Complete Inversion of Agricultural Vegetation Parameters by Pol-InSAR: Multibaseline and Δk-radar Approaches

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Abstract—The oriented volume over ground (OVoG) is a simple model applied to the retrieval of biophysical parameters of agricultural crops from polarimetric SAR interferometry data. To date, only a part of the model parameters has been retrieved because the inversion of the model is indeterminate when a single-baseline interferometer is used. In this work, we propose two approaches to overcome this limitation. First, a multibaseline configuration is used to increase the observation space. Second, an expression of the frequency correlation function is obtained for this model, and its inversion is also solved. Both algorithms derive the topography, assuming a linear distribution of the parameters (e.g. topography in [5]), or b) a range is assumed for some parameters are assumed to be known from ancillary data (e.g. vegetation height in [6]), thus retrieving parameters (e.g. extinction in [6]), or c) a range is assumed for some parameters are assumed to be known from ancillary data (e.g. vegetation height in [6]), thus retrieving parameters as solutions within a range.

The solution of the full set of parameters of the OVoG model was firstly addressed in [7], where a two-step procedure was proposed. First, the common line fitting [3] was used to derive the topography, assuming a linear distribution of the coherences in the complex plane, as in the RVoG case. Then, a numerical optimization algorithm was employed to obtain the remaining six parameters: vegetation height, extinction at H and V polarizations, and ground-to-volume ratios at all three linear polarimetric channels (HH, HV and VV). In rigour, the assumption of the linear arrangement of coherences fails for this model, so we decided to investigate other alternative strategies.

In this paper we propose two methods to solve the complete inversion of the OVoG model. In both cases, the observation space is enlarged with information from additional independent channels. The first approach consists of a typical multibaseline acquisition, which has been also proposed previously for PolInSAR. We have implemented this multibaseline inversion by means of a numerical minimization with a genetic algorithm.

The second complete inversion approach consists of a generalization of the multibaseline technique, but substituting the incidence diversity by frequency diversity. This scheme is known as frequency correlation function (FCF) or Δk-radar approach [8], [9]. For this purpose, data must be gathered with a wide-band radar. Note that this function was already used for inverting crop parameters in [10], but assuming a random volume model. We have now assumed an oriented volume, and we have also added the phase information of the FCF and the full polarimetric information, which were previously ignored.

The performance of both techniques is investigated in this paper. Wide-band fully polarimetric data collected at the EMSL at JRC-Ipsara, Italy, with scenes corresponding to maize (1.8 m high) and rice (0.75 m high) samples have been used for this purpose. Details about the experiment setup can be consulted in [6].

II. MULTIBASELINE APPROACH

A. Formulation

The formulation of the RVoG and OVoG direct models of a vegetation scene is well described in the literature [2], [11], so we will recall here the main expressions. In the OVoG case, the interferometric coherence at a certain polarization w can be expressed as a function of: ground-to-volume ratio μ(σw, w); vegetation height (i.e. the depth of the layer) hw; mean incidence angle θ0; vertical wavenumber kw; topographic phase φ0; and extinction σw. The expression of the interferometric coherence is:

$$\gamma = e^{i\phi_0} \cdot \left[ \gamma_0(\sigma_w) + \frac{\mu(\sigma_w, w)}{1 + \mu(\sigma_w, w)} (1 - \gamma_0(\sigma_w)) \right],$$

(1)
\[ \gamma_v(\sigma_w) = \frac{2\sigma_w}{\cos \theta_0} e^{\left(\frac{2\sigma_w h_v}{\cos \theta_0} + jk_z\right) h_v} - 1, \]  

(2)

and the ground-to-volume ratio is:

\[ \mu(\sigma_w, w) = \frac{w^T e^{-\frac{2\sigma_w h_v}{2\sigma_w}} T_g w}{w^T e^{-\frac{2\sigma_w h_v}{2\sigma_w}} T_g w \left(1 - e^{-\frac{2\sigma_w h_v}{2\sigma_w}}\right) T_v w}. \]  

(3)

where \( T_g \) and \( T_v \) represent the coherency matrices of ground and vegetation volume, respectively.

In the derivation of these expressions, it is assumed that polarization remains unchanged as the wave propagates through the volume. This is correct for a random volume because extinction does not depend on polarization \( \sigma_w = \sigma \). In the case of an oriented volume, however, the only polarizations which do not change as the wave propagates through the volume are the eigenpolarizations [11]. Since most agricultural plants exhibit a preferred vertical orientation, mainly dominated by the stems, the eigenpolarizations can be identified as vertical and horizontal polarizations. In such a situation, the only three polarimetric channels valid in the cited formulation are HH, HV and VV.

In summary, the observables delivered by a single–baseline polarimetric interferometer are the complex coherences for the three channels in linear basis (HH, HV and VV), summing a total of six real data (real and imaginary parts, or amplitude and phase). However, there are seven model parameters to be inverted: topographic phase \( \phi_0 \), vegetation depth \( h_v \), vertical extinction \( \sigma_v \), horizontal extinction \( \sigma_h \), and ground-to-volume ratios for the three channels: \( \mu_{HH} \), \( \mu_{HV} \) and \( \mu_{VV} \). Consequently, the inverse problem is indeterminate because the number of independent observations is lower than the number of parameters (unknowns). The first solution proposed in this paper is to increase the observation space by adding a second baseline. When we change the baseline, we can expect to maintain all electromagnetic properties of the scene: extinctions and ground-to-volume ratios. Consequently, with 2 baselines we have 12 input real data available to estimate only 7 model unknowns. The solution of the fitting between model and data has been implemented with a genetic algorithm. Due to the high nonlinearity of the model, all solutions suffer from a high dependence upon the initial guess. We have carried out 100 realizations of the inversion solution for each point, providing the algorithms with random initial values.

**B. Experimental Results**

We present in Fig. 1 the results obtained for the maize sample from 2 to 6 GHz. The maximum frequency has been limited to avoid the volumetric decorrelation produced above 6 GHz by the 0.5° baseline. Ground topography and vegetation height are estimated quite precisely in the whole frequency range. These height estimates are also very stable from the numerical point of view. In contrast, extinction estimates fluctuate when we change the initial solution of the algorithm (see the error bars). Nevertheless, as expected, vertical extinction is always higher than the horizontal one. Their difference ranges between 0.5 and 1 dB/m. Finally, all ground-to-volume ratios decrease with frequency, as we expected from the stronger vegetation response at higher frequencies.

Figure 2 shows the estimates of the multibaseline approach applied to the rice data from 5 to 9 GHz. The ground position \( z_0 \) is retrieved with extremely good accuracy, as a result of the dominance of the ground-stem contribution, which is helped by the flooded condition of the soil. Plants height is slightly underestimated and it presents a higher variance than in the maize case, but in this case vegetation is shorter and hence...
the inversion is not well conditioned. Extinction estimates are less stable than in the maize experiment. Anyway, as for the maize, the average values of \( \sigma_V \) and \( \sigma_H \) do not approach each other at high frequencies. All ground-to-volume ratios are quite constant with frequency, and \( \mu_{VV} \) is the smallest due to its lowest ground-stem response.

III. FREQUENCY CORRELATION APPROACH

A. Formulation

1) Frequency Correlation Function for a RVoG: The corresponding FCF function can be derived from [2] by considering that the sensitivity to the vertical distribution of scatterers is introduced by means of the frequency diversity, which appears in the vertical wavenumber expression \( k_z \) as:

\[
k_z = -\frac{4\pi}{c} \Delta f \cos \theta_0
\]

where \( \Delta f \) is the frequency shift.

The normalized complex correlation function for a random volume over ground (RVoG) can be expressed as [2]:

\[
\gamma_{f cfRVoG} = e^{j \frac{4\pi}{c} \Delta f r_0} \frac{f_{\text{int}_{RV}}(\sigma, k_z) + 4\mu_h v}{f(\sigma) + 4\mu_h v}
\]

where \( r_0 \) is the reference distance to the center of the resolution cell at the ground level, \( \sigma \) is the extinction coefficient (polarization independent), and \( \mu \) is the ground-to-volume ratio. Note that the topographic phase \( \phi_0 \) does not appear as a parameter in this formulation. Functions \( f_{\text{int}_{RV}}(\sigma, k_z) \) and \( f(\sigma) \) are defined as:

\[
f_{\text{int}_{RV}}(\sigma, k_z) = \frac{e^{(2\sigma/\cos \theta_0 + jk_z)h_v} - 1}{2\sigma/\cos \theta_0 + jk_z}
\]

\[
f(\sigma) = \frac{e^{(\sigma/\cos \theta_0)h_v} - 1}{2\sigma/\cos \theta_0}
\]

2) Frequency Correlation Function for an OVoG: In the OVoG model, we have to introduce two different extinction coefficients: \( \sigma_a \) for the transmitted wave and \( \sigma_b \) for the received wave, which are assumed to be the eigenpolarizations. Following the indications in [2], the integrals along \( z \) dimension of the cross-product contributions (\( V \) stands for volume, \( GV \) stands for ground-volume and \( VG \) for volume-ground) are given by:

\[
(V \ast V) \int_0^{h_v} e^{\frac{2\sigma_a}{\cos \theta_0}z + jk_z} \cdot e^{j \frac{4\pi}{c} \Delta f r_0 z} dz
\]

\[
(GV \ast GV) \int_0^{h_v} e^{\frac{2\sigma_b}{\cos \theta_0}z} \cdot e^{j \frac{4\pi}{c} \Delta f r_0 z} dz
\]

\[
(GV \ast VG) \int_0^{h_v} e^{j \frac{4\pi}{c} \Delta f r_0 z} dz
\]

\[
(VG \ast GV) \int_0^{h_v} e^{j \frac{4\pi}{c} \Delta f r_0 z} dz
\]

\[
(VG \ast VG) \int_0^{h_v} e^{-\frac{2\sigma_b}{\cos \theta_0}z} \cdot e^{j \frac{4\pi}{c} \Delta f r_0 z} dz
\]

Therefore, by calculating these integrals and grouping terms, the expression for the polarimetric complex correlation function for an oriented volume is obtained:

\[
\gamma_{f cf\mid OVoG} = e^{j \frac{4\pi}{c} \Delta f r_0} \frac{f_{\text{int}_{OV}}(\sigma_{a+b}, k_z) + 2\mu_{ab} f_2(\sigma_{a-b})}{f_1(\sigma_{a+b}) + 2\mu_{ab} f_2(\sigma_{a-b})}
\]

where \( a, b = h, v \) denote horizontal or vertical polarization and

\[
f_{\text{int}_{OV}}(\sigma_{a+b}, k_z) = \frac{e^{(\sigma_a + \sigma_b)/\cos \theta_0 + jk_z)h_v} - 1}{(\sigma_a + \sigma_b)/\cos \theta_0 + jk_z}
\]

\[
f_1(\sigma_{a+b}) = \frac{e^{((\sigma_a + \sigma_b)/\cos \theta_0)h_v} - 1}{(\sigma_a + \sigma_b)/\cos \theta_0}
\]

\[
f_2(\sigma_{a-b}) = h_v + \sinh \left( \frac{\sigma_{a-b} h_v}{\cos \theta_0} \right)
\]

Note that the main difference between RVoG and OVoG direct models appears in the additional decorrelation term \( f_2(\sigma_{a-b}) \), which induces a decrease in the absolute value of the FCF.

B. Experimental Results

Figure 3 shows the aspect of the FCF calculated from the experimental data using the linear basis for 2.5, 5.5 and 8 GHz, when it is plotted on the complex plane. The evolution of the FCF agrees with the theoretical FCF obtained for the OVoG in the previous section.

The inversion of the FCF has been performed in two ways. First, only the absolute value of the FCF function has been used. Second, the phase information of the FCF function has been incorporated. The estimates obtained for the maize sample are shown for both approaches as a function of frequency in Figs. 4 and 5, respectively.
When only the absolute value of the FCF is used in the inversion, there appear some fluctuations in the height estimates, but their error is below 20 cm, except for a frequency of 2.5 GHz. The ground-to-volume ratio at horizontal polarization $\mu_H$ is greater than at vertical polarization $\mu_V$ at low frequencies, but they approach each other at high microwave frequencies. Regarding the extinction coefficients, the retrieved values are very low and variable, which may be produced by the lack of sensitivity of the model to this parameter.

When the parameter inversion is performed by adding the phase information, the dimensionality of the problem is increased but we expect to improve the estimates. However, when observing the retrieved results in Fig. 5, no definitive conclusions can be stated. On the one hand, the estimated height error shows an increasing trend which reaches 40 cm. It must be noted that in this case the ground-to-volume ratio estimates are not provided since the inversion procedure yields extreme values for the whole frequency range. On the other hand, the estimated values for the vertical and horizontal extinction coefficients follow the theory predictions better than only using the absolute value, i.e., a higher vertical extinction for S-band, whereas the attenuation becomes similar for both polarizations as the frequency increases.

Finally, the parameter inversion has been also performed by only considering the copolar channels (i.e. ignoring the cross-polar data), but it has been also observed that the retrieved estimates are quite similar to the ones obtained previously with the complete polarimetric information.

**IV. CONCLUSIONS**

In general, the application of the two proposed techniques to the inversion of the OVoG model has resulted in acceptable estimates of vegetation height and ground topography. In contrast, the obtained values of extinction coefficients are quite variable and do not follow the expected trends. This limitation may be produced by a lack of sensitivity of the direct model to extinction, but a deeper analysis should be done before a definitive conclusion is stated.

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