Dead-Time Compensation for Systems With Multiple I/O Delays: A Loop-Shifting Approach

Leonid Mirkin, Member, IEEE, Zalman J. Palmor, Senior Member, IEEE, and Dmitry Shneiderman

Abstract—This paper studies the standard problem for a class of multiple-delay systems, where different input/output channels have different dead times. We present a procedure of converting the original problem to an equivalent delay-free problem via loop-shifting arguments. This brings about and, for the first time, explains an unorthodox form of the dead-time compensator, called the feedforward action Smith predictor (FASP), in which an inter-channel feedforward term is present alongside the conventional internal feedback. Our developments lead to a qualitative conclusion that the structure of the dead-time compensator should rely upon the structure of the regulated output and/or the way in which exogenous signals affect the measurement.

Index Terms—Control and estimation with preview, dead-time compensation, time-delay systems.

I. INTRODUCTION

AN APPROACH to cope with the infinite-dimensionality of systems with loop delays—dead-time systems—is to use the so-called dead-time compensator (DTC) controller configurations; see [1], [2], and the references therein. The idea, originated in [3], is to introduce an (infinite-dimensional albeit implementable) internal feedback in the controller, which, in a sense, compensates for the delay so that the remaining part of the controller, called the primary controller, can be designed for a rational plant.

For systems with a single loop delay, the rationale behind the DTC is currently fairly well understood. Although there is a number of DTC configurations in this case [1], most of these configurations are arguably rearrangements or problem-dependent modifications of the modified Smith predictor (MSP) scheme [2]. Moreover, it has been shown that controller designs with $H^2$ [4] and $L^1$ [5] performance criteria intrinsically result in the very same MSP controller configuration. The $H^\infty$ optimization also produces DTC-based controllers [6], although the $H^\infty$ DTC is irreducible to the MSP; see [7, Sec. IV] for interpretations and further details.

In the multiple-delay case, when different input or/and output channels have different dead times, the situation is substantially more complicated. There appears to be no clear choice of the DTC configuration, as straightforward extensions of the single-delay DTC (with infinite-dimensional internal feedback) result in closed-loop systems for which some of the properties of their single-delay counterparts do not hold [8]. It was even suggested in [9] (and advocated in [8]) to introduce artificial loop delays to equalize, in a sense, delays in different channels and thus make up for some negative effects of channels with shorter delays upon other channels.

Recently, as a byproduct of the solution to the standard $H^\infty$ problem for systems with multiple loop delays, [10] presented a novel and intriguing structure of the DTC. To the best of our knowledge, this structure did not show up in the literature before. Apart from the standard internal feedback, it also contains an infinite-dimensional block linking between different input or output channels. Yet, [10] does not present any explanation/interpretation of this structure. It is thus still not clear what might be the rationale behind these interchannel DTC blocks and whether they are also advantageous in other applications, not only in $H^\infty$ (and $H^2$) optimal control.

In this paper, we study multichannel dead-time compensation from a different viewpoint. We aim at extending the (single-delay) loop shifting approach of [4] to multiple-delay systems of the form depicted in Fig. 1, with a rational generalized plant and diagonal measurement and control delays $\Lambda_y$ and $\Lambda_u$, respectively. The loop-shifting approach for a single delay originates in [11] (which, in turn, roots in [12]) and splits, by elementary block-diagram manipulations, the impulse response of the closed-loop system from $w$ to $z$, $T_{zw}$, into two nonoverlapping parts. The first part has support in $[0, h]$ (where $h$ is the loop delay) and does not depend on the controller. The second part, which does depend on the controller, has support in $[h, \infty)$. This split bespeaks, in a constructive way, the fact that the action of any causal controller in systems with a single delay is delayed in the regulated output.

Extending the loop-shifting approach to multiple loop delays is not trivial. Several attempts that resulted in incomplete solutions [13, Sec. 8.2], [14] (see also [15], as well as related earlier approaches of [16]–[20]) attest to it. These contributions solve the stabilization problem (for the loop containing only $G_{yu}$), yet

Fig. 1. Standard problem with I/O delays $\Lambda = \text{diag}\{e^{-\bar{h}_i}I\}$. 

z

G_{zw} G_{zw}

G_{yw} G_{yw}

y

K

u

w

\Lambda_{y}

\Lambda_{u}

\Lambda_{y}

\Lambda_{u}
fail to decompose the closed-loop system $T_{zw}$ into two orthogonal parts, one of which does not depend on the controller. Intuitively, this might be thought of as a consequence of the fact that delayed channels mix up when passing through (nondiagonal) systems. Hence, delayed channels are not clearly distinguished as I/O channels in $T_{zw}$. Technically, this is manifested in the fact that the (diagonal) delay does not commute with a nondiagonal transfer function, unlike the single-delay case.

In this paper, we propose to overcome this problem by precompensating the effect of longer input delay upon the channel with shorter delays and by outputting the coordinate(s). The former is a direct counterpart of the interchannel DTC introduced in [10], so that we finally offer interpretations of this scheme. In the control channel, this interchannel DTC aims at compensating the effect of longer delays on channels with shorter delays, in the measurement channel, at compensating longer delayed measurements of exogenous signals measured via channels with shorter delay as well. Moreover, our analysis suggests that unlike the single-delay case, the multiple-delay DTC should depend not only on the plant, but also on the performance measure and the way in which exogenous signals are measured by the controller.

The paper is organized as follows. We start with a review of the single-delay loop shifting in Section II, emphasizing the reasons of why the approach is not readily extendible to the multiple-delay case. In Section III, we then consider the simplest nontrivial multiple-delay problem—the so-called input adobe delay problem, where only two input channels, delay-free and delayed, exist. We present there a motivation for our approach, a complete loop-shifting procedure, and the resulting feedback action Smith predictor (FASP) controller structure for this configuration. Section IV presents the solution to a dual, output adobe delay, problem. A general multiple-delay loop shifting is then solved via a recurrent procedure, each step of which solves an adobe problem. This procedure is presented in Section V. Concluding remarks are then provided in Section VI, and the Appendix contains the proofs.

Notation: By $n_x$ we denote the dimension of a signal $x$. The open right-half plane to the right of $\alpha \in \mathbb{R}$ is denoted as $\mathbb{C}_\alpha := \{s \in \mathbb{C} : \text{Re } s > \alpha\}$. By $M'$ and $G^*$ we represent the transpose of a matrix $M$ and the conjugate of a transfer function $G$, i.e., $G^*(s) := [G(-s)]'$. A transfer function $G(s)$ is said to be proper if $\exists \alpha > 0$ such that $\sup_{\sigma \in \mathbb{C}_\alpha} \|G(s)\| < \infty$ and strictly proper if this supremum vanishes as $\alpha \to \infty$. We frequently use the term “proper” to mean that a transfer function has a causal implementation; see [21]. The Hilbert space $H^2$ [22] comprises all analytic functions $G(s) : \mathbb{C}_0 \to \mathbb{C}^{n \times m}$ such that $\int_{-\infty}^{\infty} \|G(s + j\omega)\|^2 d\omega$ is bounded for all $\sigma > 0$, where $\|\cdot\|_F$ stands for the Frobenius matrix norm. The norm and inner product on $H^2$ are denoted by $\|\cdot\|$ and $\langle \cdot, \cdot \rangle_2$, respectively. We say that an $n \times m$ transfer function $G(s)$ is stable if $G(s) \in H^\infty$, where $H^\infty$ is the space of bounded and analytic functions $\mathbb{C}_0 \to \mathbb{C}^{n \times m}$ [22]. A transfer function $G \in H^\infty$ is inner (co-inner) if $G^*G = I$ ($GG^* = I$). Finally, the truncation and completion operators [7] are defined as

$$\tau_h\left(\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}\right) = \begin{bmatrix} A & B \\ \frac{A}{Ce^{-sh}} & B \end{bmatrix} e^{-sh}$$

and

$$\pi_h\left(\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} e^{-sh}\right) = \begin{bmatrix} A & B \\ Ce^{-Ah} & 0 \end{bmatrix} - \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} e^{-sh}$$

respectively, and produce entire transfer functions corresponding to finite impulse response (FIR) systems.

II. PRELIMINARY: SINGLE-INPUT-DELAY LOOP SHIFTING

We start with reviewing the single-input-delay case, which corresponds to the setup in Fig. 1 with

$$\Lambda_y = I \quad \text{and} \quad \Lambda_u = e^{-sh}I.$$  

We outline the main steps of the loop-shifting approach of [4], to which the reader is referred for details.

A. Review of Results in [4]

Let $G_{yu}$ be a rational transfer function and denote

$$\Pi := \tilde{G}_{yu} - G_{yu}\Lambda_u.$$  

Add and subtract the block $\Pi$ to and from the control loop, relating $u$ to $y$, and then append the added part to $G_{yu}$ (parallel connection) and the subtracted part to $K$ (feedback connection). This yields the system shown in Fig. 2(a). It can be shown that whenever $\Pi$ is stable, internal stability is preserved under this loop transformation. Also, there always exist rational $\tilde{G}_{yu}$ such that $\Pi \in H^\infty$. Thus, the stabilization problem reduces to the stabilization of the finite-dimensional $\tilde{G}_{yu}$ by a controller $\hat{K}$, connecting $\hat{y}$ and $\hat{u}$ in Fig. 2(a). Having designed $\hat{K}$, the required $K$ is implemented as

$$K = \hat{K}(I - \Pi \hat{K})^{-1},$$

which is the MSP with the primary controller $\hat{K}$ and the DTC block (internal feedback) $\Pi$.

We are not done yet, as the plant in Fig. 2(a) still contains the delay in its $u \to z$ part. What helps us now is the fact that

$$G_{zw}\Lambda_u = \Lambda_u G_{zw}$$

i.e., $G_{zw}$ and $\Lambda_u$ commute in the scalar case. With $\Lambda_u$ on the output of $G_{zw}$, the delay can be pulled out of the system. To this end, split

$$G_{zw} = \Delta + \Lambda_u \tilde{G}_{zw}$$

Fig. 2. Loop shifting with single input delay. $\Lambda_y = I$ and $\Lambda_u = e^{-sh}$.
(a) DTC in $G_{yu}$. (b) Pulling $\Lambda_u$ out of the system.
where $\Delta$ is an FIR system, the impulse response of which has support in $[0, h)$, and $\hat{G}_{zw}$ is rational (always exists). This decomposition, which splits the impulse response of $G_{zwu}$ into the parts with support in $[0, h)$ and $[h, \infty)$, leads to the system shown in Fig. 2(b). This, in turn, implies that the closed-loop system takes now the form

$$T_{zw} = \Delta + \Lambda_u T_{zwu} \tag{3}$$

where $T_{zw} = \hat{G}_{zw} + G_{zu}\hat{K}(I - \hat{G}_{yu} \hat{K})^{-1}G_{yu}$ is the closed-loop transfer function from $u$ to $\hat{z}$, i.e., is the feedback interconnection of rational

$$\hat{G} := \begin{bmatrix} \hat{G}_{zw} & G_{zu} \\ G_{yu} & \hat{G}_{yu} \end{bmatrix}$$

and $\hat{K}$. It is worth mentioning that there always exist the required $\hat{G}_{zw}$ and $G_{yu}$ such that $\hat{G}$ has the same degree and poles as $G$. Partition (3) is merely a formal expression of the fact that the effect of $u$ on $z$ is delayed by $h$ time units due to the presence of the loop delay. Indeed, the only term in the right-hand side of (3) that depends on $K$, $T_{zwu}$, is delayed there. The first term, $\Delta$, which is a truncated version of $G_{zwu}$, can then be thought of as a generalized “feedthrough.”

An important property of partition (3) is that the impulse responses of the two terms in its right-hand side do not overlap. Consequently, its first term, $\Delta$, does not affect the design of the controller in criteria based on the impulse response (namely, $H^2$ and $L^2$). This effectively reduces the original problem to the delay-free problem of minimizing a corresponding norm of $T_{zwu}$ (the multiplication by $\Lambda_u$ is just a norm-reserving time shift); see [4] and [5]. The situation is more complicated when the criterion cannot be interpreted in terms of the impulse response in the time domain, like $H^\infty$ (see [23]). Yet, even in this case, there are situations, like some classes of the robust stability problem (with $G_{zwu} = \Delta = 0$), for which the reduction to an equivalent delay-free problem is possible. Moreover, it may still make sense to design $\hat{K}$ based solely upon $\hat{G}$ in problems that are not optimization based; see, e.g., [24, §V.B].

### B. Multiple-Delay Extensions

The first stage of the loop shifting—the transition from Fig. 1 to Fig. 2(a)—extends to the multiple-delay case smoothly. Indeed, it was shown, independently, in [13]–[15] that a rational $\hat{G}_{yu}$ such that $\Pi := \hat{G}_{yu} - G_{yu} \Lambda_u \in H^\infty$ can always be found. Thus, the stabilization problem reduces1 to that for the rational $\hat{G}_{yu}$ following the same steps as in the single-delay case.

The second stage—pulling the delay out of the system—is not straightforward though. To show this, consider a simplest nontrivial extension of the single-delay case with

$$\Lambda_y = I \quad \text{and} \quad \Lambda_u = \begin{bmatrix} I & 0 \\ 0 & e^{-sh} I \end{bmatrix} \tag{4}$$

which reflects the situation where there are two input channels: One is delay-free, and another one is delayed by $h > 0$ time units. Following the terminology of [10], we refer to this case as the input adobe delay problem. A direct generalization of the single-delay result would be the extraction of a diagonal delay block, similar to that in (4) albeit with possibly different dimensions of its blocks, from $G_{zwu}$. This, however, is impossible in general as (2) is no longer valid, unless $G_{zwu}$ is itself block diagonal. Consequently, the controller-independent parts, like $\Delta$ in Fig. 2(b), are not readily detectable in $T_{zw}$.

### III. INPUT ADOBE DELAY LOOP SHIFTING

For the sake of convenience of referring, we may regard, rather arbitrarily, some parts of the output of $G_{zwu}$ as its “delay-free output channel,” and the others as its “delayed output channels.” It may then be convenient to distinguish two cases of input channels mixture in the outputs of $G_{zwu}$: the effect of the delayed input channel on the delay-free output channel, and vice versa. As we show in Section III-A, the former can be handled by pre-compensating this delay with a stably invertible filter. The latter case is more complicated—it will also require a reshape of the regulated variable $z$; see Section III-B.

#### A. Upper Triangular $G_{zu}$

Thus, assume for the time being that

$$G_{zu} = \begin{bmatrix} G_{zu,0} & G_{zu,0,\Phi} \\ 0 & G_{zu,h} \end{bmatrix} \tag{5}$$

with square and invertible $G_{zu,h}$, where the column partitioning is compatible with that in (4). For this $G_{zu}$, the split of its output to delay-free and delayed channels is rather natural. Moreover, we have only the first kind of interchannel mixes as the delayed output channel is not affected by the delay-free input channel, whereas the nondelayed output channel is mixed.

The analysis of the system of the form (5) is facilitated by the fact that

$$\begin{bmatrix} G_{zu,0} & G_{zu,0,\Phi} \\ 0 & G_{zu,h} \end{bmatrix} \Lambda_u = \Lambda_u \begin{bmatrix} G_{zu,0} & G_{zu,0,\Phi} e^{-sh} \\ 0 & G_{zu,h} \end{bmatrix} \tag{6}$$

where $\Lambda_u$ is the same as $\Delta_u$ modulo a possibly different dimension of its delayed channel (as $G_{zu,h}$ is not necessarily square). In other words, the delay block can be pulled through the output of $G_{zu}$. This is not a direct counterpart of the single-delay case as the remaining part of $G_{zu}$

$$\hat{G}_{zu} := \begin{bmatrix} G_{zu,0} & G_{zu,0,\Phi} e^{-sh} \\ 0 & G_{zu,h} \end{bmatrix}$$

still contains a delay in its off-diagonal element. This delay, however, can be compensated for by using the procedure described below.

Let us factor $\hat{G}_{zu}$ as

$$\hat{G}_{zu} = \begin{bmatrix} G_{zu,0} & G_{zu,0,\Phi} \\ 0 & G_{zu,h} \end{bmatrix} \begin{bmatrix} I & 0 \end{bmatrix} =: \hat{G}_{zu,0} \Pi_{u,0,h}^{-1} \tag{7}$$

where

$$\Pi_{u,0,h} := G_{zu,0,\Phi}^{-1}(G_{zu,0,\Phi} - G_{zu,0,\Phi} e^{-sh}) \tag{8}$$

for some rational $G_{zu,0,\Phi}$. As $G_{zu,0}$ is assumed to be invertible, $G_{zu,0,\Phi}$ is proper and then, by the single-delay DTC theory [2], there always exists a rational $\hat{G}_{tu}$ such that

$$\hat{G}_{tu} - G_{zu,0}^{-1} G_{zu,0,\Phi} e^{-sh} \in H^\infty.$$
Hence, the pick \( \tilde{G}_{zu,0h} \equiv G_{zu,0h}/\Delta_z \) renders \( \Pi_{tu,0h} \in H^\infty \). This implies that the factor \( \Pi_{tu} \), which is the only irrational factor in (7), is bistable (i.e., \( \Pi_{tu}, \Pi_{tu}^{-1} \in H^\infty \)). Therefore, it can always be canceled by the control law \( u = \Pi_{tu} \hat{u} \) for some \( \hat{u} \) with the internal stability of the closed-loop system preserved.

With this observation in mind, we can present the system in Fig. 1 with the loop delays as in (4) in an equivalent form depicted in Fig. 3(a). The (1, 2) subblock of the generalized plant comprises a rational part \( \tilde{G}_{zu} \) and an adobe delay block on its output, which makes it possible to pull \( \Lambda_z \) out of the system following the logic of the single-delay case. Namely, partition \( G_{zu} \) according to the output partitioning of (5) as follows:

\[
G_{zu} = \begin{bmatrix} G_{zu,0} & G_{zu,h} \end{bmatrix}
\]

Then, we can always find a rational transfer function \( \tilde{G}_{zu,h} \) such that

\[
G_{zu} = \begin{bmatrix} 0 & \tilde{G}_{zu,h} \end{bmatrix} = \Delta + \Lambda_z \tilde{G}_{zu}
\]

for some FIR \( \Delta_z \), the impulse response of which has support in \([0, h]\). This leads to the system in Fig. 3(b), where the closed-loop behavior is again separated into two parts having nonoverlapping impulse responses and such that one part cannot be affected by the controller.

The plant in Fig. 3(b) is not yet what we need as its (2, 2) part is still infinite-dimensional. This part appears more complicated than that of the original system in Fig. 1 because it includes the bistable irrational block \( \Pi_{tu} \), which was added to compensate for the part pulled through the control input. To see the structure of the resulting (2, 2) block, partition \( G_{yu} \) according to the partitioning of \( \Lambda_u \) as follows:

\[
G_{yu} = \begin{bmatrix} G_{yu,0} & G_{yu,h} \end{bmatrix}
\]

Taking into account (7) and (8), we then have that

\[
G_{yu}\Lambda_u \Pi_u = \begin{bmatrix} G_{yu,0} & G_{yu,0} \tilde{G}_{zu,0h}^{-1} \end{bmatrix} + e^{sh} \begin{bmatrix} 0 & G_{yu,h} - G_{yu,0} \tilde{G}_{zu,0h}^{-1} \end{bmatrix}
\]

Only the second term in the right-hand side above involves a delay to be compensated. This reduces the DTC problem for the (2, 2) block to the by-now familiar DTC problem for a single-delay system. Just pick any \( \tilde{G}_{yu} \) such that

\[
\Pi_{th} := \tilde{G}_{yu} - \left( G_{yu,0} - G_{yu,0} \tilde{G}_{zu,0h}^{-1} \right) e^{-sh}
\]

is stable (such \( \tilde{G}_{yu} \) always exists), and then add to and subtract from the control loop in Fig. 3(a) relating \( \hat{u} \) to \( y \) the stable block

\[
\Pi := \begin{bmatrix} 0 & \Pi_{th} \end{bmatrix}
\]

Conventional single-delay reasonings then lead us to the system in Fig. 3(c) for

\[
\tilde{G}_{yu} := \begin{bmatrix} G_{yu,0} & G_{yu,0} \tilde{G}_{zu,0h}^{-1} \tilde{G}_{zu,0h} + \tilde{G}_{yu} \end{bmatrix}
\]

This is effectively a generalization of the structure in Fig. 2(b). Like in the latter case, we may now design a rational \( \tilde{K} \) [dotted box in Fig. 3(c)] for the rational generalized plant and then construct \( \tilde{K} \) by the relation

\[
\tilde{K} = \Pi_u \tilde{K} (I - \Pi \tilde{K})^{-1}
\]

Also in line with the single-delay case, the closed-loop transfer function from \( y \) to \( z \) is now split as

\[
T_{zu} = \Delta + \Lambda_z T_{zu}
\]

where the two summands in the right-hand side above have nonoverlapping impulse responses. Hence, the first term does not affect the design of \( \tilde{K} \) for both \( H^2 \) and \( L^1 \) optimality criteria. In fact, in the \( H^2 \) case the two terms in the right-hand side of (12) are orthogonal.

Remark 3.1: The choices of the irrational blocks \( \Pi_{tu} \) and \( \Pi \) are not unique. For example, we may evidently replace (7) with \( \tilde{G}_{zu,0h}^{-1} \Pi_{tu} G_{yu}^{-1} \) for any bistable rational \( G_{yu}^{-1} \). This freedom of choice is similar to that in the single-delay DTC; see the discussion in [24, Sec. IV]. Our main incentive behind the choices of \( \Pi_{tu} \) and \( \Pi \) in the next subsection is to keep the state-space structures of \( \tilde{G}_{zu} \) and \( \tilde{G}_{yu} \) close to those of \( G_{zu} \) and \( G_{yu} \), respectively.

B. General \( G_{zu} \)

The procedure proposed in the previous subsection does not work if \( G_{zu} \) is not of the form (5). Technically, moving the adobe delay block to the output of a general \( G_{zu} \) would leave a noncausal second factor in the right-hand side of (6), the delays...
in which can no longer be compensated for by a causal controller. To overcome this complication, we apparently have to give up on an attempt to split ζ into parts, one of which is affected by the controller immediately, and the other one only after some delay. We argue that a possible workaround here is to reshape ζ to regain a channel separation and, at the same time, to keep quantitative properties of the original regulated signal. The latter will be understood hereafter as preserving the energy ($L^2$ signal norm).

With this in mind, a natural approach would be to factor $G_{zu}$ into inner (i.e., norm-preserving) and block upper triangular factors, similarly to the QR matrix decomposition [25]. The latter factor could then be treated by the approach of the previous subsection, while the former would remain in the final decomposition of the closed-loop system, effectively replacing a diagonal adobe delay element as the extracted irrational factor. Such a QR factorization, however, would require two spectral factorizations to be performed, which we prefer to avoid. Instead, we will be looking for a factorization

$$G_{zu} \Lambda_u = \Psi_z \hat{G}_{zu} \Pi_u^{-1}$$  

for some inner $\Psi_z$, rational (not necessarily triangular) $\hat{G}_{zu}$, and bistable $\Pi_u$ (i.e., $\Pi_u[1] \in H^\infty$). We shall show below, by construction, that the factorization as in (13) can always be carried out provided the “delay-free part” of $G_{zu}(\infty)$ is left invertible and that the resulting $\hat{G}_{zu}$ has the same structure (degree and poles) as the original $G_{zu}$. This, in turn, will facilitate the dead-time compensation for the (2, 2) subblock of the generalized plant and the extraction of $\Psi_z$ from its (1, 1) subblock, so that the whole resulting rational generalized plant will have the same structure as the original G.

To present the solution, let

$$G = \begin{bmatrix} A & B_w & B_u \\ C_z & 0 & D_{zu} \\ C_y & D_{yw} & 0 \end{bmatrix} = \begin{bmatrix} A & B_w & B_u \\ C_z & 0 & D_{zu} \\ C_y & D_{yw} & 0 \end{bmatrix}$$  

where the control input partitioning is compatible with that of $\Lambda_u$ in (4). Assuming that $D'_{zu,0}D_{zu,0} = I$ (see Remark 3.2 for a discussion on how to relax this assumption), define the Hamiltonian matrix

$$H_0 = \begin{bmatrix} H_{11,0} & H_{12,0} \\ H_{21,0} & H_{22,0} \end{bmatrix} := \begin{bmatrix} A & 0 \\ -C_z^0C_z & -A' \end{bmatrix} - \begin{bmatrix} B_{u,0} \\ 0 \end{bmatrix} \left[ D'_{zu,0}C_z & B'_{u,0} \right]$$

and its matrix exponential

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} := \exp \left( \begin{bmatrix} H_{11,0} & H_{12,0} \\ H_{21,0} & H_{22,0} \end{bmatrix} h \right)$$

Fig. 4. Loop shifting for general G and input adobe delay (4). (a) Decomposed original system; (b) rational part.

which is symplectic and such that $\Sigma_{22}$ is invertible (see [26, Prop. 3.1]). Finally, by

$$P_{z,0} := D_{zu,0}D'_{zu,0},$$

we denote the orthogonal projection onto the image of $D_{zu,0}$.

Our main result is then formulated as follows.

**Theorem 3.1:** Let $D'_{zu,0}D_{zu,0} \Pi_{u,0} = [0]$ for $G$ in (14) and $\Lambda_u$ and $\Lambda_u'$ be given by (4). Then, the system in Fig. 1 can be presented as shown in Fig. 4(a) with

$$\tilde{C} = \begin{bmatrix} \tilde{G}_{zw} \\ \tilde{G}_{yw} \end{bmatrix} = \begin{bmatrix} A \\ C_z \\ C_y \end{bmatrix} \begin{bmatrix} B_w & \tilde{B}_u \\ 0 & D_{zu} \\ D_{yw} & 0 \end{bmatrix}$$  

where $\tilde{C}_z := (I - P_{z,0})C_z \Sigma_{22} + P_{z,0}C_z - D_{zu,0}B'_{u,0} \Sigma_{21} \Sigma_{22}'$ and $\tilde{B}_u := [B_{u,0} \Sigma_{22}'C_z D_{zu,0} + \Sigma_{22}' \Sigma_{21}]$, inner FIR

$$\Psi_z = P_{z,0} + (I - P_{z,0})e^{-sh}$$

and FIR $\Pi_{u,0}$ and $\Pi_{u',0}$ given by (7) and (10), respectively, with

$$\Phi = \begin{bmatrix} H_{11,0} & H_{12,0} \\ H_{21,0} & H_{22,0} \end{bmatrix} - \begin{bmatrix} \Sigma_{22}'B_w \\ 0 \end{bmatrix} \begin{bmatrix} B_{u,0}D'_{zu,0} \\ 0 \end{bmatrix} e^{-sh}$$

and $\Phi$ (A, $\Phi_u$) are those of (A, $\Phi_u$), proper $K$ internally stabilizes the system in Fig. 1 iff proper $K$ internally stabilizes the system in Fig. 4(b), and

$$\langle \Delta, \Psi_z T_{zu} \rangle = 0$$

for every $T_{zu} \in H^2$.

**Proof:** See Appendix.
Theorem 3.1 is an extension of [4, Lemma 1] for general input adobe delay problems. In contrast to earlier results [13]–[15], it not only reduces the stabilization problem to the stabilization of the rational \( \hat{G} \), but also splits the closed-loop transfer function into two orthogonal parts

\[
T_{zw} = \Delta + \Psi_z T_{zw}
\]

one of which \( (\Delta) \) is controller-independent. The orthogonality in (16) can be exploited to solve the corresponding \( H^2 \) problem; see [27] for details. Unlike the single-delay case, however, we cannot split the closed-loop impulse response into two nonoverlapping parts. This implies that the reduction of the \( L^1 \) problem to an equivalent finite-dimensional one is no longer a direct consequence of (16). Still, it can be argued that in some situations the rational \( T_{zw} \) does reflect properties of the closed-loop dynamics and may replace \( T_{zw} \) for the analysis and design purposes.

Remark 3.2: To simplify the final expressions and their derivation, we assumed that \( D_{zw} = 0 \), \( D_{yu} = 0 \), and \( D_{zu0}[D_{zu0}, D_{zu0}] = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \). These assumptions are not crucial, in fact, only the left invertibility of \( D_{zu0} \) is required. The formulas can be modified to account for nonzero \( D_{zw} \) and \( D_{yu} \) and nonnormalized \( D_{zu0} \). These modifications follow the standard scaling and loop shifting procedures [22, Sec. 4.6] with light modifications required to handle the delay element.

In particular, the scaling factor required to normalize \( D_{zu0} \) and orthogonalize \( D_{zu0} \) and \( D_{yh} \) has to be block upper triangular. Namely, we start with factorizing

\[
\begin{bmatrix} D_{zu0} & D_{yu} \end{bmatrix} = \begin{bmatrix} R_0 & 0 \\ R_{yu} & R_y \end{bmatrix} \begin{bmatrix} R_0 & R_{yu} \\ 0 & R_y \end{bmatrix}
\]

where \( R_0 \) is square and nonsingular and the partitioning is compatible with that of \( D_{zu0} \) in (14). Form now the matrix

\[
R = \begin{bmatrix} R_0 & R_{yu} \\ 0 & I \end{bmatrix}
\]

which is also nonsingular. Then, the formulas of Theorem 3.1 are applicable modulo the scalings

\[
\begin{bmatrix} B_{yu} \\ D_{zu} \\ D_{yu} \end{bmatrix} \rightarrow \begin{bmatrix} B_{yu} \\ D_{zu} \\ D_{yu} \end{bmatrix} R^{-1} \text{ and } \tilde{B}_u \rightarrow \tilde{B}_u R
\]

the substitutions

\[
\Pi_{yu}(s) \rightarrow \Pi_{yu}(s) - D_{yu} R_0^{-1} \Pi_{yu0}(s) + (D_{yu} R_0^{-1} R_{yu})(I - e^{-sh}I),
\]

\[
\Pi_{yh}(s) \rightarrow \Pi_{yh}(s) + R_0^{-1} R_{yu} (I - e^{-sh}I), \quad \Delta(s) \rightarrow \Delta(s) + (I - \Psi_z(s)) D_{zw}
\]

and by replacing zeros in \( \hat{G}(\infty) \) as follows: \( \hat{G}_{zu}(\infty) = D_{zw}, \Delta_{yu}(\infty) = D_{yu} \).

C. Controller Structure: The FASP

Hitherto, our discussion was focused on properties of the closed-loop system under the loop-shifting transformation. Not less important is that the proposed loop-shifting procedure induces a DTC-like controller configuration, which is qualitatively different from conventional multiple-delay DTC schemes available in the literature.

Consider the controller given by (11). Taking into account the structures of \( \Pi_{yu} \) and \( \Pi \) from (7) and (10), respectively, this equation corresponds to the block-diagram depicted in Fig. 5, where \( u = \begin{bmatrix} u_0 \\ 0 \end{bmatrix} \) is partitioned according to the partition in (4). This configuration includes both the internal feedback \( \Pi_{yu} \) resembling conventional DTCs, albeit only from the delayed channel, and the interchannel feedforward DTC \( \Pi_{yu0} \). We call this controller a FASP. To the best of our knowledge, the only earlier development of this scheme in the literature is [10, § IV.D] (it has been then utilized in some applications [28], [29]). In [10], the FASP configuration is a technical byproduct of the \( IT^\infty \) optimization procedure, and no explanations of the rationale behind the interchannel DTC are provided.

In contrast, our loop-shifting arguments offer an explanation of the role of the interchannel component in the FASP. To see it, let us compare the effect of the control channels on the signal \( z_{u} := G_{zu} \tilde{A}_{yu0} u \) in the original and compensated systems. If no feedforward compensation block is present, we have

\[
z_{u} = G_{zu0} u_0 + e^{-sh} G_{zu0} u_y \cdot
\]

As \( G_{zu0} \) and \( G_{yu0} \) are not orthogonal in general, the delay-free and delayed components are mixed up in \( z_{u} \). To explore the compensated system, note that

\[
\Pi_{yu0} := \tilde{\Pi}_{yu}(G_{zu0})^{-1} G_{yu0} e^{-sh}
\]

which can be verified by a direct algebra (as a matter of fact

\[
\Pi_{yu0} := \tilde{\Pi}_{yu}(G_{yu0})^{-1} G_{zu0} e^{-sh}
\]

which also follows from the formulas of Theorem 3.1). Using this form, it is readily seen that

\[
G_{zu} \tilde{A}_{yu} \Pi_{yu} = G_{zu0} [I - \Delta(s)] + P_{zu00} [0 - G_{yu0}] e^{-sh}
\]

where \( \Delta(s) := \Pi_{yu0} + (G_{zu0})^{-1} G_{yu0} e^{-sh} \) is rational (i.e., delay-free) and

\[
P_{zu0} := I - G_{zu0} (G_{zu0} G_{zu0})^{-1} G_{zu0}
\]

so that \( P_{zu0} e^{-sh} \) at each \( \omega \) is the orthogonal projection onto \( C^\infty \ominus \text{Im} G_{zu0}(j\omega) \). Thus, for the compensated system

\[
z_{u} := G_{zu0} (u_0 + G_{yu0} u_y) + e^{-sh} P_{zu0} G_{zu0} u_y \cdot
\]

As \( G_{zu0} P_{zu0} = 0 \), the delayed and delay-free components of \( z_{u} \) in representation (18) are orthogonal in \( L^2 \) for all control in-
puts \(y\). This means that the interchannel feedforward component compensates for the effect of the delayed control channel on all directions of \(z_{i,u}\) that are affected by the delay-free control signal as well. In other words, the effects of delay-free and delayed control channels on the regulated signal are now separated.

Remark 3.3 (Preview Tracking): The FASP scheme has also a curious connection with preview-tracking applications. Consider the problem of minimizing a norm, either \(H^2\) or \(H^\infty\), of the system \(G_{\text{uf,c}} := G_{\text{zu},u}e^{-sh} - G_{\text{zu},0}K\) by a \(K \in H^\infty\) such that \(G_{\text{uf,c}} \in H^\infty\). This is a general one-sided preview-tracking problem; see [30] and the references therein. By (13)

\[
G_{\text{uf,c}} = G_{\text{zu}}\Lambda_u\left[\begin{array}{c} -K \\ I \end{array}\right] = \Psi_z \hat{G}_{zu}\Pi_u^{-1} \left[\begin{array}{c} -K \\ I \end{array}\right] \\
\Psi_z \left(\hat{G}_{zu,K} - \hat{G}_{zu,0}(K + \Pi_u0)\right).
\]

Because \(\Psi_z\) is inner, it affects neither the \(H^2\) nor \(H^\infty\) norm of the error system \(G_{\text{uf,c}}\). Moreover, the arguments of the proof of Lemma A.6 can be used to show that \(\Psi_z\) does not affect the stability of the error system either. Thus, by “shifting” the controller \(K\) by the stable FIR system \(\Pi_u0\), the problem reduces to an equivalent finite-dimensional open-loop tracking problem (this is a nontrivial generalization of the result of [31], where the case of \(G_{\text{zu},0} = I\) is addressed). Hence, our interchannel DTC is also the infinite-dimensional part of the \(H^2\) and \(H^\infty\) preview-tracking solutions.

The other available DTC schemes for systems with multiple time delays contain no interchannel dead-time compensation components. Starting from [32], multiple-delay extensions of the Smith controller were concerned with choosing the form of the internal feedback prediction element; see [8], [13]–[15], and [33]. The main rationale in most of these developments was to end up with finite-dimensional stability conditions (i.e., a delay-free characteristic equation). As we saw in Section II-B, stabilization can indeed be handled within the conventional dead-time compensation. The same line was followed in parallel research on the finite spectrum assignment controllers [16], [18], [19], which would also produce just internal feedback prediction schemes if applied to our setup.

Jerome and Ray [8] were apparently the first who pointed out that the simplification of the stability analysis via the use of conventional DTCs might conflict with the closed-loop performance (formalized, following [9], in terms of decoupling requirements). They, however, did not question the conventional DTC structure. Rather, the problem in [8] was resolved by adding extra delays to “problematic” input channels if the plant does not pass the so-called rearrangement test.\(^2\) This way the decoupling is regained, although by acquiescing to delays in the characteristic equation and imposing extra limitations on the achievable closed-loop bandwidth.

By bringing about the FASP controller configuration, the loop-shifting procedure effectively prompts a paradigm shift in dead-time compensation for systems with multiple loop delays. The FASP configuration not only compensates for the delay loops, but also for the delays in the “performance” and “measurement” (the latter will be shown in Section IV-A) channels. Consequently, performance no longer conflicts with stabilization. We earn both a delay-free characteristic equation and a norm-preserving factor accumulating loop delays extracted from the regulated channel. The only deviation from the “wish list” in [8] is that when the plant does not pass the rearrangement test, the inner factor is not diagonal any more. We argue that this is a bargain.

Another characteristic of the FASP is that the structure of its DTC blocks depends not only on properties of the plant \((G_{yu})\), but also on the way in which the control signal affects the performance measure \((G_{zu})\). Hence, for the very same plant, we may end up with different FASP structures, depending on the performance measure chosen. For example, let \(P\) be a plant and consider two, somewhat simplistic, control problems in which the closed-loop performance is measured by the output and input sensitivity functions \(S_{\text{out}} = (I + PA_uK)^{-1}\) and \(S_{\text{in}} = (I + \Lambda_u(\Pi P)^{-1})^{-1}\), respectively, and \(\Lambda_u\) is as in (4). These problems correspond to the standard problem in Fig. 1 with the rational generalized plants

\[
G_{\text{out}} = \left[\begin{array}{c} I \\ -P \end{array}\right] \quad \text{and} \quad G_{\text{in}} = \left[\begin{array}{c} I \\ -P \end{array}\right]
\]

respectively. If \(P\) is not block-diagonal, the loop shifting for \(G_{\text{out}}\) always produces FASP with an interchannel \(\Pi_u0\). However, in the case of \(G_{\text{in}}\), we intrinsically have a diagonal \(G_{zu}\), so that we end up with a conventional DTC (which is a special case of FASP with \(\Pi_u0 = 0\) and \(\Pi_u = \pi_u\{P(\Pi P)^{-1}e^{-sh}\})\).

The dependence of the structure of multiple-delay DTCs on the performance measure is the prime qualitative outcome of our analysis. To the best of our knowledge, this aspect was not previously noticed in the literature. Our conclusion is clearly a consequence of the analysis of the standard problem, rather than of its stabilization part only. It is worth emphasizing, however, that in the single-delay case, the analysis of the standard problem produced the standard MSP, which depends only upon the “\(G_{yu}\)” part of the generalized plant.

IV. OUTPUT ADOBE DELAY LOOP SHIFTING

A dual problem to that studied in the previous section is the output adobe delay problem with

\[
\Lambda_y = \left[\begin{array}{c} I \\ 0 \end{array}\right] e^{-sh} I \quad \text{and} \quad \Lambda_u = I.
\]

In other words, now we have two measurement channels, one of which is delay-free and another one is delayed. The solution in this case requires the following partitioning of the measurement channel:

\[
G = \left[\begin{array}{c|c|c} A & B_u & B_w \\ \hline C_z & 0 & D_{zu} \\ \hline C_y & D_{yw} & 0 \end{array}\right] = \left[\begin{array}{c|c|c} A & B_u & B_w \\ \hline C_z & 0 & D_{zu} \\ \hline C_y0 & D_{yw,0} & 0 \\ \hline C_yh & D_{yw,h} & 0 \end{array}\right]
\]

\(^2\)In the input adobe delay case, a plant passes the rearrangement test iff it is block upper triangular, like \(G_{zu}\) in (5).
where the measured output partitioning is compatible with that of \( \Lambda_y \) in (19). Assuming that \( D_{yw}(0)D_{yw}(0) = I \), define the Hamiltonian matrix

\[
\Pi_0 = \begin{bmatrix}
\bar{H}_{11,0} & \bar{H}_{12,0} \\
\bar{H}_{21,0} & \bar{H}_{22,0}
\end{bmatrix}
\]

and its matrix exponential

\[
\Sigma = \begin{bmatrix}
\Sigma_{11} & \Sigma_{12} \\
\Sigma_{21} & \Sigma_{22}
\end{bmatrix} := \exp\left( \begin{bmatrix}
\bar{H}_{11,0} & \bar{H}_{12,0} \\
\bar{H}_{21,0} & \bar{H}_{22,0}
\end{bmatrix} t \right)
\]

which is symplectic and such that \( \Sigma_{22} \) is invertible (see [26, Proposition 3.1]). Finally, by

\[
P_{w,0} := D_{yw}(0)D_{yw}(0)
\]

we denote the orthogonal projection onto the image of \( D_{yw}(0) \).

Our main result here is as follows.

**Theorem:** Let \( D_{yw}(0)D_{yw}(0)D_{yw}(0) = [I \; 0] \) for \( G \) in (20) and \( \Lambda_y \) and \( \Lambda_h \) be given by (19). Then, the system in Fig. 1 can be presented as shown in Fig. 6(a) with

\[
\begin{align*}
\tilde{G} &= \begin{bmatrix}
\tilde{G}_{zw} & \tilde{G}_{zy} \\
\tilde{G}_{yw} & \tilde{G}_{yz}
\end{bmatrix} = \begin{bmatrix}
A & \tilde{B}_w & 0 \\
0 & 0 & D_{zw} \\
D_{yw} & 0 & 0
\end{bmatrix} \\
\tilde{G}_y &= \begin{bmatrix}
\tilde{C}_{yw} \\
\tilde{C}_{yz}
\end{bmatrix} = \begin{bmatrix}
\begin{bmatrix}
\bar{H}_{11,0} & \bar{H}_{12,0} \\
\bar{H}_{21,0} & \bar{H}_{22,0}
\end{bmatrix} & \begin{bmatrix}
\tilde{B}_w & B_u \\
0 & D_{zw} \\
D_{yw} & 0 & 0
\end{bmatrix}
\end{bmatrix}
\end{align*}
\]

(21)

where \( \tilde{B}_w := \Sigma_{22}^{-1}B_w(I - P_{w,0}) + B_wP_{w,0} - \Sigma_{22}^{-1}\Sigma_{12}C_{y0}D_{yw}(0) \)

and

\[
\begin{align*}
\tilde{G}_y &= \begin{bmatrix}
\tilde{C}_{yw} \\
\tilde{C}_{yz}
\end{bmatrix} = \begin{bmatrix}
\begin{bmatrix}
\bar{H}_{11,0} & \bar{H}_{12,0} \\
\bar{H}_{21,0} & \bar{H}_{22,0}
\end{bmatrix} & \begin{bmatrix}
\tilde{B}_w & B_u \\
0 & D_{zw} \\
D_{yw} & 0 & 0
\end{bmatrix}
\end{bmatrix}
\end{align*}
\]

inner FIR

\[
\Psi_w = P_{w,0} + (I - P_{w,0})e^{-sh}
\]

\[
+ \tau_h \begin{bmatrix}
\tilde{H}_{11,0} & \tilde{H}_{12,0} \\
\tilde{H}_{21,0} & \tilde{H}_{22,0}
\end{bmatrix} \begin{bmatrix}
B_w(I - P_{w,0}) \\
-C_{y0}D_{yw}(0)
\end{bmatrix}
\]

FIR

\[
\Delta = \tau_h \begin{bmatrix}
\begin{bmatrix}
\bar{H}_{11,0} & \bar{H}_{12,0} \\
\bar{H}_{21,0} & \bar{H}_{22,0}
\end{bmatrix} & \begin{bmatrix}
B_w(I - P_{w,0}) \\
-C_{y0}D_{yw}(0)
\end{bmatrix}
\end{bmatrix} e^{-sh}
\]

and FIR \( \Pi_y \) and \( \Pi \) given by

\[
\Pi_y = \begin{bmatrix}
I & 0 \\
0 & I
\end{bmatrix} \quad \text{and} \quad \Pi = \begin{bmatrix}
0 & I \\
I & 0
\end{bmatrix}
\]

with

\[
\begin{align*}
\Pi_{y,0} &= \tau_h \begin{bmatrix}
\begin{bmatrix}
\bar{H}_{11,0} & \bar{H}_{12,0} \\
\bar{H}_{21,0} & \bar{H}_{22,0}
\end{bmatrix} & \begin{bmatrix}
B_w(I - P_{w,0}) \\
-C_{y0}D_{yw}(0)
\end{bmatrix}
\end{bmatrix} e^{-sh}
\end{align*}
\]

Moreover, unobservable modes of \( \tilde{G}_y, A \) are those of \( (C_y, A) \), proper \( \tilde{K} \) internally stabilizes the system in Fig. 1 iff proper \( \tilde{K} \) internally stabilizes the system in Fig. 6(b), and for every \( T_{x0} \in H^2, (\Delta, T_{x0}, \Psi_w) \) = 0.

**Proof:** A dual version of Theorem 3.1.

### A. Controller Structure

As in the input adobe delay case, the loop shifting reduces the original system to the rational system in Fig. 6(b). Once the rational primary controller \( \tilde{K} \) is designed, \( K \) can be obtained via

\[
K = (I - \tilde{K} \Pi)^{-1} \tilde{K} \Pi_y
\]

which corresponds to the block diagram in Fig. 7. We again have an interchannel DTC, this time going from the delay-free measurement channel to the delayed one. Similarly to the input adobe delay case, this structure has already appeared in [10, §IV.D]. Yet, unlike [10], our derivation offers an explanation of the rationale behind the interchannel block \( \Pi_{y,h,0} \).

Indeed, in the original, uncompensated, system, the effect of the exogenous input \( w \) on the measured output \( y \) was

\[
y_w = [y_{w,0} + \int y_{w,h} e^{-sh} + \tau_h \begin{bmatrix}
\tilde{H}_{11,0} & \tilde{H}_{12,0} \\
\tilde{H}_{21,0} & \tilde{H}_{22,0}
\end{bmatrix} \begin{bmatrix}
B_w(I - P_{w,0}) \\
-C_{y0}D_{yw}(0)
\end{bmatrix} e^{-sh}]
\]

The problem here is that both delay-free and delayed measurement channels in generally affected by the same components of \( w \), i.e., \( w \) is mixed up in \( y \). In the compensated system, the primary controller \( \tilde{K} \) obtains the compensated measurement \( \tilde{y} \). Using the relation

\[
\begin{align*}
\Pi_{y,h,0} &= \tau_h \begin{bmatrix}
\begin{bmatrix}
\bar{H}_{11,0} & \bar{H}_{12,0} \\
\bar{H}_{21,0} & \bar{H}_{22,0}
\end{bmatrix} & \begin{bmatrix}
B_wD_{yw}(0) \\
-C_{y0}D_{yw}(0)
\end{bmatrix}
\end{bmatrix} e^{-sh}
\end{align*}
\]

\[
\begin{align*}
\Pi_{y,h,0} &\quad \begin{bmatrix}
\tilde{y}_{w,h} \\
\tilde{y}_{w,0} + \int \tilde{y}_{w,h} e^{-sh} + \tau_h \begin{bmatrix}
\tilde{H}_{11,0} & \tilde{H}_{12,0} \\
\tilde{H}_{21,0} & \tilde{H}_{22,0}
\end{bmatrix} \begin{bmatrix}
B_wD_{yw}(0) \\
-C_{y0}D_{yw}(0)
\end{bmatrix} e^{-sh}
\end{bmatrix}
\end{align*}
\]
it is readily seen that the effect of \( w \) on this signal (thus, \( \hat{y}_{w,h} \) represents the effect of \( w \) on \( \hat{y}_y \) in Fig. 7) is

\[
\hat{y}_w = \begin{bmatrix} I & e^{-sh} \end{bmatrix} P_{y,0} \begin{bmatrix} G_{y,0} & 0 \\ G_{y,0} & 0 \end{bmatrix} P_{y,0} \begin{bmatrix} I \end{bmatrix}
\]

where \( G_{y,0} \) is the rational part of \( \Pi_{y,h,0} \) and

\[
P_{y,0} := I - G_{y,0}^{-1} \begin{bmatrix} G_{y,0} & 0 \end{bmatrix} P_{y,0} \begin{bmatrix} I \\ G_{y,0} & 0 \end{bmatrix}.
\]

As \( P_{y,0}(j\omega) \) at each frequency \( \omega \) is the orthogonal projection onto ker \( G_{y,0}(j\omega) \), the delayed component of the compensated measurement contains only the parts of \( w \) that are in the kernel of the \( G_{y,0} \), i.e., that cannot be measured without delay. The other parts of the exogenous input are measured delay free. Thus, the delay-free and delayed channels are now separated. In other words, the interchannel DTC for the output adobe delay problem compensates for the delayed measurements of those exogenous inputs that can be measured through the delay-free channel. The feedback component \( \Pi_{h} \) of the FASP scheme in Fig. 7 is then doing the same with the effect of the control signal \( u \) on the measurement.

**Remark 4.1 (Fixed-Lag Smoothing):** The key technical idea behind the proof of Theorem 4.1 is the factorization

\[
\Lambda_\gamma G_{yw} = \Pi_{y,0}^{-1} \tilde{G}_{yw} \Psi_w
\]

(22)

where the inner \( \Psi_w \), bistable \( \Pi_y \), and rational \( \tilde{G}_{yw} \) are defined in Theorem 4.1. This factorization leads to a connection with non-causal estimation, which is dual to that discussed in Remark 3.3.

Consider the problem of finding a \( K \in H^\infty \), which stabilizes \( G_{yw} := e^{-sh} \tilde{G}_{yw} = \tilde{G}_{yw} - KG_{yw} \) and minimizes its norm, either \( H^2 \) or \( H^\infty \). This is an estimation problem, called the fixed-lag smoothing [34]. Using (22), we have that

\[
G_{yw} = [-K] \Lambda_\gamma G_{yw} = [-K] \Pi_{y,0}^{-1} \tilde{G}_{yw} \Psi_w
\]

\[
= \left( \tilde{G}_{yw} - (K + \Pi_{h,0}) \tilde{G}_{yw} \right) \Psi_w
\]

which effectively reduces the smoothing problem to an equivalent filtering, i.e., delay-free, problem. This means that the interchannel block \( \Pi_{y,h,0} \) is the infinite-dimensional part of the \( H^2 \) and \( H^\infty \) fixed-lag smoothing solution.

V. MULTIPLE DELAY LOOP SHIFTING

The adobe delays solutions discussed in the previous two sections are instrumental in handling the case of general diagonal I/O delays. The idea, borrowed from [10], is to split the general problem into a recurrent sequence of problems, at each step of which an adobe problem is solved. In this section, we outline this approach. The reader may refer to [10, Sec. V] for more details.

A. Multiple Input Delays

We start with the input-delay case. Assume for now that \( \Lambda_u = I \) and consider the standard problem in Fig. 1 with \( q \) input delay channels for some \( q \in \mathbb{N} \). Without loss of generality, we may presume that the delay channels are ordered ascendingly, i.e.,

\[
\Lambda_u = \Lambda_q := \text{diag} \{ I_{n_0}, e^{-sh_1} I_{n_1}, \ldots, e^{-sh_0} I_{n_q} \}
\]

To this end, let \( \Lambda_q \) be the 

for \( 0 < h_1 < \cdots < h_q \). Here, \( \sum_{q=0}^q n_i = n_u \) and the dimension of the delay-free channel, \( n_0 \), might be zero.

Let us present the delay block as \( \Lambda_q = \Lambda_{\text{ind},1} \Lambda_{\text{q},-1} \), where

\[
\Lambda_{\text{ind},1} := \begin{bmatrix} I_{n_0} & 0 \\ 0 & e^{-sh_1} I_{n_u - n_0} \end{bmatrix}
\]

is an input adobe delay element and

\[
\Lambda_{\text{q},-1} := \text{diag} \{ I_{n_0 + n_1}, e^{-s(h_2 - h_1)} I_{n_2}, \ldots, e^{-s(h_q - h_1)} I_{n_q} \}
\]

is a multiple-delay block with \( q - 1 \) delay channels. We can now carry out the loop-shifting procedure of Theorem 3.1 with respect to \( \Lambda_{\text{ind},1} \) considering \( \Lambda_{\text{q},-1} \) as a part of the controller \( K \). Eventually, we will arrive at the equivalent system in Fig. 4(a) modulo replacing \( K \) with \( \Lambda_{\text{q},-1} K \). An important observation here is that \( \Lambda_{\text{q},-1} \) can be pulled out of the block \( K \). Indeed, partition

\[
\Pi_{\text{u}} := \begin{bmatrix} I_{n_0} & \Pi_{\text{u},0,h_1} & \Pi_{\text{u},0,h_2} \\ 0 & I_{n_1} & 0 \\ 0 & 0 & I_{n_u - n_0 - n_1} \end{bmatrix}
\]

to see that

\[
\Pi_{\text{u},1} := \Lambda_{\text{u},-1} \Pi_{\text{u}} \Lambda_{\text{u},-1} = \begin{bmatrix} I_{n_0} & \Pi_{\text{u},0,h_1} & \Pi_{\text{u},0,h_2} \Lambda_{\text{q},-1} \\ 0 & I_{n_1} & 0 \\ 0 & 0 & I_{n_u - n_0 - n_1} \end{bmatrix}
\]

where \( \Lambda_{\text{q},-1} := \text{diag} \{ e^{-s(h_2 - h_1)} I_{n_2}, \ldots, e^{-s(h_q - h_1)} I_{n_q} \} \) is the delayed part of \( \Lambda_{\text{q},-1} \). Thus, although \( \Pi_{\text{u}} \) does not commute with \( \Lambda_{\text{q},-1} \), \( \Pi_{\text{u},1} \) is still bistable. Thereby, we obtain the block diagram in Fig. 8 with \( \Pi_{1} := \Pi_{\text{q},-1} \). The system from \( w \) to \( z \) is a standard feedback interconnection of a rational plant (\( \tilde{G} \)), a rational controller (\( \tilde{K} \)), and an input delay (\( \Lambda_{\text{q},-1} \)).

Thus, we just saw that the loop shifting with respect to \( \Lambda_{\text{ind},1} \) reduces the standard problem with \( q \) input delay channels to that with \( q - 1 \) input delay channels. Repeating this procedure \( q \) times, we end up with a standard problem with no input delays. To formalize this recursion, we only need to see how the components of the system in Fig. 4(a) change from step to step. To this end, let the \( \ell \)th step of this recursion start with a rational plant \( G_{\ell} \), an FIR \( \Delta_{\ell} \), an inner \( \Psi_{\ell,i} \), and DTCs \( \Pi_{\ell} \) and \( \Pi_{\text{u},\ell} \). Applying Theorem 3.1 to \( G = G_{\ell} \), we can
calculate rational $\tilde{G}$ and FIR $\Psi_z, \Delta, \Pi$, and $\Pi_{\Delta}$. The data for the next iteration is then calculated as follows:

\[
\Psi_{z,i+1} = \Psi_{z,i} \Psi_z \quad \Psi_{z,0} = I \tag{23a}
\]

\[
\Delta_{i+1} = \Delta_i + \Psi_{z,i} \Delta \quad \Delta_0 = 0 \tag{23b}
\]

\[
\Pi_{i+1} = \Pi_{i} \Lambda_{q-i}^{-1} \Pi_{i} \Lambda_{q-i} + \Pi_{i} \Lambda_{q-i} \quad \Pi_0 = 0 \tag{23c}
\]

\[
\Pi_{n,i+1} = \Pi_{n,i} \Lambda_{q-i}^{-1} \Pi_{n,i} \Lambda_{q-i} \quad \Pi_{n,0} = I \tag{23d}
\]

and $G_{z,i+1} = \tilde{G}_i$. It is readily seen that $\Psi_{z,i+1}$ is kept inner at each step of this recursion. Moreover, if $\langle \Delta_i, \Psi_z, T \rangle_2 = 0$ for every $T \in H^2$, then

\[
\langle \Delta_{i+1}, \Psi_{z,i+1}, T \rangle_2 = \langle \Delta_i, \Psi_{z,i}, T \rangle_2 + \langle \Psi_{z,i}, \Delta, \Delta, \Psi_z, T \rangle_2 = \langle \Delta_i, \Psi_{z,i} \cdot \Psi_z, T \rangle_2 + \langle \Delta, \Psi_z, T \rangle_2 = 0
\]

also (mind that $\Psi_z T \in H^2$ for every $T \in H^2$ since $\Psi_z \in H^\infty$). In other words, the properties of $\Psi_z$, $\Delta$ established by Theorem 3.1 are preserved in the multiple-input-delay case.

Remark 5.1: If there are no delay-free input channels, we just take $n_0 = 0$. In this case, we still have $q$ steps in our recursion, with the only difference that the first step is the single-delay loop shifting of [4], as described in Section II-A.

B. Adding Output Delays

Output delays can be added to the recursion above seamlessly. If $\Lambda_q \neq I$, it can be considered as a part of $K$ in the input-delay recursion of the previous subsection. Then, after the $q$th step is completed and all input delays are eliminated, $\Lambda_q$ is pulled out of $K$. This only requires the following change in the feedback part of the DTC: $\Pi_q \rightarrow \Lambda_q \Pi_q$. Once the output delay is extracted from $K$, we may start a recursion, dual to that described in Section V-A.

If there are $p$ output delay channels, we need $p$ such recursion steps, and the resulting equivalent system is of the form depicted in Fig. 9. The general problem inherits from adobe delay problems the properties of the blocks $\Psi_z q$ and $\Psi_{w,p}$ to be inner and co-inner, respectively, and the orthogonality of $\Delta_{p+q}$ to the rest of the closed-loop system in the sense that

\[
\langle \Delta_{p+q}, \Psi_{z,q} T_{2\Theta}, \Psi_{w,p} \rangle_2 = 0
\]

for all $T_{2\Theta} \in H^2$.

We conclude this section with a short discussion on the structure of the multiple-delay FASP. In general, the FASP components $\Pi_{\Delta,q}$ and $\Pi_{\Delta,p}$ contain a number of interchannel DTC blocks, connecting between different input and output channels, respectively. As follows from (23d), $\Pi_{\Delta,q}$ is always block upper triangular with identity diagonal entries. This means that its feedforward terms always go from channels with shorter delays to those with longer delays. This, actually, implies (cf. the discussion in Section III-C) that $\Pi_{\Delta,q}$ compensates for the effect of the control channels with longer delays on all directions of the regulated signal that are affected by the control channels with shorter delays as well. Similarly, it can be seen that $\Pi_{\Delta,p}$ is always lower triangular, so that the interchannel links always go from the measurement channels with shorter delays to those with longer delays. Thus, we may conclude that $\Pi_{\Delta,p}$ compensates for the delayed measurements of those exogenous inputs that can be sensed through measurement channels with shorter delays.

VI. CONCLUDING REMARKS

In this paper, a loop-shifting procedure, which converts the standard problem with multiple input/output delays to a delay-free problem, has been developed. Unlike previously available solutions, which were able to cope only with closed-loop stability, the proposed procedure addresses closed-loop performance as well. In doing this, our approach brings about and, for the first time, explains an unorthodox form of the dead-time compensator, termed the feedforward action Smith predictor (FASP), which contains interchannel feedforward terms alongside the conventional internal feedback. We have shown that the FASP’s interchannel components do the following:

- at the plant input, compensate for the effect of the control channels with longer delays on all directions of the regulated signal that are affected by the control channels with shorter delays as well;
- at the plant output, compensate for the delayed measurements of those exogenous inputs that can be sensed through the measurement channels with shorter delays.

We believe that our results pave the way for resolving the long-standing question of the right generalization of the single-delay Smith predictor to systems with multiple I/O delays.

Our developments have led to a qualitative conclusion that the structure of the dead-time compensator should rely upon required control performance manifested by the structure of the regulated output and/or the way in which exogenous signals affect the measurement. This challenges the conventional paradigm in the design of dead-time compensators/finite spectrum assignment controllers, where the controller structure is brought about by stability considerations only. Our arguments also demonstrate that, in contrast to some previous researches, the reduction of the stabilization of dead-time systems to that of rational systems needs not conflict with the closed-loop performance. We believe that these conclusions are valid not only for the class of systems with diagonal loop delays considered in this paper, but also for more general classes of I/O delay systems. This is a subject of current research.

As a byproduct of our results, we have also proposed (see Remarks 3.3 and 4.1) a simple procedure of reducing the $H^2$
and $H^\infty$ preview tracking and fixed-lag smoothing problems to equivalent tracking and filtering problems without preview.

**APPENDIX**

**PROOF OF THEOREM 3.1**

As indicated in Section III-B, the main idea behind the proof of Theorem 3.1 is to construct factorization (13). We address it in two steps.

**Lemma A.1:** Let the conditions of Theorem 3.1 hold. Then

$$\Lambda_u \gamma \bar{G}_{zu} \Lambda_u = \left( P_u^{-1} \right)^t \bar{G}_{zu} \Lambda_u P_u^{-1}$$

where $\bar{G}_{zu}$ and $P_u$ are as defined in Theorem 3.1.

**Proof:** Introduce the rational transfer function

$$\Phi = \left[ \begin{array}{cc} \Phi_0 & \Phi_{\theta h} \\ \Phi_{\theta 0} & \Phi_h \end{array} \right] = \bar{G}_{zu} G_{zu}$$

where the partitioning is compatible with that of $\Lambda_u$ in (4). In this case

$$\Lambda_u \gamma \bar{G}_{zu} \Lambda_u = \left[ \begin{array}{cc} \Phi_0 & \Phi_{\theta 0} e^{-m h} \\ \Phi_{\theta 0} e^{m h} & \Phi_h \end{array} \right]$$

contains delays only in its nondiagonal terms. We thus may use the approach in [6, pp. 276–277] (known as the MZ-trick) with simplifications proposed in [35, § III-E] to factor it as

$$\Lambda_u \gamma \bar{G}_{zu} \Lambda_u = \left( P_u^{-1} \right)^t \bar{P} u^{-1}$$

where $P_u$ is as defined in Theorem 3.1 and $\bar{P}$ is a rational transfer function having the state-space realization

$$\bar{P} = \left[ \begin{array}{cc} A & 0 \\ -C_1 \bar{C}_z & -A' \end{array} \right] \begin{bmatrix} B_{u0} & B_0 \\ -C_z D_{zu0} & B_3 \end{bmatrix}$$

$$+ \left[ \begin{array}{cc} D_{zu0} \bar{C}_z & B_{u0} \\ -D_{zu0} & B_{u0} \end{array} \right] \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} B_0 & 0 \\ -C_z D_{zu0} & B_3 \end{bmatrix}$$

$$\begin{bmatrix} B_{u0} \\ B_{u0} \end{bmatrix} = \Sigma^{-1} \begin{bmatrix} B_{u0} \\ -C_z D_{zu0} \end{bmatrix} = \begin{bmatrix} \Sigma \gamma_2 \Sigma D_{zu0} + \Sigma \gamma_2 B_{u0} \\ -\Sigma \gamma_1 \Sigma D_{zu0} - \Sigma \gamma_1 B_{u0} \end{bmatrix}$$

is obtained using the relation

$$\Sigma^{-1} = \left[ \begin{array}{cc} \Sigma \gamma_2 & -\Sigma \gamma_2 \\ -\Sigma \gamma_1 & \Sigma \gamma_1 \end{array} \right]$$

follows by the fact that $\Sigma$ is symplectic [34]. Applying the similarity transformation with the matrix

$$\begin{bmatrix} I & 0 \\ \Sigma \gamma_{21} & \Sigma \gamma_{21} \end{bmatrix}$$

(see [36, § A.1] for the rationale behind this choice) and going through some tedious albeit straightforward algebra, we end up with

$$\Phi = \left[ \begin{array}{cc} A & 0 \\ -C_{z} \bar{C}_{z} & -A' \end{array} \right] \begin{bmatrix} B_{u0} & B_0 \\ -C_{z} D_{zu0} & \bar{C}_{z} D_{zu0} \end{bmatrix}$$

which together with the fact that $\bar{B}_{u0} = [B_{u0}, B_0]$ implies that $\bar{C}_{zu}$ from (15) satisfies $\bar{\Phi} = \bar{C}_{zu} \bar{G}_{zu}$. $\blacksquare$

**Lemma A.2:** $\Psi_e$ defined in Theorem 3.1 is inner and verifies

$$G_{zu} \Lambda_u P_u = \Psi_e \bar{G}_{zu}$$

**Proof:** The equality above and the fact that $\Psi_e^* \Psi_e = I$ are verified by direct state-space manipulations. Tiedious details, which are omitted because of space limitation, can be found in [36]. To prove that $\Psi_e$ is inner, it suffices to show that $\Psi_e$ is stable. This follows from the facts that it is causal and has a bounded and finite impulse response. $\blacksquare$

Having constructed $\bar{G}_{zu}$ and $\Psi_e$, we effectively obtained a counterpart of the scheme in Fig. 3(a) for a general $G_{zu}$. The difference now is that instead of $\Lambda_u$ we have $\Psi_e$. The next step is to pul $\Psi_e$ out of the system in order to arrive at a counterpart of Fig. 3(b). The following result solves this step.

**Lemma A.3:** $\Delta$ and $\gamma \bar{C}_{zu}$ given in Theorem 3.1 verify

$$G_{zu} = \Delta + \Psi_e \bar{G}_{zu}$$

and $\Psi_e^* \Delta$ is anti-causal.

**Proof:** It can be shown by direct algebra (see [36] for details) that $\bar{G}_{zu}$ is the causal part of $\Psi_e \bar{G}_{zu}$. The anti-causal part of the latter

$$\Xi := \Psi_e^* G_{zu} - \bar{C}_{zu}$$

$$= \tau_h \begin{bmatrix} H_{11,0} & H_{12,0} & B_{u0} D_{zu0} \\ H_{21,0} & H_{22,0} & \bar{C}_{zu} - C_z D_{zu0} \end{bmatrix} \sim 0$$

is an FIR impulse response, whose impulse response has support in $[-h, 0]$. The proof is completed by verifying (see [36] again) that $\Psi_e \Xi = \Delta$. $\blacksquare$

Lemma A.3 actually proves the orthogonality part of the theorem. Indeed, as $\Psi_e \in H^\infty$, we have that $\Psi_e T_{zu} \in H^2$ for every $T_{zu} \in H^2$ and then $\langle \Delta, \Psi_e^* T_{zu} \rangle = \langle \Psi_e^* \Delta, T_{zu} \rangle = 0$ because $\Psi_e^* \Delta = \Xi$ and $T_{zu}$ have nonoverlapping impulse responses for every $T_{zu} \in H^2$.

The final step of the loop shifting is a counterpart of the transition from Fig. 3(b) to (c):

**Lemma A.4:** $\bar{G}_{yu}$ and $\Pi$ given in Theorem 3.1 verify

$$G_{yu} \Lambda_u \Pi_u = \bar{C}_{yu} - \Pi$$

**Proof:** Follows by direct substitutions; see [36]. $\blacksquare$

We thus just proved that the system in Fig. 1 can be equivalently presented in the form depicted in Fig. 4(a). The next step is to prove the controllability part.

**Lemma A.5:** $\lambda \in \mathbb{C}$ is an uncontrollable mode of $(A, B_{u0})$ if it is an uncontrollable mode of $(A, \bar{B}_{u0})$.

**Proof:** As the first column blocks of $B_{u0}$ and $\bar{B}_{u0}$ coincide, we only need to check the eigenvalues of $A$ that are not controllable through $B_{u0}$. Let $\lambda \in \mathbb{C}$ be such an eigenvalue of $A$ and $\eta^*$ be the corresponding right eigenvector. Because $\lambda$ is not controllable through $B_{u0}$, $\eta^* B_{u0} = 0$. It is readily seen that $\lambda$ is then also an eigenvalue of $H_{u0}$ with the right eigenvector $[\eta^* 0]$. This implies that

$$[\eta^* 0] \Sigma^{-1} = e^{-\lambda h} [\eta^* 0]$$

where $\Sigma$ is the block diagonal matrix of the transmission zeros of $G_{zu}$. $\blacksquare$
or, equivalently [cf. (24)], that \( \eta \Sigma_{22} = e^{-\lambda h} \eta \) and \( \eta \Sigma_{12} = 0 \). Thus, we have that

\[
\eta \tilde{B}_u = [0 \quad e^{-\lambda h} \eta B_{u,h}]
\]

and, hence, that \( \eta B_{u,h} = 0 \iff \eta \tilde{B}_u = 0 \).

To complete the proof, we only need to address the internal stability issue. Remember [22] that the system in Fig. 1 with the delays as in (4) is said to be internally stable if the transfer function \( T_{03} \) from three exogenous inputs \( w, v_1, v_2 \) to three outputs \( z, y, u \) in Fig. 10(a) is stable.

We have the following result.

**Lemma A.6:** Let \( \tilde{T}_0 \) be the transfer function from \( w, \tilde{v}_1, \tilde{v}_2 \) to \( \tilde{z}, \tilde{u}, \tilde{y} \) for the system in Fig. 10(b). Then

\[
T_0 \in H^\infty \iff \tilde{T}_0 \in H^\infty
\]

provided \( \mathcal{K} \) and \( \tilde{\mathcal{K}} \) are related via (11) and \( \tilde{\mathcal{K}} \) is proper.

**Proof:** To prove the statement of the lemma, we use the arguments from the proof of [4, Lem. 1], where the single-delay case is studied. First, note that \( \mathcal{K} \) is proper iff so is \( \tilde{\mathcal{K}} \) (this follows from (11) and the strict properness of \( \Pi \)). Now, the relations between corresponding inputs and outputs for the systems in Figs. 10 are

\[
\begin{bmatrix}
    w \\
    v_1 \\
    v_2 \\
    z \\
    y \\
    u
\end{bmatrix}
= \begin{bmatrix}
    I & 0 & 0 \\
    0 & I & \Pi \\
    0 & 0 & \Pi w
\end{bmatrix}
\begin{bmatrix}
    \tilde{w} \\
    \tilde{v}_1 \\
    \tilde{v}_2 \\
    \tilde{z} \\
    \tilde{y} \\
    \tilde{u}
\end{bmatrix} + \begin{bmatrix}
    \Delta & 0 & 0 \\
    0 & 0 & 0 \\
    0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    \tilde{w} \\
    \tilde{v}_1 \\
    \tilde{v}_2
\end{bmatrix}.
\]

Hence

\[
T_0 = G_2 \tilde{T}_0 G_2^{-1} + G_3 \quad \text{and} \quad \tilde{T}_0 = G_2^{-1} T_0 G_1 + G_2^{-1} G_3 .
\]

Because \( G_2^{-1}, G_2, G_3 \in H^\infty \), the stability of \( \tilde{T}_0 \) implies that of \( T_0 \). The other direction is not true in general as \( G_2^{-1} \notin H^\infty \) (its (1, 1) subblock, \( \Psi_z \), is not stably invertible). We, however, are only concerned with proper controllers and, hence, with proper \( T_0 \) and \( \tilde{T}_0 \) (as \( G_{u,n} \) and \( G_{y,n} \) are strictly proper). To see how this helps, note that \( e^{-sh} \Psi_z^{-1} \in H^\infty \) because \( \Psi_z^{-1} = \Psi_z^{-1} \) for FIR with the support in \([-h,0] \). Hence, \( T_0 \in H^\infty \) implies that \( e^{-sh} \tilde{T}_0 \in H^\infty \). Assume that \( \tilde{T}_0 \) is nonetheless unstable. As \( \tilde{T}_0 \) is proper, there is an \( \alpha > 0 \) such that \( \tilde{T}_0(s) \) is uniformly bounded on \( \mathbb{C}_\alpha \). Therefore, to be unstable, \( \tilde{T}_0(s) \) must have a singularity in the strip \( \mathbb{C}_\alpha \). In this strip \( \alpha e^{-h} > e^{-\alpha h} > 0 \), hence, if \( \tilde{T}_0 \) has a singularity there so does \( e^{-\alpha h} \tilde{T}_0 \). This is a contradiction.

**REFERENCES**


Leonid Mirkin (M’99) is a native of Frunze, Kirghiz SSR, USSR (now Bishkek, Kyrgyz Republic). He received the Electrical Engineer degree from Frunze Polytechnic Institute in 1989, and the Ph.D. (candidate of sciences) degree in automatic control from the Institute of Automation, Academy of Sciences of Kyrgyz Republic in 1992.

From 1989 to 1993, he was with the Institute of Automation, Academy of Sciences of Kyrgyz Republic. In 1994, he joined the Faculty of Mechanical Engineering, Technion—Israel Institute of Technology, Haifa, Israel, first as a Post-Doctoral Researcher and then as a faculty member. His research interests include systems theory, control and estimation of sampled-data systems, dead-time compensation, systems with preview, the application of control to electromechanical and optical devices, and robustifying properties of corruption.

Dmitry Shneiderman was born in Ukraine in 1975. He received the Mechanical Engineer degree from Kryvyi Rih Technical University, Kryvyi Rih, Ukraine, in 1997, and the MSc degree in mechanical engineering from the Technion—Israel Institute of Technology, Haifa, Israel, in 2005, and is currently pursuing the Ph.D. degree in mechanical engineering at the Technion.

His research interests are focused on control of systems with delays and dead-time compensators.

Zalman J. Palmor (SM’92) received the B.Sc. and M.Sc. degrees in mechanical engineering from the Technion—Israel Institute of Technology (Technion—IIT), Haifa, Israel, in 1966 and 1972, respectively, and the Ph.D. degree in chemical engineering from City College, City University of New York (CUNY), New York, in 1976.

He is the Danciger Professor of Engineering and the Deputy Executive Vice President for Academic Affairs at the Technion—IIT. Until 1978, he worked for the Taylor Instrument Company, Rochester, NY. In 1979, he joined the Faculty of Mechanical Engineering at the Technion—IIT, where he is a Full Professor and, until December 2006, was the Dean of the Faculty for five years. He is actively involved with industry and international engineering entities, and some of his control algorithms were installed and distributed worldwide by leading vendors of process control equipment. In 1984 and 1993, he was a Visiting Professor with the University of Rochester, Rochester, NY, and he was a Visiting Professor with the University of Melbourne, Melbourne, Australia, in 2000. In 1994, he was a Visiting Scientist with ABB Kent-Taylor, Inc., and ABB Process Automation, Albany, NY. His research interests include control of systems with delays, sampled-data systems, auto-tuning, and the application of control to mechanical devices and to chemical processes.

From 1989 to 1993, he was with the Institute of Automation, Academy of Sciences of Kyrgyz Republic. In 1994, he joined the Faculty of Mechanical Engineering, Technion—Israel Institute of Technology, Haifa, Israel, first as a Post-Doctoral Researcher and then as a faculty member. His research interests include systems theory, control and estimation of sampled-data systems, dead-time compensation, systems with preview, the application of control to electromechanical and optical devices, and robustifying properties of corruption.

Dmitry Shneiderman was born in Ukraine in 1975. He received the Mechanical Engineer degree from Kryvyi Rih Technical University, Kryvyi Rih, Ukraine, in 1997, and the MSc degree in mechanical engineering from the Technion—Israel Institute of Technology, Haifa, Israel, in 2005, and is currently pursuing the Ph.D. degree in mechanical engineering at the Technion.

His research interests are focused on control of systems with delays and dead-time compensators.