Efficient and Low-Complexity User Selection for the Multiuser MISO Downlink

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Abstract—In this work an efficient low-complexity user selection algorithm for the multiuser MISO downlink wireless channel is presented. The algorithm combines Zero-Forcing linear precoding with low-complexity correlation-based Space Division Multiple Access (SDMA) and aims at maximizing the sum rate of all system users. The complexity of the algorithm is analyzed and is shown to be lower than previously proposed schemes. Moreover, simulation results indicate that the performance penalty is small.

Index Terms—MISO, SDMA, Zero-Forcing Beamforming, Multiuser Diversity

I. INTRODUCTION

THE use of Multiple-Input Multiple-Output (MIMO) transmission can result to a significant increase in the link reliability and spectral efficiency of multiuser wireless systems. In such systems, the different channel conditions of each user can be exploited to boost system performance metrics such as the aggregate user throughput (sum rate) [1]. In [2] it was shown that Dirty Paper Coding (DPC) can achieve transmission on the boundary of the capacity region of the MIMO Broadcast Channel (BC). An iterative algorithm for the computation of the sum rate was presented in [3]. Despite its optimality, the data-dependent nature of the encoding process renders DPC highly complex, and, therefore, unattractive in some practical scenarios. A Zero-Forcing DPC-based technique (ZF-DPC) and a greedy version of ZF-DPC were proposed in [4] and [5], respectively. These techniques combine DPC with QR decomposition to completely eliminate the interference among transmitting users. It is shown that the performance that is achieved is close to the performance of the optimal, capacity-achieving scheme. Nevertheless, both techniques employ DPC. Hence, although their complexity is lower than the optimal scheme, it can still be prohibitively high in realistic scenarios.

In practical resource allocation schemes, the available system resources are typically assigned to users in an orthogonal fashion, either in time or frequency (TDMA/FDMA). This approach maximizes the sum rate in some special cases, i.e., when the SNR is very low or when the channel is the SISO, Gaussian BC. However, it is suboptimal in the case of the MIMO BC because it does not exploit fully all inherent degrees of freedom of the system. Space Division Multiple Access (SDMA) using transmit beamforming has been proposed as a promising solution for resource allocation that retains the benefits of MIMO and is less complex than DPC-based techniques, [6]. In SDMA, a group of compatible users share the common resources in a way that improves the efficiency of the system. In order to use SDMA in downlink transmission, user separation is required. This may be a challenge, because adaptation of the transmission to one user affects the transmission of all others. In [7], Zero-Forcing transmit beamforming is used for a multiuser system where each receiver has one antenna (MISO case). A user selection policy is proposed that increases gradually the number of users sharing the resources as long as the insertion of a new member in the group of simultaneously transmitting users increases the sum rate. The idea is extended to the MIMO case in [8], where Zero Forcing for multiple receiver antennas is used, also known as Block Diagonalization (BD) [9].

The Achilles’ heel of the above strategies is their complexity, mostly because they require the calculation of the sum rate for each candidate user in every iteration. A way to reduce the complexity is to take into consideration the correlation between the users. A SDMA algorithm that is based on spatial correlation was presented in [10], but it requires solving an optimization problem using a priori knowledge on the number of users sharing the channel. Spatial correlation was also used in [11] for the case of the MIMO BC, but BD without antenna selection is not so efficient. In [12], a semi-orthogonal basis is constructed for the MISO case using the channel vectors of the users. However, the method is based on a parameter that can only be computed heuristically and its value depends on the number of users and the transmitted power.

In this paper, the problem of user selection for the MISO BC is addressed when the goal is to maximize the sum rate of all users. A new user selection algorithm is proposed that takes into consideration both the correlation of the channels of the users and the improvement of the sum rate in each step. The proposed method offers considerable complexity savings. Moreover, it is shown using simulations that the performance of the proposed method is comparable to the performance of more complex previously proposed methods [7], [12].

The remainder of the paper is organized as follows: In Sec. II, the model under consideration is introduced. The
problem that is addressed in this paper is formulated in Sec. III and the optimal, capacity-achieving scheme is reviewed briefly. In Sec. IV, the low-complexity, suboptimal proposed scheme is described followed by the evaluation of its complexity. Simulation results are provided in Sec. V and concluding remarks are given in Sec. VI.

Notation: In the following, lowercase bold letters denote column vectors and bold uppercase denote matrices. \([x]^\dagger = \max\{0, x\}\), \((\cdot)^T\) denotes transpose, \((\cdot)^\dagger\) the conjugate transpose and \(\|\cdot\|\) the norm of a matrix or vector. Set difference is denoted by \((\cdot) \setminus (\cdot)\).

II. SYSTEM MODEL

The downlink of a multiuser system is considered consisting of a single Base Station (BS) and \(K\) users. The BS is equipped with \(M\) transmit antennas, whereas each user has a single receive antenna. Perfect Channel Side Information (CSI) is assumed at the transmitter, which means that each user estimates perfectly its wireless channel and informs instantly the BS through an error-free feedback channel. In this work, only the case of \(K > M\), which is of interest in practical systems, is examined.

If the channel between the BS and user \(k\) is represented by the vector \(h_k = [h_{k,1} \ldots h_{k,M}]^T\), the signal, \(y_k\), received by the \(k\)-th user is given by

\[
y_k = h_k^T x + z_k, \tag{1}
\]

where \(x \in \mathbb{C}^{M \times 1}\) is the transmitted signal and \(z_k \in \mathbb{C}\) denotes circularly symmetric complex Gaussian additive noise with zero mean and variance \(\sigma^2\). Stacking together all \(K\) users, the general model can be described by

\[
y = Hx + z, \tag{2}
\]

where each row of \(H = [h_1 \ldots h_K]^T \in \mathbb{C}^{K \times M}\) corresponds to each user channel, \(y = [y_1 \ldots y_K]^T \in \mathbb{C}^{K \times 1}\) and the vector \(z = [z_1 \ldots z_K]^T \in \mathbb{C}^{K \times 1}\) contains \(K\) independent identically distributed (i.i.d.) samples of additive noise. The transmitted signal is subject to an average power constraint \(P_{\text{tot}}\). If \(R_{x} = \mathbb{E}[|x|^2]\) is the covariance matrix of the transmitted signal, then the constraint on the average power can be written as \(\text{trace}(R_{x}) \leq P_{\text{tot}}\).

In linear beamforming, the normalized vector \(w_k = [w_{k,1} \ldots w_{k,M}]^T\) is used for mapping the signal of user \(k\) on the \(M\) transmitting antennas. If \(W = [w_1 \ldots w_K] \in \mathbb{C}^{M \times K}\) is the beamforming matrix, (2) can be written as

\[
y = H W s + z, \tag{3}
\]

where \(s = Ws\) and the uncorrelated entries of \(s \in \mathbb{R}^{K \times 1}\) contain the symbols destined to each user. The received signal of user \(k\) is given by

\[
y_k = \sum_{i=1}^{K} h_{k,i}^T w_i s_i + z_k = h_{k,k}^T w_k s_k + \sum_{i=1, i \neq k}^{K} h_{k,i}^T w_i s_i + z_k, \tag{4}
\]

where the middle term is the undesirable interference to user \(k\) caused by the simultaneous transmission of data to other users. The corresponding Signal to Interference plus Noise Ratio (SINR) is equal to

\[
\text{SINR}_k = \frac{|h_{k,k}^T w_k|^2 p_k}{\sum_{i \neq k} |h_{k,i}^T w_i|^2 p_i + \sigma^2}, \tag{5}
\]

where \(p_k = \mathbb{E}[s_k s_k^\dagger]\) is the transmit power of user \(k\).

III. PROBLEM STATEMENT

The focus of the paper is the maximization of the throughput of the system under a transmit power constraint. Specifically, the problem to be solved is the following

\[
\max_{\{W_1, \ldots, W_K\}} \sum_{k=1}^{K} R_k \tag{6}
\]

subject to \(\sum_{k=1}^{K} p_k \leq P_{\text{tot}}\),

where \(R_k = \log_2(1 + \text{SINR}_k)\) is the rate of user \(k, k = 1, \ldots, K\).

It is well known that the solution to the optimization problem (6) involves use of DPC, [4]. Practical implementation of DPC still remains a challenging task, mainly because it requires sophisticated random and interference-dependent coding/decoding. Therefore, Zero-Forcing Beamforming (ZFB) may be an interesting alternative that combines practicability with high performance.

In ZFB the inter-user interference is completely eliminated at the transmitter. The beamforming vectors are selected such that the zero interference condition be valid, meaning \(h_{i,j}^T w_j = 0\) for \(i \neq j\) and each user is encoded individually. However, interference elimination limits the number of users that can be supported simultaneously and a user selection procedure must take place. Additionally, a power allocation strategy to the data streams of users must be deployed. With \(M\) transmitting antennas, complete elimination of the interference is possible only if up to \(M\) single-antenna users receive data. Given that the total number of users is \(K\), there are \(\sum_{m=1}^{M} C_{m}^{K}\) different candidate groups with cardinality smaller than or equal to \(M\). Obviously, the set of users that leads to sum rate maximization can be found by exhaustive search. However, when \(K \gg M\), low-complexity SDMA approaches are required. If \(Q\) is the group of users to whom the BS is transmitting, the beamforming matrix \(W(Q)\) for ZFB is equal to \(W(Q) = H(Q)(H(Q)H(Q)^\dagger)^{-1}\), where \(H(Q)\) is the part of matrix \(H\) that contains the rows corresponding to the users in \(Q\). The corresponding sum rate is given by, [1]

\[
R_{\text{sum}}^{ZF}(Q) = \sum_{k \in Q} \log_2(\mu c_k(Q)), \tag{7}
\]

where \(c_k(Q) = \{[(H(Q)(H(Q)^\dagger)^{-1})]_{k,k}\}^{-1}\) is the effective channel of user \(k\). The power allocated to user \(k, \forall k \in Q\) is

\[
p_k = c_k(Q) \left[\mu - \frac{1}{c_k(Q)}\right]^+. \tag{8}
\]

where \(\mu \in \mathbb{R}\) is obtained by solving the water-filling equation

\[
\sum_{k \in Q} \left[\mu - \frac{1}{c_k(Q)}\right]^+ = P_{\text{tot}}.
\]
IV. THE PROPOSED ALGORITHM

A low-complexity SDMA approach is described in this section that is based on the spatial correlation between different users. The normalized spatial correlation between users \(i\) and \(j\) is given by \(\rho_{ij} = \frac{|h_i^* h_j|}{\|h_i\| \|h_j\|}, 0 \leq \rho_{ij} \leq 1\). The approach is based on the intuitive observation that, when 2 users are strongly correlated, elimination of the interference between them requires significant power and does not lead to a large sum rate. The proposed user selection algorithm takes advantage of the multiuser diversity of the system and outputs a set of transmitting users with low correlation that leads to a large sum rate.

A. Algorithm Description

Let \(\mathcal{U} = \{1, \ldots, K\}\) denote the set of all \(K\) users and \(Q \subset \mathcal{U}\) denote the set of selected users. Obviously, \(|Q| \leq M\). The proposed algorithm iteratively selects users based on the spatial correlation between the users who have already been selected and the remaining ones. In each iteration, it forms a set of candidate users, \(\mathcal{A}\), of size \(L\) drawn from the set \(\mathcal{U} \setminus \mathcal{Q}\). The members of \(\mathcal{A}\) have the smallest spatial correlation with the users that are already in \(Q\). In each iteration, one more user is added to the transmission group if its insertion leads to an increase on the sum rate. Specifically, the proposed algorithm is the following:

A) Initialization:
- Set \(Q = \emptyset\), \(|A| = L\) and \(n = 1\).

B) Step 1:
- Find user \(k^* = \arg\max_{k \in \mathcal{U}} \|h_k\|^2\).
- Set \(Q = \{k^*\}\), \(C_n = R_{\text{sum}}(Q)\) and \(n = n + 1\).

C) Step 2: While \(n \leq M\)
- For each \(i \in Q\) and \(k \in \mathcal{U} \setminus \mathcal{Q}\) compute \(\rho_{i,k}\).
- Let \(\text{Cor}_{k} = \sum_{i \in Q} \rho_{i,k}\) be the average correlation between already selected users and candidate \(k\), \(k \in \mathcal{U} \setminus \mathcal{Q}\).
- Form the group, \(\mathcal{A}\), of candidates that contains the \(L\) users with the lowest values \(\text{Cor}_{k}, k \in \mathcal{U} \setminus \mathcal{Q}\).
- For each user \(a \in \mathcal{A}\), compute \(R_{ZF}(Q \cup \{a\})\) using (7) and choose \(a^* = \arg\max_{a \in \mathcal{A}} R_{ZF}(Q \cup \{a\})\).
- If \(R_{\text{sum}}(Q \cup \{a^*\}) > C_{n-1}\) then set \(Q = \{Q \cup a^*\}\), \(C_n = R_{\text{sum}}(Q)\) and \(n = n + 1\). Else, \(n = M + 1\).

D) Output:
- \(Q, W(Q) = H(Q)(H(Q)H^\dagger(Q))^{-1}\) and \(p_k, \forall k \in Q\) using (8).

Obviously, in the first iteration, the user with the largest channel gain should be selected. During each of the following iterations, the available users are sorted with respect to their average correlation with users of the transmission group. The \(L\) users that have the smallest average correlation with users in \(Q\) form a temporary set of candidates from where the new insertion will come. The member of \(\mathcal{A}\) that leads to the maximum sum rate is inserted in the transmission group \(Q\) if the new sum rate is larger than the sum rate of the previous iteration. Otherwise, the transmission group remains the same and the procedure terminates.

Intuitively, users that have small average correlation lead to large sum rates. However, it is not always true that the user with the smallest correlation will yield the largest sum rate increase. In this paper, the size of the set \(\mathcal{A}\) is set heuristically equal to the number of transmitting antennas, \(M\), because it was shown to lead to good performance in most simulated cases.

B. Complexity

As was already mentioned, the complexity of DPC can be prohibitively high. The complexity of the algorithm that is described in [3] is \(O(K^2M^2)\) per iteration, and the number of iterations depends on the desirable accuracy. In each iteration of the algorithm proposed in this paper, the average correlation between each available user and the already selected users must be computed and the effective channels of at most \(L\) users must be formed. The computation of \(\rho_{i,j}\) for each pair \((i,j)\) can be done within time \(O(M)\), as it mainly requires simply an inner product. Additionally, if the values \(\rho_{i,j}\) are kept in memory, the computation of \(\text{Cor}_{k}\) for subsequent iterations needs time \(O(KM)\). Using the matrix inversion lemma as described in [7], the inversion that is required for each effective channel computation in the \(n\)-th iteration \((n \leq M)\) can be done in time \(O(n^2)\). Given that at most \(L\) such inversions are needed in each iteration, and that there are at most \(M\) such iterations, the overall complexity of the proposed algorithm is \(O(KM^3 + LM^2)\), which is one order of magnitude smaller with respect the variable \(K\) than the complexity of DPC. Given that \(LM < K\), the complexity becomes \(O(KM^2)\) and is smaller than \(O(KM^3)\), which is the complexity of the scheme in [7] and the worst-case complexity of the scheme in [12]. Both of them have almost the same performance with the proposed algorithm. Table I summarizes the complexity of the schemes mentioned above.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>ALGORITHM COMPLEXITY</th>
</tr>
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<tbody>
<tr>
<td>DPC [3]</td>
<td>(O(K^2M^2)) per iteration</td>
</tr>
<tr>
<td>[7],[12]</td>
<td>(O(KM^3))</td>
</tr>
<tr>
<td>Proposed Algorithm</td>
<td>(O(KM^2))</td>
</tr>
</tbody>
</table>

If step 2 is modified by adding an upper threshold \(c\) on the allowable average correlation, a reduced-complexity version of the algorithm can be employed. This means that every user \(k\) that has \(\text{Cor}_{k} > c\) will not be examined in subsequent iterations. The processing gain achieved by this modification is shown in the following section.

V. SIMULATION RESULTS

In this section, the performance of the proposed algorithm is evaluated and compared with other previously proposed techniques. In all the presented simulations, only small-scale fading is considered because it is assumed that power control compensates for the path loss and large-scale fading. All the wireless channels and the additive noise are circularly symmetric complex Gaussian with zero mean and unit variance,
which means that the transmitted power is aggregate to the total SNR. In all cases $K = 50$ and the results are averaged over 1000 experiments.

Fig. 1 compares the average throughput of the proposed algorithm with that of Zero-Forcing with Exhaustive search (ZFEx) and DPC as a function of the SNR, where 50 iterations are used for DPC, [3]. As can be seen, the performance of the proposed algorithm is close to the best that can be achieved with linear processing, namely ZFEx. For example, for $M = 4$ and $SNR = 10$dB it achieves the 95% of the performance of ZFEx. It is conjectured that the small gap that exists between ZFEx and the proposed algorithm is related to the greedy nature of user selection and cannot be avoided without backward correction of previous selections.

In all simulations, a value for the parameter ($\alpha = 0.5$) that guarantees the best performance was used.

when $L = |A| = 1$. A similar approach has been proposed in [11] for the MIMO case. As can be seen, such a choice results in performance degradation by a constant gap from the performance of the proposed method (where $|A| = M$).

Fig. 3 depicts the performance for $SNR = 10$dB and $M = 4$ as a function of the number of users, $K$. Again, it can be noticed that the proposed algorithm follows closely the performance of ZFS and exploits efficiently the existence of multiuser diversity that is available in the system.

Each time a user is added to the transmitting set the level of the interference to users already in the set increases. As was already mentioned, the minimum average correlation between the new and the existing users does not always translate to a maximum gain in the sum rate. Table II shows the percentage
of the times that the user with the $[1^{st}, 2^{nd}, 3^{rd}, 4^{th}]$ lowest average correlation is included in $Q$ per iteration. The number of antennas is $M = 4$ and different values of SNR are tested. For all SNRs, almost identical percentages are noticed for the second iteration. Moreover, the $2^{nd}$, $3^{rd}$ and $4^{th}$ least correlated users are frequently selected during the third iteration. This explains the improvement in performance when $|A| > 1$ compared to the GC approach.

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>PERCENTAGE OF TIMES THAT USER WITH L-TH LOWEST CORRELATION ($l = 1, 2, 3, 4$) IS ADDED TO THE TRANSMITTING SET $Q$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
<td>0 dB</td>
</tr>
<tr>
<td>---------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>1st iteration</td>
<td>100, 0, 0, 0</td>
</tr>
<tr>
<td>2nd iteration</td>
<td>26, 26, 24, 23</td>
</tr>
<tr>
<td>3rd iteration</td>
<td>26, 26, 20, 17</td>
</tr>
<tr>
<td>4th iteration</td>
<td>100, 0, 0, 0</td>
</tr>
</tbody>
</table>

As was mentioned in Sec. IV-B, an upper threshold on the average correlation can be used for complexity reduction. The performance for different values of the threshold, $c$, $0 \leq c \leq 1$ and $M = 4$ is shown in Fig. 4. Other approaches that use the mean value and the variance of the average correlation of the available users can be employed. The performance of the proposed algorithm appears to be very robust for values of $c$ larger than 0.45. The processing gain stemming from the reduction in the cardinality of set $U$ is shown in Table III for $SNR = 10$dB. The gain is almost equal for every SNR because the rejection of users depends only on the correlation. For $c = 0.4$, both good performance and complexity reduction (a division by 2 of the cardinality of the set $U$ in every iteration) can be noticed. For $c \geq 0.5$ there is no loss in performance compared to the original proposed algorithm.

Fig. 4. Sum Rate vs SNR for different values of $c$ and Sum Rate vs $c$ for $SNR = 10$ and 20 dB.

VI. CONCLUSION AND FUTURE WORK

In this paper a new user selection algorithm for the maximization of the sum rate of a multiuser downlink system was proposed. The algorithm is based on Zero-Forcing Beamforming and uses a correlation-based metric to decide on the simultaneously transmitting users. The complexity of the algorithm is smaller than the complexity of other previously proposed methods with almost the same performance, as was verified through simulations. An efficient, parameterized version that reduces the complexity further by rejecting strongly correlated users was also presented. Given the potential reduction in complexity, work is currently under way on a proof of optimality as the number of users tends to infinity and a possible extension of the approach to frequency-selective cases, under the presence of Quality of Service (QoS) constraints for the users.

REFERENCES


<table>
<thead>
<tr>
<th>TABLE III</th>
<th>NUMBER OF AVAILABLE USERS BEFORE EACH ITERATION.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>0.3</td>
</tr>
<tr>
<td>1st iteration</td>
<td>50</td>
</tr>
<tr>
<td>2nd iteration</td>
<td>12.14</td>
</tr>
<tr>
<td>3rd iteration</td>
<td>3.65</td>
</tr>
<tr>
<td>4th iteration</td>
<td>1.22</td>
</tr>
</tbody>
</table>