A noise model for InSAR time series

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Abstract Interferometric synthetic aperture radar (InSAR) time series methods estimate the spatiotemporal evolution of surface deformation by incorporating information from multiple SAR interferograms. While various models have been developed to describe the interferometric phase and correlation statistics in individual interferograms, efforts to model the generalized covariance matrix that is directly applicable to joint analysis of networks of interferograms have been limited in scope. In this work, we build on existing decorrelation and atmospheric phase screen models and develop a covariance model for interferometric phase noise over space and time. We present arguments to show that the exploitation of the full 3-D covariance structure within conventional time series inversion techniques is computationally challenging. However, the presented covariance model can aid in designing new inversion techniques that can at least mitigate the impact of spatial correlated nature of InSAR observations.

1. Introduction

Differential synthetic aperture radar interferometry is now regularly used to generate hundred-kilometer-scale surface deformation maps with centimeter-to-millimeter accuracy [e.g., Rosen et al., 2000]. While single interferograms have been successfully used to study large deformation events [e.g., Massonnet et al., 1993; Peltzer et al., 1994; Zebker et al., 1994; Simons et al., 2002; Pritchard and Simons, 2002], their application to studying smaller amplitude surface deformation events has been hampered due to effects of temporal and geometric decorrelation [Zebker and Villasenor, 1992; Bamber and Just, 1993].

An extensive and ever-expanding archive of SAR data acquired over the last two decades and data from future SAR missions with shorter repeat periods allow us to consider the temporal evolution of surface deformation by combining information from multiple interferograms. Time series interferometric synthetic aperture radar (InSAR) techniques [e.g., Ferretti et al., 2001; Berardino et al., 2002; Hooper et al., 2004; Shanker and Zebker, 2007; Hooper, 2008; Doin et al., 2011; Hetland et al., 2011] estimate the temporal evolution of surface deformation in areas that are characterized by reasonably large signal-to-noise ratio and are less affected by temporal and geometric decorrelation. Simple models for the effect of decorrelation phenomena [Zebker and Villasenor, 1992; Bamber and Just, 1993] and the atmospheric phase screen [e.g., Hanssen, 1998; Emardson et al., 2003; Lohman and Simons, 2005; Onn and Zebker, 2006; Knope and Jónsson, 2010] in individual interferograms have been well studied. Atmospheric phase noise is spatially correlated [Hanssen, 2001; Emardson et al., 2003; Lohman and Simons, 2005; González and Fernández, 2011], and phase noise in interferograms with common image acquisition are also correlated [Emardson et al., 2003].

Hanssen [2001] developed a simple mathematical framework to describe most common sources of error in individual interferograms using independent and identically distributed (i.i.d) random variables in the temporal domain and introduced a simple functional model focusing on three-pass and four-pass differential interferometry. Guarnieri and Tebaldini [2007] and Rocca [2007] proposed similar noise models for interferogram networks and derived associated Cramer-Rao bounds on velocity estimates from time series methods. In this work, we attempt to extend the ideas from these works and focus on building a simple covariance model for interferogram networks over space and time in order to analyze the techniques used in time series InSAR. In particular, we show that the covariance structure of interferometric phase observations in the temporal domain is significantly more correlated than previously assumed. We derive a method to estimate the contribution of temporal and spatial decorrelation to the overall noise covariance, thus extending previously published models which assumed high-coherence resolution elements to resolution elements with moderate signal-to-clutter ratio. Using our derived covariance model as a reference, we also discuss the effects of various processing steps in time series analysis on the noise covariance structure of the
interferograms. We summarize some of the typical processing steps in time series InSAR analysis and their effect on the noise covariance in Table 1.

This paper is organized as follows. In section 2, we discuss various aspects of existing approaches to modeling uncertainties in interferometric observations and point out their respective shortcomings. In section 3, we lay out the mathematical framework that we use to describe the covariance structure of interferometric phase noise over space and time. We derive our covariance model from first principles and from single-interferogram phase models in section 4. We finish with a discussion on computational tractability and implications of using the proposed covariance models in time series InSAR techniques.

### 2. Previous Models

When multilooked to a resolution of few hundreds of meters, as is the case for most geophysical applications, wrapped InSAR phase data even when acquired using a perfect imaging radar instrument suffers from lack of "closure over a circuit." For example, in a network of multilooked interferograms generated using three SAR scenes labeled A, B, and C, the multilooked interferometric phase for interferogram BC cannot be recreated exactly using the multilooked interferometric phase observations from interferograms AB and AC (see Appendix B). Consequently, effective reduction of a set of interferometric phase observations to geophysical parameters of interest such as relative deformation or deformation velocity requires knowledge or a model of the uncertainty associated with each phase observation. Besides multilooking, other common interferometric signal processing operations like range slope filtering, azimuth common band filtering, and adaptive filtering result in nonclosure of phase over a circuit as these are typically implemented on a pair-by-pair basis without accounting for spectral overlap with other interferograms sharing common SAR acquisitions in the network.

InSAR coherence is a widely used statistical measure developed to quantify the associated uncertainty with every interferometric phase observation. Reduction of InSAR coherence, also known as decorrelation, with increasing geometric and temporal baselines or with change in surface scattering properties due to vegetation or precipitation [Zebker and Villasenor, 1992], is well documented. However, coherence is insufficient as the only quantitative estimate of noise as it fails to capture the effect of spatially correlated long-wavelength noise sources like atmospheric propagation delay. Consequently, all interferogram network models to date [Hanssen, 2001; Guarnieri and Tebaldini, 2007; Rocca, 2007; González and Fernández, 2011] use a combination of decorrelation noise and an atmospheric phase term.

Hanssen [2001] advocated the use of empirical covariance functions derived from each interferogram to model the contribution of turbulent atmosphere. He also provided a framework where any generalized stochastic covariance model could be incorporated. Emardson et al. [2003] and Guarnieri and Tebaldini [2007] also derived similar expressions for the covariance of the atmospheric propagation delay taking into account the correlation of interferograms with common SAR acquisitions. Rocca [2007] modeled the atmospheric phase contribution as additive noise in each interferogram.

Hanssen [2001] assumed the decorrelation noise component to be independent for each interferogram. Guarnieri and Tebaldini [2007] and Rocca [2007] argued that if temporal decorrelation could be modeled by a Brownian motion process in urban areas, the temporal decorrelation noise terms need to be temporally correlated. We build on this idea and further show that decorrelation noise for a given pixel could be correlated between different interferogram pairs even if they are not composed of common SAR acquisitions, as this term represents the effect of changing scattering behavior of the pixel over time and imaging geometry. The covariance model presented in this work builds on ideas from all these interferogram network models and generalizes many aspects of modeling the contributions from various noise sources.

### Table 1. Summary of Typical Processing Steps in Time Series InSAR Analysis and Their Effect on the Noise Covariance of the Interferograms and Time Series Products

<table>
<thead>
<tr>
<th>Processing Step</th>
<th>Implications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multilooking of interferograms</td>
<td>Breaks the closure of interferometric phase over a closed circuit in the interferogram network</td>
</tr>
<tr>
<td>Adaptive filtering of interferograms</td>
<td>Decreases the impact of decorrelation noise but at the cost of resolution</td>
</tr>
<tr>
<td>Empirical stratified-troposphere corrections</td>
<td>Decreases bias in estimated time series. Covariance largely unaffected</td>
</tr>
<tr>
<td>Wavelet transforms in spatial domain</td>
<td>Exploits spatial correlation. Can reduce uncertainty, only when large parts of the interferograms are coherent</td>
</tr>
</tbody>
</table>
3. Mathematical Notation

We consider a set of \( N \) synthetic aperture radar (SAR) images acquired using a similar geometry at time epochs \( \{ t_1, \ldots, t_N \} \). A network of \( M \) coregistered and unwrapped interferograms is generated using the ensemble of SAR scenes. We also assume that there are \( P \) pixels for which we estimate a deformation time series. We also assume, without loss of generality, that the SAR acquisitions are part of at least one interferogram in a network of interferograms (i.e., \( M \geq \frac{N}{2} \)). Similar to Hanssen [2001] and Guarnieri and Tebaldini [2007], we model the individual phase terms and not the complex interferogram for tractability. The unwrapped phase \( \{ \Delta \phi_{\text{fg}} \} \) of a pixel \( x \), in an interferogram with master acquisition index \( i \) and slave acquisition index \( j \) can be represented as

\[
\Delta \phi^{x,ij} = \Delta \phi^{x,ij}_{\text{defo}} + \Delta \phi^{x,i,j}_{\text{aps}} + \Delta \phi^{x,i,j}_{\text{decor}} + \Delta \phi^{x,i,j}_{n} \tag{1}
\]

where \( \Delta \phi_{\text{defo}} \) represents the phase contribution due to the cumulative surface deformation in the time spanning the SAR acquisitions, \( \Delta \phi_{\text{aps}} \) represents the phase contribution due to the difference in propagation delay through the atmosphere between SAR acquisitions, \( \Delta \phi_{\text{decor}} \) represents the phase noise due to degradation in surface scattering properties of the resolution element, and \( \Delta \phi_{n} \) represents the phase contribution from all other uncorrelated noise sources (see Table A1 for list of all symbols used to set up the mathematical model). Note that we have not included explicit terms for an orbital phase ramp and digital elevation model (DEM) error in equation (1).

In the case of spaceborne InSAR data, phase ramps introduced by orbital errors can be reasonably estimated using a best fitting planar or bilinear model or data from other geodetic networks (e.g., GPS) [e.g., Pollitz et al., 2000; Simons et al., 2002], and any residual ramps cannot be distinguished from the atmospheric phase screen [Hooper et al., 2007]. In any case, an orbit error term can be included in the physical modeling process. Hence, we do not include an orbital ramp term in equation (1). Moreover, long-wavelength deformation features can be recovered in the post processing stage, assuming that orbital errors are temporally uncorrelated, by inverting (like Short Baseline techniques (SBAS)) the estimated interferogram ramp coefficients for acquisition-by-acquisition contributions and adding the low-pass-filtered component back to the estimated time series. We also note that a DEM error term [Hanssen, 2001; Berardino et al., 2002] represents a systematic effect in our formulation and that the range to the scattering centers of the resolution elements are precisely known. Subpixel range offset between the geometric and scattering center of a resolution element contributes a DEM-error-like term to the observed interferometric phase. We model the phase contribution due to orbit errors and DEM errors as deterministic or bias terms that can be reasonably estimated for data acquired using sensors with precise orbit information (e.g., Envisat, TerraSAR-X) [Bähr and Hanssen, 2012; Pepe et al., 2011; Fattahi and Amelung, 2014] and assume their contribution to the noise covariance model to be negligible. Equation (1) deals only with the stochastic components of the observed InSAR phase, and we will discuss the effects of the bias terms again in section 5.

Comparing with the model in Hanssen [2001], equation (1) does not include an integer ambiguity term as we assume that the wrapped phase observations can be unwrapped accurately. For realistic modeling of phase unwrapping errors, the spatial distribution of the coherent scatterers and the gradients in time and space of the deformation signal needs to be taken into account. The former is terrain dependent, and estimating the latter is the goal of our time series technique. Hence, without complicating our model, we assume that we have a reasonably dense network of coherent scatterers and that the SAR images are acquired sufficiently often over an area that is deforming at a reasonable rate, allowing the interferogram network to be unwrapped consistently and accurately in space and time. This assumption may not be valid under all circumstances but is needed at present to allow our model to be mathematically tractable [Guarnieri and Tebaldini, 2007; Rocca, 2007]. We discuss the implications of ignoring phase unwrapping errors in section 5.

In typical InSAR time series algorithms, the deformation phase \( \{ \Delta \phi_{\text{defo}} \} \) represents the primary signal of interest and is parametrized as a combination of individual SAR phases depending on the connectivity of the interferogram network [Berardino et al., 2002] or using a predetermined dictionary of temporal functions [Hetland et al., 2011]. All other phase terms in equation (1) are considered to be nuisance or noise terms.
We represent the unwrapped SAR phase contribution at pixel for an network of interferograms. Following  
Berardino et al. [2002], we formulate our time series inversion problem for all the P pixels in our network as

\[
\begin{align*}
\Delta \phi_{1/x_1}^{x_1} & = \mathbf{A} \cdot \phi_{\text{defo}} + \phi_{\text{aps}} + \phi_{\text{decor}} \\
\end{align*}
\]

where \(\phi_{\text{SAR}}^{x_i}\) represents the unwrapped SAR phase contribution from each SAR scene to an interferogram (with respect to the mean SAR phase), \(\mathbf{A}\) represents the interferogram network incidence matrix \((M \times N)\), and \((i_k, j_k)\) represent the master and slave scene indices for interferogram \(k\). We note that the formulation in equation (2) is inclusive of the Persistent Scatterer (PS) problem [e.g., Ferretti et al., 2001] which corresponds to the specific case where only a common-master interferogram network is considered. We also note that equation (2) is a simplified form of the generic functional model described in section 3.1.3 of Hanssen [2001] for a network of interferograms.

We represent the unwrapped SAR phase contribution at pixel \(x\) in SAR acquisition \(i\) as the sum of deformation, atmospheric phase screen, and decorrelation components.

\[
\phi_{\text{SAR}}^{x_i} = \phi_{\text{defo}} + \phi_{\text{aps}} + \phi_{\text{decor}}
\]

Subsequently, we rewrite equation (2) as

\[
\begin{align*}
\Delta \phi_{1/x_1}^{x_1} & = \mathbf{A} \cdot \phi_{\text{defo}} + \mathbf{A} \cdot \phi_{\text{aps}} + \mathbf{A} \cdot \phi_{\text{decor}} \\
\end{align*}
\]

where \(\mathbf{A}\) is the block diagonal matrix resulting from the Kronecker delta product, \(\otimes\), of an identity matrix of size \(P\), \(I_P\), and the network incidence matrix \(\mathbf{A}\). The \(\phi_{\text{defo}}\) represents the SAR phase contribution due to surface deformation, \(\phi_{\text{aps}}\) represents the phase contribution from the atmospheric phase screen, and \(\phi_{\text{decor}}\) represents the phase noise due to decorrelation in the SAR phase of each of the acquisitions. The SAR phase terms in equation (4) represent deviations from the pixel-wise averages and not the absolute phase contributions. The representation of phase terms other than deformation as zero mean random variables is reasonable as the common bias terms for each of these cancel out when computing the interferometric phase. Following Zebker and Villasenor [1992], we vectorize equation (4) in the absence of surface deformation and rewrite phase noise in an interferometric network as

\[
\Delta \phi_{n}^{x} = \mathbf{A} \cdot \phi_{\text{aps}} + \mathbf{A} \cdot \phi_{\text{decor}} + \Delta \phi_{n}
\]

We tabulate all the mathematical symbols and notation used here in Appendix A.
4. Covariance Model

All the terms in equation (5) have distinct spatiotemporal characteristics [Hooper, 2006] that allow us to estimate their relative contributions from a set of observed interferometric phases. The atmospheric phase screen \((\phi_{aps})\) in every SAR scene (not just interferograms) is correlated over space but uncorrelated in time for scenes with time separation longer than approximately a day [Emardson et al., 2003]. Phase noise due to the scatterers \((\phi_{decor})\) in a resolution element is correlated for interferograms with common SAR acquisitions but is uncorrelated spatially. The \(\phi_n\) represents the combined contribution of all uncorrelated noise sources in each interferogram and is uncorrelated over space and time. The phase components in equation (5) are statistically independent as they represent unrelated physical processes. Consequently, the total covariance matrix (for all pixels in all the interferograms), \(\Sigma_{fig}\) (Equation (6) is the same stochastic model presented in Hanssen [2001]. Hanssen [2001] referred to the atmospheric phase screen component as \(C_s\) and combined \(\Sigma_{decor}\) and \(\Sigma_n\) into \(C_s\), as he did not distinguish between decorrelation phase noise and processing errors. \(C_s\) and \(C_r\) represent the spatially correlated path delay component and the scatterer noise components of the interferometric phase error, respectively), is

\[
\Sigma_{fig} = \Sigma_{aps} + \Sigma_{decor} + \Sigma_n .
\]  

4.1. Atmospheric Phase Screen

The dominant contribution to the path delay component of phase error in many interferograms is from the spatial heterogeneity of the wet component of atmospheric refractivity, resulting in excess path length of the radar signal propagating through the neutral atmosphere [Goldstein, 1995; Emardson et al., 2003; Onn and Zebker, 2006]. The atmospheric phase signal varies gradually over space and is often modeled as a long-wavelength component in unwrapped phase [Onn and Zebker, 2006; Hooper, 2006]. The tropospheric phase delay component can be further broken down into two components: stratified tropospheric delay and stochastic delay. We treat the stratified tropospheric delay component as a systematic contribution or bias which can be reasonably modeled out [Jolivet et al., 2014] (further discussion in section 5.6) and consider the stochastic delay component only for modeling uncertainties. The spatial covariance function for interferometric phase has been studied in detail [Hanssen, 2001; Lohman and Simons, 2005; Knospe and Jónsson, 2010] and can be derived from the data itself. The structure function can be easily derived for coherent interferograms with short temporal baselines that are not affected by significant deformation. However, their method cannot be applied to all types of terrain and interferograms affected by large deformation events.

Alternately, Emardson et al. [2003] used GPS data to empirically determine that the average delay structure function due to the troposphere can be modeled as

\[
\sigma_{aps}^x y = c \cdot L_x^x y + k \cdot H_x y ,
\]  

where \(\sigma_{aps}^x y\) is the variance of the difference in SAR atmospheric phase between pixels \(x, y\), \(L_x^x y\) represents the distance between the pixels, and \(H_x y\) represents the difference in altitude between the pixels. The structure function (equation (7)) can be directly related to the covariance function \((\eta_{aps}^x y)\) of atmospheric phase of two pixels \(x, y\) as

\[
\eta_{aps}^x y = \frac{1}{2} \cdot \left[ (\sigma_{aps}^x y)^2 + (\sigma_{aps}^x y)^2 - (\sigma_{aps}^x y)^2 \right] .
\]  

If the interferograms were calibrated using a set of independent geodetic observations like a GPS network and without an explicit reference region, we would use the covariance model directly derived using techniques suggested by Hanssen [2001] and Lohman and Simons [2005] to compute \(\eta_{aps}^x y\), in equation (8).

Given a functional form for \(\eta_{aps}^x y\), the covariance matrix for the atmospheric phase components in an individual SAR acquisition can then be written as

\[
\Sigma_{aps}^{\text{diff}} = \begin{bmatrix} \eta_{aps}^1 & \cdots & \eta_{aps}^p \\ \eta_{aps}^2 & \cdots & \eta_{aps}^p \\ \vdots & \ddots & \vdots \\ \eta_{aps}^p & \cdots & \eta_{aps}^p \end{bmatrix} .
\]
where $\Sigma_{\text{aps}}$ is of size $P \times P$. Assuming that the atmospheric conditions are temporally uncorrelated and the spatial covariance structure remains the same in any SAR acquisition, the atmospheric phase covariance matrix for the entire network can be written as

$$
\Sigma_{\text{aps}} = \mathbf{A} \cdot \left[ \Sigma_{\text{aps}} \otimes \mathbf{I}_{L,N} \right] \cdot \mathbf{A}^T
$$

(10)

This equation can be computed if the parameters governing the functional covariance model (equation (8)) for each SAR acquisition are known. Alternatively, the interferograms themselves can be directly analyzed to estimate the covariance function ($\eta_{\text{aps}}$) using a network approach [Biggs et al., 2007; González and Fernández, 2011]. The derivation of equation (10) is similar to the one presented in Emardson et al. [2003] and has been extended to include all the coherent pixels simultaneously in order to exploit the spatially correlated nature of the atmospheric phase screen.

InSAR phase measurements are typically correlated over a scale of a kilometer or two [Hanssen, 2001; Emardson et al., 2003; Lohman and Simons, 2005]. Consequently, $\Sigma_{\text{aps}}$ matrix is made up of a large number of nonzero elements and cannot be efficiently represented as a sparse matrix. Our atmospheric phase covariance is consistent with the model suggested by Hanssen [2001].

### 4.1.1. Ionospheric Effects

In deriving our covariance model for the atmospheric phase screen, we have currently neglected path delay introduced by ionospheric heterogeneities which can behave significantly differently compared to the troposphere [Chapin et al., 2006; Meyer, 2010; Meyer and Watkins, 2011]. A fundamental difference is that the ionospheric contributions are strongly dependent on the sensor wavelength, whereas the tropospheric delay component is almost independent of wavelength. In the future, we expect to be able to mitigate ionospheric effects to a large extent using a multifrequency approach [Meyer, 2010; Rosen et al., 2010].

### 4.2. Decorrelation

The achievable accuracy of any SAR interferogram is affected by temporal decorrelation caused by change in surface scattering properties over time, geometric decorrelation, or spectral misalignment of received echoes due to different imaging angles and radar receiver noise [Zebker and Villasenor, 1992; Bamler and Just, 1993]. The amount of phase noise affecting the interferometric phase measurement at pixel $x$ in an interferogram composed of SAR acquisitions with indices $i$ and $j$ is commonly characterized by its coherence ($\gamma^{x,i,j}$) defined as

$$
\gamma^{x,i,j} = \frac{||E\left(z_{x,i}^* z_{x,j}^\circ\right)||}{\sqrt{E\left(||z_{x,i}||^2\right) \cdot E\left(||z_{x,j}||^2\right)}}
$$

(11)

where $z_{x,i}$ and $z_{x,j}$ represent the complex return for pixel $x$ in SAR acquisitions with indices $i$ and $j$, and $E(\cdot)$ represents the expectation function. A coherence value of 1 indicates noise-free observations, whereas a value of 0 indicates pure noise observations. The coherence as estimated using equation (11) has been shown to be biased toward higher values [Touzi et al., 1990] and often needs to be corrected before use for practical applications. Assuming Gaussian scatterers, the coherence values can be related to interferometric phase standard deviation using the Cramer-Rao bound relation [Rodriguez and Martin, 1992]

$$
\gamma^{x,i,j} = \frac{1}{\sqrt{1 + 2 \cdot L \cdot \sigma_{\Delta \phi}^2}}
$$

(12)

where $L$ represents the number of looks used to estimate the coherence and $\sigma_{\Delta \phi}$ is the associated interferometric phase standard deviation. We prefer to use the observed phase standard deviation to characterize phase noise directly as it can be directly used for building covariance matrices. To reduce the effect of gradients introduced by deformation or orbital errors, the observed phase values are corrected for a local slope component over the estimation window before the standard deviations are estimated [Zebker and Chen, 2005]. The coherence estimate is assumed to be independent of the atmospheric phase screen as the physical scale of the atmospheric signal (1–2 km) is much larger than the estimation window (100–200 m).

Hanssen [2001] preferred to model decorrelation noise ($\phi_{\text{decor}}$) terms using spatially and temporally uncorrelated random variables. This assumption also lies in the heart of short baseline techniques [e.g., Berardino...
Figure 1. Example geometry of multibaseline tandem SAR acquisitions; A, B, C, and D represent the receiving antenna centers. We use this example configuration to argue that noise in interferograms AC and BD are correlated even though they do not share any common acquisitions due to the overlap in baseline space. Interferograms AB and CD would not share common decorrelation noise terms as they do not overlap in baseline space.

et al., 2002]. However, not all components of decorrelation noise are uncorrelated, as illustrated with the following example. Assume a set of interferograms generated from four SAR images (labeled A, B, C, and D) representing a set of tandem multibaseline acquisitions as shown in Figure 1. For resolution cells characterized by distributed scatterers, Zebker and Villasenor [1992] showed that coherence for any pair is a function of the separation between the antenna centers of the receivers and is not affected by temporal decorrelation. The total spatial decorrelation affecting the pair BC also affects the pair AD, though they do not share any common SAR acquisitions. This simple example illustrates that the phase noise terms affecting the interferograms BC and AD are correlated. The covariance in the decorrelation phase for two interferograms is given by

\[
\text{cov}(\phi_{ij},\phi_{kl}) = \sigma_{\phi_{pq}}^2 \cdot I(B_{\perp,ij} \cap B_{\perp,kl})\]  

(13)

where \(i, j, k, l\) represent SAR scene indices, \(B_{\perp,ij}\) represents the geometric overlap in the baselines of pairs \((i,j)\) and \((k,l)\), \(B_{\perp,ij}\) represents the perpendicular baseline of interferogram \((i,j)\) with respect to a single master acquisition, \(\sigma_{\phi_{pq}}^2\) represents the phase noise variance of pair \((i,j)\), and \(I(\cdot)\) represents the indicator function which is 1 if the baselines overlap or 0 otherwise. The covariance matrix corresponding to our example in Figure 1 as given by equation (13) is

\[
\mathbf{\Sigma}_{\text{decor}} = E[\hat{\mathbf{\phi}} \cdot \mathbf{\phi}^T]
\]

\[
= \begin{bmatrix}
\sigma_{AB}^2 & \sigma_{AC}^2 & \sigma_{AD}^2 & 0 & 0 & 0 \\
\sigma_{AB}^2 & \sigma_{AC}^2 & \sigma_{AD}^2 & 0 & 0 & 0 \\
\sigma_{AB}^2 & \sigma_{AC}^2 & \sigma_{AD}^2 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_{CD}^2 & 0 & 0 \\
0 & 0 & 0 & \sigma_{CD}^2 & 0 & 0 \\
0 & 0 & 0 & \sigma_{CD}^2 & 0 & 0 \\
\end{bmatrix} \]  

(14)

The same spatial decorrelation covariance structure can also be derived by considering the wave number shift [Gatelli et al., 1994] for each of the interferograms. The estimates of the spatial decorrelation covariance matrix is a function of the local terrain as the corresponding wave number shifts for each of the interferograms depend on the local terrain slope.
While the contribution from spatial decorrelation can be reasonably modeled considering only the imaging geometries, modeling the contributions of temporal decorrelation is significantly more challenging. Temporal decorrelation effects on data acquired over urban areas can be explained using a model similar to the one used for spatial decorrelation above. Consider the temporal decorrelation noise term when four SAR scenes (A, B, C, and D) are acquired over an urban area from the same point in space a few days apart such that they are not affected by spatial decorrelation. Assuming a Brownian motion model [Zebker and Villasenor, 1992; Rocca, 2007] for urban areas, the temporal decorrelation noise affecting the interferometric pair BC also affects the interferometric pair AD as they share a common time interval. Consequently, the decorrelation noise for these two pairs are correlated even though they share no common SAR scenes. For this example, the covariance matrix for the temporal decorrelation noise has the same structure as equation (14). However, the Brownian motion model does not necessarily apply to all terrain types. Often, temporal decorrelation patterns are modulated by weather and precipitation effects [Lauknes et al., 2010]. In the absence of knowledge of terrain type and weather patterns, it may be simpler to model the temporal decorrelation term as an independent noise term in each of the interferograms following [Hanssen, 2001]. In this section, we derive a model for phase noise covariance that is consistent with observed baseline and temporal decorrelation effects.

Hanssen [2001] used the Gaussian signal model to describe the interferometric phase variance as a function of the observed interferometric coherence and assumed that the decorrelation phase terms were independent in all the interferograms. We have already argued that the SAR decorrelation phase terms must be correlated to be consistent with observed geometric baseline effects, i.e., the scatterer noise increases with increasing baseline and possibly with the time separation between acquisitions [Zebker and Villasenor, 1992]. We show that it is possible to derive an approximate covariance matrix that is consistent with geometric and temporal baseline effects in three steps:

1. We first populate a SAR decorrelation phase correlation matrix \( \Omega_{\text{sar}}^x \) of size \( N \times N \) for a single pixel \( x \) using a coherence model as follows

\[
\Omega_{\text{sar}}^x = \begin{bmatrix}
1 & \gamma_{x,1,2} & \cdots \\
\gamma_{x,1,2} & 1 & \cdots \\
\vdots & \vdots & 1
\end{bmatrix}.
\] (15)

Zebker and Villasenor [1992] suggested the following form for the coherence model

\[
\gamma_{x,i,j} = \gamma_{\text{spatial}} \cdot \gamma_{\text{temporal}} \cdot \gamma_{\text{thermal}}.
\] (16)

Our formulation can accommodate any general form of the coherence function (not necessarily stationary) to account for time-dependent phenomena like seasonal variation in the nature of scatterers [Lauknes et al., 2010]. In general, we assume coherence to be a function of the two SAR acquisition times and the perpendicular baseline for each pixel.

\[
\gamma_{x,i,j} = \zeta(t_i, t_j, B_{ij}).
\] (17)

We use the singular value decomposition to generate a positive semidefinite approximation of \( \Omega_{\text{sar}}^x \) if needed. Note that (17) is an extremely simplistic representation of coherence which may in reality depend on numerous other factors like backscatter coefficient, signal-to-noise ratio, and soil moisture. Characterization of all sources of decorrelation in a radar imaging system is beyond the scope of this manuscript.

2. Assuming that the scatterer noise levels (ignoring change in backscatter coefficients with time) for a given pixel are the same in every SAR acquisition, we transform this matrix into a pseudocovariance matrix for interferometric phase of a single pixel \( x \) using the incidence matrix as

\[
\tilde{\Omega}_{\text{ifg}}^x = \frac{1}{2} \cdot A \cdot \Omega_{\text{sar}}^x \cdot A^T.
\] (18)

The diagonals of \( \tilde{\Omega}_{\text{ifg}}^x \) (size \( M \times M \)) is a measure of the phase noise and can be shown to be equal to \( 1 - \gamma_{x,i,j} \) for a single-interferogram network.
3. We again use the pseudocorrelation estimates to scale the InSAR correlation matrix to the InSAR covariance matrix using

\[
\Sigma^{\text{decor}}_{\text{ifg}} = D \cdot \Omega^{\text{ifg}} \cdot D
\]

\[
D = \begin{bmatrix}
\sqrt{\Omega^{(1,1)}_{\text{ifg}}} & 0 & 0 \\
\sqrt{\Omega^{(k,k)}_{\text{ifg}}} & 0 & 0 \\
\sqrt{\Omega^{(M,M)}_{\text{ifg}}} & 0 & 0 \\
\end{bmatrix},
\]

where \( \sigma^{x,l}_k \) represents the InSAR phase standard deviation in the \( k \)th interferogram (equation (12)) and \( \Omega^{x}_{\text{ifg}}(k,k) \) represents the \( k \)th diagonal element of \( \Omega^{x}_{\text{ifg}} \). This step ensures that the diagonal terms of \( \Sigma^{\text{decor}}_{\text{ifg}} \) correspond to \( \sigma^{x,l}_k \) and, hence, are consistent with our coherence model. If any of the eigenvalues of \( \Sigma^{\text{decor}}_{\text{ifg}} \) are negative, the underlying correlation structure is replaced by the nearest correlation matrix \[ \text{Higham}, 2002 \] to ensure that the covariance matrix is positive definite.

The complete covariance matrix for all the pixels in the network can be then be written as

\[
\Sigma^{\text{decor}}_{\text{ifg}} = \begin{bmatrix}
\Sigma^{\text{decor}}_{1,1} & 0 & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \Sigma^{\text{decor}}_{p,p}
\end{bmatrix},
\]

From equation (20), it is clear that \( \Sigma^{\text{decor}}_{\text{ifg}} \) has a block diagonal structure and can be efficiently represented as a sparse matrix. This approach is only true when the resolution of the SAR system is same as the pixel spacing \[ \text{Hansen}, 2001 \]. If the pixel spacing is smaller than the imaging resolution, neighboring pixels are bound to be correlated. Also, the impulse response of a point target on the ground in a SAR image is represented by a sinc function \[ \text{Cumming and Wong}, 2005 \] and the observations from adjacent pixels may be correlated. Such an effect is particularly observed in urban areas with strongly reflecting structures \[ \text{Cumming and Wong}, 2005 \]. However, we currently ignore this effect in our simple model.

\( \Sigma^{\text{decor}}_{\text{aps}} \) term is commonly used in the covariance model when setting up parametric inversions of InSAR phase observations \[ \text{Biggs et al.}, 2007; \text{Hetland et al.}, 2011 \], but \( \Sigma^{\text{decor}}_{\text{ifg}} \) is often ignored primarily because

1. Conventional time series techniques, e.g., persistent scatterers \[ \text{Ferretti et al.}, 2001 \] and small baseline subset \text{Berardino et al.} [2002], operate on a subset of pixels that are coherent throughout an interferogram network. \( \Sigma^{\text{decor}}_{\text{aps}} \) is negligible for a pixel that is consistently coherent over a network of interferograms because the entries of matrix \( \Omega^{x}_{\text{ifg}} \) and \( \sigma^{x,l}_k \) all tend to be nearly 0. We demonstrate this behavior with a numerical example in Appendix C.

2. The interferograms are often filtered in the spatial domain before analysis for many geophysical applications that focus on estimating relative deformation at a scale of a few kilometers while compromising the finer details on the scale of a few meters \[ \text{Goldstein and Werner}, 1998; \text{Baran et al.}, 2003 \]. If the reduced resolution and associated number of looks are not correctly accounted for, the adaptive filtering process appears to artificially boost the coherence of the interferometric phase observations. An example of this artificial boosting of coherence is shown for a C-band interferogram in Figure 2. We use the observed interferometric phase variance (equation (12)) for quantifying noise as opposed to traditional InSAR coherence (equation (11)) as a quantitative measure to compare the filtered and unfiltered interferograms, as we cannot quantify the effect of the adaptive filtering process on the amplitudes of the SAR images themselves. Though the filtered interferograms are often wrongly interpreted to have the same spatial resolution as the original unfiltered interferograms, the improvement in the quality of the deformation estimates can be attributed to the reduction of the effect of \( \Sigma^{\text{decor}}_{\text{ifg}} \). Moreover, when using filtered interferograms for time series analysis, an appropriate coherence measure that is representative of the phase noise characteristics of the filtered phase should be used for deriving the covariance model rather than the unfiltered correlation estimate.
Figure 2. Example of improvement in perceived coherence due to adaptive filtering of interferograms. The data corresponds to pairs of ALOS PALSAR scenes (Track 220, Frame 710) on 9 March 2007 and 15 December 2009 acquired over the Central San Andreas fault in California. The interferogram was filtered using a Goldstein filter [Goldstein and Werner, 1998] of strength 0.4. The estimated phase variance over a window of $5 \times 5$ pixels was transformed to equivalent coherence assuming Gaussian scatterers (equation (12)).

Modeling and using $\boldsymbol{\Sigma}_{\text{deco}}$ is most useful in the case of estimating deformation parameters from partially coherent scatterers [Perissin and Wang, 2011], which is beyond the scope of this manuscript. $\boldsymbol{\Sigma}_{\text{deco}}$ represents a natural weighting matrix that values coherent InSAR phase observations more than the noisy observations (see Appendix C for an example). Our model of decorrelation noise motivates the beneficial effect of filtering interferograms for time series InSAR studies over large areas, when possible.

4.3. Uncorrelated Noise
The $\boldsymbol{\phi}_n$ represents the vector of uncorrelated phase noise terms affecting interferometric phase measurements at all pixels and in all the interferograms of the network (equation 5)). Ideally, from an information theory point of view, a given redundant interferogram network contains the same amount of information as any other connected interferogram network, as the corresponding incidence matrices ($\mathbf{A}$ or $\mathbf{A}^\top$) have the same rank.

However, in practice, the same differential interferogram produced in different master geometries differ by a small random phase component that is spatially uncorrelated. Same interferograms produced using different InSAR processors also differ by a small random phase component, due to the difference in implementation of various steps in the processing chain. Hence, we include the uncorrelated noise term in our interferometric phase model (equation (5)). Hanssen [2001] attributed this error term to coregistration and interpolation errors during processing and assumed it to be uncorrelated in space and time.

$\boldsymbol{\Sigma}_n$ in equation (6), thus, has a diagonal matrix structure in our model. The magnitudes of the diagonal entries in this work have been determined empirically by comparing the phase difference between an interferogram and another obtained by switching the slave and master. Table 2 tabulates the phase noise values for various interferograms computed for a set of European Remote Sensing (ERS) satellite tandem pair interferograms over Parkfield, CA, as estimated using the ISCE software [Gurrola et al., 2010].
Table 2. Standard Deviation in Radians of Uncorrelated Phase Noise Sources in an Interferogram for a Set of ERS Tandem Pair Interferograms Over Parkfield, CA, Processed With the ISCE Software [Gurrolo et al., 2010]

<table>
<thead>
<tr>
<th>Track/Frame</th>
<th>Master Date (yyyyymmdd)</th>
<th>Slave Date (yyyyymmdd)</th>
<th>Bperp (m)</th>
<th>Phase Noise (Radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>256 / 2889</td>
<td>19951020</td>
<td>19951019</td>
<td>168</td>
<td>0.10</td>
</tr>
<tr>
<td>256 / 2889</td>
<td>19951124</td>
<td>19951123</td>
<td>247</td>
<td>0.10</td>
</tr>
<tr>
<td>256 / 2889</td>
<td>19960202</td>
<td>19960201</td>
<td>139</td>
<td>0.04</td>
</tr>
<tr>
<td>256 / 2889</td>
<td>19960412</td>
<td>19960411</td>
<td>108</td>
<td>0.04</td>
</tr>
<tr>
<td>256 / 2889</td>
<td>19960517</td>
<td>19960516</td>
<td>108</td>
<td>0.09</td>
</tr>
<tr>
<td>27 / 2871</td>
<td>19951108</td>
<td>19951107</td>
<td>222</td>
<td>0.09</td>
</tr>
<tr>
<td>27 / 2871</td>
<td>19960117</td>
<td>19960116</td>
<td>308</td>
<td>0.13</td>
</tr>
<tr>
<td>27 / 2871</td>
<td>19960501</td>
<td>19960430</td>
<td>100</td>
<td>0.08</td>
</tr>
</tbody>
</table>

The phase noise values in the table can be interpreted as the cumulative contribution of three different interpolation operations: (1) master-slave interferogram, (2) slave-master interferogram, and (3) resampling of slave-master interferogram into master-slave interferogram geometry before cross multiplication. All the resampling and interpolation operations were performed at a single look. The difference of the interferograms was multilooked to 100 m (4 looks in range, 20 looks in azimuth), and pixels with coherence greater than 0.5 were used to estimate noise statistics.

4.4. Properties of $\Sigma_{\text{ifg}}$

Following are the characteristic properties of $\Sigma_{\text{ifg}}$:

1. $\Sigma_{\text{ifg}}$ cannot be efficiently represented as a sparse matrix, primarily due to the nonsparseness structure of $\Sigma_{\text{aps}}$.
2. $\Sigma_{\text{aps}}$ (section 4.1) and $\Sigma_{\text{decor}}$ (section 4) are not full rank for a connected network of interferograms since they are computed using linear transformations involving matrices $A$ or $\tilde{A}$ which are not full rank. However, the diagonal structure of $\Sigma_n$ (section 4.3) ensures that the total covariance matrix $\Sigma_{\text{ifg}}$ is full rank and, hence, invertible for any connected subset of all possible interferograms.
3. Like any general covariance matrix, $\Sigma_{\text{ifg}}$ is symmetric. Its full-rank property also ensures that it is positive definite. These properties allow us to design a simple and efficient method to efficiently prune and augment interferogram networks [Reeves and Zhe, 1999; Broughton et al., 2010].

5. Discussion

In sections 3 and 4, we described our proposed noise covariance model in detail. In this section, we discuss the implications of the various aspects of our noise covariance model on InSAR time series estimates.

5.1. Persistent Scatterer Techniques

All the examples and interferogram networks presented here so far have dealt with distributed scatterers and the Gaussian signal model [Lee et al., 1994; Just and Bamler, 1994]. Our technique can easily be extended for persistent scatterer analysis by using the phase statistics corresponding to the constant signal model [Ferretti et al., 2001; Agram, 2010] and suitably modifying the decorrelation model in equation (17).

Nevertheless, the motivation behind the design of PS processing algorithms (e.g., use of a common-master network) can be easily explained using the proposed covariance model.

The decorrelation component ($\Sigma_{\text{decor}}$) of the total covariance matrix is negligible for PS due to their high signal-to-clutter ratio (SCR) property, and baseline (geometric and temporal) has little impact on the quality of the observed InSAR phase. Atmosphere ($\Sigma_{\text{aps}}$) and processing noise ($\Sigma_n$) are the only contributing components to the covariance matrix. Ignoring the spatial correlation and considering pixel-by-pixel inversions, our proposed covariance model for high SCR pixels in the absence of any pair-wise filtering suggests that any connected interferogram network can be used for PS analysis as long as covariance matrix used for inversion accounts for common SAR acquisitions [Emardson et al., 2003]. For the common-master acquisition interferogram network (traditionally used for PS), the atmosphere contribution to the total covariance matrix for a single pixel reduces to a diagonal matrix since the master scene atmosphere contributes only a bias term to all the interferograms. The unwrapped phases can be directly interpreted as the sum of deformation and atmosphere contributions, and the atmosphere components can be filtered out using a temporal high-pass filter. Note that this inversion technique has not accounted for the spatially correlated nature of InSAR observations, and the uncertainties on the estimated time series are expected to be spatially correlated. Double differencing of phase, in combination with a reliable a priori temporal for deformation (often driven by other geodetic measurements like those from a GPS network), during phase unwrapping in PS techniques [Ferretti et al., 2001] helps mitigate the spatial correlation at short
spatial scales to a certain extent. As we relax the criterion for PS selection to improve spatial coverage of
the estimated deformation signal and apply the same inversion techniques to a larger set of moderate SCR
pixels, $\Sigma_{\text{decor}}$ contributes by increasing the correlation between deformation estimates at different epochs.
In such situations, temporal or spatiotemporal filtering [e.g., Hooper et al., 2007] assuming the independence
of deformation estimates in time is less valid.

5.2. SBAS Techniques
The proposed covariance model can be directly used to explain the properties of the deformation esti-
mates from SBAS-like [Berardino et al., 2002] techniques. SBAS-like techniques are geared toward finding
distributed scatterers and used to analyze interferogram networks built using a combination of small
temporal and spatial baselines. Hooper [2008] and Ferretti et al. [2011] further illustrate the need for using
networks with distributed scatterers to improve spatial coverage of time series estimates for deformation
studies. However, not all distributed scatterers that have been identified as coherent over the network of
short baseline interferograms are necessarily coherent over all possible interferograms (including long
geometric baseline pairs). The decorrelation component ($\Sigma_{\text{decor}}$) of the total covariance relies on the SAR
decorrelation phase correlation matrix ($\Omega_{\text{SAR}}^x$ in equation (18)) which takes into account coherence between
all possible pairs. If all possible pairs of interferograms are highly coherent, the scatterer under consideration
behaves like PS, and conventional SBAS inversion techniques work as expected. However, if more than
a few long baseline interferograms were identified as incoherent (low coherence values in $\Omega_{\text{SAR}}^x$), the
decorrelation component ($\Sigma_{\text{decor}}$) is a result of nontrivial interaction between the network connectivity
matrix ($A$) and coherence values ($\Omega_{\text{SAR}}^x$). In general, the decorrelation component ($\Sigma_{\text{decor}}$) cannot be ignored.
SBAS techniques rely on highly connected interferogram networks, but current implementations ignore
the covariance term introduced by common SAR acquisitions. Ignoring the covariance matrix during
inversion results in biased deformation estimates. Temporal or spatiotemporal filtering operations [e.g.,
Hooper et al., 2007] that assume the independence of deformation estimates in time are less valid for such
scatterers. Similar to PS techniques, estimation of the temporal evolution of spatial gradients (double
difference) of deformation using a reliable a priori temporal model (often driven by other geodetic
measurements like from a GPS network) and integration of these model parameters over the spatial domain
can help mitigate the spatially correlated errors at short spatial scales. However, a reliable temporal model is
often not available or is the goal of exploratory InSAR time series studies.

5.3. Computational Tractability of Full 3-D Inversion
In section 4, we showed that the full 3-D covariance matrix of InSAR phase observations in an interferogram
network has rich structure. For a network with $M$ interferograms and $P$ pixels per interferogram, the total
covariance matrix is of size $MP \times MP$. Following equation (4), the deformation time series inversion for this
network can be set up as

\[
\begin{bmatrix}
\Delta \phi_{\text{defo}}^{1,1} \\
\vdots \\
\Delta \phi_{\text{defo}}^{1,1} \\
\vdots \\
\Delta \phi_{\text{defo}}^{P,1} \\
\vdots \\
\Delta \phi_{\text{defo}}^{P,N}
\end{bmatrix} = \bar{A} \cdot 
\begin{bmatrix}
\phi_{\text{defo}}^{1,1} \\
\vdots \\
\phi_{\text{defo}}^{1,N} \\
\vdots \\
\phi_{\text{defo}}^{P,1} \\
\vdots \\
\phi_{\text{defo}}^{P,N}
\end{bmatrix} + \bar{n}
\]

\[\text{[P \times M, 1] = [P \times M, P \times N] : [P \times N, 1] + [P \times M, 1],}\]

where the noise term ($\bar{n}$) is characterized by the total covariance matrix $\Sigma_{\text{defo}}$ (equation (6)). Estimating
the deformation time series from equation (21) involves the computation of the inverse of $\Sigma_{\text{defo}}$, which could
be on the order of a few million even for fairly small interferogram networks, rendering it computationally
intractable on conventional desktop machines. However, certain aspects of the correlated nature of InSAR
observations can be mitigated using reasonably tractable approaches as discussed in the next section.

5.4. Exploiting Spatial Correlation
We described a simple model in section 4 to derive the noise covariance matrix over space and time for
interferometric phase observations. The most important aspect of our noise covariance model is that phase
noise is correlated over the spatial domain in interferograms. Conventional time series InSAR approaches,
Figure 3. LOS velocity map derived using the (left) Timefn and (middle) MInTS techniques. A temporal model consisting of a constant velocity term and sinusoidal terms with a period of 1 year was used to invert the data set. (right) The difference between the data sets is also shown. The standard deviation of the LOS velocity over the entire image is approximately 1.8 mm/yr.

both PS and SB, rely on individual pixel-based inversion techniques for computational tractability. As a result, the estimated parameters are also affected by noise with the same spatial structure as the one derived in our covariance model.

Recently developed methods, like the Multiscale Interferometric Time Series (MInTS) technique [Hetland et al., 2011], attempt to exploit the spatially correlated nature of the atmospheric signal. The 2-D spatial wavelet transform of unwrapped interferograms approximately diagonalized spatial covariances that depend on the relative separation distance of pixels and thus serves as an effective approach of accounting for the spatial covariances found in interferograms [Hetland et al., 2011], although we note that there may be nonnegligible covariance between colocated wavelet scales. Consequently, inversion of wavelet coefficients at various spatial scales is a more valid approach compared to the direct inversion of interferometric observations themselves. It is sufficient to note that the MInTS approach results in the approximate diagonalization of the covariance matrix over the spatial domain [Hetland et al., 2011] resulting in significant reduction of spatially correlated error terms in the estimated time series parameters while still allowing for a reasonably efficient implementation of parameter estimation for large data sets. Ignoring the high-frequency coefficients when reconstructing the time series from the inversion results can also contribute to reduction of decorrelation noise contributions but at the expense of spatial resolution.

We illustrate the strength of wavelets using an example network of 86 ERS interferograms (Track 27, Frame 2871, descending geometry) acquired over the creeping section of the San Andreas fault in Central California (see supporting information for acquisition and baseline details) and spanning the time period from November 1992 to July 2004 (Figure 3). We used a threshold of 0.25 on the coherence value of pixels in each interferogram and restricted our analysis to pixels that were coherent in all interferograms. The deformation was modeled as a combination of a constant velocity term and sinusoidal terms with a time period of 1 year. First, we applied a parameterized inversion in the temporal domain [Hetland et al., 2011], hereby referred to as Timefn, technique to estimate the temporal model parameters on a pixel-by-pixel basis. For the MInTS approach, we applied the same parameterized inversion in the temporal domain on the wavelet coefficients of the interferograms. To reduce the impact of the decorrelation noise on our results, we used the same set of filtered interferograms and coherent pixels for our Timefn and MInTS analysis. We also used a temporal covariance model for inverting the data in the temporal domain in both the approaches [Emardson et al., 2003] and used all wavelet scales during reconstruction of MInTS estimates. To estimate the uncertainty in our time series estimates, we used a jackknife statistical approach. Subnetworks were constructed from the original interferogram network by excluding one SAR scene at a time, and the linear velocity and seasonal sinusoidal terms were reestimated for each of the subnetworks. The standard deviation of the estimated time series of the subnetworks represents the corresponding uncertainty. All data analysis was performed using the Generic InSAR Analysis Toolbox (GIAnT) [Agram et al., 2013], available for free at http://earthdef.caltech.edu. Figure 3 shows that both techniques estimate similar line of sight (LOS) velocity fields with few differences.

Figure 4 shows the ratio of the estimated uncertainty associated with the estimated time series for our network of C-band interferograms using Timefn and MInTS on a logarithmic scale. Figure 5 shows that
Figure 4. Ratio of the estimated uncertainties in LOS time series using the SBAS and MInTS techniques for all pixels and all time epochs. A jackknife approach based on the SAR acquisitions was used to determine the uncertainties. Accounting for the spatially correlated nature of the atmospheric signal decreases the uncertainty in estimated time series by roughly 4 dB. However, we do note that such gains are only possible when large sections of the interferograms are coherent.

5.5. DEM Errors

In deriving our covariance model, we have assumed that the contribution of the DEM error term is a systematic effect and can be estimated from the data itself for tractability. However, this is not always true. In case of a sensor that exhibits a systematic drift in baseline with time, like ALOS PALSAR, the perpendicular baselines are correlated with the temporal baseline causing leakage between the velocity and DEM error estimates [Samsonov, 2010; Fattahi and Amelung, 2013]. DEM error term also has an important effect when estimating time series using partially coherent scatterers [Doin et al., 2011; Perissin and Wang, 2011]. The correlation between the temporal baseline and the perpendicular baseline vectors, an indicator of trade-off between the parameters, potentially changes for each pixel. Consequently, the uncertainty and the covariances associated with the inferred time series parameters will have different characteristics.

5.6. Stratified Tropospheric Phase Delay

Another technique that is gaining popularity in time series InSAR studies is the use of GPS wet delay and meteorological data sets to estimate the atmospheric phase screen and correction of interferograms before time series analysis [e.g., Delacourt et al., 1998; Onn and Zebker, 2006; Foster et al., 2006; Cavalié et al., 2007; Jolivet et al., 2011]. The stratified component of tropospheric phase delay is similar to orbital ramps and DEM errors and can be treated as phase bias. Auxiliary data sets like GPS or meteorological models can be used to correct biases introduced by the stratified troposphere, and the quality of these corrections are heavily dependent on the spatial resolution of the auxiliary data sets used. Figure 6 shows an ERS interferogram over Parkfield, CA, that was corrected using the North American Regional Reanalysis [Mesinger et al., 2006] weather model and PyAPS software [Jolivet et al., 2011; Agram et al., 2013]. We also show the covariance function.
Figure 6. Comparison of the spatial phase covariance structure before and after correction using a North American Regional Reanalysis-based estimate of differential path delay due to temporal variations in the stratification of the atmosphere [Jolivet et al., 2011] over Parkfield, CA for an ERS interferogram spanning 26 October 1993 to 30 November 1993. Deformation in this time period is assumed to be negligible. The interferogram was analyzed at a spatial resolution of 200 m.

estimated before and after correction as estimated from the data themselves [Jónsson, 2002; Lohman and Simons, 2005] to emphasize that auxiliary data do not account for effects of turbulence which contributes most to the covariance estimates [Hanssen, 1998]. If the corrections due to the atmospheric model are not exact, it increases the covariance between pixels at larger distances as seen in Figure 6. The increased covariance at large distances is due to the fact that the phase corrections over the entire scene are derived from a finite set of meteorological grid points. Hence, a MinTS-like approach [Hetland et al., 2011] would still be needed to diagonalize the covariance structure. Our observation of increased covariance over large distances also holds true for the case when the stratified tropospheric delay corrections [Lin et al., 2010; Lauknes et al., 2010] or the orbital ramp functions are determined empirically from the data themselves. Using a wrong set of coefficients for either type of empirical corrections introduces correlation between phase observations separated by large distances. A detailed discussion on the impact of using weather models for correcting interferogram stacks can be found in Jolivet et al. [2014].

5.7. Phase Unwrapping
From a purely statistical point of view, using all available interferograms with the correct covariance information should decrease the uncertainty in our deformation estimates. In practice, however, we are restricted by our inability to reliably unwrap highly decorrelated interferograms. Reasonable modeling of phase unwrapping errors requires a high-resolution terrain classification model and a reliable back-scatter model to capture abrupt change in SAR reflectivity and InSAR phase properties. Therefore, we restrict
ourselves to analyzing interferograms whose average coherence exceeds a certain threshold assuming that they can be reliably unwrapped. Our covariance model can potentially model the interaction between the phase noise terms in a network of interferograms, allowing us to potentially develop better statistical cost models [Chen and Zebker, 2001] for phase unwrapping. Detailed statistical models, which are beyond the scope of this manuscript, that relate interferometric coherence to the spatial density of unwrapping errors could be used to better address the uncertainties associated with phase unwrapping errors in our proposed covariance model.

5.8. Noisier SAR Acquisitions

In our examples, we assumed that the noise in all SAR images are statistically similar. Our framework allows us to modify the interferogram networks suitably to compensate for noisier SAR acquisitions. It should be noted that reducing uncertainty in velocity and estimated deformation at a particular time instance can be competing objectives. In case of the former, the interferograms involving the noisier SAR observations tend to be pruned from the network in the attempt to improve a global estimate, whereas in case of the latter, the number of interferograms involving the noisier SAR images need to be increased to reduce the uncertainty of the deformation estimate at the corresponding time epochs.

6. Conclusions

Most current time series InSAR techniques operate under the assumption that interferometric phase observations are spatially and temporally independent. In this paper, we have built on simple models that describe phase statistics in single interferograms and developed a noise covariance model that shows that phase observations are both spatially and temporally correlated. Our model extends the work of Hanssen [2001] by formally deriving the covariance for interferograms with common SAR acquisitions from first principles. The most important implication of our covariance model is that accounting for spatial correlation in the inversion process can potentially improve the InSAR time series parameter estimates [Hetland et al., 2011]. We have also extended decorrelation models of individual interferograms [Zebker and Villasenor, 1992] to describe the phase noise statistics in a network of interferograms. The decorrelation covariance model can potentially be used to improve the spatial coverage of current time series InSAR techniques by allowing us to better exploit the information from partially coherent scatterers.

Appendix A: List of Symbols in the Manuscript

Table A1 contains a list of symbols used throughout this article.

Appendix B: Interferometric Phase Over a Simple Triangular Closed Circuit in an Interferogram Network

Let A, B, and C represent three coregistered single-look complex SAR images from an interferogram network, and let \( z_A(i,j) \) represent the complex measurement at pixel \((i,j)\) in complex SAR image \( A \). Then, in general

\[
\sum_{i \in R_x, j \in R_y} (z_A(i,j) \cdot z_A^*(i,j)) \neq \left[ \sum_{i \in R_x, j \in R_y} (z_A(i,j) \cdot z_B^*(i,j)) \right] \cdot \left[ \sum_{i \in R_x, j \in R_y} (z_B(i,j) \cdot z_C^*(i,j)) \right] \quad (B1)
\]

unless the region of averaging, \( R_x \) and \( R_y \), exactly span a single pixel each or the SAR images are perfectly homogeneous (constant \( z_A, z_B, \) and \( z_C \)) over the region of interest. Thus, multilooking the interferograms breaks the closure of interferometric phase over a closed circuit in an interferogram network.

Appendix C: Coherence and Decorrelation Phase Covariance

We demonstrate the applicability of our model to high and coherence pixels using a simple set of three SAR scenes A, B, and C. In the first case, we assume that a pixel is coherent in all three interferogram pairs—AB,
Table A1. List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N)</td>
<td>Number of SAR acquisitions.</td>
</tr>
<tr>
<td>(M)</td>
<td>Number of interferograms in the network.</td>
</tr>
<tr>
<td>(p)</td>
<td>Number of coherent pixels in time series analysis.</td>
</tr>
<tr>
<td>(A)</td>
<td>Incidence matrix corresponding to the interferogram network of size (M \times N).</td>
</tr>
<tr>
<td>(I_{p,p})</td>
<td>Identity matrix of order (P).</td>
</tr>
<tr>
<td>(\hat{A})</td>
<td>Block diagonal matrix ((PM \times PN)) obtained by repeating (I_{p,p} \otimes A).</td>
</tr>
<tr>
<td>(\Delta \phi_{\text{fg}}^{i,j})</td>
<td>InSAR phase of pixel (x) in interferogram composed of SAR acquisitions with indices (i) and (j).</td>
</tr>
<tr>
<td>(\Delta \phi_{\text{defo}}^{i,j})</td>
<td>Contribution of deformation in InSAR phase of pixel (x) in interferogram composed of SAR acquisitions with indices (i) and (j).</td>
</tr>
<tr>
<td>(\Delta \phi_{\text{aps}}^{i,j})</td>
<td>Contribution of atmospheric phase screen in InSAR phase of pixel (x) in interferogram composed of SAR acquisitions with indices (i) and (j).</td>
</tr>
<tr>
<td>(\Delta \phi_{\text{decor}}^{i,j})</td>
<td>Contribution of decorrelation factors in InSAR phase of pixel (x) in interferogram composed of SAR acquisitions with indices (i) and (j).</td>
</tr>
<tr>
<td>(\Delta \phi_{\text{ifs}}^{i,j})</td>
<td>Noise from uncorrelated sources in InSAR phase of pixel (x) in interferogram composed of SAR acquisition with indices (i) and (j).</td>
</tr>
<tr>
<td>(\phi_{\text{aps}}^{i,j})</td>
<td>SAR phase that contributes toward the unwrapped interferometric phase observations for pixel (x) in SAR acquisition with index (i).</td>
</tr>
<tr>
<td>(\phi_{\text{defo}}^{i,j})</td>
<td>Contribution of deformation in SAR phase of pixel (x) in acquisition with index (i).</td>
</tr>
<tr>
<td>(\phi_{\text{aps}}^{i,j})</td>
<td>Contribution of atmospheric phase in SAR phase of pixel (x) in acquisition with index (i).</td>
</tr>
<tr>
<td>(\phi_{\text{decor}}^{i,j})</td>
<td>Contribution from decorrelation sources in SAR phase of pixel (x) in acquisition with index (i).</td>
</tr>
<tr>
<td>(\Delta \phi_{\text{ifs}})</td>
<td>Vector of InSAR phases for all coherent pixels in all interferograms ((PM \times 1)).</td>
</tr>
<tr>
<td>(\phi_{\text{defo}})</td>
<td>Vector of SAR deformation phases for all pixels and all SAR acquisitions ((PN \times 1)).</td>
</tr>
<tr>
<td>(\phi_{\text{aps}})</td>
<td>Vector of SAR atmospheric phases for all pixels and all SAR acquisitions ((PN \times 1)).</td>
</tr>
<tr>
<td>(\phi_{\text{decor}})</td>
<td>Vector of SAR decorrelation phases for all pixels and all SAR acquisitions ((PN \times 1)).</td>
</tr>
<tr>
<td>(\Delta \phi_{\text{ifs}})</td>
<td>Vector of random phase noise for all pixels and all SAR acquisitions in the IFG network ((PM \times 1)).</td>
</tr>
<tr>
<td>(\Sigma_{\text{fg}})</td>
<td>Covariance matrix of InSAR phase of all pixels in the IFG network ((PM \times PM)).</td>
</tr>
<tr>
<td>(\Sigma_{\text{defo}})</td>
<td>Covariance matrix of atmospheric InSAR phase of all pixels in the IFG network ((PM \times PM)).</td>
</tr>
<tr>
<td>(\Sigma_{\text{decor}})</td>
<td>Covariance matrix of decorrelation InSAR phase of all pixels in the IFG network ((PM \times PM)).</td>
</tr>
<tr>
<td>(\Sigma_{\text{ifs}})</td>
<td>Covariance matrix of uncorrelated InSAR phase noise for all pixels in the IFG network ((PM \times PM)).</td>
</tr>
<tr>
<td>(\sigma_{\text{aps}}^{x,y})</td>
<td>Standard deviation of difference in atmospheric phase screen between two pixels (x ) and (y).</td>
</tr>
<tr>
<td>(L_{x,y})</td>
<td>The distance between two pixels (x ) and (y).</td>
</tr>
<tr>
<td>(H_{x,y})</td>
<td>The difference in altitude between two pixels (x ) and (y).</td>
</tr>
<tr>
<td>(\Sigma_{\text{defo}}^{x,y})</td>
<td>The covariance of atmospheric phase of two pixels (x ) and (y) in the same SAR acquisition.</td>
</tr>
<tr>
<td>(\Sigma_{\text{decor}}^{x,y})</td>
<td>The covariance matrix of atmospheric phase of all pixels in the same SAR acquisition.</td>
</tr>
<tr>
<td>(\gamma_{\text{ifs}}^{x,y})</td>
<td>Interferometric coherence of pixel (x) in IFG of SAR acquisitions with indices (i) and (j).</td>
</tr>
<tr>
<td>(z_{x,y})</td>
<td>Complex signal return from a pixel (x) in SAR acquisition with index (i).</td>
</tr>
<tr>
<td>(\Omega_{\text{defo}}^{x,y})</td>
<td>Correlation matrix of decorrelation phases of pixel (x) in all SAR acquisitions with respect to master scene ((N \times N)). All the values of the matrix have been normalized to lie in the interval ([0, 1]).</td>
</tr>
<tr>
<td>(\Omega_{\text{fg}}^{x,y})</td>
<td>Pseudocovariance matrix of InSAR decorrelation phase of pixel (x) in the IFG network.</td>
</tr>
<tr>
<td>(\sigma_{\Delta \phi_{\text{ifs}}}^{x,y})</td>
<td>Observed interferometric phase standard deviation around pixel (x) in interferogram ((i,j)).</td>
</tr>
<tr>
<td>(D)</td>
<td>Diagonal matrix used to normalize (\Omega_{\text{defo}}^{x,y}).</td>
</tr>
<tr>
<td>(\zeta())</td>
<td>Decorrelation model that maps perpendicular baseline and SAR acquisition times to interferometric phase standard deviation.</td>
</tr>
</tbody>
</table>
Acknowledgments
This work was supported by the Keck Institute of Space Studies Postdoctoral fellowship. We would also like to thank Scott Hensley from the Jet Propulsion Laboratory and Howard Zebker from Stanford University for helpful discussions. We thank ESA and WIN-SAR for providing the ERS-1 and ERS-2 SAR data. We also thank JAXA and ASF AADN archive for providing ALOS PALSAR data.

References
Cumming, I., and F. Wong (2005), Digital Processing of SAR Data, Artchec House, Inc., Canton Street, Norwood, MA 02062.

Table C1. Numerical Example of Proposed Covariance Model

<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th>Case II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coherence in SAR image domain ((\Omega_{\Delta \phi}))</td>
<td>1 0.9 0.7</td>
<td>1 0.3 0.8</td>
</tr>
<tr>
<td></td>
<td>0.9 1 0.8</td>
<td>0.3 1 0.35</td>
</tr>
<tr>
<td></td>
<td>0.7 0.8 1</td>
<td>0.8 0.35 1</td>
</tr>
<tr>
<td>Standard phase deviation from coherence. ((\sigma_{\Delta \phi}))</td>
<td>0.012</td>
<td>0.503</td>
</tr>
<tr>
<td></td>
<td>0.028</td>
<td>0.423</td>
</tr>
<tr>
<td></td>
<td>0.052</td>
<td>0.119</td>
</tr>
<tr>
<td>Pseudocorrelation of the interferograms. ((\Omega_{\Delta \phi}))</td>
<td>0.1 0 0.1</td>
<td>0.7 –0.575 0.125</td>
</tr>
<tr>
<td></td>
<td>0 0.2 0.2</td>
<td>–0.575 0.65 0.075</td>
</tr>
<tr>
<td></td>
<td>0.1 0.2 0.3</td>
<td>0.125 0.075 0.2</td>
</tr>
<tr>
<td>Normalization matrix ((\mathbf{D}))</td>
<td>0.24 0 0</td>
<td>0.601 0 0</td>
</tr>
<tr>
<td></td>
<td>0 0.27 0</td>
<td>0 0.529 0</td>
</tr>
<tr>
<td></td>
<td>0 0 0.29</td>
<td>0 0 0.265</td>
</tr>
<tr>
<td>Covariance matrix ((\Sigma_{\text{decor}}))</td>
<td>0.006 0 0.007</td>
<td>0.253 –0.181 0.020</td>
</tr>
<tr>
<td></td>
<td>0 0.014 0.016</td>
<td>–0.181 0.179 0.010</td>
</tr>
<tr>
<td></td>
<td>0.007 0.016 0.026</td>
<td>0.02 0.010 0.014</td>
</tr>
</tbody>
</table>

BC, and CA. In the second case, we assume that A and C represent summer SAR acquisitions and B represents a winter SAR acquisition. Consequently, interferogram AC is more coherent than the pairs AB and BC. For our numerical example, we assume 20 looks and use the inverse of equation (12) for mapping coherence to phase standard deviation. A more accurate mapping from the coherence to the phase standard deviation can be obtained using the probability distribution functions for interferometric phase from appropriate signal models [Just and Bamber, 1994; Lee et al., 1994; Agram, 2010].

From our example (Table C1), it is clear that the contribution of decorrelation to a coherent pixel covariance (Case I) is negligible. However, for a partially coherent pixel (Case II), the noise terms cannot be ignored. Moreover, our covariance model can capture the effects like the negative correlation between interferogram AB and BC, which can potentially allow us to exploit the phase information from these noisy interferograms.


