average job finding rate of 1.355.

With the above calibration approach we can obtain the model-implied elasticities of the job finding rate, tightness, vacancies and unemployment, for a given endogenous fraction $F(R)$ and elasticity of the separation rate with respect to aggregate productivity. In order to derive the fraction $F(R)$ and the separations elasticity we need information about the distribution of productivity across employment matches. Since the exact shape of this distribution matters only for the volatility of separations, I choose not to impose a particular shape for this distribution. Instead, I set the separations elasticity equal to its empirical counterpart, which based on Table 1 in Shimer (2005) equals $-1.97$ and derive results for different values of $F(R)$. This enables me to examine whether the model can...
generate realistic fluctuations in both unemployment and vacancies for reasonable amount of variation in job separations, which is the central issue here.

I use the elasticities derived in Mortensen and Nagypál (2007b) to also compute results for the MP model, using the same calibration approach. I set equal replacement ratios, separation rates, job finding rates and separations elasticities and let the parameters $y, r, \alpha$ and $\beta$ take the same values in both models.\textsuperscript{14} This implies that any differences in the predicted volatilities of tightness, vacancies and unemployment found between the two models must come from the cyclical changes in matching efficiency that are present in the current model, but absent from the MP model. Comparing the results of the two models, derived with this calibration, helps quantify the role of worker heterogeneity in amplifying the volatility of job creation.

Table 1 reports the model-implied elasticities of the key labour-market variables both with respect to aggregate and labour productivity. I use the notation $\epsilon_{i,j}$ to denote the elasticity of the variable $i$ with respect to variable $j$. The table also reports the results of the MP model (in parentheses), the results of the canonical model and the relevant empirical responses (labeled as data) based on Table 1 in Shimer (2005).\textsuperscript{15} The model has no trouble generating a large enough (negative) change in the unemployment rate relative to changes in labour productivity and generates significantly larger volatility in tightness than both the canonical and the MP model. The response of tightness to aggregate productivity shocks in the MP model is the same as in the canonical model, while in the current model it is much larger. Hence, endogenous job destruction does not contribute to more volatility in tightness in the MP model, but has a significant impact on the volatility of tightness in the current model. The current model generates realistic fluctuations in unemployment, and at the same time, predicts a procyclical vacancy rate.

The model with worker heterogeneity clearly outperforms the MP model, but still, for the selected parameter values, it cannot explain the magnitude of variation in tightness, and as a consequence, in the vacancy rate we observe in the data. This may be due to Shimer’s replacement ratio of 0.4, being too low. From equations (20) and (21) it is evident that a higher replacement ratio in our calibrations implies a larger elasticity of market tightness. This is because a higher replacement ratio reduces the firm’s profits

\textsuperscript{14}In the current model, $F(R)$ measures the probability that a worker is unemployable, while in the MP model the probability that a match-specific productivity draw is below the reservation productivity. In deriving the results below I let $F(R)$ take the same value in both models. Moreover, it is reasonable to assume that $F(R)$ is small. I therefore choose small values for this fraction.

\textsuperscript{15}As in Mortensen and Nagypál (2007b), the empirical equivalent to the change in variable $i$ relative to the change in variable $j$ (denoted by $\epsilon_{i,j}$) is the OLS (ordinary least squares) coefficient $\rho_{ij} \frac{\sigma_i}{\sigma_j}$, where $\rho_{ij}$ is the correlation between ln $i$ and ln $j$ and $\sigma_i$ is the standard deviation of ln $i$. 

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Table 1: Model results at $\tilde{b} = 0.4$

<table>
<thead>
<tr>
<th>$F(R)$</th>
<th>$\epsilon_{\theta,y}$</th>
<th>$\epsilon_{v,y}$</th>
<th>$\epsilon_{u,y}$</th>
<th>$\epsilon_{m,y}$</th>
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<td>-3.58</td>
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<td>4.26</td>
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<table>
<thead>
<tr>
<th>$F(R)$</th>
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</table>

| data   | 7.56                  | 3.68             | -3.88            | 2.34             |
| canonical | 1.72                  | 1.27             | -0.45            | 0.48             |

so that cyclical shocks have a bigger proportional impact on profits, and thus vacancy creation. Hagedorn and Manovskii (2008) agree that Shimer’s replacement ratio is too low, because it does not include the value of leisure or home production, but they suggest a replacement ratio of 0.955, which seems implausibly large. Hall and Milgrom (2008) improve on this by estimating the value of additional leisure using evidence on the Frisch elasticity of labour supply. Their suggested replacement ratio, which includes both unemployment insurance and the value of leisure, is 0.71; a value that is commonly used in recent studies.16 As shown in Table 2, setting the replacement ratio equal to the value suggested by Hall and Milgrom improves the results considerably. The elasticities of market tightness and vacancies obtained from the present model are now very close to their empirical equivalents, while those obtained from the MP model are still much smaller than in the data.

A natural question that arises is how good an approximation to the dynamic responses are the comparative static results presented above. In Appendix A, I study this issue by deriving numerical results for the dynamic model specified in Section I. To reduce the

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16See, for instance, Pissarides (2009) and Brugemann and Moscarini (2010).
Table 2: Model results at $\tilde{b} = 0.71$

<table>
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<tr>
<th>$F(R)$</th>
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<td>(3.55 0.83 -2.72 1.05)</td>
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<table>
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<tr>
<th>$F(R)$</th>
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<th>$\epsilon_{\tilde{m},y}$</th>
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<td>7.32</td>
<td>2.43</td>
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</table>

| data   | 7.56            | 3.68            | -3.88                   | 2.34            |
| canonical | 3.55            | 2.63            | -0.93                   | 0.99            |

computational burden and increase the accuracy of the numerical solution, I assume that there are only two types of workers: the marginal workers, whose productivity is close to the threshold for separation and the more productive workers.\textsuperscript{17} The results of this exercise confirm the comparative static results presented above. The model with worker heterogeneity greatly amplifies the volatility of unemployment without violating the negative correlation between unemployment and vacancies, because it generates substantially more volatility in tightness than the MP model. The model correlation between vacancies and unemployment in the model with two worker types of workers is found to be -0.7.

\textsuperscript{17}Conventional numerical methods require that the model state variables are discretized even if they are inherently continuous. Due to storage requirements and computational complexity expanding the dimensions of state space by adding more worker types restricts the number of grid points that can be used considerably. However, the number of grid points required to accurately characterize the cyclical response of the state variables is large. The computational burden and the loss in terms of accuracy from adding more worker types is therefore large.
IV Discussion

The approach taken in this paper has been to keep the model simple in order to make the role of worker heterogeneity in generating cyclical fluctuations in matching efficiency in a model with endogenous separations more transparent. Match productivity has therefore been assumed to depend only on the worker’s ability, i.e. there is no job heterogeneity in the current model. Due to this assumption, some of the workers laid off in a recession will manage to find jobs only when aggregate economic conditions improve. In the current setting, these workers stay active, because aggregate productivity is stochastic and thus their chances of getting hired might improve by the time they find a match. Such workers may also stay attached because they want to be entitled to unemployment benefits or because they have limited information about how well their attributes match with the demands of available jobs. One of the most important reasons employers interview their applicants is to learn about their inherent abilities and other characteristics that cannot be identified prior to meeting with them. Likewise, workers may want to meet with potential employers and obtain more information about their demands before they can assess whether they would be employable in available jobs or not. Hence, it may be the case that workers cannot determine whether they are employable in available jobs unless they search for them.  

Of course, in a model with both worker and job heterogeneity such marginal workers would have more incentive to search during bad times. Suppose that, as in the MP model, jobs are subject to idiosyncratic productivity shocks so that jobs with bad productivity realizations are destroyed. In such a setting the worker-specific reservation productivity (or ability) would be lower in jobs whose job-specific productivity component is higher. Thus, the possibility of a job-specific productivity draw that is high enough to bring the overall productivity of their match above the acceptance threshold would give an additional incentive for marginal workers to stay attached. Even if jobs are subject to shocks that cause job destruction the mechanism emphasized here will still be present as

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18 This feature of the model - that workers stay active even if their employment prospects are dim - is consistent with a number of empirical evidence. For instance, Shimer (2004) shows that not only unemployment did not decline, but also the search intensity of attached workers increased by 27% during the 2002-2003 recession. The explanation he proposes is that workers whose probability of success is low may have a higher incentive to search for a job more intensely. Other examples are Barnichon and Figure (2011a), who show that attainment to the labour force is countercyclical, with workers more likely to join/stay in the labour force during recessions, Shimer (2007) who finds that a decrease in the unemployment to inactivity transition rate tends to rise unemployment during downturns and Elsby, Michaels and Solon (2009) who find that the inflow from inactivity to unemployment is comparatively acyclical, suggesting that the alternative setting, in which marginal workers exit the labour force during downturns and re-enter in booms may be inconsistent with evidence.
long as jobs matched with good workers are more likely to survive a bad shock compared to jobs matched with bad workers. If this is the case then separations will be concentrated on workers who - especially during bad times - can only match with very productive jobs (i.e. jobs that received a very high job-specific productivity draw). The current setting can be viewed as the limiting case of this generalized set up, where both worker- and job-specific heterogeneity are present. To understand why, suppose that during periods of low aggregate productivity some marginal workers cannot be hired in new jobs unless their draw of job-specific productivity turns out to be exceptionally high. As the probability of such an exceptionally high productivity draw approaches zero, the labour market volatility in this generalized setting approaches the one in the simpler setting developed here.

Appendix B examines how sensitive the model’s predictions are to the introduction of job-specific productivity shocks. It studies the dynamic response in a model where match productivity reflects both a job- and a worker-specific component and jobs are subject to idiosyncratic productivity shocks that can cause job destruction. As above, to reduce the computational burden and increase the accuracy of the numerical solution it assumes that there are only two types of workers and only three types of job-specific productivity realizations that occur with equal probabilities (for a more detailed description of the model see the Appendix). The calibration of the model is such that the least productive workers are always hired in jobs with a high job-specific productivity component, meaning that they have at least a 1/3 chance of being hired, even if aggregate productivity is still low. In other words, the workers the enter unemployment due to endogenous separations are employable in at least one third of the available jobs. As shown in the Appendix, this version of the model is also capable of increasing the volatility of unemployment through endogenous separations without violating the Beveridge curve.

V  Conclusion

This paper showns that introducing worker heterogeneity in a relatively standard search and matching model can help in amplifying the responsiveness of unemployment to productivity without violating the Beveridge curve correlation. An interesting property of the model is that it reconciles endogenous separations with the Beveridge curve without introducing complex features relative to the most widely used model of endogenous separations, due to Mortensen and Pissarides (1994). The only difference is that this model allows for match productivity to depend on workers’ ability, which seems to be a natural
assumption, while in the Mortensen and Pissarides model it is randomly drawn. The inter-
action between worker heterogeneity and an endogenous match acceptance/continuation
decision generates cyclical changes in matching efficiency that help in amplifying the re-
sponse of job creation to productivity shocks. Specifically, as the firms’ threshold for
hiring or retaining a worker becomes tighter in a recession, job destruction rises and un-
employment is increased but, at the same time, matching efficiency falls. The fall in
matching efficiency offsets the positive externality from the fall in tightness on vacancy
creation and acts to further depress vacancy creation in a recession.

This mechanism provides a solution to the standard model’s failure to reconcile en-
dogenous separations with the Beveridge curve. However, incorporating this mechanism
into more generalized settings can also shed light on some other important questions that
remain open. One such question is whether search theoretic models of the labour market
can explain the wedge between the marginal rate of substitution of consumption for leisure
and the marginal product of labour, often called the “labour wedge.” As Rogerson and
Shimer (2011) argue, in a model with search frictions the labour wedge is positively corre-
lated with employment. However, the opposite holds in the data. This is because search
frictions act as an adjustment cost that dampens fluctuations in employment. Specifically,
increasing the vacancy to unemployment ratio in response to a positive productivity shock
is costly, because doing so reduces the probability for each vacancy-posting firm to match
with a worker. As this paper has shown, the negative externality from the rise in the va-
cancy to unemployment ratio on the matching probability of firms can be mitigated when
there is worker heterogeneity in the model. Moreover, as firms become more selective
about filling vacancies during bad times and less selective during booms, some workers
are constrained from working as much as they would like to in a recession and vice-versa
in a boom. These provide potential explanations to the counter-cyclical labour wedge.
Further investigation along these lines might give new insights into the cyclical behavior
of the labour wedge in the presence of search frictions.
Appendices

A Responses in the dynamic system

The purpose of this Appendix is to illustrate how good an approximation to the dynamic responses are the comparative static results presented above. To this end, I calibrate and numerically solve the dynamic model outlined in Section I. As discussed in the text, to reduce the computational burden and increase the accuracy of the numerical solution I turn to a simplified version of the model with only two types of workers: the low productivity workers (i.e. the marginal workers, whose productivity is near the threshold for separation) and the high productivity workers. The rest of the model assumptions are as stated in the text.

To facilitate direct comparison of the quantitative results presented below and the comparative static results presented above, the calibration follows the calibration in the text as closely as possible. As above, I set \( \alpha = \beta = 0.72 \), keep the fraction of marginal workers small to 3\% and set the replacement ratio to 0.71. The model period is set to one month so that the discount rate is \( r = 0.004 \). I target an average monthly separation rate of 0.0345, which implies a quarterly separation rate of 0.10, and a monthly job finding rate of 0.45, which implies a quarterly job finding rate of 1.355. The productivity of the more productive workers is set to 1.01 and that of marginal workers to 0.72, so that the average productivity in the model is equal to 1 and the standard deviation of separations matches the one in the data, which based on Table 1 in Shimer (2005) is equal to 0.075.

The aggregate productivity component is assumed to follow a mean-zero discrete state Markov process, with 20 states. The vector of states and the transition matrix are chosen so that aggregate productivity approximates an AR(1) process with mean zero. As in Shimer (2005) the autocorrelation and the standard deviation of the process are chosen to match (after aggregating to a quarterly frequency) the autocorrelation (0.09) and the standard deviation (0.02) of quarterly US data on real average output per worker in the non-farm business sector.

As in the text, for comparability I also derive results for the dynamic responses in the MP model using the same calibration approach. I assume that idiosyncratic productivity can take two values, high and low, and that the low productivity realization occurs with a 3\% probability. The values of the low and high productivity realizations are such that the model matches the standard deviation of separations and yields an average productivity
equal to 1.

The results are summarized in Table A.1. To create these statistics 1000 samples of 736 observations were simulated (details of the computational algorithm are provided in Appendix C). The first 100 observations of each sample were discarded to eliminate sensitivity to initial conditions. For each sample, the remaining 636 observations were aggregated up to quarterly averages, yielding 212 quarters of “data” per sample, corresponding to the data from 1951 to 2003 used in Shimer (2005). Following Shimer (2005), the natural logs of quarterly data were detrended using a Hodrick-Prescott filter with smoothing parameter $10^5$. The table presents the mean of the samples’ correlations, standard deviations and elasticities with respect to average labour productivity. For comparability, the table also reports the corresponding empirical statistics (from Table 1 in Shimer, 2005). The model-generated elasticities are computed in the same way as the empirical elasticities. Specifically, the elasticity of variable $i$ with respect to variable $j$ is the OLS coefficient $\rho_{ij} \frac{\sigma_i}{\sigma_j}$, where $\sigma_j$ is the standard deviation of $\ln j$ and $\rho_{ij}$ the correlation between $\ln i$ and $\ln j$.

With only two types of workers there is a limit as to how much negative correlation between separations and aggregate productivity the model can generate and as to how large the fluctuations in unemployment, and in turn, in matching efficiency can be. For this reason, in this version of the model the standard deviation of tightness is smaller than in the data, but still much larger that in the MP model. Moreover, in the model with heterogeneity all correlations have the correct signs, and the correlation between unemployment and vacancies is negative (-0.7), while the MP model generates a counterfactually positive correlation between separations and vacancies (0.5) and between unemployment and vacancies (0.6).

### B A model with idiosyncratic shocks

This appendix examines whether the results discussed above carry through with the assumption that jobs are subject to productivity shocks that cause job destruction. I first describe a version of the model with idiosyncratic productivity shocks. I then describe the calibration procedure and present the results from simulations of the model.

Let $p_{ij}$ denote the productivity of a match between a worker of type $i$ and a job of type $j$. The worker’s type reflects his inherent abilities and is assumed to be constant over time, while each period jobs are subject to idiosyncratic productivity shocks. The
Table A.1: Responses in the dynamic system

<table>
<thead>
<tr>
<th>CURRENT MODEL</th>
<th>( \theta )</th>
<th>( v )</th>
<th>( u )</th>
<th>( \tilde{m} )</th>
<th>( \tilde{s} )</th>
<th>( y\tilde{p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity</td>
<td>7.33</td>
<td>2.61</td>
<td>-4.80</td>
<td>5.00</td>
<td>-1.19</td>
<td></td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.197</td>
<td>0.060</td>
<td>0.152</td>
<td>0.151</td>
<td>0.075</td>
<td>0.020</td>
</tr>
<tr>
<td>Correlation Matrix</td>
<td>( \theta )</td>
<td>1</td>
<td>0.828</td>
<td>-0.975</td>
<td>0.940</td>
<td>-0.233</td>
</tr>
<tr>
<td></td>
<td>( v )</td>
<td>1</td>
<td>-0.694</td>
<td>0.787</td>
<td>-0.419</td>
<td>0.923</td>
</tr>
<tr>
<td></td>
<td>( u )</td>
<td>-1</td>
<td>1</td>
<td>-0.909</td>
<td>0.175</td>
<td>-0.668</td>
</tr>
<tr>
<td></td>
<td>( \tilde{m} )</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>-0.169</td>
<td>0.707</td>
</tr>
<tr>
<td></td>
<td>( \tilde{s} )</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>-0.339</td>
</tr>
<tr>
<td></td>
<td>( y\tilde{p} )</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MP MODEL</th>
<th>( \theta )</th>
<th>( v )</th>
<th>( u )</th>
<th>( \tilde{m} )</th>
<th>( \tilde{s} )</th>
<th>( y\tilde{p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity</td>
<td>2.96</td>
<td>0.98</td>
<td>-2.18</td>
<td>0.91</td>
<td>-1.64</td>
<td></td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.062</td>
<td>0.071</td>
<td>0.081</td>
<td>0.020</td>
<td>0.075</td>
<td>0.021</td>
</tr>
<tr>
<td>Correlation Matrix</td>
<td>( \theta )</td>
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<td>0.277</td>
<td>-0.577</td>
<td>0.984</td>
<td>-0.425</td>
</tr>
<tr>
<td></td>
<td>( v )</td>
<td>1</td>
<td>0.574</td>
<td>0.163</td>
<td>0.460</td>
<td>0.289</td>
</tr>
<tr>
<td></td>
<td>( u )</td>
<td>-1</td>
<td>1</td>
<td>-0.664</td>
<td>0.732</td>
<td>-0.566</td>
</tr>
<tr>
<td></td>
<td>( \tilde{m} )</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>-0.568</td>
<td>0.981</td>
</tr>
<tr>
<td></td>
<td>( \tilde{s} )</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>-0.411</td>
</tr>
<tr>
<td></td>
<td>( y\tilde{p} )</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DATA</th>
<th>( \theta )</th>
<th>( v )</th>
<th>( u )</th>
<th>( \tilde{m} )</th>
<th>( \tilde{s} )</th>
<th>( y\tilde{p} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity</td>
<td>7.56</td>
<td>3.68</td>
<td>-3.88</td>
<td>2.34</td>
<td>-1.97</td>
<td></td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.382</td>
<td>0.202</td>
<td>0.190</td>
<td>0.118</td>
<td>0.075</td>
<td>0.020</td>
</tr>
<tr>
<td>Correlation Matrix</td>
<td>( \theta )</td>
<td>1</td>
<td>0.975</td>
<td>-0.971</td>
<td>0.948</td>
<td>-0.715</td>
</tr>
<tr>
<td></td>
<td>( v )</td>
<td>1</td>
<td>-0.894</td>
<td>0.897</td>
<td>-0.684</td>
<td>0.364</td>
</tr>
<tr>
<td></td>
<td>( u )</td>
<td>-1</td>
<td>1</td>
<td>-0.949</td>
<td>0.709</td>
<td>-0.408</td>
</tr>
<tr>
<td></td>
<td>( \tilde{m} )</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>-0.574</td>
<td>0.396</td>
</tr>
<tr>
<td></td>
<td>( \tilde{s} )</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>-0.524</td>
</tr>
<tr>
<td></td>
<td>( y\tilde{p} )</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

The probability that the next period’s job-specific productivity realization will be \( j \) is given by \( \pi_j \) and \( \sum_j \pi_j = 1 \). The rest of the notation and model assumptions are as stated in the text.
The unemployment value of a type-i worker, \( U^i_t \), and the value of a match between a type-j job and type-i worker, \( W^{ij}_t \), satisfy:

\[
U^i_t = b + \gamma E_t \left[ m(\theta_t) \sum_j \pi_j W^{ij}_{t+1} + (1 - m(\theta_t))U^i_{t+1} \right] \tag{A-1}
\]

\[
W^{ij}_t = \max\{w^{ij}_t + \gamma E_t \left[ sU^i_{t+1} + (1 - s) \sum_j \pi_j W^{ij}_{t+1} \right], U^i_t\} \tag{A-2}
\]

where \( w^{ij}_t \) the wage rate of a type-i worker in a type-j job. The value of a vacancy is given by

\[
V_t = -c + \gamma E_t \left[ q(\theta_t) \sum_i u^{i}_t \sum_j \pi_j J^{ij}_{t+1} + (1 - q(\theta_t))V_{t+1} \right] \tag{A-3}
\]

and the value of a match between a type-i worker and a type-j job is given by

\[
J^{ij}_t = \max\{y_t p_{ij} - w^{ij}_t + \gamma E_t \left[ sV_{t+1} + (1 - s) \sum_j \pi_j J^{ij}_{t+1} \right], V_t\} \tag{A-4}
\]

The wage rate, \( w^{ij}_t \), satisfies the Nash conditions, \( W^{ij}_t - U^i_t = \beta S^{ij}_t \) and \( J^{ij}_t - V_t = (1 - \beta) S^{ij}_t \). Using the Nash conditions together with the free-entry conditions, \( V_t = 0 \) for all \( t \), we can write the surplus of a match between a type-j job and a type-i worker as

\[
S^{ij}_t = \max\{y_t p_{ij} - b + \gamma E_t \sum_j \pi_j S^{ij}_{t+1} \left[ 1 - s - \beta m(\theta_t) \right], 0\} \tag{A-5}
\]

and the free-entry condition as

\[
\frac{c}{q(\theta_t)} = \gamma (1 - \beta) E_t \sum_i u^{i}_t \sum_j \pi_j S^{ij}_{t+1} \tag{A-6}
\]

The law of motion for the unemployment of type-i workers is given by

\[
u^i_{t+1} = u^i_t + s \left( f_i - u^i_t \right) - u^i_t m(\theta_t) \sum_j \pi_j \Gamma^{ij}_t + \left( 1 - s \right) \left( f_i - u_t(x) \right) \sum_j \pi_j \left( 1 - \Gamma^{ij}_t \right) \tag{A-7}
\]

where \( f_i \) denotes the share of type-i workers in the labour force and \( \Gamma^{ij}_t \) is an indicator function, which takes the value of 1 if the surplus of a match between a type-i worker
and a type- \( j \) is zero and the value of 0 otherwise. Equations (A-5) to (A-7) determine the free-entry equilibrium path of \( \theta_t \) for given realizations of the aggregate productivity process.

In order to numerically solve the model I assume that \( p_{ij} = x_i(1 + m_j) \), where \( x_i \) represents the worker’s ability (or productivity) and \( m_j \) the job-specific productivity component. As mentioned above, I assume that there are only two worker types: the high-productivity workers, whose ability is denoted by \( x_h \) and the low-productivity workers whose ability is denoted by \( x_l \). I further assume that the job-specific productivity component can take only three values, high \( (m_h) \), medium \( (m_m) \) and low \( (m_l) \), and that the three job-specific productivity realizations occur with equal probabilities so that \( \pi = \pi_j = 1/3 \). Moreover, without loss of generality, I set \( m_l = 0 \). I set the rest of the productivity parameters to \( x_h = 0.970, x_l = 0.694, m_h = 0.100 \) and \( m_m = 0.022 \) so that, 1) the model-implied standard deviation of separations matches the empirical one \((0.075)\), 2) the average productivity in the model equals 1, 3) the surplus of a match between a high-productivity worker and a low-productivity job is always positive, 4) the surplus of a match between a low-productivity worker and a high-productivity job is always positive. The third condition rules out cases where no matches are profitable. The forth condition rules out the possibility that low-productivity workers cannot be hired in any job and ultimately implies that they have at least a 1/3 chance of being hired upon meeting a vacancy. The model period is again set to one month so that \( r = 0.004 \). As above, I set \( \alpha = \beta = 0.72 \) and the fraction of low-productivity workers to \( f_l = 3\% \). The rest of the parameters and the (20-state) aggregate productivity process were calibrated to match Shimer’s targets, as explained above. The procedure discussed above was again used to generate 212 data points and the log of the model-generated data was detrended using a Hodrick-Prescott filter with smoothing parameter \( 10^5 \).

The results are summarized in Table B.1. As expected, this version of the model generates a smaller standard deviation of tightness than the model without shocks to job-specific productivity, because it generates smaller fluctuations in matching efficiency. However, the difference is fairly small and the correlation between vacancies and unemployment is still negative. Moreover, the shocks to job-specific productivity that occur each period, generate movements in job separations that are more persistent. This explains why this model generates a larger negative correlation between separations and average productivity and a larger elasticity of separations and unemployment with respect to average productivity.
Table B.1: Dynamic responses in a model with idiosyncratic shocks

<table>
<thead>
<tr>
<th>Elastics</th>
<th>$\theta$</th>
<th>$v$</th>
<th>$u$</th>
<th>$\tilde{m}$</th>
<th>$\tilde{s}$</th>
<th>$y\bar{p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>0.162</td>
<td>0.036</td>
<td>0.142</td>
<td>0.112</td>
<td>0.075</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Correlation Matrix

<table>
<thead>
<tr>
<th>Elastics</th>
<th>$\theta$</th>
<th>$v$</th>
<th>$u$</th>
<th>$\tilde{m}$</th>
<th>$\tilde{s}$</th>
<th>$y\bar{p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>1</td>
<td>0.642</td>
<td>-0.981</td>
<td>0.963</td>
<td>-0.600</td>
<td>0.855</td>
</tr>
<tr>
<td>$v$</td>
<td>-</td>
<td>1</td>
<td>-0.488</td>
<td>0.643</td>
<td>-0.603</td>
<td>0.886</td>
</tr>
<tr>
<td>$u$</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-0.938</td>
<td>0.548</td>
<td>-0.758</td>
</tr>
<tr>
<td>$\tilde{m}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-0.617</td>
<td>0.819</td>
</tr>
<tr>
<td>$\tilde{s}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
<td>-0.673</td>
</tr>
<tr>
<td>$y\bar{p}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>

C Computational algorithm

This appendix provides details on the algorithms used to solve the equilibriums described in Section I and Appendix B by numerical methods. With only two labour types there are three state variables in the model. The aggregate productivity, $y$, and the unemployment rates $u_l$ and $u_h$. As discussed above, the aggregate component of productivity is a discrete-space Markov process with 20 states. I discretize the other two state variables by assuming a uniform grid of 200 values so that the state space has a dimension of $20 \times 200 \times 200$.

Let $\chi = [y, u_l, u_h]$ denote the vector state variables. Given starting values for the $(20 \times 200 \times 200)$ matrix $\theta(\chi)$, the surplus functions are jointly solved by value function iteration. With homogenous jobs there are only two surplus functions to be solved: $S^h(\chi)$ and $S^l(\chi)$ (given by equation (5)). With three types of jobs there are six surplus functions to be solved: $S^{ll}(\chi), S^{lm}(\chi), S^{lh}(\chi), S^{hl}(\chi), S^{hm}(\chi)$ and $S^{hh}(\chi)$ (given by equation (A-1)). If for a given point in the state space the solution gives a zero value to one of the surpluses, the algorithm sets the value of the relevant indicator $I^{ij}(\chi)$ equal to one. Otherwise the algorithm sets the value of the indicator to zero and lets the surplus function take the value found in the solution. For each point in the steady space the next-period values of $u_l$ and $u_h$ are determined by the laws of motion for unemployment (given in (7) or (A-7)). The next period values, $u'_l$ and $u'_h$, will in general lie between gridpoints. For this reason, the values of the surplus functions at $u'_l$ and $u'_h$ are computed by two-dimensional linear

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$^{19}$The model was solved using the standard technique of value function iteration. A similar computational algorithm was used in Pries (2008).
interpolation. Subsequently, using the solutions for the surplus functions, the algorithm calculates the optimal matrix $\theta(\chi)$ that solves the free entry condition for each vector $\chi$.

The solution to the optimal matrix $\theta(\chi)$ and the surplus functions are then used to simulate 736 observations. To simulate each of the 736 observations, a Markov chain with 736 values of $y$ is generated. Given initial values for $\chi = [y, u_l, u_h]$ the initial surplus values and the initial values for $\theta$ are calculated, using interpolation. As above, for surplus values equal to zero the relevant indicator takes the value of 1 and the value of 0 otherwise. Given the values for $\theta$ and the indicators, the subsequent period’s values of $u_h$ and $u_l$ are given by the laws of motion and the process is repeated until forward iteration yields 736 observations.
References


