Quantitative verification of weighted Kripke structures

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Context

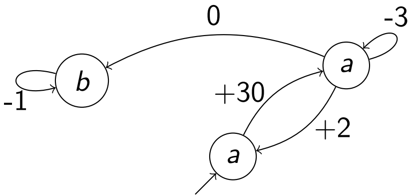
Model checking problem

- One or multi dimensional weighted Kripke structures.
- Extensions of temporal logics (LTL, CTL) with numerical constraints.
- Constraints about the sum of accumulated weights and the average of the weights seen in a path.

Syntax and semantic

Definition

$CTL(sum, avg) \ni \varphi, \phi := p|\alpha|\varphi \vee \varphi|\neg\varphi|EF\varphi|EG\varphi|E\phi U\varphi|EX\varphi$
with α a sum/avg constraint on one or more variables, and p an atomic proposition.

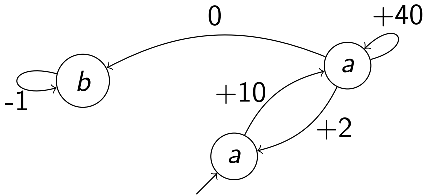


$$EF(sum(x) = 0 \wedge \neg EX(b \wedge sum(x) > 0) \wedge EF(sum(x) < 0))$$

Syntax and semantic, bis

Definition

$LTL(sum, avg) \ni \varphi, \phi := p|\alpha|\varphi \vee \varphi|\neg\varphi|F\varphi|G\varphi|\phi U\varphi|X\varphi$
with α a sum/avg constraint on one or more variables, and p an atomic proposition.



$$G(a \Rightarrow sum(x) \geq 0) \wedge F(b \wedge sum(x) < 0 \wedge \neg F(avg(x) > 30))$$

Temporal specifications with accumulative values

In [**DG09**], Demri & al. looked into LTL with sum constraints as atoms in the 1-dimensional case.

In [**BCHK11**], Boker & al., different temporal logics on multi-dimensional weighted Kripke structure with sums and avg constraints.

Multi-weighted decidable fragment : EF

Definition (EF^{Σ} fragment (multi-dimensional))

$\varphi, \phi \;:=\; \alpha | p | \neg\varphi | \phi \vee \varphi | EF(\varphi)$
with α a numerical assertion and p an atomic proposition.

Theorem ([BCHK11])

EF^{Σ} model checking is decidable

The proof is a reduction toward Presburguer arithmetic and yields a 4-EXPTIME algorithm (one for the transformation and 3 for P. Arithmetic).

Results with only sum assertions

Theorem (In the 1-dimensional case)

CTL(sum only) model checking is EXPSPACE-complete for binary storage and PSPACE-complete for unary storage.

We pass the assertions from the formula to the structure, then use the results for $\mu - calculus$ on one-counter automata. Lower bound come from the result of [**GHOW10**].

Theorem (In the 1-dimensional case, [DG09])

LTL(sum only) model checking is PSPACE-complete for both kind of storage.

Results with average assertions

Theorem

Model checking of CTL(avg only) and LTL(avg only) over 1-dimensional weighted Kripke structure is undecidable.

One can encode the 2-counter machines halting problem into the model checking of a formula (namely $E\psi U\phi$ with ψ a boolean combinaison of propositions and averages assertions, ϕ a proposition).

The length of the path is used to encode the counters :
n$=2^{c1} * 3^{c2} * 5^{a}$ -for some $a \in \aleph$-.

Structure used in the undecidability proof

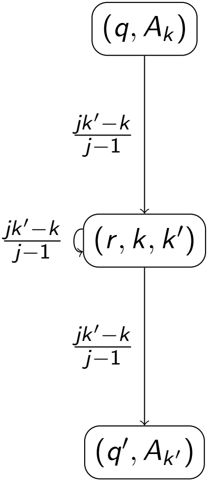


Figure:
Updating counters

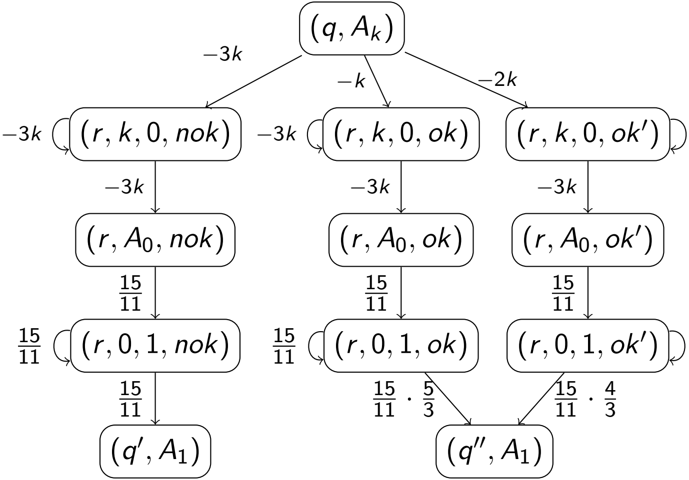


Figure: Testing counters (here c_2)

Logics with several weights, a decidable fragment

Definition (Multi-dimensional)

$FlatWLTL \ni \phi := p|\alpha|\phi \vee \phi|\phi \wedge \phi|X\phi|\ G\psi|\psi U\phi$
With $\psi \in LTL$

Such logic model checking is decidable, and yield an NEXPTIME algorithm :

- non deterministically order the numerical objectives
- to find a path following the non-deterministic choices, we build a sequence of automaton with the classical method going from LTL to a sequence of Buchi automaton.
- Find an appropriate path in the sequence of Buchi automaton by a reduction toward existential Presburger arithmetic.

Weighted ATL, ATL*

We are working on the same kind of extension for ATL and ATL*, multi-agent coalition logics.

Already developed extensions mainly focus on quantifying the amount of resource a strategy can use, but may lack expressiveness of the variation of the resource level.

The quantifications on ATL/ATL* revoke previous strategies for other agents, this limit greatly the problems we can model in ATL/ATL*, looking into ATL with strategy context (that deal with such problem) in the 1-variable case may be of interest.

Thank you !