Title	A semi-systematic procedure for producing chaos from sinusoidal oscillators using diode-inductor and FET-capacitor composites
Author(s)	Elwakil, Ahmed S.; Kennedy, Michael Peter
Publication date	2000-04
Original citation	Elwakil, A.S., Kennedy, M.P., 2000. A semi-systematic procedure for producing chaos from sinusoidal oscillators using diode-inductor and FET-capacitor composites. IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 47(4), pp.582-590. doi: 10.1109/81.841862
Type of publication	Article (peer-reviewed)
Link to publisher's version	http://dx.doi.org/10.1109/81.841862 Access to the full text of the published version may require a subscription.
Rights	**©2000 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.**
Item downloaded from	http://hdl.handle.net/10468/174

A Semi-Systematic Procedure for Producing Chaos from Sinusoidal Oscillators Using Diode-Inductor and FET-Capacitor Composites

A. S. Elwakil and M. P. Kennedy

***Abstract*—A design procedure for producing chaos is proposed. The procedure aims to transfer design issues of analog autonomous chaotic oscillators from the nonlinear domain back to the much simpler linear domain by intentionally modifying sinusoidal oscillator circuits in a semisystematic manner. Design rules that simplify this procedure are developed and then two composite devices, namely, a diode-inductor composite and a FET-capacitor composite are suggested for carrying out the modification procedure. Applications to the classical Wien-bridge oscillator are demonstrated. Experimental results, PSpice simulations, and numerical simulations of the derived models are included.**

***Index Terms*—Chaos, nonlinear dynamics, oscillators.**

I. Introduction

The realization of electronic chaos generators has for some time been a topic of increasing interest. Some contributions in this direction were based on emulating a system of ordinary nonlinear differential equations that are known to be chaotic [1]–[4], resulting in pure analog circuits. Some other contributions were based on iterating one-dimensional (1-D) maps, such as the logistic map, within their chaotic windows [5]–[8], resulting in oscillators that are generally suitable for digital applications. These contributions have a common feature of starting from a mathematical study and ending with an electronic circuit. However, the opposite direction in which a chaotic oscillator is discovered and, accordingly, a mathematical model is derived and has been reported [9]–[11]. Due to the possible applications of chaos in several areas, particularly in communications, the need for systematic methods for designing chaotic oscillators has increased. Such methods should be based on well-established electronic design techniques to make use of the enormous literature and experience already available. Based on Chua's circuit, an attempt in this direction was introduced in [12]. However, due to the nature of the nonlinearity in Chua's circuit, which is active and piecewise linear, designing such a nonlinearity is not straightforward [13]. In general, active nonlinearities are not easy to design or reconfigure using different building blocks and technology parameters.

Among the chaotic oscillators that have been recently reported, the chaotic Colpitts oscillator [11] receives special importance. This oscillator has demonstrated the fact that a classical sinusoidal oscillator can behave chaotically for a specific set of parameters. Accordingly, the question whether it is possible for other classical sinusoidal oscillators to behave chaotically naturally arises. Researchers in [14]–[19] have shown that Wien-type sinusoidal oscillators can be modified for chaos. Other types of oscillator have also been modified for chaos [20]–[22]. These contributions suggest a possible route to producing chaos starting from an existing sinusoidal oscillator circuit.

It is the aim of this work to propose a three-step procedure for designing autonomous chaotic oscillators. Four design rules that simplify this procedure are developed. Finally, the diode inductor and FET-capacitor composites are introduced to carry out the procedure in a semisystematic manner. Using these two composites, generation of chaos from the classical Wien-bridge oscillator is demonstrated.

Manuscript received June 19, 1998; revised April 27, 1999. This work was supported in part by the Enterprise Ireland Basic Research Programme under Grant SC/98/740. This paper was recommended by Associate Editor C. W. Wu.

A. S. Elwakil is with the Department of Electronics and Electrical Engineering, University College Dublin, Dublin 4, Ireland.

M. P. Kennedy is with the Department of Microelectronic Engineering, University College Cork, Cork, Ireland

Publisher Item Identifier S 1057-7122(00)02322-9.

II. Design Procedure

Once a chaotic oscillator is to be designed for a target application there are generally two sets of requirements and design limitations that should be satisfied. The first set is concerned with statistical measures of the produced chaotic signal (eigenvalues, Lyapunov exponents, space dimension, power spectrum characteristics, basins of attraction, etc.), while the second set is concerned with the nature and characteristics of the physically produced waveform and its electronic circuit realization. It is unfortunately not yet clear whether the available statistical measures are sufficient to describe and compare different types of chaos. Most of these measures are analysis oriented and no method has yet been developed to produce chaos with a set of prespecified measures, even from an abstract mathematical model. On the other hand, it is possible to realize a chaotic oscillator that fulfills (optimizes) a set of circuit-specific constraints. For this purpose, the following three-step design procedure is proposed.

1) Design a sinusoidal oscillator circuit that meets the desired requirements in terms of passive element structure, tunability, sensitivity, and active building blocks. This design is to be based on simple and well-established linear design techniques which allow an oscillator to have specific features. For example, an RC oscillator with all grounded capacitors and the minimum number of resistors can be designed. The appropriate active building block [i.e., voltage op amp (VOA), operational transconductance amplifier (OTA), current feedback op amp (CFOA), current conveyor (CCII), etc.] and the function it performs (i.e., amplifier, integrator, impedance converter, current/voltage follower, etc.) can also be specified. Formulas defining the necessary conditions for oscillation and the frequency of oscillation are then derived. Oscillators where the frequency of oscillation can be independently controlled are advantageous. A huge collection of sinusoidal oscillators with a wide variety of features can be found in the literature (see, e.x., [23]–[32] and the references therein). Researchers who are not acquainted with sinusoidal oscillator design can simply choose from the available catalogues.
2) Guided by the derived condition for oscillation formula, and by inspecting the structure of the oscillator, selection is made for a suitable position to insert a simple nonlinear element. If the designed sinusoidal oscillator is of an order less than three, an additional energy storage element (inductor or capacitor) should also be added in a suitable position.
3) The tuning parameters identified in the first step are adjusted around the same values that satisfy the condition for oscillation of the sinusoidal oscillator.

The resulting chaotic oscillator inherits the features of the sinusoidal oscillator and has a continuous noise-like power spectrum centred approximately around the operating frequency of the sinusoidal oscillator.

In order to simplify the above procedure, the following design rules are presented.

1) Nonlinearity should be introduced by a passive rather than active element. Equivalently, the sources of circuit energy should only be the linear building blocks. For example, an active nonlinear resistor, such as that in Chua's circuit, is not recommended.
2) The nonlinear element should be separated from the linear blocks such that the functionality of these blocks remains clear, ideal, and independent of any parameters of the nonlinear element.

Building blocks such as voltage-controlled voltage sources (inverting or noninverting), integrators, negative impedance converters (voltage or current controlled, etc.) should operate linearly. In the chaotic Colpitts oscillator [11] the nonlinear element can not be separated from the gain device, so it is suboptimal from this point of view.

3) It should be possible to reproduce the chaotic dynamics of the oscillator with a model that does not depend on any device-specific parasitic effect. For example the family of chaotic oscillators in [17] utilizes a voltage op amp as an amplifier and as a current-controlled negative impedance converter. Modeling the observed chaotic behavior of this family requires the internal dominant pole of the op amp to be included into analysis. Hence, the functionality of the linear building blocks cannot be reproduced other than by using a voltage op amp.
4) Simple two-terminal nonlinear resistors (diodes or diode-connected transistors) should be used where possible.

The basic advantage of adopting these design rules is that the resulting chaotic oscillator is independent of the circuit and technology. Since the active blocks operate only linearly and ideally, it should be possible to use any implementation for these blocks. However, once an implementation has been chosen, effects of any known parasitics on the chaotic behavior should be considered. Limitations on the operating bandwidth, supply voltage, and power dissipation of the chaotic oscillator are imposed only by the linear active block and not by the nonlinear element, which is strictly passive. The type of signal processing (current mode or voltage mode) is also defined by the linear element. Accordingly, benchmarks used to evaluate and compare linear designs become valid for chaotic oscillators.

It should be noted that starting with a third-order sinusoidal oscillator, such as the Twin-T oscillator, the above procedure simplifies to choosing a suitable position to insert a passive nonlinear element and adjusting the tuning parameters associated with the condition of oscillation [20]. However, since most of the available sinusoidal oscillators are second order, an additional energy storage element is required to permit chaotic behavior. The position to insert this element and its value are currently subject to designer experience, hence, we consider the proposed procedure as semisystematic. However, with the aid of the composites presented in Section III and with sufficient experience, this design methodology proves to be indeed systematic. The authors have demonstrated the flexibility of this procedure by modifying families of sinusoidal oscillators for chaos [33]–[35].

In the following section, we introduce two nonlinear composite devices that facilitate the modification process. Application to the classical Wien-bridge oscillator is demonstrated.

III. The Diode-Inductor and FET-Capacitor Composites with Application to the Wien-Bridge Oscillator

A. The Diode-Inductor Composite

Fig. 1(a) shows a diode-inductor (D–L) composite which is a parallel combination of a signal diode and an inductor. The diode switches on and off according to the voltage developed across the inductor. This voltage appears across the parasitic transit capacitance [36] of the diode C_D. Hence, the composite is described by the following equations:

$$\begin{aligned} L\dot{I}_L &= V_{CD} \\ C_D\dot{V}_{CD} &= I - I_L - I_D \end{aligned} \tag{1a}$$

where

- I composite current;
- I_L inductor current;
- I_D nonlinear diode current modeled by

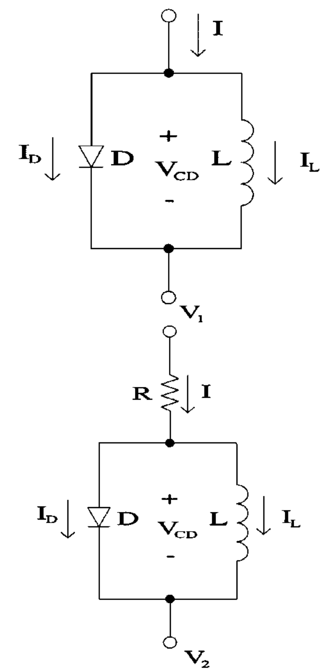


Fig. 1. (a) The diode inductor (D–L) composite. (b) A suitable position for the D–L composite in series with a resistor R.

$$I_D = \frac{1}{R_D}\begin{cases} V_{CD} - V_\gamma, & V_{CD} \ge V_\gamma \\ 0, & V_{CD} < V_\gamma. \end{cases} \tag{1b}$$

R_D and V_γ are the diode forward conduction resistance and voltage drop, respectively.

When modifying an oscillator for chaos, one must choose a position within the sinusoidal oscillator to insert this composite and a suitable value for L. The best position to insert this composite is in series with one of the resistors, as shown in Fig. 1(b). The composite current I then becomes

$$I = \frac{V_1 - V_{CD} - V_2}{R}. \tag{2}$$

Clearly, grounding the composite ($V_1 = 0$ or $V_2 = 0$) is preferable.

Consider the classical Wien-bridge configuration shown in Fig. 2 which employs a noninverting voltage-controlled voltage source (NVCVS) with gain K. With equal capacitors, the state space representation of this configuration is given by

$$\begin{bmatrix} \dot{V}_{C1} \\ \dot{V}_{C2} \end{bmatrix} = \frac{1}{C}\begin{bmatrix} \frac{K-1}{R_2} - \frac{1}{R_1} & \frac{-1}{R_2} \\ \frac{K-1}{R_2} & \frac{-1}{R_2} \end{bmatrix}\begin{bmatrix} V_{C1} \\ V_{C2} \end{bmatrix}. \tag{3}$$

Hence, the condition and frequency of oscillation are found to be

$$K = 2 + \frac{R_2}{R_1} \quad \text{and} \quad \omega_o = \frac{1}{C\sqrt{R_1 R_2}} \tag{4}$$

respectively.

Choosing $R_1 = R_2$, the theoretical gain required to start oscillation is $K = 3$. By inspecting the Wien oscillator, two positions are suitable for inserting the D–L composite: in series with R_1 or R_2. The position in series with R_1 enables the composite to be grounded and has been reported in [19]. For the position in series with R_2, the modified configuration is described by the following equations:

$$\begin{aligned} C_1\dot{V}_{C1} &= I - \frac{V_{C1}}{R_1} \\ C_2\dot{V}_{C2} &= I \end{aligned} \tag{5}$$

in addition to the set of equations in (1a) and (1b) and with $I = (K-1)V_{C1} - V_{C2} - V_{CD}/R_2$.

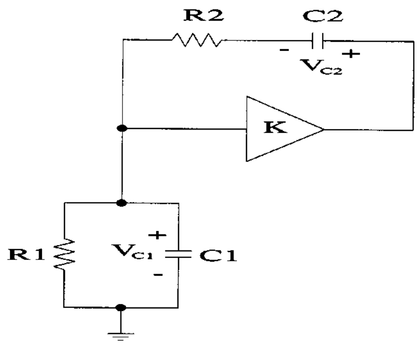


Fig. 2. The classical Wien-bridge sinusoidal oscillator configuration.

Since the D–L composite is a second-order composite, the modified for chaos Wien-bridge oscillator is a fourth-order chaotic oscillator.

For the choice of $C_1 = C_2 = C, R_1 = R_2 = R$ and by introducing the following dimensionless quantities: $\tau = t/RC, X = V_{C1}/V_\gamma, Y = V_{C2}/V_\gamma, Z = RI_L/V_\gamma, W = V_{CD}/V_\gamma, \varepsilon = C_D/C, \alpha = R/R_D$, and $\beta = R^2C/L$, the state space representation of this chaotic oscillator is given by

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \varepsilon\dot{W} \end{bmatrix} = \begin{bmatrix} K-2 & -1 & 0 & -1 \\ K-1 & -1 & 0 & -1 \\ 0 & 0 & 0 & \beta \\ K-1 & -1 & -1 & -(1+a) \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ a \end{bmatrix} \quad (6a)$$

and

$$\begin{cases} a = \alpha & W \geq 1 \\ a = 0 & W < 1. \end{cases} \quad (6b)$$

The Y–Z phase space trajectory obtained by numerically integrating (6) using a Runge–Kutta fourth-order algorithm with 0.005 step and with $K = 3$, $\varepsilon = 0.01$, $\alpha = 10$, and $\beta = 0.1$ is plotted in Fig. 3(a).

In (6), the active part of the oscillator is represented only by its gain K which is, of course, the best tuning parameter. The passive nonlinearity is characterized by α and ε. Thus, the active linear and the passive nonlinear blocks are clearly separated. For fixed values of R, C, and K, the mathematical model of (6) provides a maximum value for β which is necessary to observe chaos. Hence, a corresponding minimum value for L can be calculated. Practical minimum values of L are given in Table I for $R = 1$ kΩ, $K = 3$ and different values of C. In general, the inductor value increases linearly per decade increase of C. The case where $\varepsilon = 1$ is particularly attractive for high-frequency and monolithic implementations since the value of the physical capacitors is as small as the diode parasitic capacitance. We have integrated (6) with $K = 3.5$, $\varepsilon = 1$, $\alpha = 10$, and $\beta = 0.2$ and a trajectory similar to Fig. 3(a) was observed. It should be noted, however, that follower-based sinusoidal oscillators are more suitable for such implementations than the Wien oscillator.

A PSpice simulation of the V_{C2}—I_L trajectory is shown in Fig. 3(b) when $C_1 = C_2 = 1$ nF, $R_1 = R_2 = 1$ kΩ, $L = 10$ mH, and $K = 3.4$. The amplifier was implemented using a CFOA [37] and a general purpose diode (D1N914, D1N4148) was used.

B. The FET-Capacitor Composite

Fig. 4(a) shows the FET-capacitor (FET-C) composite which is a series combination of a FET, connected to operate as a two-terminal device and a capacitor. The composite is described by the following equation:

$$C\dot{V}_C = I_N \quad (7a)$$

where I_N is the nonlinear FET current modeled as

$$I_N = \frac{1}{R_N} \begin{cases} V_{GS} & V_{GS} \geq V_P \\ V_P & V_{GS} < V_P. \end{cases} \quad (7b)$$

R_N is the FET small signal resistance at the operating point, V_{GS} is the gate to source voltage, and V_P is the pinch-off voltage. For a positive V_P, (7b) can be written in a form similar to (1b) if I_N is shifted by V_P/R_N. For a negative V_P, V_{GS} should in addition be shifted by $2V_P$.

Since FET's (particularly junction field effect transistors) can operate as voltage controlled resistors, the FET-C composite is intended to replace any series R–C branch within a sinusoidal oscillator architecture. Starting with a second-order oscillator, it remains to add an extra capacitor or inductor. Although several chaotic oscillators have been designed using the FET-C composite after adding an inductor [33], [34], we demonstrate two configurations that require the addition of a single capacitor. The result is an inductorless chaotic oscillator which is advantageous in many respects. Consider the configuration in Fig. 4(b) where a series R_3–C_3 branch appears in parallel with the FET-C_2 composite. The configuration is then described by

$$\begin{aligned} C_2\dot{V}_{C2} &= I_N \\ C_3\dot{V}_{C3} &= \frac{V_1 - V_{C3} - V_2}{R_3} \end{aligned} \quad (8a)$$

and

$$I_N = \frac{1}{R_N} \begin{cases} V_1 - V_{C2} - V_2, & V_1 - V_{C2} - V_2 \geq V_P \\ V_P, & V_1 - V_{C2} - V_2 < V_P. \end{cases} \quad (8b)$$

Using this configuration to replace the R_2–C_2 branch of the Wien oscillator in Fig. 2, one obtains the following equation that describes the system along with the set of (8):

$$C_1\dot{V}_{C1} = \frac{V_1 - V_{C3} - V_2}{R_3} - \frac{V_{C1}}{R_1} + I_N \quad (9)$$

and the VCVS forces the voltage difference V_1–V_2 to equal $(K - 1)V_{C1}$.

Guided by the Wien oscillator design equations (4), we choose the parameter set: $R_1 = R_3 = R_N = R$ and $C_1 = C_2 = C_3 = C$. Setting: $\tau = t/RC, X = V_{C1}/V_P, Y = V_{C2}/V_P$ and $Z = V_{C3}/V_P$, the state space representation of the configuration is given by

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} -2-a+(1+a)K & -a & -1 \\ a(K-1) & -a & 0 \\ K-1 & 0 & -1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} b \\ b \\ 0 \end{bmatrix} \quad (10a)$$

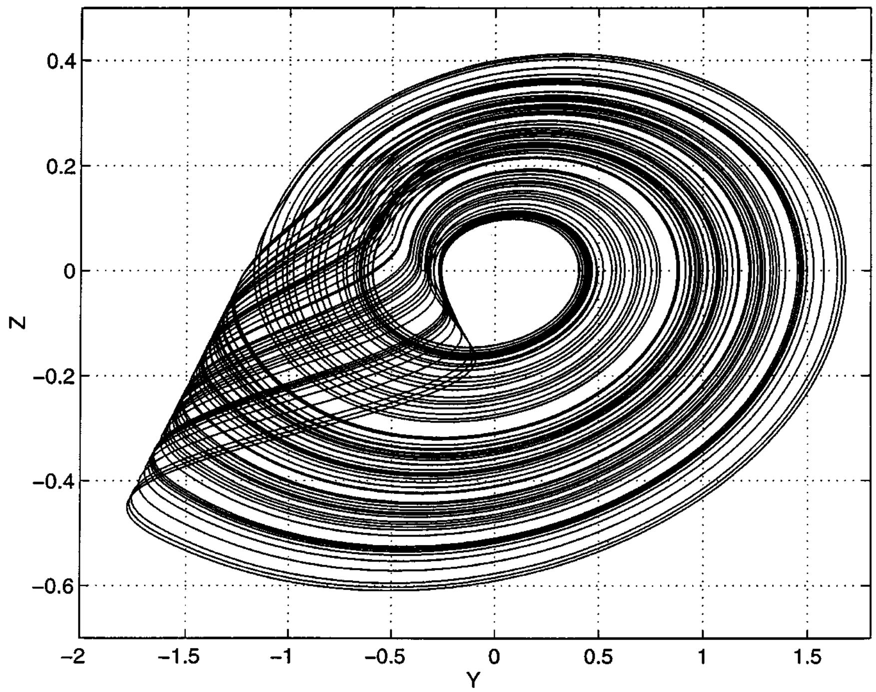
where

$$\begin{cases} a = 1, b = 0, & (K-1)X - Y \leq 1 \\ a = 0, b = 1, & (K-1)X - Y > 1. \end{cases} \quad (10b)$$

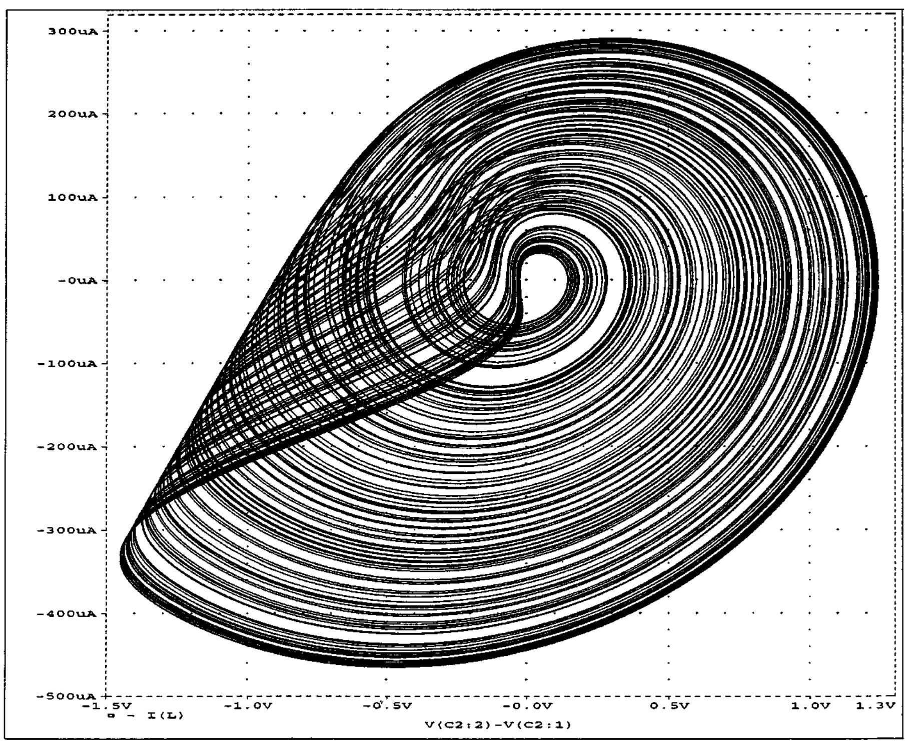
The X–Y phase space trajectory obtained by numerically integrating (10) with $K = 2.38$ is plotted in Fig. 5(a). Note that (10) is tuned via a single parameter (the VCVS gain K) and represents a circuit-independent model. When a specific active device is used to implement this VCVS, (10) should be modified to include the effects of any known parasitics or nonidealities.

PSpice simulations using a J2N4338 type JFET ($R_N = 750\,\Omega, V_P = -0.7$ V) were performed taking $R_1 = R_3 = 750\,\Omega, C_1 = C_2 = C_3 = 1$ nF, and $K = 2.5$. The resulting V_{C1}–V_{C2} trajectory is shown in Fig. 5(b). The VCVS was realized as in [19] using an AD844 CFOA biased with ±9 V supplies. Of course, the CFOA should not be allowed to saturate and, hence, the voltage across C_1 should not exceed V_{sat}/K.

The circuit was experimentally constructed with the above components and with K varied via a 30-KΩ pot. Fig. 6(a) represents the limit cycle just before the period-two orbit, shown in Fig. 6(b), is born. The period-doubling cascade continues as K is increased and the chaotic attractor is shown in Fig. 6(c).



(a)



(b)

Fig. 3. (a) The Y–Z phase space trajectory obtained by numerically integrating (6) with $K = 3$, $\varepsilon = 0.01$, $\alpha = 10$, and $\beta = 0.1$. (b) PSpice simulation of the V_{C2}–I_L trajectory using $C_1 = C_2 = 1$ nF, $R_1 = R_2 = 1$ kΩ, $L = 10$ mH, a D1N914 diode and with $K = 3.4$. The amplifier is implemented using an AD844 CFOA biased with ± 9 V supplies.

Next consider the configuration in Fig. 7(a) which can be described by

$$\begin{aligned} C_2\dot{V}_{C2} &= I_N \\ C_3\dot{V}_{C3} &= \frac{V_1 - V_{C3} - V_2}{R_3} - I_N \end{aligned} \tag{11a}$$

and

$$I_N = \frac{1}{R_N}\begin{cases} V_{C3} - V_{C2}, & V_{C3} - V_{C2} \geq V_P \\ V_P, & V_{C3} - V_{C2} < V_P. \end{cases} \tag{11b}$$

Using this configuration to replace the R_2–C_2 branch of the Wien

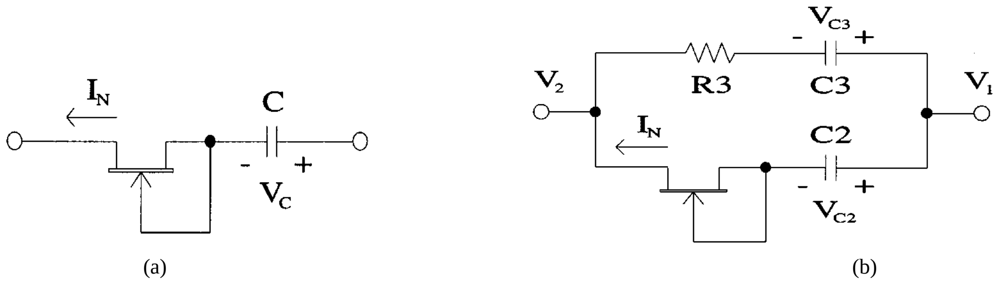


Fig. 4. (a) The FET-capacitor (FET-C) composite. (b) A proposed RC configuration using the FET-C composite.

TABLE I
TYPICAL MINIMUM INDUCTOR VALUES OF THE D–L COMPOSITE IN Fig. 1(b) FOR A GIVEN VALUE OF C

R=1kΩ, K=3, R_D=50Ω, C_D=2pF	
C	L
10pF	33μH
100pF	450μH
1nF	4.5mH
10nF	40mH

oscillator, the resulting circuit can be described by (11) in addition to the following equation:

$$C_1\dot{V}_{C1} = \frac{V_1 - V_{C3} - V_2}{R_3} - \frac{V_{C1}}{R_1} \tag{12}$$

where V_1–V_2 is forced to equal $(K-1)V_{C1}$.

Choosing the parameter set $R_3 = R_N = R$, $R_1 = 2R$, $C_2 = C_3 = C$, $C_1 = 2C$ and using the same dimensionless settings as above, the following set of equations is obtained:

$$\begin{bmatrix} 2\dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} K-\frac{3}{2} & 0 & -1 \\ 0 & -a & a \\ K-1 & a & -(1+a) \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} 0 \\ b \\ -b \end{bmatrix} \tag{13a}$$

where

$$\begin{cases} a=1, b=0, & Z-Y \le 1 \\ a=0, b=1, & Z-Y > 1. \end{cases} \tag{13b}$$

The X–Y trajectory shown in Fig. 7(b) was obtained by integrating (13) with $K = 3.1$. A PSpice simulation is also shown in Fig. 7(c) with $C = 1$ nF and $K = 3.17$. The model of (13) is also a single-parameter tuned circuit-independent model.

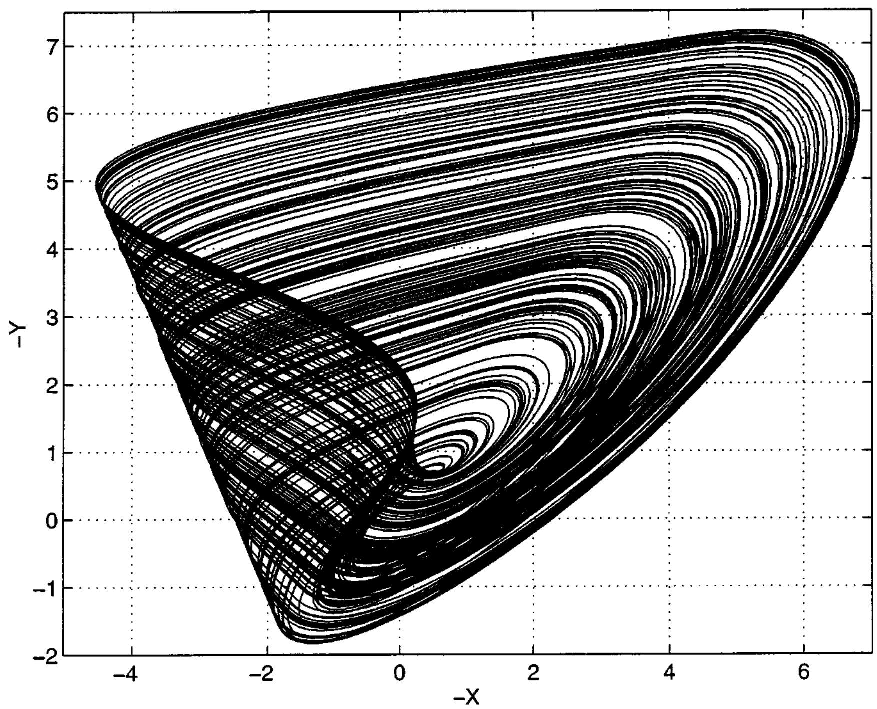
It is worth noting that the design sets which we have demonstrated here are based on the equal C design equations of (4). However, other design sets can be used as well. A particularly useful design set using the composite of Fig. 7(a) in any oscillator is to take $C_1 = m(C_2+C_3)$ and $R_1 = n(R_3+R_N)$ where m and n are guided by the oscillator's design equations. In some cases, and with good knowledge of the existing parasitic elements of a specific implementation, one might replace R_3 or C_3 with these parasitics [34].

IV. CONCLUDING REMARKS

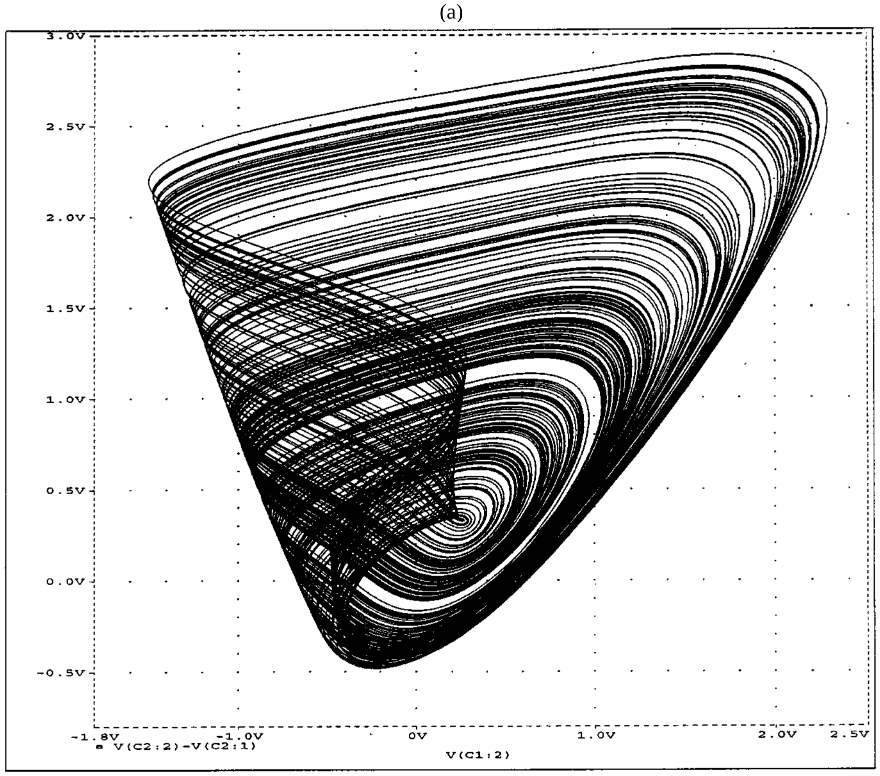
We have introduced a methodology for designing autonomous chaotic oscillators. The aim of this methodology is to transfer chaotic oscillator design issues from the nonlinear domain back to the well-established linear circuit theory of design by specifying a linear starting point for the design process along with a set of design rules. Following these design rules, a chaotic oscillator automatically inherits the main features of a mother simple harmonic oscillator. Thus, optimization techniques performed on the harmonic oscillator result in an optimized chaotic oscillator as well.

We have also introduced two passive nonlinear composites, namely, a second-order diode-inductor composite and a first-order FET-capacitor composite, as tools to carry out the design procedure. Indeed, with some experience, designers would find using these tools rather systematic. It should be noted that all areas of analog circuit design still depend on designers' experience to a great extent. In this sense, chaotic oscillator design is no exception. In conclusion, we conjecture that following the proposed three-step design procedure, at least one chaotic oscillator can always be derived from any simple harmonic oscillator. Several important points should be mentioned.

1) All circuits that have been modified using the D–L and FET-C composites show a Colpitts-like chaotic attractor [11], [38] and are governed by dynamics similar to the chaotic Colpitts oscillator [39]. In fact, there is enough evidence to believe that this attractor is universal and will naturally arise in many low-dimensional chaotic systems [40].
2) Chaotic oscillators based on sinusoidal oscillators have a broad power spectrum which is concentrated around the sinusoidal oscillator's operating frequency. For this reason, such oscillators can not have a spectrum with a flat magnitude over a wide bandwidth. This also applies to Chua's circuit which we have recently shown to have a core sinusoidal oscillator [41], [42]. Chaotic oscillators based on voltage-controlled oscillators (VCO's) which have a varying center frequency appear to be more attractive when flat spectra are required.
3) Our design methodology strictly recommends using passive nonlinear elements. This remains desirable as long as there is no evidence that chaotic signals produced when active nonlinearities are used possess any statistical features that are not possessed by chaotic signals produced when passive nonlinearities are used. One of the reviewers has noted that diodes and FET's suffer from large parameter spreads and temperature sensitivity. If active nonlinear devices, such as Chua's diode, prove to be better in this respect for a specific application, they should be used. We note, however, that when passive antisymmetric voltage-controlled diode and FET characteristics are replaced with active (or passive) odd symmetrical characteristics in any of the designed oscillators [21], [35], a chaotic attractor still results and can be directly related to two of the basic Colpitts-like attractors by simple flip, mirror, and merge operations.
4) Our proposed design methodology applies also to hysteresis chaotic oscillators which can be obtained by modifying sinusoidal oscillators using hysteresis nonlinear resistors [43].



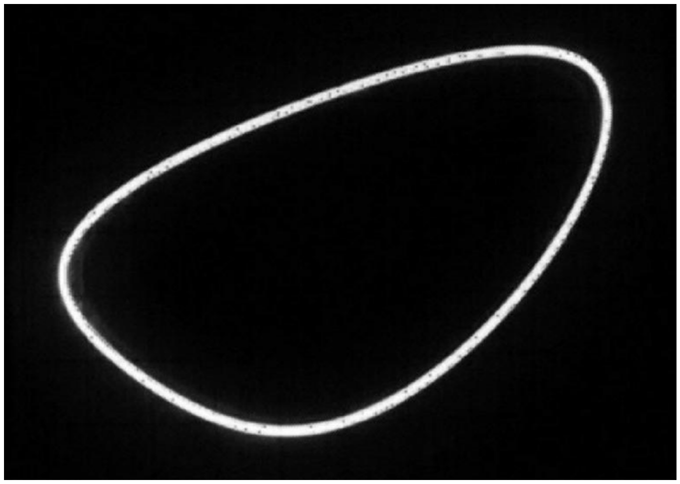
(a)



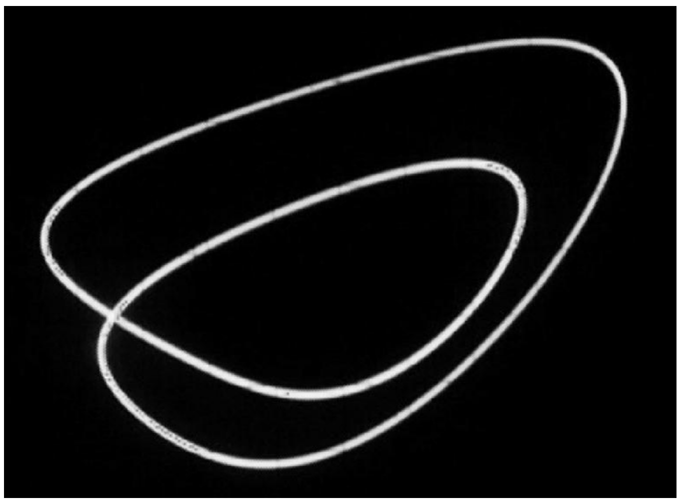
(b)

Fig. 5. Results of replacing the R_2–C_2 branch of Fig. 2 with the configuration of Fig. 4(b). (a) The X–Y phase space trajectory obtained by numerically integrating (10) with $K = 2.38$. (b) PSpice simulations of the V_{C1}–V_{C2} trajectory using a J2N4338 JFET, $R_1 = R_3 = 750\,\Omega$, $C_1 = C_2 = C_3 = 1$ nF, $K = 2.5$ and using ±9 V supplies.

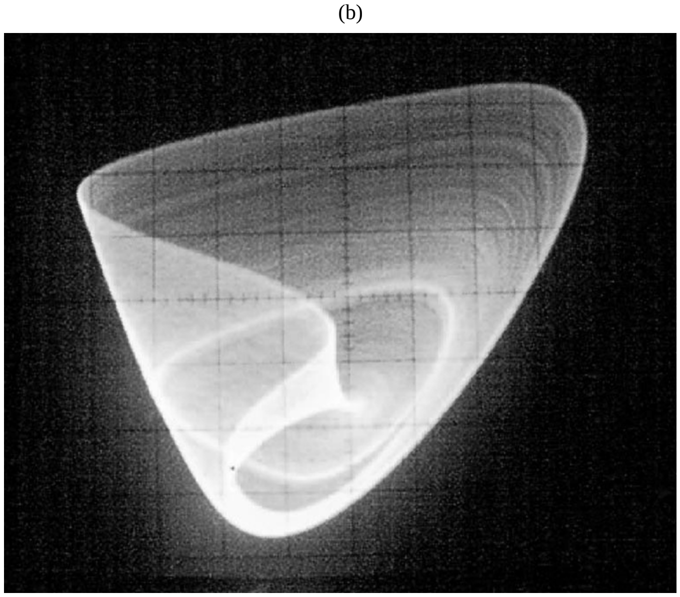
Passive hysteresis nonlinear resistors have been introduced in [44] and we have successfully used them to systematically design a class of hysteresis chaotic oscillators [45]. Hysteresis resistors do not require the addition of an extra energy storage element (inductor/capacitor) as required by Step 2 of our design procedure since there always exists a parasitic inductor (capacitor) associated with the fast dynamics of their nonmonotone current-controlled (voltage-controlled) characteristics [46]. The observed chaotic attractor from this class of oscillators is a single screw which can also develop into a Colpitts-like attractor [45].



(a)



(b)

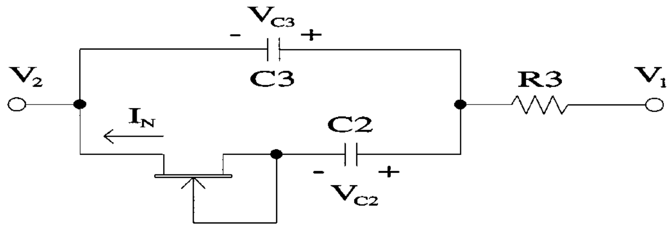


(c)

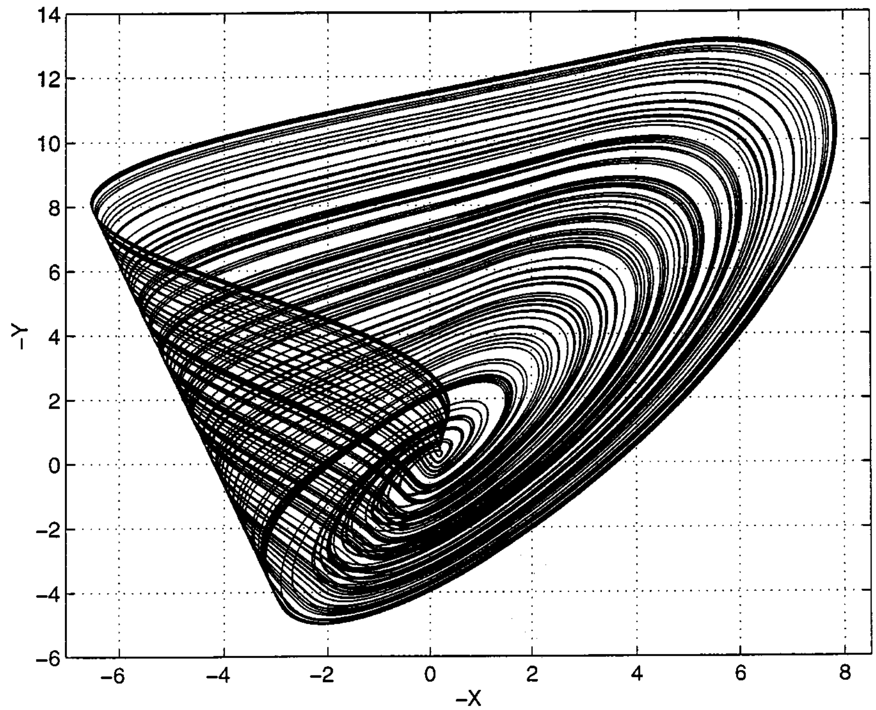
Fig. 6. (a) Limit cycle from an experimental setup: $K = 2.48$. (b) Birth of the period two orbit: $K = 2.5$. (c) Chaotic attractor: $K = 2.59$. All photographs represent the V_{C1}–V_{C2} trajectory. V_{C2} is measured by a differential probe. X axis: 0.1 V/div, Y axis: 0.1 V/div.

5) It is possible to design higher order chaotic and hyperchaotic oscillators by coupling at least two sinusoidal oscillators using passive nonlinearities [47].

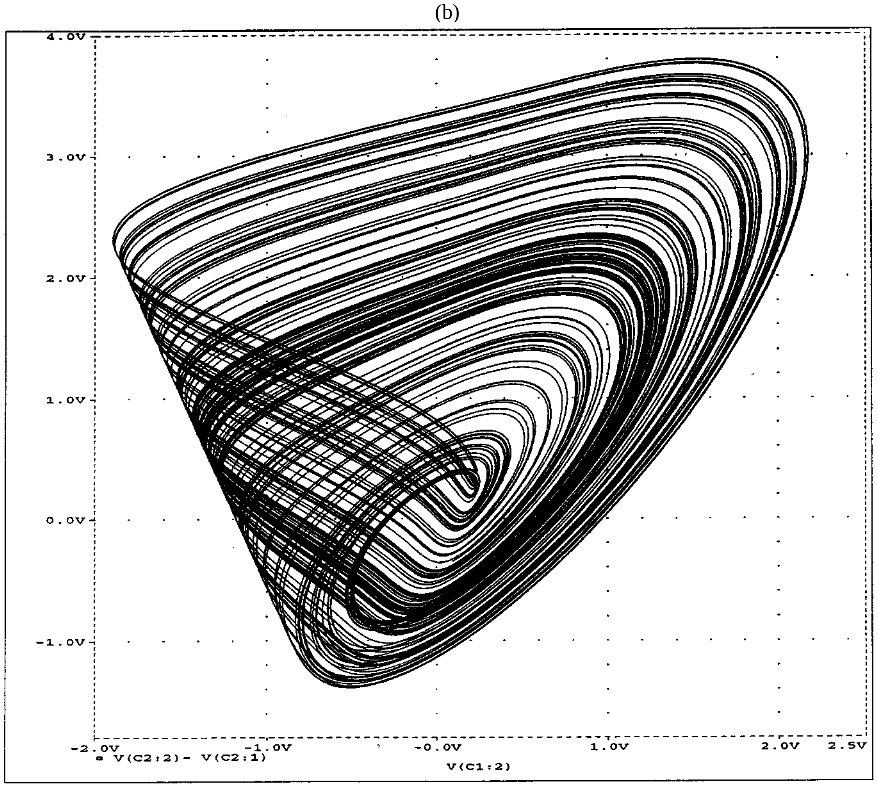
Finally, we believe that the topic of chaotic oscillator design has advanced significantly guided by a linear circuit design perspective. By contrast, design-oriented tools developed under the theory of nonlinear



(a)



(b)



(c)

Fig. 7. (a) Another RC configuration using the FET-C composite. (b) The X–Y trajectory obtained by integrating (13) with $K = 3.1$. (c) A PSpice simulation with $R_1 = 1.5$ kΩ, $R_3 = 750\,\Omega$, $C_1 = 2$ nF, $C_2 = C_3 = 1$ nF, and $K = 3.17$.

dynamics have not advanced as much. On the application front, more effort is needed to define the required properties of chaotic signals. Clear specifications are required in order to attract the interest of analog circuit designers to chaotic electronics (chaotronics).

ACKNOWLEDGMENT

The authors wish to thank the reviewers for their constructive criticism. The AD844 amplifiers used in this work were provided by Analog Devices.

REFERENCES

[1] M. P. Kennedy and L. O. Chua, "Van der Pol and chaos," *IEEE Trans. Circuits Syst. I*, vol. 33, pp. 974–980, Oct. 1986.
[2] R. Tokunaga, M. Komuro, T. Matsumoto, and L. O. Chua, "Lorenz attractor from an electrical circuit with uncoupled continuous piecewise-linear resistor," *Int. J. Circuit Theory Appl.*, vol. 17, pp. 71–85, 1989.
[3] N. Inaba and S. Mori, "Chaos via torus breakdown in a piecewise linear forced van der pol oscillator with a diode," *IEEE Trans. Circuits Syst. I*, vol. 38, pp. 398–409, Apr. 1991.
[4] A. Namajunas, K. Pyragas, and A. Tamasevicius, "An electronic analog of the Mackey-Glass system," *Phys. Lett. A*, vol. 201, pp. 43–46, 1995.
[5] G. C. McGonigal and M. I. Elmasry, "Generation of noise by electronic iteration of the logistic map," *IEEE Trans. Circuits Syst. I*, vol. 34, pp. 981–983, Aug. 1987.
[6] A. R. Vazquez, J. L. Huertas, and L. O. Chua, "Chaos in a switched capacitor circuit," *IEEE Trans. Circuits Syst. I*, vol. 32, pp. 1083–1084, Oct. 1985.
[7] M. D. Restituto, A. R. Vazquez, S. Espejo, and J. L. Huertas, "A chaotic switched capacitor circuit for 1/f noise generation," *IEEE Trans. Circuits Syst. I*, vol. 39, pp. 325–328, Apr. 1992.
[8] J. Zhu, H. Takakubo, and K. Shono, "Observation and analysis of chaos with digitalizing measure in a CMOS mapping system," *IEEE Trans. Circuits Syst. I*, vol. 43, pp. 444–452, June 1996.
[9] R. W. Newcomb and S. Sathyan, "An RC op amp chaos generator," *IEEE Trans. Circuits Syst. I*, vol. 30, pp. 54–56, Jan. 1983.
[10] T. Matsumoto, "A chaotic attractor from Chua's circuit," *IEEE Trans. Circuits Syst. I*, vol. 31, pp. 1055–1058, Dec. 1984.
[11] M. P. Kennedy, "Chaos in the Colpitts oscillator," *IEEE Trans. Circuits Syst. I*, vol. 41, pp. 771–774, Nov. 1994.
[12] A. Rodriguez-Vazquez and M. Delgado-Restituto, "CMOS design of chaotic oscillators using state variables: A monolithic Chua's circuit," *IEEE Trans. Circuits Syst. II*, vol. 40, pp. 596–613, Oct. 1993.
[13] M. P. Kennedy, "Robust op amp realization of Chua's circuit," *Frequenz*, vol. 46, pp. 66–80, 1992.
[14] A. Namajunas and A. Tamasevicius, "Modified Wien-bridge oscillator for chaos," *Electron. Lett.*, vol. 31, pp. 335–336, 1995.
[15] O. Morgul, "Wien bridge based RC chaos generator," *Electron. Lett.*, vol. 31, pp. 2058–2059, 1995.
[16] A. Namajunas and A. Tamasevicius, "Simple RC chaotic oscillator," *Electron. Lett.*, vol. 32, pp. 945–946, 1996.
[17] A. S. Elwakil and A. M. Soliman, "A family of Wien-type oscillators modified for chaos," *Int. J. Circuit Theory Appl.*, vol. 25, pp. 561–579, 1997.
[18] ——, "Current mode chaos generator," *Electron. Lett.*, vol. 33, pp. 1661–1662, 1997.
[19] A. S. Elwakil and M. P. Kennedy, "High frequency Wien-type chaotic oscillator," *Electron. Lett.*, vol. 34, pp. 1161–1162, 1998.
[20] A. S. Elwakil and A. M. Soliman, "Two Twin-T based op amp oscillators modified for chaos," *J. Franklin Instit.*, vol. 335B, pp. 771–787, 1998.
[21] ——, "Two modified for chaos negative impedance converter op amp oscillators with symmetrical and antisymmetrical nonlinearities," *Int. J. Bifurcat. Chaos*, vol. 8, pp. 1335–1346, 1998.
[22] A. S. Elwakil and M. P. Kennedy, "Three-phase oscillator modified for chaos," *Microelectron. J.*, vol. 30, pp. 863–867, 1999.
[23] S. Celma, P. A. Martinez, and A. Carlosena, "Current feedback amplifiers based sinusoidal oscillators," *IEEE Trans. Circuits Syst. I*, vol. 41, pp. 906–908, Dec. 1994.
[24] R. Senani and V. K. Singh, "Novel single-resistance-controlled oscillator configuration using current feedback amplifiers," *IEEE Trans. Circuits Syst. I*, vol. 43, pp. 698–700, Aug. 1996.
[25] ——, "Synthesis of canonic single-resistance-controlled oscillators using a single current feedback amplifier," *Proc. Inst. Elect. Eng.*, vol. 143, pp. 71–72, 1996.
[26] S. S. Gupta and R. Senani, "State variable synthesis of single-resistance-controlled grounded capacitor oscillators using only two CFOAs," *Proc. Inst. Elect. Eng.*, vol. 145, pp. 135–138, 1998.
[27] ——, "State variable synthesis of single-resistance-controlled grounded capacitor oscillators using only two CFOAs: Additional new realizations," *Proc. Inst. Elect. Eng.*, vol. 145, pp. 415–418, 1998.
[28] A. M. Soliman, "Novel generation method of current mode Wien-type oscillators using current conveyors," *Int. J. Electron.*, vol. 85, pp. 737–747, 1998.
[29] ——, "Current mode oscillators using grounded capacitors and resistors," *Int. J. Circuit Theory Appl.*, vol. 26, pp. 431–438, 1998.
[30] ——, "Novel oscillators using current and voltage followers," *J. Franklin Inst.*, vol. 335B, pp. 997–1007, 1998.
[31] A. M. Soliman and A. S. Elwakil, "Wien oscillators using current conveyors," *Comp. Elect. Eng.*, vol. 25, pp. 45–55, 1999.
[32] M. T. Abuelma'atti and H. A. Alzaher, "Current-mode sinusoidal oscillators using single FTFN," *IEEE Trans. Circuits Syst. II*, vol. 46, pp. 69–73, Jan., 1999.
[33] A. S. Elwakil and M. P. Kennedy, "A family of Colpitts-like chaotic oscillators," *J. Franklin Inst.*, vol. 336, pp. 687–700, 1999.
[34] ——, "Chaotic oscillators derived from sinusoidal oscillators using the current feedback op amp," Analog Integrated Circuits Signal Processing, to be published.
[35] ——, "Chaotic oscillator configuration using a frequency-dependent negative resistor," *Int. J. Circuit Theory Appl.*, vol. 28, pp. 69–76, 2000.
[36] L. O. Chua and P. M. Lin, *Computer-Aided Analysis of Electronic Circuits*. Englewood Cliffs, NJ: Prentice-Hall, 1975.
[37] A. M. Soliman, "Applications of the current feedback operational amplifiers," *Analog Integrated Circuits Signal Processing*, vol. 14, pp. 265–302, 1996.
[38] M. P. Kennedy, "On the relationship between the chaotic Colpitts oscillator and Chua's oscillator," *IEEE Trans. Circuits Syst. I*, vol. 42, pp. 376–379, June 1995.
[39] G. M. Maggio, O. De Feo, and M. P. Kennedy, "Nonlinear analysis of the Colpitts oscillator and applications to design," *IEEE Trans. Circuits Syst. I*, vol. 46, pp. 1118–1129, Sept. 1999.
[40] R. Gilmore, "Topological analysis of chaotic dynamical systems," *Rev. Mod. Phys.*, vol. 70, pp. 1455–1529, 1998.
[41] A. S. Elwakil and M. P. Kennedy, "Improved implementation of Chua's chaotic oscillator using the current feedback op amp," *IEEE Trans. Circuits Syst. I*, vol. 47, pp. 76–79, Jan. 2000.
[42] ——, "Chua's circuit decomposition: A sinusoidal oscillator coupled to a voltage-controlled nonlinear resistor". *J. Franklin Inst*, to be published.
[43] ——, "Chaotic oscillators derived from Saito's double-screw hysteresis oscillator," *IEICE Trans. Fundamentals*, vol. E82, pp. 1769–1775, 1999.
[44] L. O. Chua, J. Yu, and Y. Yu, "Negative resistance devices," *Int. J. Circuit Theory Appl.*, vol. 11, pp. 161–185, 1983.
[45] A. S. Elwakil and M. P. Kennedy, "Systematic realization of a class of hysteresis RC chaotic oscillators," Int. J. Circuit Theory Appl., to be published.
[46] M. P. Kennedy and L. O. Chua, "Hysteresis in electronic circuits: A circuit theorist's perspective," *Int. J. Circuit Theory Appl.*, vol. 19, pp. 471–515, 1991.
[47] A. S. Elwakil and M. P. Kennedy, "Inductorless hyperchaos generator," *Microelectron. J.*, vol. 30, pp. 739–743, 1999.