STRIPE-BASED MSE CONTROL IN IMAGE CODING

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ABSTRACT

It is well known that compression of very large images (e.g. medical imaging, microscopy, satellite images) requires stripe-based or tiling processing. In some applications, the transmission of compressed data is performed through a rate constrained channel. Thus, rate allocation and control procedures have to be used to fit the channel characteristics. On the contrary, most of applications require high quality image coding without any real time rate constraint (e.g. off-line compression for storage or for broadcasting over IP, ADSL, HDTV...). Therefore, we propose a new stripe-based compression algorithm based on quality control. Our method computes first an optimal subband MSE allocation and then, the corresponding quantization steps. The proposed algorithm provides both the accurate local MSE control and a global rate-distortion improvement when compared to a rate constrained compression scheme. Furthermore, it performs better than JPEG2000.

1. INTRODUCTION

The compression of very large images (e.g. medical imaging, microscopy, satellite images, professional photography) requires scan-based or tiling processing. In some applications, the transmission of compressed data is performed through a rate constrained channel. Thus, rate allocation and control procedures have to be used to fit the channel characteristics [1]. However, most of applications require high quality image coding without any real time rate constraint (e.g. off-line compression for storage or for broadcasting over IP, ADSL, HDTV...) since they use adapted buffers for streaming.

In this paper, we propose a new stripe-based compression algorithm based on quality control. Our algorithm ensures a local image quality control using a novel mean squared error (MSE) allocation and control procedure.

In section 2, we present the new stripe-based compression scheme. Section 3 is devoted to the MSE allocation and regulation procedures used to control the local image quality and section 4 shows some experimental results.

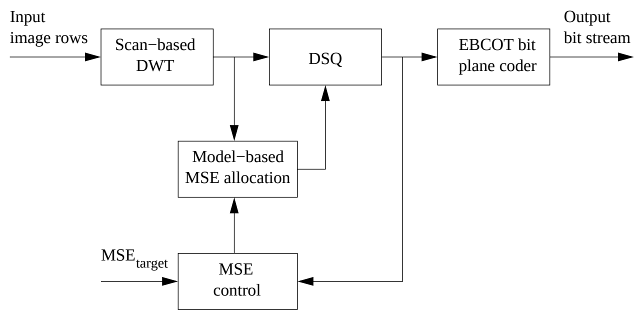

Fig. 1. Flow chart of the compression scheme.

2. GENERAL COMPRESSION SCHEME

Fig. 1 shows the flow chart of our stripe-based compression scheme. The first step of the method is the scan-based or tiled computation of the image wavelet decomposition. In this paper we use scan-based DWT. The method can be straightforwardly adapted to tiling for JPEG 2000 [2]. Then, we use statistical information of the wavelet coefficients to compute an optimized model-based MSE allocation. In the third step, we apply optimal Deadzone Scalar Quantizers (DSQ) in each subband [3]. Finally, the quantized wavelet coefficients are lossless entropy encoded using EBCOT's bit plane context-based arithmetic coder [4]. This compression scheme is modular and once the MSE allocation has been computed, the quantization and encoding of subbands are processed separately and can be performed concurrently for each subband.

We have selected the 9-7 biorthogonal wavelet transform [5] which is known to be almost orthogonal and gives the best results for dyadic sampled images [6]. We use a three level decomposition DWT and compute it on-the-fly to avoid blocking artifacts introduced by spatial tiling. Further information on scan-based wavelet transform computation are available in [7, 8, 9, 10].

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3. MODEL-BASED MSE ALLOCATION

3.1. Principle of MSE allocation

The purpose of MSE allocation is to determine the quantizers in each subband which minimize the total bitrate for a given output MSE. The 9-7 biorthogonal filter bank is nearly orthogonal. Therefore, the mean squared error between the original image and the decoded one can be computed by a weighted sum of the mean squared quantization errors undergone in each subband. We have:

$$MSE_{output} = \sum_{i=1}^{\#SB} \pi_i \sigma_{Q_i}^2 \tag{1}$$

with $\#SB$ the number of subbands, $\sigma_{Q_i}^2$ the mean squared quantization error for subband i and $\{\pi_i\}$ the weights used to take account of the non-orthogonality of the filter bank [11]. The output bitrate can be expressed as the following weighted sum:

$$R_{output} = \sum_{i=1}^{\#SB} a_i R_i \tag{2}$$

with R_i the output bitrate for subband i and a_i the weight of subband i in the total bitrate (a_i is the ratio of the size of subband i divided by the size of the image).

The subband quantizers are scalar quantizers with optimized deadzone size. They are defined by the size of their zero quantization bin z and the size of the other quantization bins q. Therefore, the solution of our constrained problem is obtained thanks to Lagrangian operators by minimizing the following criterion:

$$J = \sum_{i=1}^{\#SB} a_i R_i\left(z_i, q_i\right) + \lambda \left(\sum_{i=1}^{\#SB} \pi_i \sigma_{Q_i}^2 \left(z_i, q_i\right) - D_T \right) \tag{3}$$

where D_T denotes the target output MSE and both R and σ_Q^2 depend on the quantization step and the deadzone size we are looking for.

3.2. Rate and distortion models

An efficient way to minimize (3) without pre-quantizing subbands is to model the distribution $p(x)$ of the wavelet coefficients and use theoretical models for both distortion and bitrate. In each subband, the probability density function of the wavelet coefficients can be approximated with a generalized Gaussian [5]. Therefore, we have

$$p(x) = a e^{-|bx|^\alpha} \tag{4}$$

with $b = \frac{1}{\sigma}\sqrt{\frac{\Gamma(3/\alpha)}{\Gamma(1/\alpha)}}$ and $a = \frac{b\alpha}{2\Gamma(1/\alpha)}$. We also assume that wavelet coefficients are independent and identically distributed (i.i.d.) in each subband.

We use $\Pr(m)$, the probability of the quantization level m to compute the bitrate produced by the deadzone scalar quantizer $\{z, q\}$. We have $\Pr(m) = \int_{\frac{z}{2}+(|m|-1)q}^{\frac{z}{2}+|m|q} p(x)\,dx$ for $m \neq 0$ and $\Pr(0) = \int_{-\frac{z}{2}}^{+\frac{z}{2}} p(x)\,dx$.

Then, the bitrate is approximated by the entropy of the output quantization levels:

$$R = -\sum_{m=-\infty}^{+\infty} \Pr(m) \log_2 \Pr(m) \tag{5}$$

According to [12], the best decoding value for the quantization level m is the centroid of its quantization bin $\hat{x}_m = \frac{\int_{\frac{z}{2}+(m-1)q}^{\frac{z}{2}+mq} xp(x)dx}{\Pr(m)}$ for $m > 0$, $\hat{x}_m = -\hat{x}_{|m|}$ for $m < 0$ and $\hat{x}_0 = 0$. Then, the mean squared quantization error is

$$\begin{aligned} \sigma_Q^2 = \; & \int_{-\frac{z}{2}}^{+\frac{z}{2}} x^2 p(x)\,dx \\ & + 2\sum_{m=1}^{+\infty} \int_{\frac{z}{2}+(m-1)q}^{\frac{z}{2}+mq} \left(x - \hat{x}_m\right)^2 p(x)\,dx. \end{aligned} \tag{6}$$

For generalized Gaussians, we can show that the bitrate depends only on the shape parameter α and the ratios $\frac{z}{\sigma}$ and $\frac{q}{\sigma}$. We have $R = R\left(\alpha, \frac{z}{\sigma}, \frac{q}{\sigma}\right)$. We can also show that the quantization distortion can be written as $\sigma_Q^2 = \sigma^2 D\left(\alpha, \frac{z}{\sigma}, \frac{q}{\sigma}\right)$ where the function D depends only on α and the ratios $\frac{z}{\sigma}$ and $\frac{q}{\sigma}$.

Therefore, (3) becomes

$$\begin{aligned} J = \; & \sum_{i=1}^{\#SB} a_i R\left(\alpha_i, \frac{z_i}{\sigma_i}, \frac{q_i}{\sigma_i}\right) \\ & + \lambda \left(\sum_{i=1}^{\#SB} \pi_i \sigma_i^2 D\left(\alpha_i, \frac{z_i}{\sigma_i}, \frac{q_i}{\sigma_i}\right) - D_T \right) \end{aligned} \tag{7}$$

3.3. Solution

Let f be any function and x_k its k^{th} variable and define $\frac{\partial f}{\partial x_k}$ as the derivative of f with respect to its k^{th} variable. To find the minimum of J, we differentiate it with respect to z_i, q_i and λ. This provides the three following equations:

$$a_i \frac{\partial R}{\partial x_2}\left(\alpha_i, \frac{z_i}{\sigma_i}, \frac{q_i}{\sigma_i}\right) + \lambda \pi_i \sigma_i^2 \frac{\partial D}{\partial x_2}\left(\alpha_i, \frac{z_i}{\sigma_i}, \frac{q_i}{\sigma_i}\right) = 0 \quad (8a)$$

$$a_i \frac{\partial R}{\partial x_3}\left(\alpha_i, \frac{z_i}{\sigma_i}, \frac{q_i}{\sigma_i}\right) + \lambda \pi_i \sigma_i^2 \frac{\partial D}{\partial x_3}\left(\alpha_i, \frac{z_i}{\sigma_i}, \frac{q_i}{\sigma_i}\right) = 0 \quad (8b)$$

$$\sum_{i=1}^{\#SB} \pi_i \sigma_i^2 D\left(\alpha_i, \frac{z_i}{\sigma_i}, \frac{q_i}{\sigma_i}\right) - D_T = 0 \quad (8c)$$

Thus, the quantizers parameters $\{z_i, q_i\}$ we are looking for must verify the following system of $2 \times \#SB + 1$ equations and $2 \times \#SB + 1$ unknowns:

$$\frac{\frac{\partial D}{\partial x_2}\left(\alpha_i, \frac{z_i}{\sigma_i}, \frac{q_i}{\sigma_i}\right)}{\frac{\partial R}{\partial x_2}\left(\alpha_i, \frac{z_i}{\sigma_i}, \frac{q_i}{\sigma_i}\right)} = \frac{\frac{\partial D}{\partial x_3}\left(\alpha_i, \frac{z_i}{\sigma_i}, \frac{q_i}{\sigma_i}\right)}{\frac{\partial R}{\partial x_3}\left(\alpha_i, \frac{z_i}{\sigma_i}, \frac{q_i}{\sigma_i}\right)} \quad (9a)$$

$$\frac{\frac{\partial D}{\partial x_2}\left(\alpha_i, \frac{z_i}{\sigma_i}, \frac{q_i}{\sigma_i}\right)}{\frac{\partial R}{\partial x_2}\left(\alpha_i, \frac{z_i}{\sigma_i}, \frac{q_i}{\sigma_i}\right)} = -\frac{a_i}{\lambda \pi_i \sigma_i^2} \qquad (9b)$$

$$\sum_{i=1}^{\#SB} \pi_i \sigma_i^2 D\left(\alpha_i, \frac{z_i}{\sigma_i}, \frac{q_i}{\sigma_i}\right) = D_T \qquad (9c)$$

The solution of Eq. $(9a)$ provides the optimal relation between z_i and q_i for the MSE allocation procedure. We get $\frac{z_i}{\sigma_i} = g_{\alpha_i}\left(\frac{q_i}{\sigma_i}\right)$ with g_{α_i} the function which provides the solution of $(9a)$. Inserting this in Eq. $(9b)$ and $(9c)$ gives

$$h_{\alpha_i}\left(\frac{q_i}{\sigma_i}\right) = \frac{\frac{\partial D}{\partial x_3}\left(\alpha_i, g_{\alpha_i}\left(\frac{q_i}{\sigma_i}\right), \frac{q_i}{\sigma_i}\right)}{\frac{\partial R}{\partial x_3}\left(\alpha_i, g_{\alpha_i}\left(\frac{q_i}{\sigma_i}\right), \frac{q_i}{\sigma_i}\right)} = -\frac{a_i}{\lambda \pi_i \sigma_i^2} \quad (10a)$$

$$\sum_{i=1}^{\#SB} \pi_i \sigma_i^2 D\left(\alpha_i, g_{\alpha_i}\left(\frac{q_i}{\sigma_i}\right), \frac{q_i}{\sigma_i}\right) = D_T. \quad (10b)$$

The function h is introduced in $(10a)$ in order to simplify the notations. Therefore, the quantization steps q_i can be found with the following two equations:

$$h_{\alpha_i}^{-1}\left(-\frac{a_i}{\lambda \pi_i \sigma_i^2}\right) = \frac{q_i}{\sigma_i} \qquad (11)$$

$$\sum_{i=1}^{\#SB} \pi_i \sigma_i^2 \quad D\left(\alpha_i, g_{\alpha_i}\left(h_{\alpha_i}^{-1}\left(-\frac{a_i}{\lambda \pi_i \sigma_i^2}\right)\right), h_{\alpha_i}^{-1}\left(-\frac{a_i}{\lambda \pi_i \sigma_i^2}\right)\right) = D_T \qquad (12)$$

where Eq. (12) depends only on λ and the set of equations (11) provides the optimal quantization steps once λ has been found with Eq. (12).

Unfortunately, there is no analytical formula for h^{-1}. To clear up this difficulty, we plot the parametric curve $\left[D\left(\frac{q}{\sigma}\right); h\left(\frac{q}{\sigma}\right)\right]$ for α fixed. It results from (11) that it is equivalent to the curve $\left[D\left(h^{-1}\left(-\frac{a_i}{\lambda \pi_i \sigma_i^2}\right)\right); -\frac{a_i}{\lambda \pi_i \sigma_i^2}\right]$. Therefore, the proposed MSE allocation procedure is the following:

1. Lambda is given. For each subband i, compute $\ln\left(\frac{a_i}{\lambda \pi_i \sigma_i^2}\right) = \ln(-h)$ and read the corresponding normalized mean squared error D_i from the curves shown in Fig. 2.

2. Compute $\left|\sum_{i=1}^{\#SB} \pi_i \sigma_i^2 D_i - D_T\right|$. If it is lower than a given threshold, the constraint (12) is verified and the current λ is optimal. Else, compute a new λ and go back to step 1.

3. For each subband i, compute $\ln\left(\frac{a_i}{\lambda \pi_i \sigma_i^2}\right) = \ln(-h)$ with the optimal λ and read $\frac{q_i}{\sigma_i}$ from the curves shown in Fig. 3. q_i is the optimal quantization step for subband i.

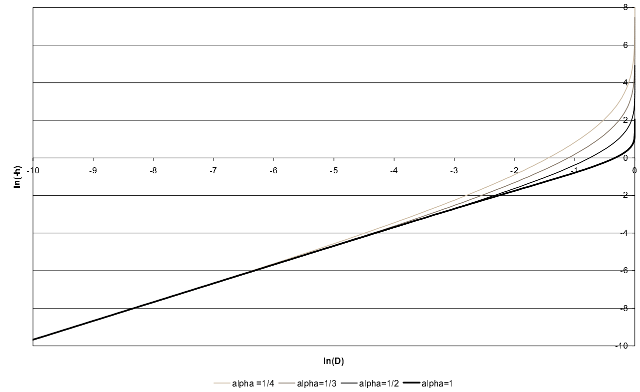

Fig. 2. Curves of $h(\frac{q}{\sigma})$ versus $D(\frac{q}{\sigma})$ for different shape parameters α of the generalized Gaussian distribution.

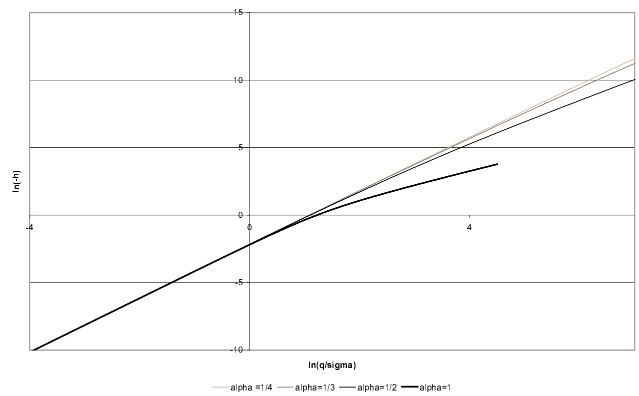

Fig. 3. Curves of h for different shape parameters α of the generalized Gaussian distribution.

4. For each subband i, read the optimal deadzone size from the curves shown in Fig. 4

4. EXPERIMENTAL RESULTS

We compare the proposed compression scheme with a similar compression scheme based on rate regulation. Both coders use optimized deadzone quantizers and EBCOT's bit plane context-based arithmetic coder. We apply both coders on *Woman* and *Txtur1* from the JPEG 2000 test images database [13]. From Fig. 5, we notice that for the same global bitrate, the rate regulation provides local PSNR variations of up to 12 dB. On the contrary, the proposed method allows an accurate local PSNR control with variations lower than 1 dB. Fig. 6 shows an improvement of about 0.2 dB for the image *Textur1* between the proposed controled MSE algorithm and a rate controled version of the same coder. For the image *Woman* coded at 2 bpp with stripes size of 8 image rows, we get an improvement of 1 dB. Therefore, it is

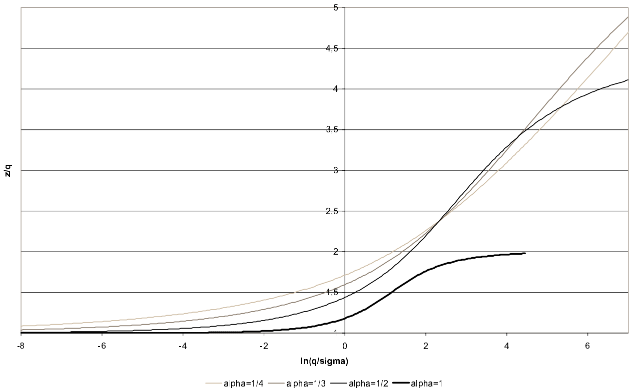

Fig. 4. Curves of the optimal $\frac{z}{q}$ ratio (solution of Eq. (9a))

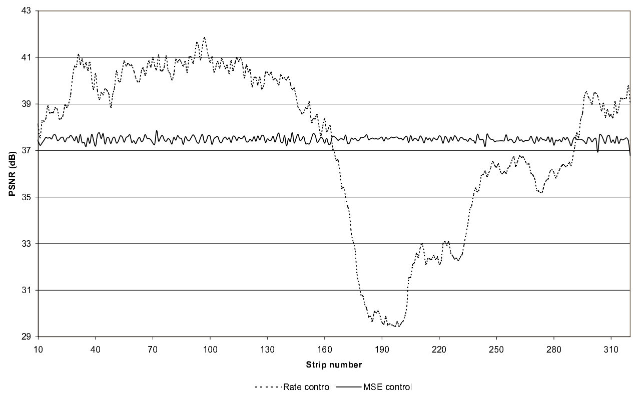

Fig. 5. PSNR of each stripe for *Woman*. Stripe size is 8 image rows. Global bitrate of 1 bpp in both cases.

better to perform MSE control than rate control when stripe-based processing is required. Furthermore, Fig. 6 shows that our stripe-based controled MSE image coder performs better than JPEG2000 (Verification Model 8.6 [13]) applied on the full image. Therefore, the proposed method provides local PSNR control and better global rate-distortion performances when compared to stripe-based rate controled algorithms or JPEG2000 applied on the full image.

5. CONCLUSION

The method proposed in this paper provides high efficiency local quality control. Furthermore, our simulations show a global rate-distortion improvement compared to stripe-based rate controled image coders or JPEG2000 applied on the full image. Therefore, our method is well adapted to all applications which require high quality with no real time rate constraint (off-line coders or real time acquisition systems with buffering).

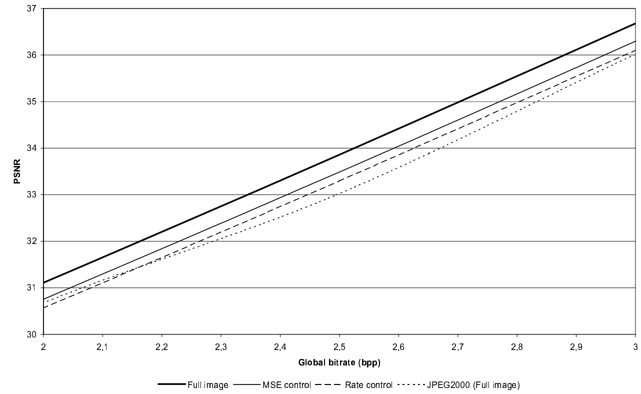

Fig. 6. Global PSNR versus bitrate performances of the proposed MSE control procedure compared to a rate controled version of our algorithm and JPEG2000 (VM8.6). Stripe size is 16 image rows. Image *Txtur1*.

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