Correspondence

A Noise-Filtering Method Using a Local Information Measure

Azeddine Beghdadi and Ammar Khellaf

***Abstract*— A nonlinear-noise filtering method based on the entropy concept is developed and compared to the well-known median filter and to the center weighted median filter (CWM). The performance of the proposed method is evaluated through subjective and objective criteria. It is shown that this method performs better than the classical median for different types of noise and can perform better than the CWM filter in some cases.**

I. Introduction

There are two basic approaches for noise filtering, namely, spatial methods and frequency methods. Most of the spatial smoothing processes as the mean and the median filters, which are widely used [1]–[5], generally tend to remove noise without explicitly identifying it. It is, though, possible to filter selectively the noise signal by comparing the gradient grey level in a neighborhood to a fixed threshold [2], [6]. The frequency smoothing methods [7], [8] remove the noise by designing a frequency filter and by adapting a cut-off frequency when the noise components are decorrelated from the useful signal in the frequency domain. Unfortunately, these methods are time consuming and depend on the cut-off frequency and the filter function behavior. Furthermore, they may produce artificial frequencies in the processed image. This correspondence introduces a new nonlinear filter based on the entropy concept. Since the pioneer work of Frieden [9], the use of entropy [10], [11] in image analysis has attracted a great number of researchers, especially in image reconstruction [12], [13] and segmentation [14], [15]. In our previous work [16] that followed the idea of Shiozaki [17], which consists in defining a local entropy and thus performing a local treatment, we have shown that by amplifying the local entropy of the contrast, one can get a contrasted image. It has also been suggested that a noise-filtering treatment can be obtained by decreasing the entropy of the local contrast in a given neighborhood. In this correspondence, we develop this point in detail in Section II, and in Section III give some examples to show the effectiveness of the proposed filtering method. A comparison with filters of comparable complexity—the well-known median filter [3] and the CWM filter [18], [19]—followed by a general discussion is also given. The proposed method is not compared to the weighted median (WM) filter, which is more complex, since it requires the optimization of the weights using some error criterion under certain constraints [19], [21]. Finally, Section IV is devoted to the conclusion and perspective.

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II. Noise Filtering—A New Approach

In the present approach, we propose a method of identifying the noise by using the local contrast entropy. A picture element is considered as noise when the associated local contrast is very different from those of its neighboring pixels. Therefore, a local contrast threshold allowing this discrimination is defined according to the local contrast entropy. Thus, a pixel is identified as noise according to its contribution weight to the local contrast entropy. Given a pixel i, center of a window W_i, of grey level g_i, the associated contrast is defined according to the Weber–Fechner law by

$$C_i = \frac{|g_i - \bar{g}_i|}{\bar{g}_i} = \frac{\Delta_i}{\bar{g}_i} \tag{1}$$

where $\bar{g}_i$ is the mean grey level of the surround region of the center pixel in the window W_i and Δ_i is the gradient level. In contrast to other contrast definitions [6], [20], this quantity gives the same local contrast for grey levels situated at the same distance from the mean grey level. One can associate to the local contrast the probability

$$P_i = \frac{C_i}{\sum_{i=1}^{n} C_i} = \frac{C_i}{C_s} \quad \text{or} \quad P_i = \frac{\Delta_i}{\sum_{i=1}^{n} \Delta_i} \tag{2}$$

where n is the window size. Once the probability of the local contrast is defined, one can estimate the probability to find a noise point. In fact, a zero contrast zone, i.e., a homogeneous region, corresponds to a zero probability. That means that the probability to find a noise point in a homogeneous region is equal to zero. In information context, we say that the degree of uncertainty to find a noisy point is minimum and is equal to zero. Therefore, to give a measure of the degree of uncertainty that a point in a region is a noise, we associate to the given region a local contrast entropy defined as follows:

$$H = -\sum_{i=1}^{n} P_i \log P_i. \tag{3}$$

Therefore, a noise pixel or isolated point heavily contributes to the contrast entropy since its probability is high. The basic idea of the proposed technique is to modify the grey level of the pixel according to its contrast value or its contrast probability. Then, the grey level of the current pixel is transformed with respect to a threshold contrast probability P_C corresponding to a window where all the local contrasts are equally distributed; this results in a maximum entropy. From (2), one can show that this case occurs when all the local contrasts are equal and different from zero. It corresponds to a region composed of two homogeneous subregions of the same area, for example, a well-contrasted sharp symmetrical edge. Thus, a pixel is considered as noise signal when the corresponding probability P_i is greater than or equal to the critical value $P_C = 1/n$, where n is the size of the analysis window. We give to the pixel i the mean or better the median grey level if the associated probability P_i is greater than or equal to P_C. In the following we quantitatively justify the choice of this critical value for the contrast probability.

A. Noise Model—How to Identify the Undesirable Information

It could be noticed that the proposed method is data dependent. It becomes thus difficult to perform a complete analytical analysis without assuming some *a priori* knowledge of the signal and the noise. In the following, for the sake of simplicity we choose a size of

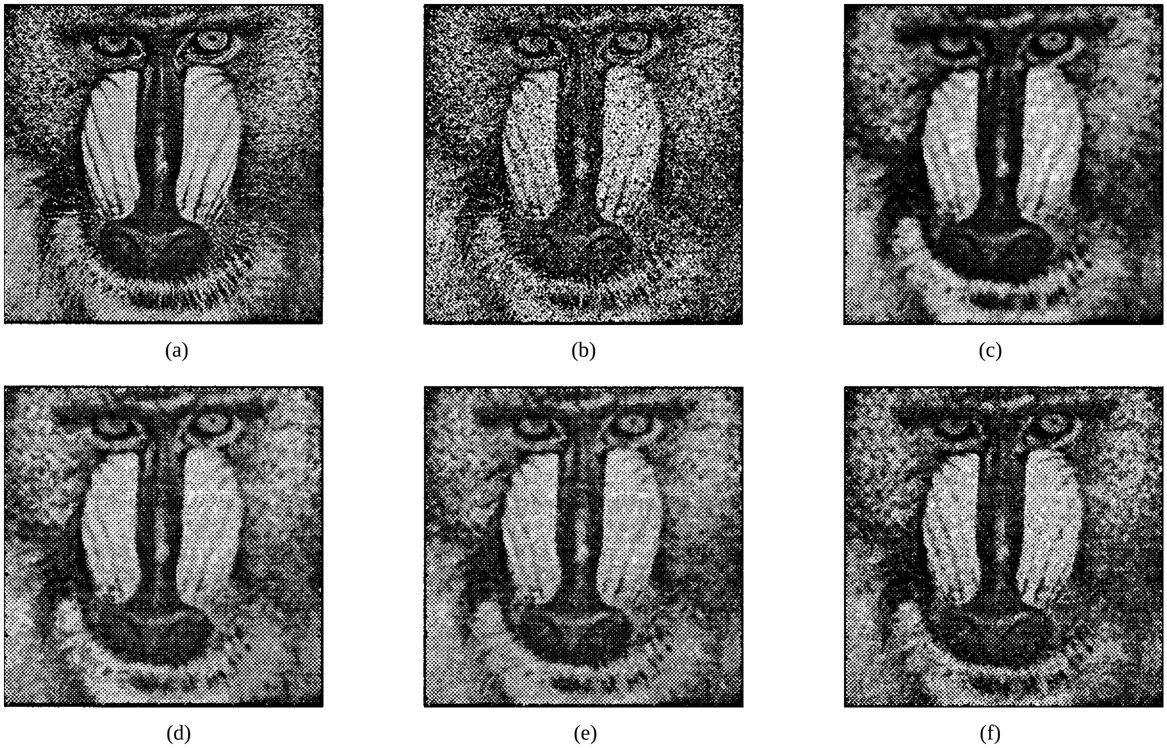

Fig. 1. (a) Original image. (b) Noisy image (Gaussian and impulsive noise). (c) Median filter. (d) CWM filter (weight $W = 3$. (e) CWM ($W = 5$). (f) Our method.

3 × 3 pixels for the analysis window and we consider some typical cases of spatial grey-level distribution in the window. Let us now analyze the following decision rule: g_i is a noise grey level iff

$$P_i > P_C. \tag{4}$$

First Case—A Spot Point: This is the easiest case. It consists of a well-localized spot point with a grey level B surrounded by its eight neighbors of grey level A such that $B = A \pm \Delta$. It is easy to show that $P_B > P_C$. In other words, the degree of uncertainty to identify this point as noise is zero.

Second Case—A Homogeneous Region: An example of a homogeneous and noise free region consists of 3 × 3 pixels having almost the same grey level, say A. It can be easily shown that the contrast entropy associated to this case is zero or very low and thus $P_i < P_C$. In this case we are sure, with a degree of uncertainty equal to zero, that there is no noise in the given window.

Third Case—Critical Situation: This case corresponds to a sharp transition or a ramp. Indeed, in such a situation, the number of pixels having a grey level greater than the mean grey level is the same as that of the complementary set. It results in a contrast probability equal to P_C. It is easy to establish this result from (3). If n is the window size, then $P_C = \frac{1}{n}$, and the considered pixel is not a noise point.

Fourth Case—A Transition Region: To simplify the analysis, let us consider a window of size n, where n_1 pixels have nearly the same grey level A_1 and n_2 pixels have the grey level A_2. Let B be the grey level associated to the central pixel. One can easily obtain the mean grey level $\bar{g}$ and the local contrasts C_B, C_1, and C_2 corresponding, respectively, to the central point, a pixel of the first class, and a pixel belonging to the second class. It is easy to show that if the grey level of the two regions 1 and 2 are identical, then it results in a zero probability. It corresponds to $P_B = 1$, and $P_1 = P_2 = 0$. This case has been already considered (a spot point).

A similar analysis for the case when $n1 = n2 = k$ yields the following decision rule. The central point of grey level B is considered as noise if

$$B < \min(A_1, A_2) \quad \text{or} \quad B > \max(A_1, A_2). \tag{5}$$

Now, let us consider the case where A_1 is different from A_2 and n_1 is not equal to n_2. The size of the window is $n = n_1 + n_2 + 1$. A similar computation leads to

$$P_B > \frac{1}{n} \Leftrightarrow \left| B - \frac{n_1 A_1 + n_2 A_2}{n_1 + n_2} \right| > \frac{2 n_1 n_2}{(n_1 + n_2)^2} |A_1 - A_2|. \tag{6}$$

It can be noticed that the first side of the inequality is nothing else than the difference between the grey level of the center pixel and the mean grey level $\bar{A}$ of the surround pixels. The term on the right side of the relation is proportional to the interclass variance. Indeed, the interclass variance is

$$\sigma_{1,2}^2 = \frac{n_1 n_2}{(n_1 + n_2)^2} (A_1 - A_2)^2 \tag{7}$$

Therefore, condition (6) can be written

$$|B - \bar{A}| > \frac{2\sqrt{n_1 n_2}}{(n_1 + n_2)^2} \sigma_{1,2}. \tag{8}$$

In summary, conditions (5) and (8), which are equivalent to condition (4), state that a pixel is considered as a noise element if its grey level is far from those of its neighbors. This distance is measured through statistical parameters. This is debatable but so are other similar noise testing models.

Fifth Case—Actual Noisy Window: This case corresponds to the most encountered configuration in nontextured images. It consists in a noisy point embedded in a quasihomogeneous region as shown below.

$$\begin{matrix} A_1 & A_2 & A_3 \\ A_4 & B & A_5 \\ A_6 & A_7 & A_8. \end{matrix}$$

The grey levels of the surrounding points can be written in the form $A_j = A_m \pm \varepsilon_j$, where A_m is the mean grey level and ε_j is a small

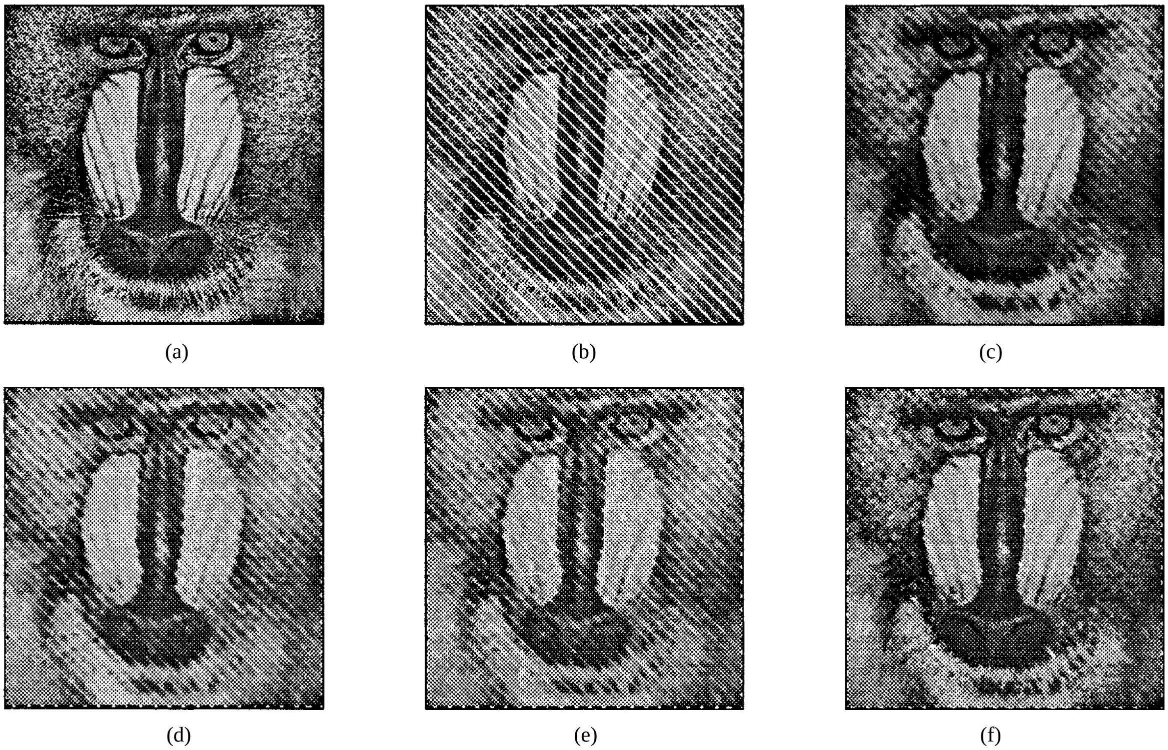

Fig. 2. (a) Original image. (b) Noisy image (structured noise). (c) Median filter. (d) CWM filter (weight $W = 3$). (e) CWM ($W = 5$). (f) Our method.

value. In this case, B is different from the mean grey level, that is B can be written as follows: $B = A_m \pm \Delta_i$ where Δ_i is much greater than ε_j. Using the contrast definition and the associated probability, one can easily show that the central point gives the major contribution to the local contrast entropy. The associated probability is greater than the threshold value P_C. Indeed, from (3) the probability P_B of the center pixel is greater than P_C if the following condition is satisfied:

$$\Delta_i > \frac{1}{n} \sum_{j \neq i}^{n} \Delta_j = \bar{\Delta}. \tag{9}$$

The condition (9), equivalent to the inequality $P_i > \frac{1}{n}$, is intuitively appealing. Indeed, this condition states that the examined pixel is considered as a noise point if the corresponding gradient grey level is greater than the average gradient level computed in its neighboring.

III. Experimental Results

To test the efficiency of the proposed method, two types of additive noise have been considered. The first one is a mixture of a zero mean Gaussian noise (with $\sigma^2 \approx 215$) and a impulsive noise (with probabilty $p = 0.10$), and the second one is a structured noise. The method is compared to the classical median filter and the CWM filter with 5 × 5 square window. The degraded signal with the additive structured noise is generated following the expressions: $g(x,y) = f(x,y) - I\{\eta(x,y) > \eta_0\}[f(x,y) - N_0]$, where $f(x,y)$ is the original grey level, I the indicator function, η the interference noise given by: $\eta(x,y) = \eta_0 + \eta_1 \cos(2\pi\nu(x\cos\theta - y\sin\theta))$ and N_0 the desired maximum noise amplitude. In the experiments, the following values are chosen for the noise parameters: $N_0 = 255,\ \eta_0 = 128,\ \eta_1 = 127,\ \nu = 2^{-10}$ and $\theta = \frac{\pi}{4}$. With these values the fraction of corrupted pixels is 18%. The size of the test images is 256 × 256 pixels quantized with 256 grey levels. For subjective comparison only subjective criteria, namely, the visual perception quality, are used. For objective comparison the well-known normalized mean square error (NMSE) and the mean absolute error (MAE) are used as in [18], [19] and [21].

Fig. 1(a) shows a digitized image of mandrill. This image presents a typically difficult case for filtering purposes. In fact, many interesting small structures have size and tone values comparable to those of the noise. Thus, filtering such an image seems to be a rather difficult task. Fig. 1(b) shows the image of Fig. 1(a) after adding a Gaussian and impulsive noise. Fig. 1(c) is the result of applying a 5 × 5 median filter and Fig. 1(d) and (e) corresponds to the CWM filter with, respectively, a weight of 3 and 5. Fig. 1(f) displays the result obtained with the proposed method. Through these results, it can be noticed that the median filter smooths out the noise as well as the image details. This results in a blurred image. This undesirable effect is less important when using the CWM filter. Whereas, our method cleans the image without blurring the contours. Furthermore, Table I clearly shows that the proposed method performs better than the median filter and the CWM with the lower weight. However, for the high weight, the CWM yields lower errors (NMSE and MAE) and, thus, objectively performs better than our method. However, a simple visual comparison clearly shows that the proposed method preserves better the image contrast and details than the CWM filter. This disagreement with the quantitative comparison is essentially due to the fact that the NMSE and MAE measures cannot distinguish between a few large deviations and many small ones. Consequently, one has to develop other quantitative measures taking into account the visual criteria to compare the obtained results. Unfortunately, to our knowledge, it is difficult to find such measures at present time.

The second comparison concerns the structured noise. One can observe in Fig. 2(b) a periodical structure with bands of the same orientation and size. This noise is easily distinguishable from the actual structure of the image. Thus, it is easy to follow the filter effects on the noise. This noise can obviously be smoothed out by using frequency filtering as described in [7] and [8]. But the frequency methods require orthogonal transformations to decorrelate the image components. This approach is time consuming and complicated.

TABLE I
OBJECTIVE COMPARISON—NMSE'S AND MAE'S ASSOCIATED WITH THE DIFFERENT FILTERS APPLIED TO THE DEGRADED IMAGE OF FIG. 1(b) (ZERO MEAN GAUSSIAN NOISE WITH $\sigma^2 \approx 215$ AND IMPULSIVE NOISE WITH THE PROBABILITY $p = 0.10$)

Method	Median filter	CWM (W=3)	CWM (W=5)	CWM (W=7)	Our method
NMSE	0.4457	0.5129	0.3957	0.3218	0.4118
MAE	23.9699	26.5925	23.0711	20.6408	23.3697

TABLE II
OBJECTIVE COMPARISON—NMSE'S AND MAE'S ASSOCIATED WITH THE DIFFERENT FILTERS APPLIED TO THE DEGRADED IMAGE OF FIG.2(b) (STRUCTURED NOISE)

Method	Median filter	CWM (W=3)	CWM (W=5)	CWM (W=7)	Our method
NMSE	0.3103	0.4193	0.3476	0.2997	0.1961
MAE	21.7761	24.0214	20.3661	17.4875	10.0960

Furthermore, the result depend on the filter function behavior which may create artificial frequencies at the output.

Fig. 2 shows the obtained results corresponding to the median filters and to the proposed method for a 5 $\times$ 5 window size. The superiority of our technique is clearly demonstrated on this example. The median is successful in eliminating the grid, but it blurs the image, and an ondulation effect due to the grid appears in the processed image of Fig. 2(c). In contrast, the CWM does not blur the image but the interference noise is not completely removed. Whereas, our method [see Fig. 2(f)] removes a large fraction of the noise without sensibly modifying the contours and other details. Furthermore, it is noticed that the classical median filter performs better than the CWM filter in smoothing out this structured noise. This result is not surprising since it was shown by Ko and Lee [12] that the CWM filter tends to preserve lines and more details at the expense of less noise suppression. Table II confirms these comparison results. Indeed, the proposed method yields the lowest NMSE and MAE.

IV. CONCLUSION

A simple method for noise filtering has been presented and compared to the well-known median filter and the CWM filter. The obtained results on actual images corrupted by two types of noises confirm the superiority of the proposed technique over the well-known median filter and the CWM filter. The usefulness of the entropy concept for image enhancement purposes is demonstrated. This superiority is justified by the fact that the proposed method is simple and successful in smoothing out different noises. It was shown that for noise smoothing, the CWM can objectively perform better than the proposed method for some weights. But the central weight should be carefully selected depending on both the characteristics of the original image and the added noise. In the proposed method, such constraints do not exist. This makes the proposed method more flexible than the median filters, as it does not necessitate sorting the data. The derivation of a detailled analytical analysis taking into account the data-dependent character of the method is under study. Furthermore, the separability of the filter will be considered in a near future, making thus the method faster.

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