2013

VISUAL ATTITUDE PROPAGATION FOR SMALL SATELLITES

Samir Ahmed Rawashdeh

University of Kentucky, SAMIR.RAWASHDEH@GMAIL.COM

Recommended Citation
http://uknowledge.uky.edu/ece_etds/30

This Doctoral Dissertation is brought to you for free and open access by the Electrical and Computer Engineering at UKnowledge. It has been accepted for inclusion in Theses and Dissertations--Electrical and Computer Engineering by an authorized administrator of UKnowledge. For more information, please contact UKnowledge@lsv.uky.edu.
STUDENT AGREEMENT:

I represent that my thesis or dissertation and abstract are my original work. Proper attribution has been given to all outside sources. I understand that I am solely responsible for obtaining any needed copyright permissions. I have obtained and attached hereto needed written permission statements(s) from the owner(s) of each third-party copyrighted matter to be included in my work, allowing electronic distribution (if such use is not permitted by the fair use doctrine).

I hereby grant to The University of Kentucky and its agents the non-exclusive license to archive and make accessible my work in whole or in part in all forms of media, now or hereafter known. I agree that the document mentioned above may be made available immediately for worldwide access unless a preapproved embargo applies.

I retain all other ownership rights to the copyright of my work. I also retain the right to use in future works (such as articles or books) all or part of my work. I understand that I am free to register the copyright to my work.

REVIEW, APPROVAL AND ACCEPTANCE

The document mentioned above has been reviewed and accepted by the student’s advisor, on behalf of the advisory committee, and by the Director of Graduate Studies (DGS), on behalf of the program; we verify that this is the final, approved version of the student’s dissertation including all changes required by the advisory committee. The undersigned agree to abide by the statements above.

Samir Ahmed Rawashdeh, Student
James E. Lumpp, Jr., Major Professor
Cai-Cheng Lu, Director of Graduate Studies
VISUAL ATTITUDE PROPAGATION FOR SMALL SATELLITES

Dissertation

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the College of Engineering at the University of Kentucky

By
Samir A. Rawashdeh
Irbid, Jordan

Director: Dr. James E. Lumpp, Jr.,
Associate Professor of Electrical and Computer Engineering
Lexington, Kentucky
2013

Copyright © Samir A. Rawashdeh 2013
As electronics become smaller and more capable, it has become possible to conduct meaningful and sophisticated satellite missions in a small form factor. However, the capability of small satellites and the range of possible applications are limited by the capabilities of several technologies, including attitude determination and control systems. This dissertation evaluates the use of image-based visual attitude propagation as a compliment or alternative to other attitude determination technologies that are suitable for miniature satellites. The concept lies in using miniature cameras to track image features across frames and extracting the underlying rotation.

The problem of visual attitude propagation as a small satellite attitude determination system is addressed from several aspects: related work, algorithm design, hardware and performance evaluation, possible applications, and on-orbit experimentation. These areas of consideration reflect the organization of this dissertation.

A “stellar gyroscope” is developed, which is a visual star-based attitude propagator that uses relative motion of stars in an imager’s field of view to infer the attitude changes. The device generates spacecraft relative attitude estimates in three degrees of freedom. Algorithms to perform the star detection, correspondence, and attitude propagation are presented. The Random Sample Consensus (RANSAC) approach is applied to the correspondence problem to successfully pair stars across frames while mitigating false-positive and false-negative star detections. This approach provides tolerance to the noise levels expected in using miniature optics and no baffling, and the noise caused by radiation dose on orbit. The hardware design and algorithms are validated using test images of the night sky. The application of the stellar gyroscope as part of a CubeSat attitude determination and control system is described. The stellar gyroscope is used to augment a MEMS gyroscope attitude propagation algorithm to minimize drift in the absence of an absolute attitude sensor.
The stellar gyroscope is a technology demonstration experiment on KySat-2, a 1-Unit CubeSat being developed in Kentucky that is in line to launch with the NASA ELaNa CubeSat Launch Initiative. It has also been adopted by industry as a sensor for CubeSat Attitude Determination and Control Systems (ADCS).

KEYWORDS: Small Satellites, Attitude Determination, Egomotion Estimation, RANSAC, Image Processing
VISUAL ATTITUDE PROPAGATION FOR SMALL SATELLITES

By

Samir A. Rawashdeh

Dr. James E. Lumpp, Jr.
Director of Dissertation

Dr. Cai-Cheng Lu
Director of Graduate Studies

July 8th, 2013
To my mother, the epitome of kindness and courage,
My father, the epitome of dedication and diligence,
And to those that do their part in making the world a better place.
Acknowledgements

I would like to express my gratitude for the role my advisor Dr. James Lumpp has played in my professional and personal development. What I have learned from Dr. Lumpp directly and through the fantastic lab environment and opportunities he facilitated is limitless. I would also like to thank my PhD Committee Members, Dr. Suzanne Smith, Dr. Daniel Lau, and Dr. Sen-ching Samson Cheung for their invaluable advice and direction throughout the years.

The synergetic team environment at the Space Systems Laboratory made much of my work possible, I thank my colleagues at the Space Systems Laboratory over the years. In addition, our lab’s collaborators made many endeavors successful, including NASA, NASA Ames Research Center, NASA LSP and the ELaNa program, NASA EPSCoR, Morehead State University, the Kentucky Science and Technology Corporation, and the NASA Kentucky EPSCoR program and Space Grant Consortium. I also greatly value the collaboration with James Barrington-Brown and SSBV Space and Ground Systems in the United Kingdom.

My PhD would not have been possible without my family’s support and encouragement. Thank you, Diana, for being a wonderful wife and for facing the nocturnal wild life with me during data collection. And finally, my endless gratitude goes to my parents, brothers, sister, and their families, for being role models, advisors, friends, and the best family one could have.
## Table of Contents

ACKNOWLEDGEMENTS ........................................................................................................ III

LIST OF TABLES ................................................................................................................ VII

LIST OF FIGURES ............................................................................................................... VIII

CHAPTER 1 INTRODUCTION ............................................................................................. 1

1.1 NANOSATELLITES ...................................................................................................... 1
   1.1.1 The CubeSat Standard .................................................................................... 1
   1.1.2 Other Small Satellite Form Factors ................................................................. 3
   1.1.3 Challenges of Miniaturization ........................................................................ 4
   1.1.4 Traditional Attitude Determination Systems and their Scalability ................. 6

1.2 STELLAR GYROSCOPE ............................................................................................. 8
   1.2.1 Concept ........................................................................................................... 8
   1.2.2 Motivation for Visual Attitude Propagation ................................................... 9
   1.2.3 Stellar Gyroscope Challenges ....................................................................... 10

1.3 PROBLEM STATEMENT .......................................................................................... 11

CHAPTER 2 BACKGROUND ............................................................................................ 13

2.1 REFERENCE FRAMES ............................................................................................ 13

2.2 ASTRODYNAMICS AND ATTITUDE REPRESENTATIONS ..................................... 14
   2.2.1 Direction Cosine Matrix and Euler Angles .................................................. 15
   2.2.2 Eigen Axis rotations ...................................................................................... 16
   2.2.3 Euler Symmetric Parameters (Quaternions) ................................................ 17

2.3 RELEVANT ASTRONOMY CONCEPTS .................................................................. 17
   2.3.1 Stellar Magnitudes ........................................................................................ 17
   2.3.2 Stellar Parallax ............................................................................................. 19
   2.3.3 Proper Motion ............................................................................................... 20

2.4 RANDOM SAMPLE CONSENSUS .......................................................................... 20

2.5 RELATED WORK ..................................................................................................... 22
   2.5.1 Star Trackers/Mappers ................................................................................. 23
   2.5.2 Related work on Random Sample Consensus ............................................... 23
   2.5.3 Ego-motion Estimation .................................................................................. 24
CHAPTER 3 STELLAR GYROSCOPE ALGORITHM ........................................... 27
3.1 STAR DETECTION ............................................................................................................................... 27
  3.1.1 Camera Modeling .................................................................................................................. 27
  3.1.2 Centroiding .......................................................................................................................... 28
  3.1.3 Filtering to Reduce False Positives ....................................................................................... 29
3.2 ROTATION ESTIMATION USING Q-METHOD ................................................................................. 31
3.3 CORRESPONDENCE BY RANDOM SAMPLE CONSENSUS (RANSAC) ......................................... 33
  3.3.1 RANSAC Considerations for 3DOF Rotation ........................................................................ 33
  3.3.2 RANSAC Implementation for 3DOF Rotation ....................................................................... 34
  3.3.3 Parameters and Algorithm .................................................................................................. 37
  3.3.4 Lens Distortion Correction .................................................................................................. 40

CHAPTER 4 PERFORMANCE EVALUATION......................................................................................... 44
4.1 EMBEDDED CAMERA DESIGN ................................................................................................. 44
4.2 SPIN TABLE DESIGN .................................................................................................................... 45
4.3 ALGORITHM DEVELOPMENT USING NIGHT-SKY TESTS ....................................................... 47
  4.3.1 Experiment Setup and Collected Data .................................................................................. 47
  4.3.2 Alternative Hypothesis Testing Approach .......................................................................... 47
  4.3.3 Hypothesis Generation Guided by Proximity ...................................................................... 48
  4.3.4 Experiment Results .............................................................................................................. 48
4.4 NIGHT SKY TESTS OF EMBEDDED HARDWARE ......................................................................... 52
  4.4.1 Experiment Setup and Collected Data .................................................................................. 52
  4.4.2 Improved Hypothesis Generation and Consensus Test ...................................................... 52
  4.4.3 Analysis of Detected Star Magnitudes .................................................................................. 53
  4.4.4 Nomenclature of Result Metrics ......................................................................................... 55
  4.4.5 Analysis of Panning Motion ............................................................................................... 56
  4.4.6 Analysis of Spinning Motion ............................................................................................. 64
  4.4.7 Analysis of Motion Blur ....................................................................................................... 70
4.5 OPERATION IN ORBITAL ENVIRONMENT ................................................................................... 72

CHAPTER 5 APPLICATIONS ................................................................................................................. 74
5.1 DRIFT CONTROL IN MEMS GYROSCOPE INTEGRATION .......................................................... 74
5.2 ROBUST STAR DETECTION AND RELIABILITY FOR LONG MISSION DURATIONS .... 78
5.3 DISCUSSION .............................................................................................................................. 78
  5.3.1 Camera Alignment ............................................................................................................. 78
  5.3.2 Random Number Generation ............................................................................................ 79
  5.3.3 Operation Limits ................................................................................................................ 79

CHAPTER 6 SYSTEM IMPLEMENTATIONS ......................................................................................... 81
  6.1 DESIGN FACTORS FOR STAR IMAGING ............................................................................. 81
  6.2 KYSAT-2 ................................................................................................................................. 82
    6.2.1 Satellite and Mission Overview ....................................................................................... 82
    6.2.2 Concept of Operations ...................................................................................................... 86
  6.3 SSBV CUBESat ADCS SYSTEM ......................................................................................... 88
    6.3.1 SSBV System Overview ................................................................................................. 88
    6.3.2 Experiment on TechDemoSat-1 ..................................................................................... 90

CHAPTER 7 CONCLUSION ............................................................................................................... 92

REFERENCES ..................................................................................................................................... 96

VITA .................................................................................................................................................. 102
List of Tables

Table 2-1: Summary of Attitude Representations .................................................. 15
Table 2-2: Apparent magnitudes of celestial objects in the visual spectrum. Star magnitude values retrieved from Ref. [44] ........................................................................ 18
Table 4-1: Specifications Summary of Camera Hardware. ..................................... 44
Table 4-2: Summary of stellar gyroscope algorithm performance in point-and-shoot camera night sky test. ................................................................. 51
Table 4-3: Precision and accuracy of stellar gyroscope from night sky tests – panning motion........................................................................................................ 63
Table 4-4: Precision and accuracy of stellar gyroscope from night sky tests – spinning motion.................................................................................................... 70
Table 6-1: Summary of stellar gyroscope hardware of SSBV ADCS experiment .... 91
List of Figures

Figure 1-1: CubeSat satellite trend (1U, 1.5U, 2U and 3U), for satellites known as of July 2013. Data compiled from Ref [7]........................................................................................................... 2

Figure 1-2: KySat-1 (top) is a 1-U CubeSat (measuring 10x10x10 cm$^3$) developed in Kentucky, launched on the NASA Glory Mission [11]. CAD drawing and 3D-printed model of KySat-2 in the deployed state (bottom), another 1-U CubeSat currently under development. ...................................................................................................... 3

Figure 1-3: A set of images of the Cassiopeia constellation. The stars appear to be panning and rotating in a way that indicates a unique attitude maneuver of the camera in three degrees of freedom.......................................................................................... 9

Figure 2-1: ECEF, ECI, and body-fixed reference frames ............................................... 14

Figure 2-2: Illustration of star annual parallax angle ($P$) .................................................. 19

Figure 2-3: Example data set of a line with a large number of outliers (left). Fitted line using RANSAC (right), where the outliers are rejected and do not contribute towards the line fit. Image credit: Ref [46]. ............................................................................. 21

Figure 3-1: Camera Model. A celestial shell is mapped onto the image plane. ............... 27

Figure 3-2: Resolving star vector from pixel coordinates. ............................................... 28

Figure 3-3: Close up of Gienah ($\epsilon$ Cygni, magnitude 2.48) as detected in a night test using the prototype hardware described in Section 4.1 (top), and illustration of the centroiding process (bottom)..................................................................................... 30

Figure 3-4: Image filter kernel used to maximize dim star detection and minimizing false positive detections. ................................................................................................................................. 31

Figure 3-5: Star field image overlaid by star detections in five consecutive frames using a handheld point-and-shoot camera, with a three degree rotation between each frame. This figure illustrates the tracking challenge where the data consists of reliable stars, false positives, and false negatives. Colors are adjusted for clarity. .......................... 34

Figure 3-6: Stellar gyroscope algorithm flow chart ........................................................ 36

Figure 3-7: Recipe (in MATLAB syntax) for the RANSAC application to establish correspondences between vector pairs over and underlying rotation in three degrees
of freedom. ................................................................................................................. 39

Figure 3-8: Recipe (in MATLAB syntax) for the rotation matrix hypothesis generation for the RANSAC implementation. The search is guided by similarity in brightness. 40

Figure 3-9: Photos of the calibration checkerboard (top), and a diagram (bottom) depicting the camera on the left and the registered checkerboard pattern from the calibration photo set. ........................................................................................................... 41

Figure 3-10: The calculated distortion model for an S-Mount 16mm lens....................... 42

Figure 4-1: Prototype models of embedded Linux board and Camera assembly. ......... 45

Figure 4-2: Spin Table photo and design diagram............................................................ 46

Figure 4-3: Processing and star pairing of two images 9.98439° apart. ......................... 49

Figure 4-4: Processing and star pairing of two images 24.960975° apart. ..................... 49

Figure 4-5: Processing and star pairing of two images 44.929755° apart. ..................... 50

Figure 4-6: Camera response to two stars of apparent magnitudes of 2.48 and 4.92. HD 198134 is near the noise floor, and was reliably acquired using the filtering approach described................................................................................................................................... 54

Figure 4-7: Illustration of detected stars in first photo set of the Cygnus constellation. A photo was taken every minute as Earth rotated in inertial space, every color represents star detections in a single photo and star apparent magnitudes are marked. .................................................................................................................................... 54

Figure 4-8: Star pairing of two star field images 0.2507° apart (top), and the projection error plot (bottom). 19 photo pairs like this were analyzed. ............................... 57

Figure 4-9: Star pairing of two star field images 0.5014° apart (top), and the projection error plot (bottom). 18 photo pairs like this were analyzed. ............................... 58

Figure 4-10: Star pairing of two star field images 1.0027° apart (top), and the projection error plot (bottom). 16 photo pairs like this were analyzed. ............................... 59

Figure 4-11: Star pairing of two star field images 2.005476° apart (top), and the projection error plot (bottom). 12 photo pairs like this were analyzed. ............................... 60

Figure 4-12: Star pairing of two star field images 2.506844° apart (top), and the projection error plot (bottom). 10 photo pairs like this were analyzed. ............................... 61

Figure 4-13: Projection Error for all analyzed photo pairs – panning motion.................. 62

Figure 4-14: Consensus Set Size and Mean Projection Error for analyzed rotation angles
Figure 4-15: Star pairing of two star field images 0.9984375° apart (top), and the projection error plot (bottom). 6 photo pairs like this were analyzed. .......................... 65
Figure 4-16: Star pairing of two star field images 4.99921875° apart (top), and the projection error plot (bottom). 13 photo pairs like this were analyzed. ................... 66
Figure 4-17: Star pairing of two star field images 9.9984375° apart (top), and the projection error plot (bottom). 9 photo pairs like this were analyzed. ...................... 67
Figure 4-18: Star pairing of two star field images 19.9968749° apart (top), and the projection error plot (bottom). 6 photo pairs like this were analyzed. ...................... 68
Figure 4-19: Projection Error for all analyzed photo pairs – spinning motion. .............. 69
Figure 4-20: Consensus Set Size and Mean Projection Error for analyzed rotation angles – spinning motion. .................................................................................. 69
Figure 4-21: Acquisition of 61 Cygni (magnitude 5.21) at various speeds of the spin table. The blue square indicates the calculated centroid for the star. ................. 71
Figure 4-22: Simulated image of Ursa Minor, which includes the North Star, using the SKY2000 Star Catalog. ....................................................................................... 72
Figure 5-1: Simulink® model of the Attitude and Orbit propagator, as well as the models for the MEMS rate gyroscopes and the stellar gyroscope. ......................... 75
Figure 5-2: Euler angles representation of attitude difference between estimated and actual attitudes for an unassisted MEMS gyroscope rate integrator. Beginning with perfect knowledge, the plot illustrates attitude loss in eclipse. ................... 77
Figure 5-3: Euler angles representation of attitude difference between estimated and actual attitudes for a MEMS gyroscope rate integrator assisted by a stellar gyroscope. The stellar gyroscope resets the drift every 10 seconds. .......................... 77
Figure 6-1: Side view of KySat-2 internal components (top). Close up of the camera structure (bottom left). KySat-2 in the deployed configuration (bottom right). ....... 83
Figure 6-2: Simulation of the expected orbit of KySat-2 and the expected camera view at various positions in orbit. Passive magnetic attitude stabilization aligns the camera axis with the magnetic field direction. ......................................................... 85
Figure 6-3: Steps of the filtering and file size reduction approach of star field images on the KySat-2 mission. ................................................................................. 87
Figure 6-4: Top: photos of the CubeSat ADCS board of both faces with daughterboard installed (prototype hardware with test connectors). Bottom: block diagram of the overall system. Photos courtesy of: SSBV Space and Ground Systems, United Kingdom.

Figure 6-5: Camera assembly and SSBV CubeSat ADCS experiment on TechDemoSat-1. Photos courtesy of SSBV Space and Ground Systems, United Kingdom.
Chapter 1 Introduction

This chapter provides an introduction to relevant concepts to motivate the work in this dissertation. This dissertation discusses the application of imaging to the problem of satellite attitude determination. The motivation at the time of the writing arises in the trend in satellite miniaturization and the challenges that occur in attitude determination systems, as well as the trend in camera miniaturization. The commercially available processing capabilities, imaging sensors, and optics make attitude determination approaches based on image processing feasible.

1.1 Nanosatellites

Nanosatellites are spacecraft that are below 10kg in mass. Small spacecraft technology has been shown to be capable of carrying out a wide range of missions and scientific experiments. Small satellites are also considered to be suitable platforms for low-cost experiments for high-risk short-term missions [1, 2, 3, 4, 5, 6].

1.1.1 The CubeSat Standard

The CubeSat Standard was developed by Stanford University and California Polytechnic State University (CalPoly), originally as a means to deploy sub-satellites from a larger mother satellite, namely the OPAL satellite which was launched in the year 2000 [7]. A standard CubeSat measures 10x10x10 cm³, and the classic CubeSat deployer allows two or three cubes to be “stacked” to construct larger 2-Unit and 3-Unit CubeSats. There are many CubeSat deployers (or Launch Vehicle Interfaces) currently available, and some allow larger combinations of CubeSat Units, such as a 6-U and a 12-U [8].

The standardized system and isolation between the satellite and launch vehicle via the standardized deployer has had wide success. Launch vehicles manifest a deployer, and CubeSats must adhere to the deployer requirements. This split in interfaces allows launch vehicle designers to proceed independently of the CubeSats’ development schedule, and vice versa.
The wide adoption of the CubeSat standard increased the availability of satellite launches and opened space exploration to smaller organizations, in particular university student teams that would not otherwise have the opportunity to build, launch, and operate spacecraft [10, 11, 12]. Figure 1-1 shows the number of CubeSats that are known to have launched since 2003, and CubeSats currently in development with expected future launch dates. The data was compiled in July 2013 from reference [9], and future launch projections will likely vary with launch delays and new CubeSat missions.

Figure 1-2 shows two examples of 1-U CubeSats developed in Kentucky. The small size of the CubeSat imposes substantial mass, volume, and power constraints. This drives designers away from 1-Unit CubeSats towards 2-U and 3-U CubeSats, as well as the 6-U (organized in a 2 by 3 arrangement) form factor. The utility of small satellites remains limited until novel alternatives are presented that are of low mass, volume, and power requirements that can match the capabilities of larger spacecraft. Attitude Determination and Control Systems are a notable aspect in need of novel approaches to increase the utility of small satellites.
Figure 1-2: KySat-1 (top) is a 1-U CubeSat (measuring 10x10x10 cm$^3$) developed in Kentucky, launched on the NASA Glory Mission [13]. CAD drawing and 3D-printed model of KySat-2 in the deployed state (bottom), another 1-U CubeSat currently under development.

1.1.2 Other Small Satellite Form Factors

CubeSats are classified as Nanosatellites (1kg – 10kg). Microsatellites (10kg – 100kg) have been used extensively since the beginning of space exploration. Sputnik in 1956 (83.6 kg) is considered to be a microsatellite. A currently popular mechanism for the launch of microsatellites on the Evolved Expendable Launch Vehicles (EELV) Atlas V and Delta IV rockets is the ESPA ring (EELV Secondary Payload Adapter). The ESPA
ring can accommodate 6 microsatellites as secondary satellites on a launch vehicle. A notable example is the lunar LCROSS mission, where all the ports on an ESPA ring were fitted with equipment to turn the upper stage of an Atlas V Centaur rocket into a spacecraft to search for water ice in a permanently shaded region at the lunar pole [14].

A few Picosatellite (100g – 1kg) standards have been proposed, such as the PocketQub which is approximately 1/8th of a 1-U CubeSat. Single board satellites have been proposed as well as Femtosatellites (10-100g), namely the “MatchbookSat”. In terms of utility, picosatellites and femtosatellites are severely limited in the possible applications, partly because power generation is limited, and few attitude control schemes are conceivable.

To provide an overview of the satellite miniaturization trend and the challenges associated with it, this chapter overviews common attitude determination and control systems on recent missions and introduces the image-based attitude propagation approach.

### 1.1.3 Challenges of Miniaturization

As described earlier, miniaturization of spacecraft faces several challenges that limit the range of missions that can be done using a small spacecraft. There is significant room for innovation and technology development in the following aspects:

- **Power Generation**: Small satellites have a limited surface area limiting the amount of power that can be generated using solar panels. Higher efficiency solar cells and lightweight deployable structures to increase the available surface area are possible approaches to address power generation on small spacecraft.

- **Processing**: Field-Programmable Gate Arrays (FPGA) provide superior data processing capabilities at the cost of high power consumption. Low-power FPGA technologies and low-power high-performance microprocessor systems are possible solutions to support highly capable small satellites.

- **Radiation tolerance**: Long-duration and inter-planetary satellite missions must endure the expected radiation dose. Radiation shielding using heavy metals is infeasible
from a satellite miniaturization standpoint. Possible solutions include software single-event-upset mitigation techniques, redundant systems, and coating.

- **Communications:** Communication systems face numerous challenges on board a CubeSat, including limited available power, volume, and surface area for antennae. There is significant room for improvement and several efforts are underway to boost data throughput from orbit [15, 16, 17, 3].

- **Solar Sailing:** Propulsion systems based on stored propellant are difficult to miniaturize without significant reduction in capability. Utilizing solar radiation pressure for orbit maintenance and adjustment is a viable alternative for small spacecraft [18]. Challenges in this work include the packing and deployment of the solar sail, as well as attitude determination and control to articulate the solar sail properly.

- **Attitude Control:** Miniaturization has a significant effect on satellite attitude control systems. The low inertia of small satellites causes them to have a greater response to disturbance torques. Attitude control devices based on momentum exchange (reaction wheels, control moment gyros) are challenging to miniaturize because of the mass and volume limits on miniature satellites [19, 20]. Another aspect that introduces room for development is that miniature satellites can be efficiently controlled using methods previously infeasible on larger spacecraft, such as magnetic torqueing and passive attitude control schemes [21, 22, 23].

- **Attitude Determination:** The low available volume and power are the major challenges of attitude determination systems on miniature satellites. High accuracy rate gyroscopes are physically large and impractical on miniature spacecraft. Detector cooling for star detection or Earth albedo detection is impractical as well. The power consumption of devices based on FPGAs is also unattractive on low power budgets, such as some star tracker search algorithm implementations. Baffles to block the sun are challenging to incorporate on miniature spacecraft as well.

Satellite subsystems with novel solutions for miniature spacecraft continue to develop in these areas. The focus here is miniaturization of attitude determination systems.
1.1.4 Traditional Attitude Determination Systems and their Scalability

Spacecraft attitude determination sensors suites vary greatly depending on the pointing requirements, the operating environment, and desired pointing accuracy among other factors. Complete attitude knowledge in inertial space can require a large set of attitude determination sensors, such as star mappers, Earth sensors, Sun sensors, and magnetic field sensors, along with rate gyroscopes. Other systems of limited attitude knowledge and range of control can also be conceived, for example a sun-pointing system where attitude knowledge relative to the Sun are estimated using the current and voltage measurements from the solar panels, and the attitude is controlled to maximize the generated power. Here I discuss several technologies and highlight challenges and work being done on scaling them to be used on miniature spacecraft.

- Sun Sensing: Sun sensors that measure the direction of the sun are often implemented using photo-diode assemblies or as structures with slits or apertures to detect the inclination of the Sun relative to the device. An obvious limitation is that sun sensors will not generate the sun vector while in the Earth’s shadow, in eclipse. However, miniature Sun sensors of high accuracy are effective solutions for miniature satellites and are commercially available [24, 25].

- Earth Sensing: Earth horizon sensors can be used to provide the nadir vector. Challenges in horizon sensing have made these devices traditionally large and complex. A major challenge is a result of the variability of the atmosphere causing a variable blurring effect on the Earth’s edge as seen from space. Also, in the visible spectrum, the Earth will have phases (like the moon) as the Sun illumination angle and the satellite observation angle change. Many Earth sensor technologies will observe Earth in the infra-red spectrum, to operate independently of the illumination and operate in eclipse as well. Infra-red detector cooling may also be necessary, making miniaturization challenging. Several approaches for miniature satellites have been proposed, for example, a CubeSat Earth sensor based on an array of infra-red thermometer devices has been developed [25].

- Magnetic Field Sensing: Magnetic field sensors can be used in LEO (Low Earth Orbit) to find the magnetic field vector. Like Sun sensors, they are not inherently large or difficult to miniaturize. However, noise is a concern where the magnetometer would
ideally not be affected by the magnetic fields generated by current flow in the spacecraft systems. This can be done using shielding, or by utilizing a deployable boom to distance the magnetometer from the spacecraft body. This may be a significant challenge as spacecraft become smaller, and compensation techniques may be effective [26]. Miniaturized magnetic field sensors are available commercially [24]. Another consideration factor is the need for an Earth magnetic field model and position knowledge to find the magnetic field direction in inertial space.

- **Gyroscopes**: Rate gyroscopes generally are used to propagate a satellite’s attitude. Optical gyroscopes (laser ring or fiber optic) contain two counter-propagating coherent light beams over the same path to detect rotation. The operation is based on the Sagnac effect where the nulls of the standing wave shift in response to the angular rotation. They are highly accurate, but the concept is difficult to miniaturize because there is a physical limit restricting the radius of the laser ring. Given that three orthogonal gyros are required for complete information, the technology is uncommon on nanosatellites. Rate gyroscopes based on Micro-electro-mechanical Systems (MEMS), however, are convenient solutions for miniature satellites. The accuracy of MEMS devices is improving, but they are significantly noisy compared to optical devices (section 5.1 discusses the use of MEMS rate gyroscopes in more detail). The challenges in miniaturizing optical gyroscopes and the unavailability of highly accurate alternatives motivated the search for alternatives that lead to the visual approach presented in this work.

- **Star Mapping**: Star Mappers/Trackers are devices that image the sky and identify the constellation in view to solve for the satellite’s attitude in inertial space. A search algorithm is used to identify the star patterns in view. The search algorithms are often sensitive to false star detection and require reliable imagers [27], which results in the need to use high quality sensors that are possibly temperature controlled, wide aperture precision optics, and a large baffle to block reflected sun light. Miniaturization of star imagers is mostly affected by the use of small optics reducing the signal to noise ratio of the images, and also by the variability in the images resulting from temperature effects, radiation dose, and stray light. Several miniature star trackers are in development [28, 29, 30, 31], often with operation constraints. The inherent noise-tolerance of the approach
presented in this work may be beneficial to future work in this area.

Achieving highly accurate attitude determination in a small form factor remains to be a challenge. This motivates the search for novel solutions that target the small satellite domain, instead of relying on the scaling of classic technologies.

1.2 Stellar Gyroscope

The majority of Small Satellites are launched as ride-share satellites on rockets with major objectives, such as resupplying the ISS or launching a major scientific instrument into orbit. The small satellite trend is based on being of marginal mass compared to the launch vehicle and main payload such that the satellite is launched as a secondary payload at relatively low cost. Several nanosatellite and CubeSat missions have been developed to date with increasingly interesting applications and scientific return [32, 33]. With the mass, volume, and power constraints of small satellites, the utility of these satellites is often contingent on the ability to design low-power subsystems of low volume and mass. Precise attitude determination and control systems conventionally used on large spacecraft are challenging to miniaturize. Many technologies do not scale down well, which drives the need to develop new approaches specifically for the small satellite domain in order to enable new applications by increasing the utility of small satellites [34, 35].

1.2.1 Concept

The stellar gyroscope presented in this work offers an image based approach to propagate a spacecraft’s attitude by tracking the motion of stars in an imager’s field of view. Between star field images, the motion of the stars indicates a unique change in orientation. Given at least two stars tracked across photos, the relative attitude (in three degrees of freedom) between the image instances can be calculated.

The mathematical model to find the attitude change between two image frames is based on identifying the stars’ motion between frames. A single star moving in the field of view would imply a panning motion of the camera, however a single star cannot imply anything regarding the camera’s rotation about its optical axis. Therefore, intuitively, two
stars are needed at a minimum to estimate the camera’s motion in three degrees of freedom. The sequence of images in Figure 1-3 illustrates this concept.

Figure 1-3: A set of images of the Cassiopeia constellation. The stars appear to be panning and rotating in a way that indicates a unique attitude maneuver of the camera in three degrees of freedom.

1.2.2 Motivation for Visual Attitude Propagation

Normally, in the absence of an absolute attitude measurement, attitude is propagated by integrating gyroscope angular rate data (typically MEMS based for small satellites). This results in a drift in the attitude estimate, which is essentially a loss of attitude knowledge after a sufficient amount of time. The stellar gyroscope’s image-based approach can propagate attitude without drift while sufficient stars are common across frames [36, 37, 38]. As the camera pans the sky, all the stars will leave the frame after sufficient time. In that case, some error accumulates as rotation estimates are stacked. However, this happens over a significantly longer period of time compared to a MEMS rate integrator. Therefore, the image-based rotation estimates can complement a set of MEMS rate gyroscopes to maintain a high accuracy attitude estimate at low angular rates (where MEMS gyroscope drift is most severe).

A high-accuracy attitude controller requires highly accurate attitude knowledge. Maintaining a high quality attitude estimate throughout the orbit, including eclipse, is challenging. In eclipse, in the absence of the sun vector measurement, attitude determination on small satellites is often addressed by propagating rate information from the rate gyroscopes at the cost of drift. In order to maintain a high quality attitude
estimate in eclipse, two alternatives are often employed. A star tracker/mapper can be used to identify star constellations and retrieve absolute attitude. However, star trackers add cost and complexity requiring a star database, high update rates, and consequently high quality optics, sensors and a baffle. The second method is to use an Earth horizon sensor together with a magnetometer to generate absolute attitude measurements. For the horizon sensor to work when the Earth is not lit, the sensor must operate in infra-red (IR). This typically requires a specialized IR sensor, a detector cooling system, or a chopping or rotation mechanism to generate differential readings of the Earth and space temperatures. Such a system requires significant power, volume, and the mechanical systems have reliability concerns. The stellar gyroscope offers an alternative when combined with a MEMS gyroscope where the relative attitude measurements can reset the drift from the MEMS-based rate gyroscope propagation in eclipse [38]. This provides the attitude controller with attitude knowledge at a high update rate while limiting the drift. The stellar gyroscope does not require a star database and can be realized using low cost sensors and optics, where the algorithms can tolerate a large amount of noise.

Given the continued improvement of miniature imaging on smart phones and the increasing low-light imaging capabilities, it is conceivable to create highly integrated small satellites with multiple general purpose cameras capable of star and Earth imaging, interchangeably. Visual attitude propagation and Earth sensing using a camera array has the potential to offer attitude determination solutions for miniature satellites without the need to use devices that are traditionally large. This work is meant as a step in that direction, where I investigate visual attitude propagation using the stars using a single camera.

1.2.3 Stellar Gyroscope Challenges

The stellar gyroscope operates as a relative attitude measurement sensor by observing how stars move in the image plane. The star correspondence problem across frames is challenging due to spurious false-star detections (false-positives) and missed stars (false-negatives). Correspondence of stars across frames can be done with limited success by proximity for small angular changes, where for short time intervals the stars are assumed
to not have moved much. However, for large attitude changes the star association algorithm between frames must overcome false stars, missed stars, stars leaving the field of view, and new stars entering the field of view. The problem is essentially to fit a mathematical model over data with a large number of outliers, for which the Random Sample Consensus (RANSAC) approach is effective [39, 40]. RANSAC is a popular algorithm in machine vision and stereo vision, and has been proposed for satellite based image registration for geographic applications [41].

In our work on the stellar gyroscope concept at the University of Kentucky [36, 37], the concept is described, the camera model is developed, and solutions for star detection, star correspondence, and the relative attitude determination problem are discussed, as well as evaluating the performance on simulated images and photos of the night sky using a point-and-shoot camera. In other collaborative work [38, 24], the integration of the stellar gyroscope into a CubeSat attitude determination and control system is discussed. The system being developed by SSBV Space and Ground Systems, in the United Kingdom, utilizes a stellar gyroscope in its sensor suite for a CubeSat pointing solution. In reference [42], we presented an improved version of the algorithm and presented an embedded camera system being developed for KySat-2, a CubeSat being developed in Kentucky that will demonstrate the stellar gyroscope in orbit. Prototype hardware is used to take pictures of the night sky. The image set is used to measure the performance of the camera in acquiring stars and the accuracy of the algorithm.

1.3 Problem Statement

As electronics become smaller and more capable, it has become possible to conduct meaningful and sophisticated satellite missions in a small form factor. However, the capability of small satellites and the range of possible applications are limited by the capabilities of several technologies, including attitude determination and control systems. This dissertation evaluates the use of image-based visual attitude propagation, as a compliment or alternative to other attitude determination technologies that are suitable for miniature satellites. The concept lies in designing cameras specifically for attitude determination or in using a multi-purpose generic imager to aid in attitude determination.
Repurposing a satellite imager for attitude knowledge after a sensor failure is also conceivable using the proposed approach.

The problem of visual attitude propagation as a small satellite attitude determination system is addressed from several aspects: related work, algorithm design, performance evaluation, possible applications, and hardware implementation. These areas of consideration reflect the organization of this dissertation. Chapter 2 discusses relevant background information and related work. Chapter 3 develops an algorithm to establish feature correspondences and find relative attitude in three degrees of freedom between successive images of a star field. In Chapter 4, an experiment is developed to evaluate the performance of the described algorithm and hardware. A simulation is developed in Chapter 5 to present an application of the visual attitude propagation approach on board a small satellite. Imaging hardware systems capable of star imaging and upcoming flight demonstrations of the work are described in Chapter 6. Finally, Chapter 7 concludes the work.
Chapter 2  Background

This chapter discusses theoretical background relevant to this work and includes the literature review of related works. Specifically, attitude reference frames and attitude representation forms used in the algorithm development, analysis, and illustrations are described. Relevant astronomy and machine vision concepts are also discussed. Finally, research and previous work related to the various aspects of this work are outlined.

2.1 Reference Frames

The Earth Centered Inertial frame is taken as the main reference to observe and study the body-fixed frame (satellite attitude). The Earth-Centered Earth-Fixed reference frame is a body-fixed coordinate system centered in earth, and rotates relative to the Earth-Centered Inertial frame. Figure 2-1 illustrates the following reference frames.

*Earth-Centered Earth-Fixed (ECEF).* This reference frame is earth-centered, having a z-axis that lines up with the earth spin axis pointing towards the celestial north pole. The x-axis extends to the zero latitude and longitude point, i.e. the intersection of the Equator and the prime meridian passing through Greenwich, UK. The y-axis is such that it completes the right hand rule. The ECEF frame is convenient to describe phenomena that are earth-fixed, such as ground stations, earth targets, and the geomagnetic field.

*Earth-Centered Inertial (ECI).* This reference frame is earth-centered, with the z-axis towards the celestial north pole. The x-axis points towards the Vernal Equinox, which is the intersection of the ecliptic plane with the equatorial plane at the ascending node. The y-axis completes the right hand rule.

The ECI frame is considered to be fixed relative to the celestial sphere. The ECEF frame rotates once around ECI approximately every 24 hours. ECEF is convenient for earth referenced phenomena. For example, the translation from latitude and longitude to ECEF is a direct calculation independent of time, and the Earth’s magnetic dipole is also fixed in ECEF and rotating with respect to ECI. With the time of day factored into the transformations, the rotation between the ECI and ECEF frames can be calculated.
Finally, the body-fixed frame, as the name suggests, is defined by the satellite geometry. The rotation between the body-fixed frame and ECI is considered to be the attitude of the satellite, the estimation of which is considered to be the attitude determination problem.

2.2 Astrodynamics and Attitude Representations

Astrodynamics is the study of the motion of man-made objects in space subject to both naturally and artificially induced forces [43]. The definition combines both Orbital Dynamics and Attitude Dynamics. Orbital Dynamics describe an object’s translation through orbit under gravitational pull from earth and other celestial objects, and changes in orbit due to spacecraft maneuvers or orbit decay from atmospheric drag. Attitude Dynamics pertain to the representation and dynamics of rotational changes of a satellite.
about its center of mass. There are numerous mathematical representations for satellite attitudes. Table 2-1 summarizes commonly used mathematical models [44, 45], and the definitions follow below.

Table 2-1: Summary of Attitude Representations

<table>
<thead>
<tr>
<th>Parameterization</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direction Cosine Matrix</td>
<td>-No singularities</td>
<td>-Six redundant parameters</td>
</tr>
<tr>
<td></td>
<td>-No trigonometric Functions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Clear physical interpretation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-Convenient product rule for successive rotations</td>
<td></td>
</tr>
<tr>
<td>Euler Angles</td>
<td>-No redundant parameters</td>
<td>-Singularities at some angles</td>
</tr>
<tr>
<td></td>
<td>-Clear physical interpretation</td>
<td>-Trigonometric functions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-No convenient product rule for successive rotations</td>
</tr>
<tr>
<td>Eigen Axis</td>
<td>-Clear physical interpretation</td>
<td>-Axis undefined when rotation is 0º</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Trigonometric Functions</td>
</tr>
<tr>
<td>Quaternions</td>
<td>-No singularities</td>
<td>-One redundant parameter</td>
</tr>
<tr>
<td></td>
<td>-No trigonometric functions</td>
<td>-No obvious physical interpretation</td>
</tr>
<tr>
<td></td>
<td>-Convenient product rule for successive rotations</td>
<td></td>
</tr>
</tbody>
</table>

2.2.1 Direction Cosine Matrix and Euler Angles

The Direction Cosine Matrix (DCM) is a 3 by 3 matrix that defines the rotations between two reference frames. For example, the rotation matrix $C^{ba}$ describes the rotation between frame $a$ and frame $b$. The following equation rotates the vector $\vec{v}$ is from frames $a$ to $b$:

$$\vec{v}^b = C^{ba} \vec{v}^a$$

The rotation between two frames can be broken down into a sequence of rotations about the three body orthogonal axes such that:

$$C = R_1(\theta_1) R_2(\theta_2) R_3(\theta_3)$$

$$R_1(\theta_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_1) & \sin(\theta_1) \\ 0 & -\sin(\theta_1) & \cos(\theta_1) \end{bmatrix}$$
\[ R_2(\theta_2) = \begin{bmatrix} \cos(\theta_2) & 0 & -\sin(\theta_2) \\ 0 & 1 & 0 \\ \sin(\theta_2) & 0 & \cos(\theta_2) \end{bmatrix} \]

\[ R_3(\theta_3) = \begin{bmatrix} \cos(\theta_3) & \sin(\theta_3) & 0 \\ -\sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

These rotation angles \( \theta_1, \theta_2, \theta_3 \) are referred to as Euler rotation angles. The order of the rotations matters and affects the definition of the satellite rotations. In this work, the rotations are chosen to be around the three orthogonal body axes: roll, pitch, and yaw.

Euler rotation angles efficiently describe a rotation (or an object’s orientation) with three parameters. However, dynamic equations suffer from singularities when described in Euler Angles, i.e., trigonometric functions appear in the denominator of some dynamic and kinematic equations which become undefined for certain values of rotation angles when a zero appears in the denominator. On the other hand, Euler angles are intuitive and frequently used outside the dynamic and kinematic equations.

The Direction Cosine Matrix (DCM) describes a rotation with 9 parameters, making it inefficient. It is also non-intuitive. However, vector rotations under this representation are simply a matrix multiplication by the DCM.

### 2.2.2 Eigen Axis rotations

The Eigen Axis representation of a rotation between two frames defines the transformation as a single rotation about a complex Eigen-axis. The Eigen-axis is the unique solution to the following equality for the rotation between the vectors \( a \) and \( b \):

\[
e_1 \vec{a}_1 + e_2 \vec{a}_2 + e_3 \vec{a}_3 = e_1 \vec{b}_1 + e_2 \vec{b}_2 + e_3 \vec{b}_3
\]

\[
e = (e_1, e_2, e_3)^T
\]

The Eigen-axis’s orientation relative to both frames remains unchanged [45]. Intuitively, it can be thought of as the axis around which the object rotates to perform an attitude maneuver with a single rotation, as opposed to a sequence of rotations around the body axes (Euler Angles). The rotation angle about the Eigen-axis can be calculated from:
\[
\cos(\theta) = \frac{1}{2} (C_{11} + C_{22} + C_{33} - 1)
\]

where \(C_{11}, C_{22}, C_{33}\) are the diagonal elements in the Direction Cosine Matrix.

### 2.2.3 Euler Symmetric Parameters (Quaternions)

Quaternion elements do not carry a direct intuitive meaning. The Quaternion representation however simplifies the kinematic and dynamic equations and does not suffer from singularities which occur in other representations (such as Euler rotation angles).

The quaternion vector that defines the rotation between two frames is defined based on elements of the Eigen Axis rotations representation, as:

\[
\mathbf{q} = (q_1, q_2, q_3)^T = \mathbf{e} \sin \left( \frac{\theta}{2} \right)
\]

\[
q_1 \equiv e_1 \sin \left( \frac{\theta}{2} \right)
\]

\[
q_2 \equiv e_2 \sin \left( \frac{\theta}{2} \right)
\]

\[
q_3 \equiv e_3 \sin \left( \frac{\theta}{2} \right)
\]

\[
q_4 \equiv \cos \left( \frac{\theta}{2} \right)
\]

The attitude representations described here are used throughout the dissertation in algorithm development, the experiment, and in illustrations.

### 2.3 Relevant Astronomy Concepts

#### 2.3.1 Stellar Magnitudes

A celestial body’s apparent magnitude (m) is a measure of its brightness as seen from Earth. The value is normalized to compensate for atmospheric attenuation. The apparent magnitude has a logarithmic relationship with the object’s brightness, and the brighter the object appears, the lower the value of its magnitude.

The modern system used for star magnitudes is based on the observations made by the
Greek astronomer Hipparchus (190 - 120 BC), who assigned brightness values between 1 and 6 to the stars by their observed sizes, where magnitude 1 stars are the brightest in the sky and magnitude 6 stars are the dimmest stars visible to the naked eye [46]. The modern system more accurately assigns fractions and allows zeros and negative values for objects brighter than magnitude 1. It was also found that the difference in star magnitude between two stars is proportional to the logarithm of the ratio of their brightness.

The system was formalized by Pogson in 1856, by assigning a brightness ratio of 100 to correspond to a magnitude difference of 5. Therefore, because $100^{1/5}$ is equal to 2.512, the magnitude relationship can be written as:

$$\frac{I_1}{I_2} = 2.512^{-(m_1-m_2)}$$

$$m_1 - m_2 = -2.5 \log_{10} \left( \frac{I_1}{I_2} \right)$$

where $m_1$ and $m_2$ are the apparent magnitudes of two stars, and $I_1$ and $I_2$ are their corresponding brightness/flux values (measured in Watts, or a linearly related unit) [47].

The formulation of apparent magnitude defines a relative relationship, and reference stars are used when measuring star magnitudes. Table 2-2 shows the apparent magnitudes of a few example celestial objects.

<table>
<thead>
<tr>
<th>Apparent Magnitude</th>
<th>Object</th>
</tr>
</thead>
<tbody>
<tr>
<td>-12.74</td>
<td>Full moon [49]</td>
</tr>
<tr>
<td>-0.01</td>
<td>Alpha Centauri A</td>
</tr>
<tr>
<td>0.03</td>
<td>Vega</td>
</tr>
<tr>
<td>2.005</td>
<td>Polaris</td>
</tr>
<tr>
<td>3.44</td>
<td>Andromeda Galaxy</td>
</tr>
<tr>
<td>6</td>
<td>Typical limit of naked eye under optimal conditions</td>
</tr>
<tr>
<td>32</td>
<td>Limit of Hubble Space Telescope</td>
</tr>
</tbody>
</table>

We note that the apparent magnitude describes how bright a star appears from Earth,
regardless of the star’s distance or actual emitted energy. The “absolute magnitude” defines the star brightness at a normalized distance. However, this measure is not used in this work.

### 2.3.2 Stellar Parallax

Imaging the stars on board an orbiting satellite to estimate satellite rotation raises the concern of star motion that is not attributed to the rotation. Namely the parallax caused by the satellite’s translational motion, and the stars’ proper motions.

The effect of parallax on the stellar gyroscope operation is discussed first. Considering the closest star to our solar system, \( \alpha \) Centauri A which is 1.32 parsecs away (40.7 trillion kilometers), and taking the Earth orbital radius to be \( 1.496 \times 10^8 \) km, the annual parallax angle \( P \) that corresponds to Earth’s translation around the Sun, as shown in Figure 2-2, can be calculated as:

\[
P = \tan^{-1} \left( \frac{1.496 \times 10^8}{4.07 \times 10^{13}} \right) = 0.00021° = 0.76''
\]

![Figure 2-2: Illustration of star annual parallax angle (P)](image)

Parallax causes a change in the direction of a star in the ECI reference system. The calculated angle for \( \alpha \) Centauri A can be considered to be the worst-case star parallax angle caused by a translation of 1 Earth orbital radius. From a star imaging perspective on board a spacecraft for attitude determination purposes, where images are analyzed within
an hour (and significantly less translation occurs), the parallactic motion is considered to be negligible in this work.

### 2.3.3 Proper Motion

The proper motion of a star can be described as its motion perpendicular to the line of sight, which changes the direction of the star in inertial space. This motion is at most a few arcseconds per year ("/yr), and like parallactic motion, it is considered to negligible in this work.

### 2.4 Random Sample Consensus

The Random Sample Consensus (RANSAC) approach was first introduced by Fischler and Bolles [40] in 1981. The approach aids in interpreting data that contains a significant percentage of gross errors. It is often used in automated image analysis where the data is likely to contain many errors. Fischler and Bolles applied RANSAC to the Location Determination Problem, estimating the point in space from which an image was taken given the known locations in space of landmarks in the image.

RANSAC is an iterative algorithm to estimate the parameters of a mathematical model from the observed data. The data is assumed to contain inliers that fit the true model, and can be contaminated by a large number of outliers. The main advantage of the approach is the ability to consistently reject the outliers and find a solution. The algorithm is iterative and non-deterministic, where the model parameters are hypothesized using random samples from the data and checked for consensus, and the process is repeated until consensus is found. The computational cost of RANSAC depends on the nature of the data and the underlying model, which are described in this work. I use RANSAC to establish correspondence between detected stars across images of a star field with common stars. False correspondences severely affect the resulting estimate, motivating the use of RANSAC in this application in place of simpler means of establishing correspondence, like proximity or similarity in brightness.

Figure 2-3 demonstrates RANSAC for a line-fitting problem, where the data contains data points that fit the actual model, called inliers, as well as a significant number of data
points that do not fit the line, called outliers. A simple least-squares fit using the entire data set would result in a poor estimate of the line. RANSAC is an iterative algorithm that randomly selects data points to generate hypotheses that are tested using the remaining data set for consensus. The algorithm involves the following steps:

1. A minimum number of hypothetical inliers required to estimate the model parameters are selected from the data set (called the Minimum Sample Set (MSS)). The model is fitted to these hypothetical inliers and the model parameters are calculated.

2. The hypothetical model is tested against the remaining data points. A point that fits well to the estimated model is counted towards the Consensus Set (CS).

3. If enough inliers are registered in the Consensus Set, the hypothesized model is considered to be a good fit. If the data does not show sufficient consensus towards the hypothesized model, the process is repeated at step 1.

4. When a model shows sufficient consensus, the model parameters are recalculated from the entire Consensus Set.

Figure 2-3: Example data set of a line with a large number of outliers (left). Fitted line using RANSAC (right), where the outliers are rejected and do not contribute towards the line fit. Image credit: Ref [50].
Several aspects of RANSAC depend on the specific application, and several modifications to improve the algorithm efficiency and accuracy are proposed. Specifically:

- A mechanism to estimate the model parameter has to be identified, first given the Minimum Sample Set, and at a later step using the entire data points in the Consensus Set.
- The random selection process in the hypothesis generation step can be replaced with a guided selection to reduce the number of iterations required to find a hypothesis that finds enough consensus.
- In the consensus testing step, a mechanism to test a data point’s alignment with the hypothesized model has to be identified to decide whether that point shall be counted towards the consensus set or not.
- Evaluation of the consensus set is classically done by evaluating the number of data points that showed consensus. This step can be replaced by a more sophisticated probability model to evaluate the fitness of the data points in the consensus set. Such RANSAC variants include MSAC and MLESAC [51].
- Various parameters and threshold must be selected and tuned, such as the threshold value in evaluating data points for consensus and the termination criterion.

### 2.5 Related Work

Research by Liebe et al. from the NASA Jet Propulsion Laboratory (JPL) studies the feasibility of using a stellar gyroscope to estimate high rotation rates [52]. The basis of operation depends on using long exposure images of the star field and then analyzing the circular arcs caused by the stars’ motion. The concept is optimized for high rotation rates outside regular gyroscope operation ranges, and with the single-exposure method results in a noisy image even with a high quality sensor. The research by JPL, despite the difference in scope, presents the concept and motivation to infer the rotation rate and spin rate visually. The approach in this work adopts the idea while eliminating the requirement of taking long exposure images (which have a low Signal to Noise Ratio) by taking a sequence of snap shots and effectively processing a “video” instead of a single long-exposure image. This also enables the device to operate at low rotation rates.
2.5.1 Star Trackers/Mappers

As described in the background section, Star Trackers/Mappers are traditionally attitude determination systems carrying star catalogs that are used to identify star constellations in order to calculate the spacecraft’s attitude in inertial space. Recent research efforts have been focused on improving the hardware and search algorithms of star trackers to increase the update rates to a level where the angular rates can be approximated. Such a high update rate star tracker is sometimes referred to as a stellar gyroscope [53]. Another effort to estimate the angular rate of a satellite using star sensors implements a Kalman filter that models the environmental torques as a random process and depends on the absolute attitude measurements of a high update rate star tracker [54]. The concept proposed in this dissertation aims to provide an alternative that does not require absolute attitude measurement using a star database and a star identification algorithm for attitude propagation.

From a small satellite perspective, few star imaging solutions exist at the time of the writing. Sinclair Interplanetary offers a miniature Star Tracker, the S3S, which measures 59mm x 56mm x 32.5mm and weighs 90 grams [30, 55, 31]. This mass and volume range is a significant volume of a 1U CubeSat, but not prohibitively large, and is considered a convenient solution for Nanosatellites (1kg - 10 kg spacecraft). Another small satellite star tracker being developed measures 29mm x 26mm x 37 mm and weighs 74 grams [28]. The hardware and optical designs in this dissertation fall in the same volume and mass range. However, the algorithms developed herein aim to support further miniaturization as higher resolution and quality sensors become available (primarily driven by the smartphone industry). Performing reliable star field imaging using narrow aperture and wide field of view lenses requires a robust star detection approach, which is addressed in this dissertation.

2.5.2 Related work on Random Sample Consensus

RANSAC is a popular approach in image analysis because feature detectors are prone to errors. The approach has been successfully used in several domains, including stereo-vision, ego-motion estimation, and image registration.
In stereo-vision, RANSAC is often applied to epipolar geometry and used to establish correspondence between features across images [56, 57, 58]. In the stellar gyroscope problem, where stars are tracked across images, reliable correspondence between stars across images is a critical aspect of the problem. A significant difference arises in replacing the RANSAC epipolar geometry model with rotational kinematics. In stereo vision, the relationship in space between the two cameras is known, and depth information of the object in the scene is estimated. In the stellar gyroscope, depth is not a factor because features are considered to be infinitely far away, however the relationship in space between the camera positions from which the images were taken is unknown.

RANSAC has also been considered for camera ego-motion estimation (also referred to as visual odometry). It is often applied to estimate the motion in highly dynamic environments, while the camera moves through an environment which has moving elements itself [59, 57, 60]. Typically, the images involved are feature rich and estimation accuracy is below the accuracies typically sought in aerospace systems. Some approaches do apply towards to the stellar gyroscope problem, and these are outlined in the following section (2.5.3).

An aerospace application of RANSAC has been proposed in satellite image registration [41]. Image registration entails finding the relationship between the image coordinates and a reference coordinate system. The work aims to automatically register satellite Earth images and map the images in Earth coordinates.

### 2.5.3 Ego-motion Estimation

Research in Ego-motion Estimation (including the classic relative pose problem), while usually based on land systems and images of objects in close proximity, offers valuable insight to the stellar gyroscope concept [61, 62]. A star based attitude propagator has the advantage of eliminating the need to track translation because stars are considered to be infinitely far away (as discussed in section 2.3.2), and only rotation is tracked.
Ego-motion estimation techniques can be classified as either gradient methods or displacement methods [63]. Gradient methods, such as optical flow, are expected to show limited success when applied to star field images because the images lack features. However, tracking algorithms based on displacement methods that track distinct features across images applies to the stellar gyroscope problem when translation is assumed to be negligible.

Bazin offers an approach to perform rotation estimation from a sequence of images [64]. The work is mainly concerned with motion estimation using fish-eye images by decoupling rotation and translation. The rotation estimation portion is based on calculating the Eigen Axis rotation parameters (Spin Axis and Angle) from two features tracked between the two frames. Incorporating more tracked features to improve the accuracy involves an iterative algorithm. The approach is based on calculating the Eigen-axis and rotation angle from two correspondences by performing vector manipulations on the geometry. Being based on vector cross product, the approach works on small rotations (which is expected when processing video), but may face problems when generalized to study arbitrary rotations between images. I expect accuracy issues and possible singularities to arise, for example when the features travel in parallel directions and the cross product result is very small.

A circle fitting approach has also been proposed to estimate the rotation axis and angle [52], as mentioned in section 2.5. I expect issues to arise that are a function of the motion being analyzed. For example, when the camera pans the sky, star arcs will resemble straight lines, and a circle fitting approach will face difficulty estimating the center of rotation for the arcs. Also, if rotations are small, the arcs will appear small creating ambiguity in locating the arc centers as well as in estimating the arc lengths accurately.

The approach used in this paper is based on using a camera model to find vectors associated with the features, and then using the quaternion estimation method, described in Section 3.2, to find the underlying rotation. The approach is not sensitive to the type or range of motion, as long as vector pairs are retrieved of features in the image sequence.
The main difficulty remains to be the correspondence problem, where a false correspondence can skew the estimate significantly. This approach is discussed in detail in section 3.2.
Chapter 3  Stellar Gyroscope Algorithm

3.1  Star Detection

The stellar gyroscope operation begins by detecting stars and calculating the unit vectors originating from the spacecraft pointing towards the stars, defined in body-fixed coordinates. The changes of these vectors are tracked and used to infer the rotation changes between frames.

3.1.1  Camera Modeling

An ideal pinhole camera model is used where a shell of the celestial sphere is mapped onto the camera’s sensor as shown in Figure 3-1. The field of view (FOV) is a function of the focal length and the imaging sensor’s physical dimensions.

Figure 3-1: Camera Model. A celestial shell is mapped onto the image plane.
The vectors associated with each star can be obtained by modeling the camera as shown in Figure 3-2. The mapping from pixel coordinates to vector components is done by identifying the origin (the focal point) and the star coordinates on the image plane. It should be noted that the values of the star indices on the image plane and the focal length must be in the same units of distance (mm, pixel widths, etc.). The unit vector is found by dividing by the vector magnitude. The unit vector for a star located at \((x_1, y_1)\) on the image plane is:

\[
\vec{v}_1 = \frac{1}{\sqrt{x_1^2 + y_1^2 + f^2}} \cdot \begin{bmatrix} x_1 \\ y_1 \\ f \end{bmatrix}
\]

where \(f\) is the camera focal length, and \(x\) and \(y\) are the pixel locations in space in units of distance.

![Figure 3-2: Resolving star vector from pixel coordinates.](image)

### 3.1.2 Centroiding

The camera optics are designed such that the image is slightly out of focus in order for the energy of a single star to affect multiple pixels on the sensor as illustrated in Figure 3-3. This allows the detection of the star center with sub-pixel resolution, using a process
often referred to as Centroiding. This is achieved by fitting a normal distribution curve over the data [65]. Calculating the expected value as the star location utilizes information in several pixels and results in a more accurate estimate of the star location. The expected value can be found by first thresholding the noise to ensure that the noise values will not contribute to the expectation. Also, scaling the star region to sum up to a total probability of unity is necessary. Figure 3-3 shows a star cross section to illustrate this process, and the expected value is found as:

\[
E(x) = \sum x \cdot f_x(x)
\]

where \( f_x(x) = \sum_y f_{xy}(x, y) \)

and where \( x \) and \( y \) represent the pixel coordinates and \( f_{xy}(x, y) \) is the thresholded and scaled star distribution, and \( f_x(x) \) is the marginal probability along the \( x \) image axis. The same approach applies to the \( y \) image axis to find \( E(y) \).

As it can be seen in Figure 3-3, the maximum value and the calculated centroid (expected value) deviate from each other. It is noted that the estimated star location is sensitive to improper selection of the noise threshold, which varies for a selected camera sensor and exposure time. The threshold is selected based on the histograms of the photo at a level that maximizes dim star detections while limiting false star detections.

### 3.1.3 Filtering to Reduce False Positives

Noise frequently surpasses the threshold and can be identified as stars when only using a threshold to detect stars. A filtering step can improve the signal to noise ratio in star detections. A correlation filter with a kernel that matches the shape of the star was found to be a reliable approach to detect dim stars, while providing tolerance to dark current, thermal, and read-out noise. Specifically, defective hot-pixels (which can be triggered by radiation on orbit) surpass the threshold, but do no correlate with the kernel because they have a different spread functions, resulting in a high signal to noise ratio in terms of star detection. Figure 3-4 shows the kernel that was used with the star field images taken using the hardware described in Section 4.1, a 2-dimensional sinc function with a width that matched the star width in the images. Section 6.2.2 shows an application and some results of this approach.
Figure 3-3: Close up of Gienah (ε Cygni, magnitude 2.48) as detected in a night test using the prototype hardware described in Section 4.1 (top), and illustration of the centroiding process (bottom).
The images are first filtered to identify the regions of interest where stars are expected to be. Clustering of the pixels that clear the threshold after filtering identifies the stars in the image. Then, the centroiding algorithm is applied to the original image at the location of the clusters.

![Figure 3-4: Image filter kernel used to maximize dim star detection and minimizing false positive detections.](image)

### 3.2 Rotation Estimation using Q-Method

Given the detected stars in a set of star field images, the next step in the operation of the stellar gyroscope algorithm is to estimate the ego-motion in three degrees of freedom. For this discussion, the Direction Cosine Matrix is used represent the attitude change. For example, the rotation between frame $a$ and frame $b$ for a star represented by the vector $\vec{v}$ is given by:

$$\vec{v}^b = C^{ba} \vec{v}^a$$

$$\vec{v}^b = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \vec{v}^a$$

where $\vec{v}^a$ and $\vec{v}^b$ are the unit vectors pointing at the same star in two image frames in body-fixed coordinates. The matrix $C^{ba}$ hence defines the camera attitude change between those two frames.
Given at least two measured vector pairs across frames $a$ and $b$, $C^{ba}$ can be estimated using the q-method which minimizes the sum of the square errors of all vector pairs [44].

This error can be represented by the following cost function for $L$ vector measurements:

$$ J(C^{ba}) = \sum_{k=1}^{L} w_k \left| v^b_k - C^{ba} v^a_k \right|^2 $$

where $v^b_k$ are the vectors in frame $b$, and $v^a_k$ are the corresponding vectors in frame $a$. The quantity $v^b_k - C^{ba} v^a_k$ for a certain value of $k$ (one pair of vectors) represents the rotation of the vector from frame $a$ to frame $b$ and subtracting it from the corresponding vector in frame $b$, resulting in the error vector being minimized by finding the optimal value of $C^{ba}$. $w_k$ is a weighting factor to assign relative measurement quality to individual vector pairs, and is set as a constant in this work.

The q-method is an analytical solution for the minimization of the cost function $J(C^{ba})$. This is done by representing the rotation matrix by the attitude quaternion. The literature describes the derivation and restating of the minimization problem of the cost function with the following maximization problem of the gain function in quaternion form [44]:

$$ J'(\mathbf{q}) = \mathbf{q}^T K \mathbf{q} $$

where

$$ \mathbf{q} \text{ is the attitude quaternion} $$

$$ K = \begin{bmatrix} S - \sigma I & Z \\ Z^T & \sigma \end{bmatrix} $$

$$ B = \sum_{k=1}^{N} w_k (v^b_k v^a_k^T) $$

$$ S = B + B^T $$

$$ Z = [B_{23} - B_{32}, B_{31} - B_{13}, B_{12} - B_{21}] $$

$$ \sigma = \text{tr}[B] $$

The solution is shown to be, using Lagrange multipliers, a quaternion that is the eigenvector of $K$ of the largest eigenvalue. The quaternion is next converted to its equivalent rotation matrix $C^{ba}$. To simplify the notation in the remainder of the paper, the q-method will be referred to with the following operator that returns the optimal estimate of $C^{ba}$ given two sets of vectors in frames $a$ and $b$: 

32
\[ C^{b_\alpha} = q\text{method}\left([v_1^b \ v_2^b \ldots \ v_L^b], [v_1^\alpha \ v_2^\alpha \ldots \ v_L^\alpha]\right) \]

We note that the solution requires solving the Eigenvalue and Eigenvector problems for a 4x4 matrix, which is computationally expensive. There are several available approximations and optimizations in the literature that can be used if computational cost reduction is sought, such as the Quaternion Estimation algorithm (QUEST) [44]. The Q-method was used in this work to return the most accurate results. The number of times required to solve for a quaternion was considered to be a measure of computational cost and drove the optimization efforts to the algorithm.

3.3 Correspondence by Random Sample Consensus (RANSAC)

3.3.1 RANSAC Considerations for 3DOF Rotation

Before the Q-method can be used to solve for the relative attitude between image pairs, the correspondences across frames between the detected stars have to be established. A false correspondence can severely skew the estimate. As it can be seen in Figure 3-5, the data collected (using the low cost sensor and optics) and expected on orbit contains false stars and missed stars. In addition, stars entering and leaving the field of view will appear in one frame and not the other of the two frames being analyzed. It was also noticed that hot pixels may appear as stars in both frames, which suggest that the satellite did not move. These challenges set the requirement of a robust algorithm that is not sensitive to these errors.

Random Sample Consensus (RANSAC) is an iterative method to estimate parameters of a mathematical model from a set of observed data which is contaminated by a large number of outliers that do not fit the model [40]. RANSAC is applied to the correspondence problem of the stellar gyroscope. The steps of RANSAC, in general, can be summarized as [51]:

1. Hypothesize: A Minimum Sample Set (MSS) is randomly selected from the input data and the model parameters (in this paper’s implementation: the rotation matrix) are computed using only that randomly selected set.
2. Test: The model generated in the first step is tested against the entire dataset. The data that shows consensus, to some measurement of deviation, are counted towards the Consensus Set (CS).

3. Iterate: RANSAC iterates between the above two steps until a random hypothesis finds “enough” consensus to some selected threshold.

![Figure 3-5: Star field image overlaid by star detections in five consecutive frames using a handheld point-and-shoot camera, with a three degree rotation between each frame. This figure illustrates the tracking challenge where the data consists of reliable stars, false positives, and false negatives. Colors are adjusted for clarity.](image)

In order to adopt RANSAC for the relative attitude determination problem, several elements had to be identified, namely the mathematical model to generate the hypothesis, a test process to evaluate the model’s fitness against the entire dataset, and finally, a measure of error to determine consensus.

### 3.3.2 RANSAC Implementation for 3DOF Rotation

First, the mathematical model is the rotation matrix that is generated using the q-method described earlier. Recall that two stars are required as the minimum sample set (MSS). To generate the hypothesis for $M$ detected stars in first frame and $N$ stars in second frame, we randomly select $[v^a_{H1} \ v^a_{H2}]$ as two stars from frame $a$ and select $[v^b_{H1} \ v^b_{H2}]$ as two stars from frame $b$. A hypothesis rotation matrix is generated using the randomly selected pair.
Second, the hypothesis is tested against the remaining stars. This is done by projecting each star in the first frame \((M\) stars) to the second frame using the hypothesis rotation matrix, and checking if there is a star in the second frame near that location by calculating an error vector as follows for every star in frame \(b\) \((N\) stars):

\[
\overrightarrow{\text{error}} = \overrightarrow{v}^b - C_{\text{Hypothesis}}^{ba} \overrightarrow{v}^a
\]

The magnitude of \(\overrightarrow{\text{error}}\) is minimal for correctly paired stars when the hypothesized rotation matrix represents the true rotation between frames \(a\) and \(b\). The star pair being tested counts towards the Consensus Set (CS) if the vector magnitude \(||\overrightarrow{\text{error}}||\) is below a tuned threshold.

Effectively, hypotheses will find little consensus unless they represent the actual rotation, which makes the application of RANSAC to the stellar gyroscope problem effective. Once a hypothesis finds consensus larger than 40% of the number of stars in the first frame, RANSAC terminates and returns the consensus set, which consists of the hypothesis stars in the first frame and their corresponding location in the second frame, along with the star pairs that showed consensus. The consensus set (CS) is used to generate the relative attitude solution:

\[
C^{ba} = \text{qmethod} \left( [v_{c1}^b, v_{c2}^b, \ldots v_{c}^b], [v_{c1}^a, v_{c2}^a, \ldots v_{c}^a] \right)
\]

Figure 3-6 is a flow chart of the algorithm. This description concludes the implementation of the purest form of RANSAC on the stellar gyroscope problem. Random star selection with uniform random distribution in both frames to generate the hypothesis (namely \([v_{H1}^b, v_{H2}^b]\) and \([v_{H1}^a, v_{H2}^a]\) works for an arbitrary change in orientation whether small or large. However, in this case, RANSAC requires a significantly large number of iterations to find a hypothesis which results in consensus. The number of iterations can be reduced by improving the hypothesis and test processes to require a fewer number of iterations. For example, modifying the randomization process by pairing stars across frames by proximity or brightness in the hypothesis step, the number of hypotheses required to find a solution can be drastically reduced.
Figure 3-6: Stellar gyroscope algorithm flow chart.
The time complexity of the RANSAC implementation can be represented as:

\[ T(I_{\text{thresh}}, n) = O(I_{\text{thresh}} (C_{\text{estimate}} + s^2 C_{\text{fitting}})) \]

where \( I_{\text{thresh}} \) is the maximum number of iterations allowed, set such that the probability of not finding consensus for images with sufficient common stars is very low. \( s \) is the number of stars detected per frame. \( C_{\text{estimate}} \) is the complexity associated with the \( Q \)-method in the hypothesis step, where a rotation matrix is generated given two vector pairs. \( C_{\text{fitting}} \) is the complexity associated with the fitness test in the hypothesis testing step which consists of vector manipulations to find the magnitude of the error vector. The complexity of the \( C_{\text{estimate}} \) and \( C_{\text{fitting}} \) computations are independent of \( I_{\text{thresh}} \) and \( n \), and can be considered to be constant time.

### 3.3.3 Parameters and Algorithm

In the hypothesis step, pairing stars across frames that are similar in brightness was found to be most effective in reducing the number of iterations necessary to find consensus, while making no assumptions regarding the underlying rotation. In this case, stars are randomly selected in the first frame \((v_H^a, v_H^b)\), and are paired randomly with a star from a pool of the stars closest in brightness in the second frame \((v_H^a, v_H^b)\). This was found to work well on the night-sky dataset for the field of view and star sensitivity of the camera described in Section 4.1. The sum of the star pixel values was used as a measure of brightness. Section 4.3 highlights other implementations that have been considered where pairing is done completely randomly or by proximity.

Star brightness measurement was unreliable in night tests. The atmosphere causes “twinkling” of the stars where the brightness appears to change across frames. More accurate star brightness measurement is expected to be possible in space in the absence of the atmosphere. In turn, further reduction in the number of iterations may be possible by pairing stars across frames more effectively. For example, the hypothesis step can also be improved by initially selecting the brightest stars in the first frame that have the highest signal to noise ratio. It is noted that the algorithm presented does not depend on accurate or reliable measurement of star brightness values; the information only improves the
algorithm efficiency by reducing the number of iterations required. Independence of precise star brightness measurements provides tolerance to varying noise levels caused by thermal noise (dark current) under temperature variations on orbit and by radiation dose over the satellite lifetime.

The algorithm as presented to this point does not assume any prior knowledge of the satellite rate or the attitude change between images. With knowledge of the satellite angular rates, a hypothesis rotation matrix can be generated using that information to reduce the number of required iterations. For example, MEMS gyroscope rate measurements between images can be used to generate a hypothesis by propagating the attitude between frames (with limited drift over a short period of time) to eliminate the randomization process. Alternately, when the image-based algorithm is operating continuously, it may be possible to assume that the attitude does not change suddenly between tested image pairs, and the hypothesis matrix can be assumed to be equal to the last estimate. Also, as shown in section 4.3, pairing stars by proximity is effective if the satellite rates and frame rate are such that the stars move in small steps in the images.

Figure 3-7 and Figure 3-8 provide MATLAB recipes for the RANSAC algorithm described in this chapter. The hypothesis rotation matrix is generated by pairing stars across frames by closeness in brightness. Hypotheses are then tested for consensus for all the stars in both frames until a hypothesis finds a large enough consensus. The analysis and results in section 4.4 use this implementation.
function [consensus_set i]= RANSAC(vectors1, mags1, vectors2, mags2)
% RANSAC Correspondence function to pair vectors between two reference
% frames with an unknown underlying 3DOF rotation.
% 'vectors1' an vectors2' are the vectors corresponding to the stars in
% the first and second frame, respectively. 'mags1' and 'mags2' are
% brightness measurements for the corresponding vectors.

%% Tunable parameters
ITERATIONS_TIMEOUT = 60000;   % Maximum number of iterations allowed.
CONSENSUS_THRESHOLD = 0.0005; % Error vector length threshold to
% register consensus.
PERCENTofV1_THRESH = 0.4;     % Exit criteria, when a hypothesis pairs
% at least 40% of the vectors from the
% first frame.

for i = 1:ITERATIONS_TIMEOUT
  % Generate Hypothesis (see following Figure)
  DCM_hypo = RANSAC_Hypothesize(vectors1, mags1, vectors2, mags2);

  % Test for consensus, loop through all vectors in first frame
  score = 0; %Initializing variable to hold the consensus count.
  for j = 1:length(vectors1)
    % Project vector from first frame using the Hypothesis
    % rotation matrix.
    V1j_projected = vectors1(j,:)*DCM_hypo;

    % Loop through all vectors in second frame
    for k = 1:length(vectors2)
      % Calculate error vector and register consensus if the
      % projected vector from the first frame matches a vector
      % in the second frame
      error_vector = V1j_projected - vectors2(k,:);
      if norm(error_vector) < CONSENSUS_THRESHOLD
        % Register consensus
        score = score+ 1;
        consensus_set(score,:) = [j, k];
        break % move on to next star if a match is made
      end
    end
  end

  % End search if a hypothesis finds enough consensus
  if(score >  PERCENTofV1_THRESH * length(vectors1))
    break % break out of main loop, exits function
  end
end

Figure 3-7: Recipe (in MATLAB syntax) for the RANSAC application to establish
correspondences between vector pairs over and underlying rotation in three degrees of
freedom.
function DCM_hypo = RANSAC_Hypothesize(vectors1, mags1, vectors2, mags2)
% RANSAC hypothesis function, randomly pairs vectors across frames
% guided by similarity in brightness. All inputs are expected sorted
% by brightness in descending order.

%% Tunable parameters
PAIRING_RANGE = 8;  % Pool size of closest stars in brightness for
% pairing

%% Generate hypothesis
% Select two stars randomly from first frame, with a uniform
% distribution
random_1stframe = select_two_randomly(vectors1);

%% Sort second frame stars by closeness in brightness
sorted_1 = sort_ascending(abs(mags2-mags1(random_1stframe(1))));
sorted_2 = sort_ascending(abs(mags2-mags1(random_1stframe(2))));

%% Randomly select stars in the second frame, from a pool of stars
% close in brightness
tmp1 = random_int(PAIRING_RANGE); %random # between 1:PAIRING_RANGE
tmp2 = random_int(PAIRING_RANGE); %random # between 1:PAIRING_RANGE
random_2ndframe(1,1) = sorted_1(tmp1);
random_2ndframe(1,2) = sorted_2(tmp2);

%% Done randomizing, select vectors
v_1 = vectors1(random_1stframe,:);
v_2 = vectors2(random_2ndframe,:);

%% Use two paired vectors to generate rotation matrix using
% the Q-method
DCM_hypo = q_method(v_2,v_1);

Figure 3-8: Recipe (in MATLAB syntax) for the rotation matrix hypothesis generation
for the RANSAC implementation. The search is guided by similarity in brightness.

3.3.4 Lens Distortion Correction

Correction for lens distortion was considered in this work to improve the accuracy of the
relative attitude estimates. Radial distortion is of particular concern, where the detected
stars are relocated closer or further away from the turning center, making arcs longer or
shorter than they should be and resulting in a bias in the rotation estimates.

Several lens characterization techniques are described in the literature [66, 67, 68]. These
efforts involve analyzing photos taken using the camera being calibrated of a controlled
setup to estimate the focal length and distortion parameters of the camera lens. The
distortion model typically models the radial and tangential distortion components as polynomial functions, where given the polynomial coefficients data points on a distorted image can be corrected. Camera calibration is the process of finding these polynomial coefficients.

Figure 3-9: Photos of the calibration checkerboard (top), and a diagram (bottom) depicting the camera on the left and the registered checkerboard pattern from the calibration photo set.
We evaluated this method of distortion correction for a miniature S-Mount lens and the camera system described in Section 4.1. Using the Camera Calibration Toolbox developed by Jean-Yves Bouguet [69], a set of images of a checkerboard pattern was analyzed. Figure 3-9 shows the photo set of the calibration poster (56” x 36”) that is hung on a wall and a diagram of the registered corners in the poster that are used in the calibration algorithm. The distortion model consists of a 3rd order polynomial function for radial distortion and a 2nd order polynomial for tangential distortion. Figure 3-10 shows the resulting distortion model. It was found, however, that while the distortion model did seem to provide a good fit, it did not result in a significant improvement in attitude estimates, according to the experiments outlined in Chapter 4.

Figure 3-10: The calculated distortion model for an S-Mount 16mm lens.

For a small board lens (S-Mount) mounted manually over the chosen CMOS sensor, the application of the polynomial distortion model did not improve the results according to the metrics developed and defined in Chapter 4, which include the size of the consensus
set in the RANSAC algorithm, the accuracy of the estimate in a controlled setup, and the projection error. It is suspected that lens imperfections and misalignment are the main causes of the deficient fit of the global polynomial distortion model. A more suitable approach for the selected lens may be a distortion correction approach that does not use a global model, and instead uses an array of localized models using a set of control points distributed across the field of view [70]. This approach is considered for future work, and the results in this work do not incorporate lens correction.
Chapter 4  Performance Evaluation

In order to evaluate the performance of the system developed in the previous chapters, a controlled experiment is designed to generate data sets of known attitudes to support algorithm development and study variations, as well as studying the estimation accuracy and the computational cost. The experiments are based on images of the night sky. A point-and-shoot camera was used initially to study the feasibility of the approach and to support algorithm development (section 4.3). Prototype hardware of the KySat-2 camera system is used to evaluate the accuracy of the embedded hardware (section 4.4). Finally, analysis using a star database follows to evaluate the expected behavior in an operational environment.

4.1  Embedded Camera Design

The stellar gyroscope system being developed consists of a low-cost camera assembly and processing hardware. The imager is the Aptina MT9P031 CMOS 5 megapixel sensor, and the lens has a focal length of 16mm and an aperture of F/1.2. This configuration results in a 15° by 20.2° field of view with good low-light sensitivity with an exposure time of 100ms. The camera interface and processing is done on a Linux-based single board computer. Figure 4-1 shows prototype models of the camera module and interface board. Table 4-1 summarizes the camera specifications.

Table 4-1: Specifications Summary of Camera Hardware.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor</td>
<td>Aptina MT9P031, 5 Megapixel CMOS Sensor</td>
</tr>
<tr>
<td>Optics</td>
<td>Marshall Electronics 16mm S-Mount Lens, Aperture F/1.2</td>
</tr>
<tr>
<td>Field of View</td>
<td>15° x 20.2°</td>
</tr>
<tr>
<td>ADC Resolution</td>
<td>12 bits</td>
</tr>
</tbody>
</table>
Chapter 6 discusses camera design for star imaging in more detail, and outlines the KySat-2 CubeSat mission which will demonstrate this hardware in orbit. Prototype hardware of that system was used to collect night sky data that is analyzed in this chapter.

4.2 Spin Table Design

To measure the performance of the stellar gyroscope algorithm and hardware, datasets were collected of the night sky with known orientations. A stepper motor based spin table was created such that it could be rotated a precise number of steps. Figure 4-2 describes the components of the spin table that was created to support this research.

Due to the discrete nature of stepper motors, with 51,200 steps per revolution using a micro-stepping motor driver, the control resolution is 0.00703125 °/step. This results in a rounding error, where for a desired rotation of 1° the table moves 142 steps (as oppose to 142.222 steps) and results in a rotation of 0.9984375°. This explains the non-rounded nature of the applied rotations in some of the results.
Figure 4-2: Spin Table photo and design diagram
4.3 Algorithm Development Using Night-Sky Tests

4.3.1 Experiment Setup and Collected Data

For initial algorithm development, a dataset using a point-and-shoot camera was taken of the night sky at increments of approximately 1 degree. The images were captured using a Cannon G10 camera set to an exposure time of 500ms, an aperture of F/2.8, and an ISO sensitivity of 1600. The camera was set facing straight up. A photo was taken at 0.9984375° rotation increments using the spin table with approximately 5 seconds between photos. Earth’s motion contributes by 0.02° between photos that are 5 seconds apart. Therefore the results in this section are expected to show an overestimation in the stellar gyroscope algorithm due to earth’s motion, especially when analyzing photo pairs with long elapsed time in between. Another non-ideality in the dataset is caused by the atmosphere and is considered to attenuate and blur the stars and cause inconsistent star brightness, which will cause unreliable star detection. Also, the algorithm has shown to be sensitive to inaccurate estimates for the field of view, and barrel-distortion from the camera lens. Therefore, the field of view was measured in a laboratory setup to be 63.86° by 49.37° with an uncertainty of ±0.07° caused by uncertainty in the location of the focal point of the camera lens. The images were corrected for barrel distortion in post-processing, because the Canon G10 wide lens has significant radial distortion.

4.3.2 Alternative Hypothesis Testing Approach

As discussed in Section 3.3, the Random Sample Consensus (RANSAC) algorithm is used to solve the correspondence across frames. The hypothesis generation and consensus test steps needed to be identified to apply RANSAC to the rotation problem. This section describes some algorithm variants and optimizations. The performance of the algorithm variants is compared using the night sky data set.

The star detection, rotation estimation, and correspondence calculation are as described in Chapter 3, except for the following difference in the RANSAC implementation (because an earlier version of the algorithm was used). In the hypothesis test step, instead of calculating the projection error, the q-method was used by augmenting the two hypothesis pairs with the star pair being tested for consensus. This augmentation will
skew the estimate of the rotation matrix unless the pair of stars being tested shows consensus.

\[ C_{\text{TEST}}^{BA} = \text{qmethod} ([v_{H1}^B v_{H2}^B \ v_N^B], [v_{H1}^A v_{H2}^A \ v_M^A]) \]

To test the consensus the angular error between the hypothesized rotation matrix and the test rotation matrix is evaluated as the angle of the eigen-axis representation of the error rotation matrix.

\[ C_{\text{ERROR}} = C_{\text{HYPOTHESES}}^{BA} C_{\text{TEST}}^{BA}^{-1} \]

\[ \cos(\theta_{\text{error}}) = \frac{1}{2} (C_{\text{error}11} + C_{\text{error}22} + C_{\text{error}33} + 1) \]

Consensus is registered if \( \theta_{\text{error}} < 0.2 \) degrees. Effectively, hypotheses will find little consensus unless they represent the actual rotation.

This version of the consensus test is used for the results shown in section 4.3.4. The improved version presented in Chapter 3 is used for the results shown in section 4.4.

### 4.3.3 Hypothesis Generation Guided by Proximity

Purely random hypothesis generation by randomly selecting two stars in the first frame and paring them with two randomly selected stars in the second frame was found to require a large number of iterations until a hypothesis found consensus. To improve the algorithm’s efficiency, guided hypothesis generation is used by paring stars across frames by proximity or brightness. In this experiment, stars are paired across frames by proximity randomly with one of the two stars nearest to its location. This is expected to be effective for small rotations. In the later experiment (Section 4.4), guided hypothesis generation by pairing stars with similar brightness is used.

### 4.3.4 Experiment Results

Figure 4-3 through Figure 4-5 illustrate the stellar gyroscope operation. In Figure 4-3, two images from the spin table dataset are selected that are known to be 9.98439° apart. The images are overlaid and the colors are inverted for clarity. The paired stars using RANSAC are highlighted. Similarly, Figure 4-4 and Figure 4-5 show the stellar
gyroscope algorithm results for pairs of images that are 24.960975° and 44.929755° apart.

Figure 4-3: Processing and star pairing of two images 9.98439° apart.

Figure 4-4: Processing and star pairing of two images 24.960975° apart.
Figure 4-3 through Figure 4-5 illustrate the success of the implementation of RANSAC to the relative attitude determination problem. It is noted that despite the dataset being rotations around the optical axis, all three degrees of freedom are being estimated and the algorithm would work just as well for panning motions, as long as there are a number of stars visible in both frames. Also, only the estimated angular motion is being shown to gauge performance where the spin axis is also being estimated by the algorithm.

Table 4-2 summarizes the results of the analysis. It is noted that the algorithm appears to be consistently over estimating the rotation amount where the average rotation over multiple trials is larger than the actual applied rotation from the spin table. This is because of the time delay between the photos taken. Looking at every 10th, 25th and 45th image introduces a large time delay between the photos that includes Earth’s rotation that is added to the spin table’s rotation. This is amplified by the nature of the dataset where the photos were taken in steps of 0.9984375° with approximately 5 seconds in between. The results show the Earth’s spin in addition to the spin table rotation.
Table 4-2: Summary of stellar gyroscope algorithm performance in point-and-shoot camera night sky test.

<table>
<thead>
<tr>
<th>Applied Rotation</th>
<th>RANSAC, Random Correspondence</th>
<th>RANSAC, correspondence random of nearest two</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimated Rotation</td>
<td>Estimated Rotation</td>
</tr>
<tr>
<td>9.984°</td>
<td>10.0729°</td>
<td>10.0884°</td>
</tr>
<tr>
<td></td>
<td>σ= 0.0591°</td>
<td>σ= 0.0679°</td>
</tr>
<tr>
<td>24.96°</td>
<td>25.1609°</td>
<td>25.2565°</td>
</tr>
<tr>
<td></td>
<td>σ= 0.0991°</td>
<td>σ= 0.1822°</td>
</tr>
<tr>
<td>44.93°</td>
<td>45.1482°</td>
<td>45.2181°</td>
</tr>
<tr>
<td></td>
<td>σ= 0.0832°</td>
<td>σ= 0.1873°</td>
</tr>
<tr>
<td>Average Complexity for a 10° rotation¹</td>
<td>251640 Quaternion solutions</td>
<td>3109 Quaternion solutions</td>
</tr>
</tbody>
</table>

In this variant of the RANSAC algorithm, a quaternion solution is generated at every iteration (to find a hypothesis rotation), and then for every consensus test (for all combinations of stars in the first and second frames). Table 4-2 shows the average number of required quaternion solutions required to find consensus and calculate the relative attitude. To reduce the number of required number of quaternion solutions, guided hypothesis generation by proximity is used where a star is paired randomly with one of the two nearest detected stars to it. This method is shown to be very effective when the stellar gyroscope is expected to be operated at a rate where stars remain close across frames. Further reduction in the number of required quaternion solutions is achievable by modifying the consensus test step to not require a quaternion solution, as presented in Section 3.3.2 as the latest version of the algorithm, and as evaluated in

¹ Note: In the version of the algorithm used here, the consensus testing step uses the Q-method, i.e. M x N times for every generated hypothesis, where M and N are the number of detected stars in each frame.
Section 4.4.

4.4 Night Sky Tests of Embedded Hardware

Prototype hardware of the camera system (as described in section 4.1) that is designed as part of this work to be used on small satellites is tested next. The ability of the camera system to detect stars is evaluated, and the rotation estimation accuracy is calculated for several test cases.

4.4.1 Experiment Setup and Collected Data

Photos of the night sky were taken on a clear night using the prototype hardware shown in Figure 4-1. Two datasets were collected to study the algorithm’s ability to estimate panning and rotating motions. For the first dataset, the camera was set up pointed upward with arbitrary attitude, and a photo was taken every minute without moving the camera. The image set is shown in Figure 4-7, given the known attitude change of earth between these photos, the algorithm estimate can be compared to the actual values. As the Earth spins in inertial space at a rate of $7.292115 \times 10^{-5}$ radians/s, the rotation between successive photo pairs is $0.250684^\circ$. For the second dataset, the camera was set up pointed upward on the spin table, and photos were taken before and after the spin table was driven to turn a specified angle.

4.4.2 Improved Hypothesis Generation and Consensus Test

The experiment using the embedded camera system was used to further develop the stellar gyroscope algorithm. The results are generated using the latest version of the algorithm, as described in Chapter 3. Recall that the hypothesis generation is based on pairing stars by brightness, where two stars are randomly selected in the first frame, and are paired with two stars in the second frame that are close in brightness.

To reduce the computational cost of the algorithm used in earlier development (as described in section 4.3.2), the consensus testing step is improved to no longer require a quaternion solution. Instead, by projecting each star in the first frame to the second frame using the hypothesis rotation matrix, the hypothesis rotation can be checked for consensus by calculating an error vector as follows for every star combination across
frames:

\[
\vec{\text{error}} = \vec{v}^b - C_{\text{HYPOTHESIS}} \vec{v}^a
\]

The magnitude of \( \vec{\text{error}} \) is minimal for correctly paired stars when the hypothesized rotation matrix represents the true rotation between the two frames. The star pair being tested counts towards the Consensus Set (CS) if \( \| \vec{\text{error}} \| < 0.0005 \). This value was found to work well given the camera and lens parameters, it is expected that the optimal threshold value for other camera systems to greatly depend on the field of view.

4.4.3 Analysis of Detected Star Magnitudes

The atmosphere attenuates the stars and causes blurring. In terms of attenuation, improved response for dim stars is expected in space, and results from the night sky tests are considered to be a worst case sensitivity. Blurring is desirable to allow efficient centroiding. In the absence of atmospheric blurring in space, the lens is set slightly out of focus to achieve the spreading of the stars’ radiation over multiple pixels.

Figure 4-6 shows the camera’s response to the stars Gienah (ε Cygni) and HD 198134, which have apparent magnitudes of 2.48 and 4.92 respectively. While the dim star is not very different from the noise floor in terms of magnitude, the clustering of the pixels allows the filtering and centroiding algorithm to reliably detect it, as it can be noted in Figure 4-7. It can be seen that a star of magnitude 5.47 was intermittently registered across the images. Rayleigh scattering, aerosols, and molecular absorption in the atmosphere are contributing factors to the dimming of the stars in view. Given the parameters of the setup, the atmospheric extinction was estimated to be 0.25 magnitudes [71, 72]. It is expected that in the absence of the atmospheric attenuation in space that stars of magnitude 5.7 and brighter are reliably acquired using this camera.
Figure 4-6: Camera response to two stars of apparent magnitudes of 2.48 and 4.92. HD 198134 is near the noise floor, and was reliably acquired using the filtering approach described.

Figure 4-7: Illustration of detected stars in first photo set of the Cygnus constellation. A photo was taken every minute as Earth rotated in inertial space, every color represents star detections in a single photo and star apparent magnitudes are marked.
4.4.4 Nomenclature of Result Metrics

The following list describes the nomenclature and metrics used in this chapter’s figures and tables to evaluate the performance of the system:

1. **Sample Size**: For every angular spread across images, a number of photo pairs with the same angular spread are used to calculate statistics. Given the nature of the dataset, the Sample Size varies.

2. **Estimate (degrees)**: Applying the stellar gyroscope algorithm between any two photos, results in a relative attitude measurement in three degrees of freedom. The measurements are converted to the Eigen-axis attitude representation, where the attitude is represented as a single rotation angle about a complex rotation axis. The axis is defined by how the camera was set up during the test, which was arbitrary. The angle however, directly correlates to the rotation rate of the Earth and the amount of time elapsed between photos, or the controlled rotation using the spin table.

3. **Estimate mean bias and standard deviation**: The bias is the error angle between the estimated angular rotation and the actual rotation known from the controlled experimental setup. The mean bias and standard deviation are calculated over the Sample Size.

4. **Projection Error (pixels)**: Once the rotation for a photo pair is calculated, the vectors associated with the paired stars are projected from the first frame to the second and are compared to the stars in the second frame. Next, the image plane pixel coordinates for both vector sets are calculated. The projection error is the difference between the pixel coordinates of the stars in the second frame and the pixel coordinates of the stars in the first frame projected using the calculated rotation matrix (shown in the figures as $x$ and $y$ distance components in units of pixels, for each star pair). Sample projection error plots are included in following result figures, as an indicator of the quality of consensus. For a given photo pair and its projection error per star pair, a singular value for projection error is found as the mean (over the number of stars) of the magnitude of the projection error per star.

5. **Mean Projection Error (pixels)**: For a set of photo pairs and the projection error per photo pair, the mean projection error is found as the mean (over the Sample Size) of the projection errors.
6. Average Number of Iterations: Every rotation estimate per photo pair requires the application of RANSAC. The number of iterations is the number of hypotheses that were generated before consensus was found. The average number of iterations is the mean over the Sample Size for a certain trial.

7. Consensus Set Size (%): The stars in the consensus set are the stars that were successfully paired. The number of stars in the Consensus Set is divided by the average number of stars detected per frame, to find the percent value of the Consensus Set Size.

4.4.5 Analysis of Panning Motion

Figure 4-8 through Figure 4-12 illustrate the results of applying the stellar gyroscope algorithm to several photo pairs of panning motion of the camera relative to the sky. The data was obtained by fixing to camera pointed upward on a clear night and taking photos at precise times. Essentially, the rotation of the Earth is being estimated and compared to the documented rotation rate of Earth relative to inertial space.
Figure 4-8: Star pairing of two star field images 0.2507° apart (top), and the projection error plot (bottom). 19 photo pairs like this were analyzed.
Figure 4-9: Star pairing of two star field images 0.5014° apart (top), and the projection error plot (bottom). 18 photo pairs like this were analyzed.
Figure 4-10: Star pairing of two star field images 1.0027° apart (top), and the projection error plot (bottom). 16 photo pairs like this were analyzed.
Figure 4-11: Star pairing of two star field images $2.005476^\circ$ apart (top), and the projection error plot (bottom). 12 photo pairs like this were analyzed.
Figure 4-12: Star pairing of two star field images 2.506844˚ apart (top), and the projection error plot (bottom). 10 photo pairs like this were analyzed.
Figure 4-13: Projection Error for all analyzed photo pairs – panning motion.

Figure 4-14: Consensus Set Size and Mean Projection Error for analyzed rotation angles – panning motion.
Twenty photos were taken of the sky at 1 minute intervals. Comparing successive photo pairs (each 1 minute, or 0.250684°, apart), results in 19 estimates whose statistics are outlined in Table 4-3. The estimate bias is shown as the mean of the difference between the estimates and the actual values. The standard deviation describes the distribution of the estimates about that mean. The table shows accuracy and precision statistics of the algorithm when applied to photo pairs that are 1, 2, 4, 8, and 10 minutes apart, which correspond to angular rotations of Earth by 0.2507°, 0.5014°, 1.0027°, 2.0055°, and 2.5068°, respectively. The estimate bias is below 18 arc seconds (0.005°), and the standard deviation is below 72 arc seconds (0.02°).

Table 4-3: Precision and accuracy of stellar gyroscope from night sky tests – panning motion

<table>
<thead>
<tr>
<th>Actual angular distance between photos</th>
<th>Number of photo pairs</th>
<th>Estimate Bias</th>
<th>Standard Deviation</th>
<th>Detected stars in Consensus Set</th>
<th>Average number of iterations (also number of quaternion solutions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.250684°</td>
<td>19</td>
<td>0.00047°</td>
<td>0.0113°</td>
<td>80.4%</td>
<td>434</td>
</tr>
<tr>
<td>0.501369°</td>
<td>18</td>
<td>-0.00103°</td>
<td>0.0092°</td>
<td>79.2%</td>
<td>506</td>
</tr>
<tr>
<td>1.002738°</td>
<td>16</td>
<td>-0.00418°</td>
<td>0.0108°</td>
<td>74.4%</td>
<td>487</td>
</tr>
<tr>
<td>2.005476°</td>
<td>12</td>
<td>-0.00369°</td>
<td>0.0148°</td>
<td>65.4%</td>
<td>424</td>
</tr>
<tr>
<td>2.506844°</td>
<td>10</td>
<td>-0.00231°</td>
<td>0.0141°</td>
<td>60.5%</td>
<td>793</td>
</tr>
</tbody>
</table>

Figure 4-13 shows the projection error for all the analyzed photo pairs. The projection error can be considered to be a measure of the rotation estimate’s quality, where a small projection error indicates that the estimated rotation correctly projects stars from one frame to the other. Also, as Figure 4-14 shows, the projection error inversely correlates with the consensus set size. It is observed that for larger rotation angles between photos, the estimates’ quality and consensus set sizes decrease. This is caused by unpaired stars that have left the field of view. Some unpaired stars can be attributed to lens distortion, where stars that are in close proximity on the image plane are likely to be displaced (because of the distortion) by the same amount in the same direction, and more likely to show consensus. On the other hand, distortion has a worse effect for larger rotation angles. The distortion correction approach described in Section 3.3.4 did not show significant improvement on this trend, and as discussed, a localized approach for
distortion correction may be more appropriate for the small board mount lens that was used instead of a global distortion model. It is noted that this effect did not prevent the algorithm from operating at arbitrarily large angles; it only results in smaller consensus set sizes. It was found that the described algorithm consistently generated estimates while the consensus set size was larger than 40%.

4.4.6 Analysis of Spinning Motion

To demonstrate the algorithm’s independence of the type of rotation and its ability to estimate rotations about the camera focal axis, Figure 4-15 through Figure 4-18 illustrate the results of applying the stellar gyroscope algorithm to several photo pairs of spinning motion of the camera relative to the sky. The data was obtained using the spin table with the camera pointed upward on a clear night and taking photos between initiated rotations. It is noted that the rotation of the Earth in this setup is an error in the estimate that will appear as a bias, depending on the elapsed time between the photos taken.
Figure 4-15: Star pairing of two star field images $0.9984375^\circ$ apart (top), and the projection error plot (bottom). 6 photo pairs like this were analyzed.
Figure 4-16: Star pairing of two star field images 4.99921875° apart (top), and the projection error plot (bottom). 13 photo pairs like this were analyzed.
Figure 4-17: Star pairing of two star field images 9.9984375° apart (top), and the projection error plot (bottom). 9 photo pairs like this were analyzed.
Figure 4-18: Star pairing of two star field images 19.9968749° apart (top), and the projection error plot (bottom). 6 photo pairs like this were analyzed.
Figure 4-19: Projection Error for all analyzed photo pairs – spinning motion.

Figure 4-20: Consensus Set Size and Mean Projection Error for analyzed rotation angles – spinning motion.
The spin table was set up to move in specified increments, as outlined in Table 4-4. The estimate bias is shown as the mean of the difference between the estimates and the values applied using the spin table (not accounting for Earth rotation). The standard deviation describes the distribution of the estimates about that mean. The table shows accuracy and precision statistics of the algorithm when applied to photo pairs that are approximately 1°, 5°, 10°, 20°, and 10° apart. Even with the Earth rotation introducing bias and variability, the estimate bias is below 38 arc seconds (0.105°), and the standard deviation is below 360 arc seconds (0.1°). The time elapsed between photos was approximately 10 seconds, which corresponds to approximately 0.04° of Earth’s motion, accounting for the major difference in bias estimates compared to the previous section.

Table 4-4: Precision and accuracy of stellar gyroscope from night sky tests – spinning motion

<table>
<thead>
<tr>
<th>Actual angular distance between photos</th>
<th>Number of photo pairs</th>
<th>Estimate Bias</th>
<th>Standard Deviation</th>
<th>Detected stars in Consensus Set</th>
<th>Average number of iterations (also number of quaternion solutions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9984375°</td>
<td>6</td>
<td>0.0578°</td>
<td>0.0843°</td>
<td>74.9%</td>
<td>700</td>
</tr>
<tr>
<td>4.99921875°</td>
<td>13</td>
<td>0.0633°</td>
<td>0.0626°</td>
<td>73.3%</td>
<td>307</td>
</tr>
<tr>
<td>9.9984375°</td>
<td>9</td>
<td>0.0949°</td>
<td>0.0836°</td>
<td>69.2%</td>
<td>253</td>
</tr>
<tr>
<td>19.9968749°</td>
<td>6</td>
<td>0.1013°</td>
<td>0.0433°</td>
<td>59.3%</td>
<td>820</td>
</tr>
</tbody>
</table>

Like the previous section, Figure 4-19 and Figure 4-20 show the projection error for all the analyzed photo pairs and the consensus set size. Similar to the previous section, it is observed that for larger rotation angles between photos, the estimates’ quality and consensus set sizes decrease, which can be attributed to stars leaving the view and to lens distortion. This experiment shows that the stellar gyroscope algorithm is capable of reliable star correspondence without knowledge of the underlying rotation, and of finding the relative attitude between images is calculated in three degrees of freedom.

4.4.7 Analysis of Motion Blur

Motion blur causes the star energy to be spread across multiple pixels and become indistinguishable from the noise floor. Using the spin table rotating at constant speeds, several photos of the night sky were taken. Figure 4-21 shows several detections of the
star 61 Cygni, which has an apparent magnitude of 5.21. In all these images, the star was approximately 10º (of field of view) from the spin center. It was found that the dimmest stars are lost under the noise floor, however, stars of magnitude 5 and brighter remain detectable at these rates using the algorithm and hardware presented in this work. It can be observed that the system can tolerate up to 3 º/second while maintaining reliable star detections. This agreed with simulation predictions for the specific camera resolution and field of view, where angular rates beyond 3 º/second result in a poor signal to noise ratio because of the spreading of the star’s incident energy over a large number of pixels.

Figure 4-21: Acquisition of 61 Cygni (magnitude 5.21) at various speeds of the spin table. The blue square indicates the calculated centroid for the star.
4.5 Operation in Orbital Environment

To consider the effect of the stars magnitude and distribution across the celestial sphere, simulated images are created using the SKY2000 Star Catalog [73]. The database is an extensive compilation of information on almost 300000 stars brighter than magnitude 8.0, meant to be used to create derivative catalogs for specific missions. Figure 4-22 shows a sample simulated image of Polaris that was created for a camera with a field of view of $20^\circ \times 32^\circ$. The background noise level was set based on measurements from night sky tests using the hardware described in section 4.1, and the stars’ pixel values were set to result in a signal to noise ratio in line with measurements for known stars [30].

![Simulated image of Ursa Minor, which includes the North Star, using the SKY2000 Star Catalog.](image)

The simulated images are used to assess the availability of sufficient stars in the camera’s view over the entire sky. The more that dim stars are visible the more stars the camera will have in view on average. There may be patches in the sky where a camera with a narrow field of view would not acquire sufficient stars to apply the presented algorithm. Replacing the optics to widen the field of view is an option at the cost of reduction of the spacial sampling frequency and an inability to distinguish stars that are close to each other.
To perform the analysis, simulated images are created that pan the entire sky. Specifically, the simulated camera is rotated in inertial space with roll angles from 0° to 180°, and pitch angles from 0° to 360°. This is done at increments of half a field of view, and results in a sweep of the entire sky.

Given the 20.2° × 15° field of view of the embedded camera designed for this work, and assuming a threshold where stars brighter than magnitude 5.7 (as discussed in section 4.4.3) are visible, the camera would acquire an average of 22.9 stars per frame over the entire sky. And at least 8 stars are visible in the least star-populated regions.

Using the designed camera and optics with a conservative threshold where only stars of magnitude 5 and brighter are visible, simulations show that 10.5 stars are acquired per frame on average. Also, at least 4 stars are visible in 98.2% of the sky, at least 3 stars are visible in 99.3% of the sky, and at least 2 stars are visible the entire sky.
Chapter 5 Applications

The presented algorithm is capable of calculating the relative attitude between two star field photos with common stars. The RANSAC approach provides tolerance to noise, which enables implementations using small, low cost, and low quality sensors and optics. This allows miniature implementations that are feasible for CubeSat class satellites.

In essence, a three degree-of-freedom implementation of RANSAC applied to angular motion was presented, which can be utilized in several systems. This chapter discusses possible applications of the visual attitude propagation approach presented to this point.

5.1 Drift control in MEMS Gyroscope integration

One application uses the stellar gyroscope relative attitude measurements to maintain attitude knowledge in eclipse for a CubeSat attitude determination system. Many satellites depend on gyroscopes to propagate the attitude in the absence of other means to estimate attitude (in the absence of the sun-vector in eclipse for example). As discussed and motivated in the introduction chapter, MEMS gyroscopes are often the only feasible alternative for attitude propagation under the mass, volume, and cost constraints of small satellite and CubeSat missions. Also, gyroscope integration in three degrees of freedom causes drift in the attitude estimate which can cause loss of attitude knowledge for the noise levels of common MEMS based gyroscopes.

This application of the algorithm has been adopted as a solution for CubeSat attitude determination systems [38, 74, 75]. It is assumed that attitude determination on the miniature spacecraft in the sun cycle is based on a suite of sensors that are based on technologies that can be miniaturized. This is likely to include Sun sensing, which cannot aid in attitude determination in eclipse. For example, attitude knowledge for the system in the illuminated part of the orbit can be based on sun and magnetic field vector measurements combined with the MEMS rate gyroscope data in a Kalman Filter. In eclipse, as it is common in many CubeSat systems that lack star trackers or IR Earth sensors, the loss of the sun vector eliminates the ability to generate absolute attitude estimates and the satellite relies on integrating rate data to maintain attitude knowledge, at the cost of drift.
To demonstrate how the stellar gyroscope can be integrated into an attitude determination system, a simulation was developed that models an attitude determination system entering eclipse. The simulation is based on the SNAP (Smart Nanosatellite Attitude Propagator) tool [76], which I developed in other work [23]. A CubeSat in Low Earth Orbit was modeled in six degrees of freedom under gravity gradient torques. These dynamics provide a test case where the spacecraft body wobbles with a maximum rate of approximately 1.5 °/second to model the MEMS rate gyroscopes and the stellar gyroscope and compare the computed estimates with the actual attitude. Figure 5-1 shows the Simulink® block diagram of the simulation. The top portion implements the satellite orbital and attitude dynamics. The detailed lower portion describes the implementations of a MEMS gyroscope rate integrator, and a stellar-gyroscope assisted rate integrator. The two propagator estimates are compared to the simulated “actual” attitude by finding the attitude error. The estimate errors are visualized in the following figures using the Euler rotation angles representation.

Figure 5-1: Simulink® model of the Attitude and Orbit propagator, as well as the models for the MEMS rate gyroscopes and the stellar gyroscope.
The simulation models the eclipse part of the orbit, beginning with perfect knowledge of the attitude before entering eclipse. The following quaternion kinematic equation is used to propagate the attitude \([45]\):

\[
\dot{q} = \frac{1}{2}(q_4 \omega - \omega \times q)
\]

\[
\dot{q}_4 = -\frac{1}{2} \omega^T q
\]

where \(\omega\) are the angular rates of the spacecraft as measured by the MEMS rate gyroscopes. The rate gyroscopes produce rate measurements at 50Hz with a noise level of 0.1 \(^\circ\)/second RMS, sampled and quantized by a 12-bit analog to digital converter with a range of \(\pm 80\) \(^\circ\)/second. No measurement bias is assumed in this simulation, to provide a best-case result.

The following figures show the drift associated with that integration process, and show how the stellar gyroscope can assist the system in resetting the drift. The discretization in time and magnitude (sampling and quantization), as well as the measurement noise, cause the attitude estimate to drift with time. Figure 5-2 shows the attitude estimate error for the eclipse duration (approximately 40 minutes for a 90-minute orbit). The plot shows the attitude difference between the estimated and actual attitudes in Euler angles representation. Attitude knowledge error increases up to \(5^\circ\) in the first 5 minutes.

Figure 5-3 illustrates how the stellar gyroscope can assist the rate integrator during eclipse. The system generates relative attitude estimates between photos that are taken. In this example, an attitude measurement is made every 10 seconds. A photo is taken at the beginning of the eclipse phase, and is assigned the best known absolute attitude acquired using the attitude determination filter (that uses the sun-vector). Subsequent photos are referenced against the first photo to propagate the attitude through the eclipse phase. It is modeled in the simulation as an imperfect reset to the attitude estimate every 10 seconds. The attitude estimate of the MEMS rate propagator is reset to the actual attitude at an error with a standard deviation of \(0.02^\circ\) and \(0.005^\circ\) of bias. As Figure 5-3 shows, when
stars are in common across frames throughout the period, attitude knowledge is maintained to under 1° of error. This is while maintaining the update rate of 50Hz to allow the attitude controller to maintain control.

Figure 5-2: Euler angles representation of attitude difference between estimated and actual attitudes for an unassisted MEMS gyroscope rate integrator. Beginning with perfect knowledge, the plot illustrates attitude loss in eclipse.

Figure 5-3: Euler angles representation of attitude difference between estimated and actual attitudes for a MEMS gyroscope rate integrator assisted by a stellar gyroscope. The stellar gyroscope resets the drift every 10 seconds.
5.2 Robust Star Detection and Reliability for Long Mission Durations

A major concern in using commercial off-the-shelf (COTS) components on spacecraft is the reliability and durability of the components in the space environment. Imagers suffer from degradation with exposure to radiation where the dark current increases and hot and dead pixels accumulate with time. This limits the useful operational lifetime of these sensors. Shielding is an effective solution, but it may not be feasible on small spacecraft missions. Software solutions that are tolerant to the noise are effective alternative. The RANSAC algorithm at the core of the stellar gyroscope operation searches for consensus among detected stars and is effective at rejecting false detections, which can occur under radiation dose.

The noise tolerance of the presented algorithm can aid star identification algorithms. In star identification systems (star trackers), false star detections can cause problems in correctly identifying the star pattern, limiting their long term reliability. The stellar gyroscope algorithm can be used by processing multiple images of the same star field in order to identify the features that show consensus across photos, to be used by the star identification search algorithm as the actual stars in the star field.

5.3 Discussion

5.3.1 Camera Alignment

Misalignment of the camera with respect to spacecraft body coordinates may occur because of workmanship error during integration or launch vibrations. The accuracy of the attitude propagation algorithm presented is expected to degrade for severe misalignments mainly affecting the spin axis estimates, while the magnitude of the spin angular distance should still be estimated accurately. Several approaches can be employed to eliminate the misalignment relative to the other attitude determination sensors. A suggested approach is to download and process some star field images after launch by identifying the constellations in view to estimate the attitude in inertial space alongside the data from the from the other attitude measurement sensors, and comparing the estimates over several trials. The difference between the two estimates can be attributed to misalignment. Another possible approach is to generate relative attitude estimates at a high rate and
comparing these estimates to the relative attitude calculated by propagating the MEMS rate gyroscope. At a high rate, the drift in the MEMS rate gyroscope integration will be limited, and a consistent bias between those estimates and the stellar gyroscope estimates would indicate a misalignment between those devices.

5.3.2 Random Number Generation

Random numbers are used in the RANSAC algorithm implementation in star selection to establish correspondences. A pseudorandom number generator was found to be sufficient for the algorithm operation, and a high accuracy true random number generator is not required. The algorithm has been developed using MATLAB’s random number generator, and has been tested on target hardware using the standard C library (stdlib.h) for random number generation. Random integer values are used for star selection, and the number of iterations in the RANSAC implementation is fewer than the repeating length of the random pattern of a typical pseudorandom number generator.

5.3.3 Operation Limits

The proposed algorithm reliably calculated relative attitude measurements when photos had a large number of common stars. In an attitude determination system, the stellar gyroscope can be expected to provide these relative estimates for rotations that are smaller than half the field of view for panning motion, and for any magnitude rotation about the camera focal axis. With the current implementation, the maximum number of RANSAC iterations will be exhausted and no consensus will be found if the stars that represent the model are less than 40% of the stars in the images. In a system, care should be taken that the operation profile does not include correlating star field images with too few common stars. Multiple stellar gyroscopes can be used on orthogonal axes to mitigate this issue. In addition, motion blur in imaging the stars becomes an issue at high rotation rates. In the spin table tests, it was found that rates of 3 °/second and below were tolerable.

We note that without a star catalog, the relative approach of egomotion still allows unbounded gyroscope drift when the satellite motion is such that the camera pans the sky and stars leave the frame, resulting in frames that do not have sufficient star pairs for consensus. Multiple stellar gyroscopes on orthogonal axes may also be utilized in such
cases. However, in scenarios where drift control is only required for limited time frames, such as the eclipse duration, the stellar gyroscope is an effective approach where its drift is orders of magnitude lower than an approach based purely on MEMS rate gyroscopes.
Chapter 6 System Implementations

This chapter describes upcoming flight opportunities to demonstrate the stellar gyroscope in orbit. Two systems are discussed, the first is the camera system, as described in section 4.1, which is developed at the University of Kentucky and used to develop the technology and validate the approach to support this research. This system is scheduled to fly on KySat-2, a CubeSat being developed in Kentucky. The second system is an attitude determination and control system developed by SSBV Space and Ground Systems, in the United Kingdom, that utilizes a stellar gyroscope in its sensor array.

6.1 Design Factors for Star Imaging

Star field imaging puts cameras in a low-light imaging mode with low signal to noise ratio. Several sensor and lens factors affect the resulting image quality. Ideally, a low-noise high-sensitivity high-resolution sensor would be used with a small lens with a wide aperture. Realistically however, several of these features cannot be obtained simultaneously and a balance is sought instead. The main factors that have been considered are:

- **Signal to Noise Ratio:** To be able to detect the dimmest stars possible, the camera’s response to light (known as Sensitivity, which is often measured in Volts·Lux s⁻¹) should be maximized and its noise level (dark current, read noise, etc.) should be minimized. It is noted that increasing the exposure time (reducing “shutter speed” in photography terms) allows the accumulation of more light and allows the camera to capture dimmer stars. Disadvantages of a long exposure time include the accumulation of dark current noise, especially when the sensor is set to a high gain value (“ISO sensitivity” in photography terms), and the adverse effects of motion blur where imaging while the satellite rotates might not be possible. Sensors with high signal to noise ratios tend to be large, where the larger pixel wells can accumulate more light for a certain exposure time compared to a physically smaller pixel. The image quality of small imaging sensors continue to improve which motivates further miniaturization of the visual attitude propagation concept in future work.

- **Lens Aperture:** To complete the discussion of the signal-to-noise ratio, a wide lens aperture (small F-number) allows more incident light onto the sensor. From an
Optics point of view, small lenses with large apertures are challenging to design.

- **Lens Focal Length**: The focal length and the sensor’s physical dimensions define the field of view of the camera assembly. Given the camera sensitivity that is mainly dictated by the parameters discussed so far, a field of view should be selected such that the camera is capable of acquiring sufficient stars in the frame over the entire sky, given the stars’ magnitudes and distribution in the sky. Section 4.5 discusses this calculation.
- **Resolution**: The sensor and lens resolutions should be selected to sufficiently distinguish between stars that are near each other. For example, if a large portion of the sky is mapped onto a low resolution sensor, nearby stars may blur together and become indistinguishable. This is essentially the Nyquist sampling theory.
- **Defocusing**: Above the atmosphere, stars can be considered to be point sources. A camera focused to infinity might miss stars that land in between pixels. Defocusing the lens spreads a star’s energy over a few pixels to allow for accurate centroiding. The field of view and the pixel count would determine how much to defocus the lens to get a good spread for each star.

These counteracting factors were considered when designing cameras to function as stellar gyroscopes.

### 6.2 KySat-2

#### 6.2.1 Satellite and Mission Overview

Figure 6-1 shows a 3D model KySat-2 and the location of the camera. KySat-2 is a 1-Unit CubeSat under development by the Kentucky Space Consortium that is being prepared to launch in late 2013. The CubeSat is manifested to launch with NASA’s Educational Launch of Nanosatellites (ELaNa) program, on the ELaNa-IV mission. The expected orbit is circular at 500 km, at an inclination of 40.5°.

KySat-2 will demonstrate key technologies being developed in Kentucky. The stellar gyroscope star imaging camera is an experiment on board KySat-2. The CubeSat also uses a distributed network computing architecture, as well as a power and radio systems developed in Kentucky.
Figure 6-1: Side view of KySat-2 internal components (top). Close up of the camera structure (bottom left). KySat-2 in the deployed configuration (bottom right).

Figure 6-1 overviews the internals of KySat-2. The project is managed by the Kentucky Science and Technology Corporation (KSTC), coordinating the efforts across the
consortium of universities. The Electric Power System (EPS) is designed by Morehead State University and uses Direct Energy Transfer (DET) solar array interfaces and provides power regulation for the other satellite sub-systems based on 18650 Lithium Ion batteries. The communications radio is the Lithium-1 by Astronautical Development LLC, operating in the UHF band. The Command and Data Handling (C&DH) system, Image Processing Unit, and Camera Assembly are developed at the University of Kentucky. The Image Processing Unit is based on a single board computer running Linux, which provides image capture and signal processing capabilities. The Camera is composed of a 5 megapixel CMOS sensor and a 16mm S-Mount lens. The satellite’s sensor suite includes temperature sensors, a 3-axis MEMS rate gyroscope, and a 3-axis magnetometer, along with other spacecraft telemetry.

Passive magnetic stabilization is used for attitude control in orbit, using a set of permanent magnets and magnetic hysteresis material for damping. The magnet placement along the camera axis, such that given the expected orbit the camera has a view of the Earth in the northern hemisphere, and a view of the sky in the southern hemisphere. Figure 6-2 illustrates the expected camera view at various points in the orbit.

The KySat-2 mission will entail image collection of star fields alongside other attitude determination sensor data to demonstrate the performance of the stellar gyroscope and measure its accuracy. The camera system’s ability to resolve stars given their apparent magnitude will be evaluated and a number of raw-images will be downlinked to study the imager’s quality and performance in vacuum, and without the atmospheric distortion affecting ground tests. To demonstrate the relative attitude determination algorithm, image sets will be collected with accurate time stamps to estimate rotation rates, by considering the magnitude of the rotation and the elapsed time between photos. The calculated rates will be compared to the expected rates from the attitude control system simulations, as well as measurements taken from the MEMS rate gyroscopes.
Figure 6-2: Simulation of the expected orbit of KySat-2 and the expected camera view at various positions in orbit. Passive magnetic attitude stabilization aligns the camera axis with the magnetic field direction.

The accuracy of the relative attitude estimates between photos will be checked as well. Coarse attitude knowledge can be obtained by acquiring the Sun vector, using the solar panel voltage and current measurements, and the magnetic field vector measurement using the onboard magnetometer. The change in absolute attitude between the photo pair will be compared to the relative attitude measurement obtained from the images. Also, the relative attitude can also be estimated by propagating the MEMS rate gyroscope data between the photo acquisitions which will have limited drift when the elapsed time is less than a minute.
6.2.2 Concept of Operations

A major challenge in demonstrating the technology is the communications bottle-neck. The space-to-ground data rate is 9600 baud, and given the orbit, a ground station has access to the satellite less than 30 minutes per day. In order to maximize the amount of data returned from the camera system, it is important to compress the images as much as possible, without loss of important information.

For the stellar gyroscope experiment, some forms of compression were deemed acceptable, when considering images from the night sky. Specifically, the color information is dropped and a grayscale image is produced. Also, a depth of 8-bits per pixel was considered to be sufficient. For the purpose of this discussion, a 3 megapixel image is used. Night sky images with the above properties are 3 megabytes large (one byte per pixel, at 3 megapixels). After undergoing lossless image compression (PNG format), the size is reduced to approximately 1.6 megabytes. This size is considered to be very large, which motivated the following mechanism to isolate regions of interest to further reduce the size of the data per image. Lossy image compression techniques (such as JPEG compression) applied to star field images, showed undesirable suppression of low brightness stars and desirable features.

The regions of interest are the star regions, allowing analysis of the star’s registration in the image, as well as the noise levels surrounding the star. Suppressing the remaining areas of the image would reduce the file size significantly. The approach used here is to mask the image to preserve the pixel values around the detected stars while setting the remaining pixel values to zero, then applying a lossless compression algorithm on the resulting image. The presence of the repeating zero valued pixels will result in significant compression.

The steps in this star field image compression approach are shown in Figure 6-3, and can be summarized as:

1. The “Original Image”, as described earlier to be 3 megabytes large, is filtered using the filter kernel described in section 3.1.3.
2. A threshold is applied to the filtered image, creating a mask of the regions of interest. The threshold value is a parameter passed to the function.

3. A dilation filter is applied [77, 78]. Regions surrounding the star are valuable features for centroiding and signal-to-noise ratio estimation. Therefore, the mask is dilated to pass through the surrounding pixels around the detected stars.

4. The dilated mask is applied to the original image (pixel by pixel multiplication). At this point, the image size is still 8-bits per pixel (3 megabytes). However, the image file contains long sequences of zeros, which is highly “compressible”.

5. Applying BZ2 compression onto the file results in a file size around 4 kilobytes, depending on the threshold value. For reference, applying BZ2 compression onto the original image results in a file size of around 900 kilobytes.

Figure 6-3: Steps of the filtering and file size reduction approach of star field images on the KySat-2 mission.

Four kilobytes per star field image is considered very acceptable. Given the access durations and data rates expected for KySat-2, several images can be downloaded per access window. The Open Source Computer Vision library was used to implement this algorithm on the Linux target board [79, 80].

87
6.3 SSBV CubeSat ADCS System

SSBV Space and Ground Systems, in the United Kingdom, which is part of the SSBV Aerospace & Technology Group based in the Netherlands, has been developing a CubeSat Attitude Determination and Control System that incorporates a stellar gyroscope. This section describes SSBV’s system, and highlights analysis and an experimental design I had developed under SSBV’s support.

6.3.1 SSBV System Overview

The attitude determination and control system is designed for CubeSats on a standard PC104 board. In its basic configuration, it integrates a high sensitivity magnetometer, up to 6 sun sensors, three-axis MEMS gyroscopes, and three magnetic torque rods as a three-axis magnetic attitude control system. In its full configuration for improved attitude knowledge and pointing accuracy, a GPS receiver, a stellar gyroscope and an ADCS control computer are added on a daughter board, still within the PC104 height constraints. A momentum wheel or three reaction wheels can be added from a third party supplier.

In this system, the stellar gyroscope complements the MEMS rate gyroscopes in eclipse to maintain an accurate estimate of attitude. However, in order to benefit from accurate propagation in eclipse, accurate knowledge in sunlight is necessary. The system utilizes sun sensors accurate within 0.5 degrees developed by SSBV, as well as a high-accuracy magnetometer that produces magnetic field vector measurements to around 1 degree of accuracy in combination with an IGRF magnetic model and good knowledge of the position in orbit, which is provided by the GPS receiver. This combination results in a high quality estimate of attitude in sunlight.
Figure 6-4: Top: photos of the CubeSat ADCS board of both faces with daughterboard installed (prototype hardware with test connectors). Bottom: block diagram of the overall system. Photos courtesy of: SSBV Space and Ground Systems, United Kingdom.
6.3.2 Experiment on TechDemoSat-1

Figure 6-5 shows the camera system and ADCS system as designed for the technology demonstration experiment on TechDemoSat-1 [81]. The camera on this ADCS experiment consists of a low-cost camera assembly and processing hardware and is designed to require little mass and volume. The camera is based on the OmniVision OV7725 VGA CMOS sensor and a miniature S-mount lens with a focal length of 6mm. This configuration results in a 27.6° by 36.7° field of view. Table 6-1 summarizes the camera specifications.

![Camera assembly and SSBV CubeSat ADCS experiment on TechDemoSat-1. Photos courtesy of SSBV Space and Ground Systems, United Kingdom.](image)

The camera is designed to register stars of magnitude 4 and brighter. With the selected optics, field of view, and an exposure time of 800 ms, at least 4 stars are visible in 97% of the sky, and at least 3 stars are visible in 99% of the sky. Data return from this experiment would aid in further development of the camera system and computing hardware.
Table 6-1: Summary of stellar gyroscope hardware of SSBV ADCS experiment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor</td>
<td>OmniVision OV7725 CMOS VGA Sensor (640 x 480 pixels)</td>
</tr>
<tr>
<td>Optics</td>
<td>6 mm focal length, Aperture F/2.0</td>
</tr>
<tr>
<td>Field of View</td>
<td>27.6° by 36.7°</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>3.8 V/(Lux · s)</td>
</tr>
<tr>
<td>S/N Ratio</td>
<td>50 dB</td>
</tr>
<tr>
<td>Dark Current</td>
<td>40 mV/s</td>
</tr>
<tr>
<td>Pixel Size</td>
<td>6 x 6 µm</td>
</tr>
</tbody>
</table>
Chapter 7 Conclusion

Small satellites are a growing trend and becoming highly capable, reducing the barrier to space for meaningful missions. Small satellites also face unique challenges in terms of power, mass, and volume constraints, motivating the development of miniaturized subsystems based on novel techniques. Attitude determination systems specifically are a major factor in the utility of small satellites. As discussed in chapter one, the availability of low cost and highly compact camera systems motivates attitude determination approaches based on imaging over other technologies based on physical principles that don’t support satellite miniaturization. This dissertation developed star-based visual attitude propagation algorithms and hardware, and demonstrated the feasibility of the approach for small satellites using experimentation, analysis, and simulation.

The practicality of visual attitude propagation was discussed in chapter two. The background research in astronomy and the related work on egomotion estimation and random sample consensus (RANSAC) show that image-based attitude propagation on orbit is feasible. By tracking the motion of stars in the camera field of view, and solving the relative attitude problem, the spacecraft attitude can be propagated in three degrees of freedom as long as the camera is viewing the sky. A device based on this approach is referred to as a stellar gyroscope.

The stellar gyroscope algorithm has been discussed in chapter three. Stars entering and leaving the field of view present a tracking challenge. Also, using low cost and small sensors and optics, star field images are noisy and suffer from false star detections and missed stars. Filtering was implemented to reduce the number of false stars (noise that appear as stars) and missed stars (low brightness stars near the noise floor). A centroiding algorithm has been presented to locate a star’s coordinates on the image plane with sub-pixel accuracy. The camera model described was used to find the star vectors in the body reference frame associated with the stars registered on the image plane. The Q-method has been presented, which was used to find the relative attitude between two vector sets from two images with common stars. To find correct correspondence of stars across frames, the RANSAC algorithm was implemented on rotational kinematics. The iterative
approach used the Q-method with randomly paired stars to generate hypotheses that were tested over the remaining stars for consensus. Several methods of hypothesis generation and consensus testing have been evaluated to optimize the algorithm. The application of RANSAC over three degree-of-freedom rotational kinematics for high accuracy estimation is uncommon in literature and was addressed in detail. The tests in chapter four show that the algorithm was effective at finding a rotation estimate with consensus while rejecting false data, with no assumption or prior knowledge of the underlying rotation direction or magnitude.

An embedded camera system was presented in chapter four, and the accuracy and performance of the algorithm were evaluated. The hardware and algorithms implement a stellar gyroscope for small satellites. Using a spin table and a point and shoot camera, a comparison of algorithm variants was presented. Night sky images from a controlled experiment using the embedded camera hardware and the matured algorithm were used to evaluate the accuracy of the estimates. Photo pairs of known rotation changes for panning and spinning motion were used to compare the estimates to the actual rotation values. It was found that estimation bias was within 0.005° and the standard deviation was below 0.02°. The ability of the camera to acquire dim stars was also evaluated where stars of magnitude 5.7 and brighter are expected to be registered in space. Experiments using the spin table showed that rotation rates up to three degrees per second are tolerable. Analysis using the SKY 2000 star catalog showed that the designed camera, given its field of view and star sensitivity, will register 22.9 stars on average and at least 8 stars in the least star populated regions of the sky.

As the experiments in chapter four showed, the approach successfully paired stars and found the relative attitude in three degrees of freedom with no prior knowledge of the motion. It was shown that the algorithm is unaffected by the orientation of the rotation axis or whether the center of rotation is in the frame or not, as it was shown for panning and spinning motions in sections 4.3.4 and 4.3.5. The noise rejection qualities of RANSAC were also apparent in the sample figures, entering or leaving stars, false star detections, and missed stars were not paired or used towards attitude estimates.
An application of the stellar gyroscope has been presented in chapter five as a low-cost solution to maintain attitude knowledge in eclipse on board a CubeSat, and a simulation was developed to quantify the performance for the specified arrangement. Attitude determination in the sun phase of an orbit can be achieved using miniature sensors, such as magnetometers, sun sensors. In eclipse, the Sun vector is unavailable and rotation rate propagation can be used to track attitude at the cost of drift, which is especially severe for typical MEMS devices. For accurate attitude knowledge in eclipse, star mapping using miniature cameras and optics is an option at the cost of the star database search algorithm and sensitivity to unreliable camera operation caused by temperature drift, stray lights, or increased noise over the mission lifetime. Earth horizon sensing is another option for eclipse, but the technology is challenging to incorporate on miniature spacecraft. The stellar gyroscope’s noise-tolerant relative attitude propagation approach offers an alternative by complementing a three-axis MEMS rate gyroscope integration algorithm, by resetting the accumulated drift using the relative attitude estimates. Simulations showed that attitude knowledge can be maintained below 1° of error when the stellar gyroscope is operated at 0.1 Hz.

The stellar gyroscope experiment on the KySat-2 CubeSat was described in chapter six. The Linux-based single board computer and camera were adapted for the CubeSat mission. Image processing functions have been written to reduce the size of star field images to increase data return over the narrow downlink channel. The data will be used to evaluate the sensor’s star imaging ability above the atmosphere, the survivability of the lens under launch vibration and the vacuum of space, and the sensor and board’s performance over time under the radiation dose.

Future work can utilize the implementation of RANSAC to three degree-of-freedom rotational kinematics by applying it to image features other than stars. For example, Earth and Moon features can also be tracked. Camera exposure control and parallax due to orbital motion become significant factors in this case. Also, further camera miniaturization is suggested to be pursued, as well as evaluating the use of an array of
small multi-purpose cameras on a miniature spacecraft. A camera array can support a wider variety of attitude profiles where an obscured field of view for one camera would not impede the stellar gyroscope’s operation. Also, in a multi-camera system stars can be tracked across cameras, allowing attitude propagation for larger magnitudes of rotation without drift. Finally, it is recommended to utilize the robust star detection approach based on RANSAC to address the challenges associated with the miniaturization of star trackers/mappers. The presented approach can aid by reliably identifying actual stars from the noise, therefore supporting implementations using smaller apertures and prolonging the device’s operational lifetime by tolerating the increasing noise levels as the sensor degrades under radiation dose.
References


[76] S. A. Rawashdeh, "Smart Nanosatellite Attitude Propagator," University of


Vita

Samir A. Rawashdeh

EDUCATION

University of Kentucky, Lexington, Kentucky
M.Sc. in Electrical Engineering  2010
Thesis: Passive Attitude Stabilization for Small Satellites

University of Jordan, Amman, Jordan
B.Sc. in Electrical Engineering  2007

PROFESSIONAL EXPERIENCE

Space Systems Laboratory, University of Kentucky, Lexington, KY
Research Assistant  2007–2013

SSBV Space and Ground Systems, Portsmouth, United Kingdom
Satellite Engineer  04–07/2012

Electrical and Computer Engineering Dept. - University of Kentucky, Lexington, KY
Instructor – Introduction to Embedded Systems (Fall 2010, Spring 2011)

Mechatronics Engineering Laboratory, University of Jordan, Amman, Jordan
Full-time Research Assistant  06-07/2007

IDEA Laboratory, University of Kentucky, Lexington, KY
Research Intern  06-08/2005

HONORS AND AWARDS

• 2012, Honorable Mention, Frank J. Redd Student Scholarship Competition, 26th Annual AIAA/USU Conference on Small Satellites, Logan, UT.
• 2010/2011, Research Challenge Trust Fund Fellowship, Department of Electrical and Computer Engineering, University of Kentucky, Lexington, KY.
• 2011, Honorable Mention, Frank J. Redd Student Scholarship Competition, 25th Annual AIAA/USU Conference on Small Satellites, Logan, UT.
• 2008, First Place Kentucky Academy of Sciences Graduate Research Competition – Engineering Category, Lexington, KY
• 2006, Honorable Mention Award, IEEE Computer Society International Design Competition (CSIDC), Washington, DC.

PUBLICATIONS


S. A. Rawashdeh, “Passive Attitude Stabilization for Small Satellites”, University of Kentucky Master’s Theses, 2010, Lexington, KY.


TALKS

- "Smart Nanosatellite Attitude Propagator (SNAP 2.0)", 2012 Spring CubeSat Developers’ Workshop, San Lois Obispo, CA.
- “Kentucky Space Balloon-1 Near-Space Experiment”, Kentucky Academy of Science Annual Meeting, November, 2008, Lexington, KY.