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Universal Seesaw Mass Matrix Model with an S_3 Symmetry

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Abstract

Stimulated by the phenomenological success of the universal seesaw mass matrix model, where the mass terms for quarks and leptons f_i $(i = 1, 2, 3)$ and hypothetical super-heavy fermions F_i are given by $\overline{f}_L m_L F_R + \overline{F}_L m_R f_R + \overline{F}_L M_F F_R + h.c.$ and the form of M_F is democratic on the bases on which m_L and m_R are diagonal, the following model is discussed: The mass terms M_F are invariant under the permutation symmetry S_3, and the mass terms m_L and m_R are generated by breaking the S_3 symmetry spontaneously. The model leads to an interesting relation for the charged lepton masses.

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The universal seesaw mass matrix model[1] is one of the most promising candidates of unified quark and lepton mass matrix models. The model has hypothetical fermions F_i ($F = U, D, N, E$; $i = 1, 2, 3$) in addition to the conventional quarks and leptons f_i ($f = u, d, \nu, e$; $i = 1, 2, 3$), and these fermions are assigned to $f_L = (2, 1)$, $f_R = (1, 2)$, $F_L = (1, 1)$ and $F_R = (1, 1)$ of SU(2)$_L\times$ SU(2)$_R$. The 6×6 mass matrix which is sandwiched between the fields $(\overline{f}_L, \overline{F}_L)$ and (f_R, F_R) is given by

$$M^{6\times 6} = \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} , \tag{1}$$

where m_L and m_R are universal for all fermion sectors ($f = u, d, \nu, e$) and only M_F have structures dependent on the flavors F. For $\Lambda_L < \Lambda_R \ll \Lambda_S$, where $\Lambda_L = O(m_L), \Lambda_R = O(m_R)$ and $\Lambda_S = O(M_F)$, the 3×3 mass matrix M_f for the fermions f is given by the well-known seesaw expression

$$M_f \simeq -m_L M_F^{-1} m_R \ . \tag{2}$$

Thus, the model answers the question why the masses of quarks (except for top quark) and charged leptons are so small compared with the electroweak scale Λ_L ($\sim 10^2$ GeV). On the other hand, in order to understand the observed fact $m_t \sim \Lambda_L$, we put the ansatz [2, 3] $\det M_F = 0$ for the up-quark sector ($F = U$). Then, one of the fermion masses $m(U_i)$ is zero [say, $m(U_3) = 0$], so that the seesaw mechanism does not work for the third family, i.e., the fermions (u_{3L}, U_{3R}) and (u_{3R}, U_{3L}) acquire masses of $O(m_L)$ and $O(m_R)$, respectively. We identify (u_{3L}, U_{3_R}) as the top quark (t_L, t_R). Thus, we can understand the question why only the top quark has a mass of the order of Λ_L.

For the numerical results, excellent agreements with the observed values of the quark masses and Cabibbo-Kobayashi-Maskawa [4] (CKM) matrix are obtained by putting the following assumptions [2]:

(i) The mass matrices m_L and m_R have the same structure

$$m_R = \kappa m_L \equiv m_0 \kappa Z \ . \tag{3}$$

(ii) The mass matrix M_F is given by the form

$$M_F = m_0 \lambda (\mathbf{1} + 3 b_f X), \tag{4}$$

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} , \tag{5}$$

on the basis on which the matrix Z is diagonal, i.e.,

$$Z = \mathrm{diag}(z_1, z_2, z_3), \tag{6}$$

where $z_1^2 + z_2^2 + z_3^2 = 1$.
(iii) The parameter b_f for the charged lepton sector is given by $b_e = 0$, so that in the limit of $\kappa/\lambda \ll 1$, the parameters z_i are given by

$$\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}} \tag{7}$$

(iv) Then, the up- and down-quark masses are successfully given by the choice of $b_u = -1/3$ and $b_d = -e^{i\beta d}$ ($\beta_d = 18^\circ$), respectively. The CKM matrix is also successfully obtained.

In this phenomenological success, the assumption that the mass matrix M_F is the democratic type is essential. The form of M_F, (4), is invariant under the permutation symmetry S_3 for (F_1, F_2, F_3), while the form of m_L (m_R) is not invariant under the permutation symmetry S_3 for (F_1, F_2, F_3) and (f_1, f_2, f_3). In this paper, we consider that the mass terms m_L (m_R) are generated by breaking the S_3 symmetry not explicitly, but spontaneously at $\mu = \Lambda_L$ ($\mu = \Lambda_R$). For this purpose, we introduce three $SU(2)_L$-doublet Higgs scalars $(\phi_{1L}, \phi_{2L}, \phi_{3L})$, which obey to the permutation symmetry S_3 as well as (F_1, F_2, F_3) and (f_1, f_2, f_3). (We also assume three $SU(2)_R$-doublet Higgs scalars.) The purpose of the present paper is to discuss the possible structure of m_L (m_R) under this S_3 symmetry.

The Yukawa interactions which generate the mass matrix m_L are given by

$$y_L \sum_i [(\overline{\nu}_{iL}\ \overline{e}_{iL}) \left[\begin{pmatrix} \phi_{iL}^+ \\ \phi_{iL}^0 \end{pmatrix} E_{iR} + \begin{pmatrix} \overline{\phi}_{iL}^0 \\ -\phi_{iL}^- \end{pmatrix} N_{iR} \right] + (\text{quark sectors})\ , \tag{8}$$

Hereafter, for convenience, we drop the index L. The most simple form of the S_3 invariant potential of the Higgs scalars (ϕ_1, ϕ_2, ϕ_3) is

$$V_1 = \mu^2 \sum_i (\overline{\phi}_i \phi_i) + \frac{1}{2}\lambda_1 [\sum_i (\overline{\phi}_i \phi_i)]^2, \tag{9}$$

where $(\overline{\phi}_i \phi_i) = \phi_i^- \phi_i^+ + \overline{\phi}_i^0 \phi_i^0$. Note that the term

$$V_2 = \eta_1 (\overline{\phi}_\sigma \phi_\sigma)(\overline{\phi}_\pi \phi_\pi + \overline{\phi}_\eta \phi_\eta), \tag{10}$$

is also S_3-invariant, where

$$\phi_\pi = \frac{1}{\sqrt{2}}(\phi_1 - \phi_2), \tag{11}$$

$$\phi_\eta = \frac{1}{\sqrt{6}}(\phi_1 + \phi_2 - 2\phi_3), \tag{12}$$

$$\phi_\sigma = \frac{1}{\sqrt{3}}(\phi_1 + \phi_2 + \phi_3), \tag{13}$$

and

$$\sum_i (\overline{\phi}_i \phi_i) = (\overline{\phi}_\pi \phi_\pi) + (\overline{\phi}_\eta \phi_\eta) + (\overline{\phi}_\sigma \phi_\sigma). \tag{14}$$

We assume that the potential of the Higgs scalars (ϕ_1, ϕ_2, ϕ_3) is given by

$$V = V_1 + V_2. \tag{15}$$

Then, the conditions for the vacuum expectation values $v_i \equiv \langle \phi_i^0 \rangle$ at which the potential (15) takes the minimum are

$$\mu^2 + \lambda_1 \sum_i |v_i|^2 + \eta_1(|v_\pi|^2 + |v_\eta|^2) = 0, \tag{16}$$

$$\mu^2 + \lambda_1 \sum_i |v_i|^2 + \eta_1 |v_\sigma|^2 = 0, \tag{17}$$

so that

$$|v_\sigma|^2 = |v_\pi|^2 + |v_\eta|^2 = \frac{-\mu^2}{2\lambda_1 + \eta_1}, \tag{18}$$

From the relations (13), (14) and (18), we obtain

$$|v_1|^2 + |v_2|^2 + |v_3|^3 = 2|v_\sigma|^2 = \frac{2}{3}|v_1 + v_2 + v_3|^2, \tag{19}$$

which means the relation[6]

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2, \tag{20}$$

from the relation (7). The relation (20) is excellently satisfied by the observed values of the charged lepton masses, i.e., the observed values of m_e and m_μ give the predicted value $m_\tau = 1776.97$ MeV in agreement with the observed value [7] $m_\tau^{exp} = 1777.05^{+0.29}_{-0.26}$ MeV. [We should not take this excellent agreement too rigidly, because the electromagnetic corrections to the observed values spoil the agreement of $m_\tau(\mu)$, for example, to 1.2% at the energy scale $\mu = m_Z = 91.2$ GeV. However, note that the relation (7) is an approximate one. When we define $m_L = m_0 Z_L$ and $m_R = m_0 \kappa Z_R$, the values of $Z_L(\mu)$

and $Z_R(\mu)$ are dependent of on the energy scale μ, so that the relation $Z_L(\mu) = Z_R(\mu)$ is an approximate relation even if it is exact at a unification energy scale $\mu = \Lambda_X$. In order to examine the validity of the relation (20), we must know the energy scale structures in the seesaw model (e.g., the energy scales of m_R, M_F, and so on). At present, we consider that the relation (19) is sill worth noting.]

Explicitly, from the relations (11) - (13), the charged lepton masses m_i^e are given by

$$\sqrt{m_\tau} = \sqrt{m_1^e} \propto v_1 = \left(\frac{1}{\sqrt{2}} \cos\theta + \frac{1}{\sqrt{6}} \sin\theta + \frac{1}{\sqrt{3}} \right) v_\sigma \ , \tag{21}$$

$$\sqrt{m_\mu} = \sqrt{m_2^e} \propto v_2 = \left(-\frac{1}{\sqrt{2}} \cos\theta + \frac{1}{\sqrt{6}} \sin\theta + \frac{1}{\sqrt{3}} \right) v_\sigma \ , \tag{22}$$

$$\sqrt{m_e} = \sqrt{m_3^e} \propto v_3 = \left(-\sqrt{\frac{2}{3}} \sin\theta + \frac{1}{\sqrt{3}} \right) v_\sigma \ , \tag{23}$$

where

$$v_\pi = v_\sigma \cos\theta \ , \quad v_\eta = v_\sigma \sin\theta \ , \tag{24}$$

Since the model is $\phi_\pi \leftrightarrow \phi_\eta$ symmetric, it is likely that the vacuum expectation values satisfy the relation $v_\pi \simeq v_\eta$, i.e., $\sin\theta \simeq \cos\theta \simeq 1/\sqrt{2}$. In the limit of $\sin\theta = \cos\theta = 1/\sqrt{2}$, the electron mass becomes exactly zero. In order to give $v_\pi \neq v_\eta$, we must add a small additional term to the Higgs potential (15). However, for a time, we will not touch the origin of $m_e \neq 0$.

The potential (15) is not general form which is invariant under the S_3 symmetry. The general S_3-invariant potential is given as a function of $\overline{\phi}_{\sigma\alpha}\phi_{\sigma\beta}$ and $\overline{\phi}_{\pi\alpha}\phi_{\pi\beta} + \overline{\phi}_{\eta\alpha}\phi_{\eta\beta}$ (and also $\phi_{\sigma\alpha}\phi_{\sigma\beta}$ and $\phi_{\pi\alpha}\phi_{\pi\beta} + \phi_{\eta\alpha}\phi_{\eta\beta}$, where α and β are SU(2) indices. For example, the potential

$$V = \mu^2 \left[(\overline{\phi}_\sigma \phi_\sigma) + k(\overline{\phi}_\pi \phi_\pi + \overline{\phi}_\eta \phi_\eta) \right]$$

$$+ \frac{1}{2}\lambda \left[a(\overline{\phi}_\sigma \phi_\sigma)^2 + b(\overline{\phi}_\sigma \phi_\sigma)(\overline{\phi}_\pi \phi_\pi + \overline{\phi}_\eta \phi_\eta) + c(\overline{\phi}_\sigma \phi_\sigma)(\overline{\phi}_\pi \phi_\pi + \overline{\phi}_\eta \phi_\eta)^2 \right] \ , \tag{25}$$

is S_3-invariant, while the potential (25) with $k \neq 1$ and $a \neq c$ cannot give the relation (19). In order to give the relation (19), the following condition is required: The potential is invariant under the exchange

$$\begin{aligned} \overline{\phi}_{\sigma\alpha}\phi_{\sigma\beta} &\leftrightarrow \overline{\phi}_{\pi\alpha}\phi_{\pi\beta} + \overline{\phi}_{\eta\alpha}\phi_{\eta\beta} \ , \\ \phi_{\sigma\alpha}\phi_{\sigma\beta} &\leftrightarrow \phi_{\pi\alpha}\phi_{\pi\beta} + \phi_{\eta\alpha}\phi_{\eta\beta} \ , \\ \overline{\phi}_{\sigma\alpha}\overline{\phi}_{\sigma\beta} &\leftrightarrow \overline{\phi}_{\pi\alpha}\overline{\phi}_{\pi\beta} + \overline{\phi}_{\eta\alpha}\overline{\phi}_{\eta\beta} \ . \end{aligned} \tag{26}$$

The most general form which is invariant under the exchange (26) is given by $V = V_1 + V_2 + V_3$, where V_3 is given by

$$V_3 = \frac{1}{2}\lambda_2 \sum_i \sum_j (\overline{\phi}_i \phi_j)(\overline{\phi}_j \phi_i) + \frac{1}{2}\lambda_3 \sum_i \sum_j (\overline{\phi}_i \phi_j)(\overline{\phi}_i \phi_j)$$

$$+\eta_2 \left[(\overline{\phi}_\sigma \phi_\pi)(\overline{\phi}_\pi \phi_\sigma) + (\overline{\phi}_\sigma \phi_\eta)(\overline{\phi}_\eta \phi_\sigma) \right]$$

$$+\eta_3 \left[(\overline{\phi}_\sigma \phi_\pi)(\overline{\phi}_\sigma \phi_\pi) + (\overline{\phi}_\sigma \phi_\eta)(\overline{\phi}_\sigma \phi_\eta) + h.c. \right] \ . \tag{27}$$

Then, the potential V leads to the relation

$$|v_\sigma|^2 = |v_\pi|^2 + |v_\eta|^2 = \frac{-\mu^2}{2(\lambda_1 + \lambda_2 + \lambda_3) + \eta_1 + \eta_2 + 2\eta_3} \ , \tag{28}$$

instead of (18), so that we can again obtain the relation (20).

In Table I, we give the masses of the physical Higgs bosons H_S^0, H_A^0, H_B^0, χ_A^0, χ_B^0, $\chi_A^\pm$, and $\chi_B^\pm$, which are defined by

$$\phi_i \equiv \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i\sqrt{2}\chi_i^+ \\ H_i^0 - i\chi_i^0 \end{pmatrix} , \tag{29}$$

$$\begin{pmatrix} \phi_S \\ \phi_A \\ \phi_B \end{pmatrix} = \frac{1}{v_0} \begin{pmatrix} v_1 & v_2 & v_3 \\ v_1 - \sqrt{\frac{2}{3}} v_0 & v_2 - \sqrt{\frac{2}{3}} v_0 & v_3 - \sqrt{\frac{2}{3}} v_0 \\ \sqrt{\frac{2}{3}}(v_3 - v_2) & \sqrt{\frac{2}{3}}(v_1 - v_2) & \sqrt{\frac{2}{3}}(v_2 - v_3) \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

$$= \frac{1}{v_0} \begin{pmatrix} v_\pi & v_\eta & v_\sigma \\ v_\pi & v_\eta & -v_\sigma \\ \sqrt{2} v_\eta & -\sqrt{2} v_\pi & 0 \end{pmatrix} \begin{pmatrix} \phi_\pi \\ \phi_\eta \\ \phi_\sigma \end{pmatrix} , \tag{30}$$

$$v_0^2 = v_1^2 + v_2^2 + v_3^2 = v_\pi^2 + v_\eta^2 + v_\sigma^2 = 2 v_\sigma^2 \ . \tag{31}$$

[The evaluations are analogous to those in Ref.[8], where the U(3)-family nonet Higgs scalars ϕ_i^j $(i, j = 1, 2, 3)$ were assumed. We can read ϕ_i^j in Ref.[8] as $\phi_i^i \to \phi_i$ $(i = 1, 2, 3)$ and $\phi_i^j \to 0$ $(i \neq j)$.] The Higgs components $\chi_S^\pm$ and χ_S^0 are eaten by the weak bosons $W^\pm$ and Z, respectively. The Higgs boson H_S corresponds to that in the standard one Higgs boson model. Note that the Higgs scalar H_B is massless. Also, H_A is massless if the η-terms are absent, and $\chi_A^\pm$, $\chi_B^\pm$, χ_A^0, and χ_B^0 are massless if the terms V_3 are absent.

In the present model, the flavor-changing neutral currents (FCNC) effects do not appear in the charged lepton sector, because the mass matrix of the charged leptons is diagonal. However, in the neutrino and quark sectors, the FCNC effects appear through the exchanges of the neutral Higgs bosons H_A^0, H_B^0, χ_A^0, and χ_B^0. Although the FCNC in the neutrino sectors have a possibility [9] that they can offer an alternative mechanism to the neutrino oscillation hypothesis, they, in general, bring unwelcome effects, especially, in the quark sectors. In order to avoid this problem, for example, we must distinguish the Higgs scalars ϕ_i^u which couple to the up-fermion sectors, from the scalars ϕ_i^d which couple to the down-fermion sectors. At present, this is an open question.

In conclusion, stimulated by the phenomenological success of the universal seesaw mass matrix model [**?**], we have proposed a Higgs potential which is invariant under the permutation symmetry S_3 for (f_1, f_2, f_3), (F_1, F_2, F_3) and (ϕ_1, ϕ_2, ϕ_3), and which leads to the relation (20) for the charged lepton masses. It is worth while to notice the model because of the agreement of the relation (20) with experiments, although it has a trouble in FCNC.

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Table I. Physical Higgs boson masses, where $v_0^2 = v_1^2 + v_2^2 + v_3^2 = (174\ \text{GeV})^2$.

ϕ	$\chi^\pm$	χ^0	H^0
$m^2(\phi_S)$	eaten by $W^\pm$	eaten by Z	$[2(\lambda_1 + \lambda_2 + \lambda_3) + \eta_1 + \eta_2 + 2\eta_3]v_0^2$
$m^2(\phi_A)$	$-(\lambda_2 + \lambda_3 + \eta_2 + 2\eta_3)v_0^2$	$-2(\lambda_3 + 2\eta_3)v_0^2$	$-(\eta_1 + \eta_2 + 2\eta_3)v_0^2$
$m^2(\phi_B)$	$-(\lambda_2 + \lambda_3 + \frac{1}{2}\eta_2 + \eta_3)v_0^2$	$-2(\lambda_3 + \eta_3)v_0^2$	0