Abstract

Existing linear methods [1, 2, 3] for estimating homographies, rely on coordinate normalization, to reduce the error in the estimated homography. Unfortunately, the estimated homography depends on the choice of the normalization. The proposed extension to the (linear) Taubin estimator is the perfect substitute for such methods, as it does not rely on coordinate normalization, and produces homographies whose error is consistent with existing methods. Also, unlike existing linear methods, the proposed Taubin estimator is theoretically unbiased, and unaffected by similarity transformations of the correspondences in the two views. In addition, it can be adapted to estimate other quantities such as trifocal tensors.

Homography

Given 2 images of a static scene, a homography describes a perspective mapping from one image to the other.

A homography is a non-singular 3x3 matrix that describes a linear mapping in 2D homogeneous coordinates.

\[
x'_i = \frac{h_{11}x_i + h_{12}y_i + h_{13}}{h_{31}x_i + h_{32}y_i + h_{33}};
y'_i = \frac{h_{21}x_i + h_{22}y_i + h_{23}}{h_{31}x_i + h_{32}y_i + h_{33}}.
\]

(1)

\[
x_i \approx \left[ \begin{array}{c} x \vspace{5pt} \vspace{5pt} y \end{array} \right]^\top; \quad y_i \approx \left[ \begin{array}{c} y \vspace{5pt} \vspace{5pt} 1 \end{array} \right]^\top
\]

(2)

The linear system of equations described by Eq.(1),Eq.(2) are referred to as the Direct Linear Transform [1].

Algebraic Least Squares Estimator

Given N > 4 sets of point correspondences \( \{x_i\}_{i=1}^N \) we estimate the homography relating the measurements, as the solution to the Rayleigh Quotient Problem [1]

\[
M_{LS} \approx \hat{h} = \lambda_{\min} \hat{h}
\]

The above RQP is minimized by the eigenvector corresponding to \( \lambda_{\min} \).

\[
M_{LS} \hat{h} = \lambda_{\min} \hat{h}
\]

(4)

Problem : The ALS estimator (eigenvectors of \( M_{LS} \)) is overly sensitive to noise in the measurements \( \{x_i\}_{i=1}^N \).

Cause : The large disparity in the column norms of the matrix \( M_{LS} \).

Remedy [1]: replace the raw pixel measurements in Eq.(3), with normalized measurements

\[
\left[ \begin{array}{c} x_i \\
N \vspace{5pt} y_i \end{array} \right] \approx \left[ \begin{array}{c} x_i/v_i \\
N \vspace{5pt} y_i/v_i \end{array} \right]
\]

(5)

Normalized ALS estimator [1]

\[
\min_{h} \lambda_{\min} \hat{h} = \lambda_{\min} \hat{h}
\]

(6)

\[
M_{LS} \hat{h} = \lambda_{\min} \hat{h}
\]

(7)

Problem : The estimated homography \( \hat{h} \) depends on \( \lambda \), which are inferred from noisy pixel measurements

Maximum Likelihood Estimator [4, 5, 1]

For zero-mean small gaussian noise, 
\( x_i = x + \epsilon \), \( E[\epsilon] = 0 \), \( \epsilon \sim \mathcal{N}(0, \Sigma) \),

the optimal homography (ML estimate) minimizes the sample error. In particular,

\[
\hat{h}_{ML} \approx \min \left( \lambda_{\min} \left( \sum_{i=1}^{N} E[\epsilon_i] \right) \right)
\]

(8)

Problem : iterative estimator that needs a reliable initial guess for convergence

Is there a linear method for estimating homographies that is

1. invariant to coordinate normalization
2. minimizes an approximation to the sample error for homographies? YES!!

Proposed extension to Taubin estimator [6]

The proposed estimator solves the RQP

\[
\min \lambda_{\min} \hat{h} = \lambda_{\min} \hat{h}
\]

(9)

\[
M_{LS} \hat{h} = \lambda_{\min} \hat{h}
\]

(10)

Properties:

1. minimizes an approximation to the sample error

\[
N_{LS} = \min \left( \sum_{i=1}^{N} \left( \lambda_{\min} \hat{h} \right) \right)
\]

2. invariant to coordinate normalization

\[
N_{LS} = \min \left( \sum_{i=1}^{N} \left( \lambda_{\min} \hat{h} \right) \right)
\]

The above RQP is minimized by the generalized eigenvector corresponding to

\[
M_{LS} \hat{h} = \lambda_{\min} N_{LS} \hat{h}
\]

(11)

Experimental Results

References