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# A Development Path for the Stabilized Spheromak

T. K. Fowler

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# A Development Path for the Stabilized Spheromak

T. K. Fowler

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## Abstract

In Refs. [1] - [3], I suggest a concerted computational effort to study profile control of spheromaks, in anticipation that it is timely to incorporate the  $q < 1$  regime of RFP's and spheromaks into an integrated advanced toroidal confinement program, together with improvements in tokamaks and stellarators now being pursued. For profile control of spheromaks by neutral beam injection, with care to avoid super-Alfvenic beam instability the main issue is excitation of tearing modes that can be studied using the NIMROD code already calibrated to MST and SSPX. In this note, I show that profile control on spheromaks could be demonstrated in a device the size of SSPX, leading ultimately to a very compact ignition facility, and possibly modular fusion reactors with a shorter development path.

### 1. Numerical Study of Profile Control

Following Ref. [4], profile control could first be studied merely by adding a force term to Ohm's Law of the form:

$$\partial \mathbf{A} / \partial t - \mathbf{v} \times \mathbf{B} = \mathbf{F} / e - \eta \mathbf{j} \quad (1)$$

Success depends on the existence of a stable state,  $\mathbf{j}_s(\mathbf{x})$ . That interesting stable states probably exist is discussed in Refs. [3] and [5]. Our task is to find a force  $\mathbf{F}$  giving the stable  $\mathbf{j}_s(\mathbf{x})$  as was done successfully for MST [4]. Given the existence of a stable  $\mathbf{j}_s(\mathbf{x})$ , an existence theorem for a force  $\mathbf{F}$  yielding this state is found by choosing a rising function  $a(t) \rightarrow 1$  in steady state, writing  $\mathbf{j} = \mu_0^{-1} \nabla \times \nabla \times \mathbf{A} = a(t) \mathbf{j}_s(\mathbf{x})$  in Eq. (1), and solving for  $\mathbf{F}(\mathbf{x}, t)$  [3]. More practically, we would seek a trial function for  $\mathbf{F}$  corresponding roughly to a realizable array of NBI or RF current drive. Linear analysis

discussed in Appendix A of Ref. [3] suggests that stability against tearing mainly requires that  $\lambda = (\mu_0 \mathbf{j}(\mathbf{x})/B)$  be flat in the interior (like a Taylor state), even if  $\mathbf{j}(\mathbf{x})$  then sags to zero at the separatrix bounding closed flux surfaces. Thus a suitable trial function should direct current drive mainly near the edge, but as far from the edge as is consistent with stability, in order to avoid excessive collisional dissipation of the injected current.

## 2. Buildup of the Current

To explore the potential for spheromaks stabilized by profile control, we construct a zero-D model along the lines of Ref. [6]. In Eq. (1) we drop  $\mathbf{v} \times \mathbf{B}$  and operate by  $\mu_0^{-1} \nabla \times \nabla \times \approx \mu_0^{-1} \lambda^2$  to transform  $\mathbf{A}$  to  $\mathbf{j}$  and multiply by  $\pi a^2$  to obtain the total current (with minor radius  $a$  and Taylor eigenvalue  $\lambda = 2.5/a$  in a cylinder). The result is:

$$dI/dt = (1/\tau_{L/R})[C_1(PT/na) - I] \quad (2)$$

analogous to Eq. (2) in Ref. [6]. Here  $\tau_{L/R} = (\mu_0/\eta\lambda^2) = 5a^2T^{3/2}$  with electron temperature  $T$  in KeV. Also  $n$  is in units of  $10^{20} \text{ m}^{-3}$  and  $I$  in MA for current drive power  $P$  in MW. For neutral beam current drive,  $C_1 = 0.33J(x,y)(\cos\theta)(1 - 1/Z)$  where  $\theta$  is the mean angle between neutral beams and field lines. The factor  $J$ , discussed in Ref. [7], is given by  $J(x,y) = x^2/\{7 + x^3 + 2x^2\}$  (for deuterium and  $Z = 2$  giving  $y = 1$ ), derived by fittings to  $j \propto \int dv v_{||} f$  with an analytical approximation for  $f$ , the distribution of beam ions slowing down by electron-ion collisions, with a cutoff at  $E_c = 15T$  and  $x^2 = E/E_c$  with beam energy  $E$  [6].

All numbers are evaluated for deuterium beams. The coefficient 0.33 was chosen to agree with Cordey's formula in Eq. (13) of Ref. [8], giving  $I/P = 0.33J(T/nR)(1 - 1/Z) = 0.06(T/nR)(1 - 1/Z)$  evaluated at the maximum value of a quantity  $J$  playing the role of  $J(x,y)$  in Refs. [6] and [7] (both Cordey's  $J$  and  $J(x,y)$  have maxima at  $E = 80T$  [8]). Following Ref. [7], the coefficient could also be obtained from  $I_{\text{CIRC}} = \int eS\tau_{se}v_0 xJ(x,y)$  integrated over the beam cross-sectional area  $A_b$  with beam speed  $v_0$  at injection, slowing down time  $\tau_{se} \propto T^{3/2}/n$  and source term  $eS = (P/2\pi R A_b E)$ . The latter procedure yields a coefficient smaller by a factor 1.5 (see Appendix). Better numbers, and guidance how to design beam injection to maintain profile control, can be obtained from neutral beam

injection calculations using the Corsica code already applied to calculate current drive in DIIIID and SSPX.

The electron temperature appearing in Eq. (2) is given by:

$$dT/dt = (P/3nV) - T/\tau_E = 1.06 (P/na^3) - T/\tau_E \quad (3)$$

where the coefficient 1.06 accounts for the plasma volume  $V = 2\pi^2 a^3$  and conversion to the units above. For  $\tau_E$  we take the L-mode scaling adapted for spheromaks [9], consistent with results in SSPX [3], giving:

$$\tau_E = 0.023 a^{1.83} (10n)^{0.4} I^{0.96} / P^{0.73} \quad (4)$$

As in Ref. [6], we note that  $\tau_E$  is small compared to timescales in Eq. (1), and substitute the steady state for Eq. (3) into Eq. (1). Doing this, and using Eq. (4), we obtain a condition for buildup of the current by requiring that the right hand side of Eq. (1) be positive, giving, after some algebra:

$$P > 116 (n^{1.26} a^{1.71}) \quad (\text{MW}) \quad (5)$$

$$T = 0.06 (P^{0.27} I^{0.96} / n^{0.6} a^{1.17}) \quad (6)$$

$$t = I/(dI/dt) = I/(C_1(P/na^3)) \approx 20\sqrt{E}(Ina^3/P) \quad (7)$$

In Eq. (5), for simplicity we have set  $I^{0.96} \approx I$ . Also, we should design for  $\cos\theta \approx 1$  and we note that  $J(x,y)$  varies less than a factor 2 over an order of magnitude range in  $x^2$  and take the optimum  $J = 0.18$ . With these simplifications, and  $Z = 2$ , we get  $C_1 \approx (0.33)(.18)(1 - 1/Z) = 0.03$  as in Cordey's formula cited above. In Eq. (7),  $t$  is the buildup time for the current, neglecting the dissipation term. Note that  $\beta \propto T/I^2 \propto 1/I$  decreases during buildup, so that pressure driven modes are likely to be avoided during buildup.

### 3. Sustainment of the Current

Still relying on tokamak scaling, in this section we will assume H-mode confinement in steady-state, using the ITER-98P(y,2) scaling of Ref. [9] given by:

$$\tau_E = 0.056 (2.5)^{0.19} (aA)^{1.97} A^{-0.58} (10n)^{0.41} I^{0.93} P^{-0.69} \quad (8)$$

where we have now kept scaling with the aspect ratio  $A$  and a factor  $(2.5)^{0.19}$  for DT mass. For ignition, for which  $P$  should include alpha heating, we solve Eq. (3) for  $P$  in steady state and substitute this in Eq. (8) keeping a factor  $A$  in the volume in the denominator of the first term. Substituting this  $P$  into Eq. (8) gives, after some algebra:

$$n\tau_E = 3.83 \times 10^{-3} (I^{3.0} A^{2.25} / T^{2.2}) (n^{0.1} / a^{0.3}) \quad (9)$$

This gives ignition at  $T = 20$  KeV,  $I = 47$  MA and  $A = 1$  for the Hagenson-Krakowski spheromak discussed in Ref. [3]. The larger aspect ratio for tokamaks reduces the required current, giving ignition for  $I$  around 20 MA for ITER CDA phase parameters.

Eq. (9) gives the proper  $n\tau_E$  to be applied in a heat balance equation whatever the source of heating. Given  $T$ , the power required to sustain the current is given by the steady state of Eq. (2), giving, with  $C = 0.06(1 - 1/Z) = 0.03$  with  $Z = 2$ , as in Section 2:

$$P = (1/0.03) (fInR/T) = 33(fInR)/T \quad (10)$$

where  $f$  is the fraction of the current that must be driven by NBI (after subtracting bootstrap current) and  $R = Aa$  is the major radius with aspect ratio  $A$  ( $A = 1$  for spheromaks).

Eq. (10) agrees fairly well with current drive powers found in ARIES studies [e.g. 35 MW to drive a net  $fI = 1.2$  MA (91% bootstrap) in ARIES-AT and 237 MW to drive 7.2 MA (57% bootstrap) in ARIES-I, both with  $(\langle nR \rangle / T) \approx 1$  [10].

Better estimates of the power can be obtained using the Corsica code applied to neutral beam injection, as mentioned in Section 2 (see Appendix). These calculations include approximate treatment of beam ion orbits not included in Eq. (10). Even so,

without taking account of  $\langle \cos\theta \rangle$  adjusted to confine beam ions, Eq. (10) agrees within a factor  $\approx 2$  with the power calculated by Corsica for neutral beam injection experiments proposed for SSPX (0.7 MW absorbed to produce driven  $I = 0.016$  MA at  $n = 0.9$ ,  $R = a = .25$ ,  $T = 0.3$  [11]). Similar calculations have been compared with experiments in DIIID [12, 13]. Generally speaking, regimes do exist in which calculations and experiment agree. The main discrepancy between theory and experiment occurs when instabilities are excited [12], including fast ion modes not captured in NIMROD MHD simulations but observed on NSTX [14]. Fast ion modes are discussed in the next Section.

#### 4. Fast Ion Instability

In addition to stabilizing ideal and resistive MHD modes, discussed in Section 1, we must also avoid fast ion instability created by neutral beam current drive, or analogous unwanted wave excitations with RF current drive. For neutral beams, relevant parameters are:

$$\begin{aligned}
 v_A &= 4.3 \times 10^5 (I/a)(1/nM)^{1/2} && \text{Alfven speed} \\
 v_B &= 4.3 \times 10^5 (E/M)^{1/2} && \text{beam speed} \\
 n_B &= 0.092 (fI/a^2)(1/EM)^{1/2} && \text{beam density}
 \end{aligned}$$

Here  $M$  is the beam ion mass in ratio to hydrogen. Coefficients are chosen to yield the units of Section 2. That coefficients for  $v_A$  and  $v_B$  are the same in these units is correct.

The density  $n_B$  represents the energetic tail of the ion distribution created by neutral beams, accumulating to carry a fraction  $f$  of the total current  $I$ . The formula above is  $n_B = [(fj/ev_B)/(1 - 1/Z)]$  with  $j = I/\pi a^2$  and  $Z = 2$ . To represent the energetic tail, we require also that  $n_B \leq n(\tau_S/\tau_p)$  or  $n$ , whichever is smaller, where  $\tau_p$  is the lifetime of thermal ions and  $\tau_S$  is the slowing down time of fast ions, given by  $\tau_S = 0.01(T^{3/2}/n)$  in our units. Particle lifetimes are not well known. Using instead the energy confinement time given by Eq. (9), we find that usually  $\tau_S/\tau_p > 1$  and hence adopt as our rule of thumb:

$$n_B < n \quad \rightarrow \quad I_{MAX} = 10.9 (na^2)(E/M)^{1/2} \quad (11)$$

An analysis of beam-driven instability requires more work. Here, we apply experience with NBI in NSTX to provide practical rules by which instability might be avoided, namely:

$$v_A > v_B \quad \rightarrow \quad I_{MIN} = a(nE)^{1/2} \quad (12)$$

Eq. (12) is the condition to avoid super-Alfvenic wave excitation. Even if Eq. (12) were violated, instability might be suppressed or weak if the beam free energy were sufficiently small, say  $(n_B E / 2nT) < 20\%$  [14] giving  $n_B/n < 5\% M^{-1}$  for the optimum  $E/T = 40 M$  for deuterium [8]. We find this free energy limit to be difficult to satisfy and choose instead to insist on satisfying Eq. (12), together with Eq. (11).

We note that, with neutral beams to stabilize and sustain the current, the Hagenson-Krakowski spheromak reactor design discussed in Ref. [3] only marginally satisfies Eq. (12), giving  $v_B/v_A = 1.5$ . The free energy factor is also marginal, giving  $(n_B E / 2nT) = 1$  at a nominal  $E = 1000 \text{ KeV}$  and a temperature  $T = 20 \text{ KeV}$  at the design point. Improved stability margins would require some adjustment of parameters.

Eq. (11) determines the maximum current consistent with fast ions as the current carriers, while Eq. (12) determines the minimum current needed in a target plasma created by gun injection. An additional criterion in order to confine beam ions is:

$$(a/r_L) = 43 [I/(EM)^{1/2}] > 5 \quad (13)$$

where the demand for 5 orbits across is somewhat arbitrary. Substituting the minimum allowed  $I$  from Eq.(12) gives:



$$na^2 > 0.014 M \quad (14)$$

Eq. (14), based on the minimum current in Eq. (12), is nearly always satisfied. Thus the operative criteria both to confine ions and avoid destabilizing super-Alfvenic ions are just those given by Eqs. (11) and (12).

Eq. (11) places constraints on  $na^2$  to obtain buildup of the current. We define the allowed current gain as  $G = (I_{MAX} / I_{MIN})$ , giving:

$$G = 10.9 (na^2/M)^{1/2} \quad (15)$$

$$na^2 \geq (G/10.9)^2 M \quad (16)$$

Eqs. (15) and (16) are independent of the beam energy  $E$ . Introducing Eq. (16) into Eq. (5) and solving for  $G$  gives the maximum gain  $G$  for a given power  $P$ , consistent with Eqs. (11) and (12):

$$G = 1.6 M^{-0.5} a^{0.3} P^{0.4} \quad (17)$$

Given  $G$ , one can calculate the corresponding minimum density  $n$  from Eq. (16). Given  $n$  and the beam energy  $E$ , one can calculate the current limits from Eqs. (11) and (12).

## 5. Experimental Tests

The SSPX has achieved good confinement in the core corresponding to  $\chi \approx 1$  m<sup>2</sup>/s [15] without profile control. Building on these results, in Table 1 we give parameters for a Proof of Principle (POP) experiment with profile control satisfying Eqs. (5) - (17), and also an ignition facility. All results listed are highly model dependent. Better numbers could be obtained using the Corsica code, as mentioned in Sections 2 and 3. Also, we have not yet reconciled the line density  $na$  with requirements for beam penetration (see Appendix).

The density listed is that obtained from Eq. (16), applicable during buildup. For the ignition experiment with DT, two values for the temperature are listed. The lower value of T is that given by Eq. (6) for beam heating and L-mode scaling. The higher value applies to the sustainment phase for which we assume improved H-mode confinement, yielding a correspondingly higher I requiring some increase in density in the approach to steady state. For the POP and ignition facility,  $n\tau_E$  is that for H-mode confinement, from Eq. (9), using the higher values of T and I in the table.

Table 1. Experimental Parameters

	<u>a</u>	<u>P</u>	<u>G</u>	<u>E</u>	<u>n</u>	<u>I<sub>MIN</sub></u>	<u>I<sub>MAX</sub></u>	<u>T</u>	<u>t</u>	<u>n<math>\tau_E</math></u>
POP	0.25	20	2.5	40	1.6	2	5	2.4	0.8	0.1
Ignition:										
D	0.75	40	6	80	0.6	5	33	9	37*	0.8
DT	0.75	60	5	80	0.8	5	30→50	8→10	30*	3.0

\*See text, Section 6

The SSPX employs a flux conserver for confinement, while the POP with a buildup time  $t = 0.8$  sec requires poloidal coils with duration of several seconds to maintain equilibrium. Since the physical size of the POP is the same as that for SSPX, first studies of equilibrium could be done by adding coils to SSPX. These studies could also begin to address the stabilization of tilt/shift modes now stabilized by the flux conserver in SSPX. Maintaining tilt/shift stability on POP timescales will require active feedback -- no small undertaking.

## 6. Spheromak "CIT"

The ignition facility listed in Table 1 could for spheromaks serve the role previously intended for the tokamak CIT. Because of its small size, one might hope to avoid an intermediate step.

As with TFTR and JET, we imagine this facility to be operated first with hydrogen or deuterium plasmas to explore confinement scaling in spheromaks, with full power and DT operation to follow only if warranted. With reuse of equipment in mind, beam power for the deuterium plasma phase is taken as  $P = 40$  MW, like TFTR.

Though possible in principle, buildup by neutral beams would take a long time,  $t = 30$  s as shown in the table. Alternatively it might be possible to inject most of the current by gun injection at low temperature using inexpensive homopolar energy storage [16], followed by neutral beam injection to heat the plasma rapidly and to sustain and stabilize the current.

Finally, we note that the parameters listed for DT operation in Table 1, giving  $n\tau_E = 3$  at  $T = 10$  KeV, yield an alpha power equal to thermal loss, a fully ignited condition, but not quite a steady state due to the low density required to maintain profile control with only  $P = 60$  MW. Scaling to achieve a self-consistent steady state burn with profile control is discussed in the next section.

## 7. Reactor Scaling

A dominating feature of reactor scaling is the neutron wall load  $P_{WALL}$  given by:

$$P_{FUS} \propto P_{WALL} a^2 \kappa A \propto n^2 (\sigma v)_{DT} E_{NEUTRON} (a^3 \kappa A) \quad (18)$$

with elongation  $\kappa$  and aspect ratio  $A$ . For a fixed temperature, this gives:

$$n \propto (P_{WALL}/a)^{0.5} \quad (19)$$

where in the second step we use the ignition condition  $a \propto B^m$ , like Eq.(10). With current drive, an additional important parameter is the electrical gain  $Q_E$  given by:

$$Q_E \equiv \eta_{TH}(P_{FUS}/P) \propto [(P_{WALL} a)^{0.5} / fB] \quad (20)$$

$$P \propto f I n a A \propto f B n a^2 \kappa A \quad (21)$$

where  $\eta_{TH}$  is the efficiency of conversion to electricity. In Eq. (21), we use Eq. (10) for the power required to maintain profile control in steady state,  $f$  being the fraction of current that must be driven, and we use  $B = (I/5a\kappa)$ . On the far right side of Eq. (20), we first substitute Eq. (18) for  $P_{FUS}$  in terms of  $P_{WALL}$  and then  $P$  from Eq. (21) with  $n$  from Eq. (19). Finally, we estimate reactor cost as:

$$\begin{aligned} \$/Kwe &\propto [1 - (1/Q_E)]^{-1} \{K_1(a^3\kappa A/P_{FUS}) + K_2(P/P_{FUS})\} \\ &\propto [1 - (1/Q_E)]^{-1} \{k_1(a/P_{WALL}) + k_2(1/Q_E)\} \end{aligned} \quad (22)$$

Here the first term proportional to total confinement facility volume (roughly 50-100 times the plasma volume) represents coils, structures and the blanket. The second term represents the cost of current drive.

While the compactness of spheromaks ( $\kappa, A \approx 1$ ) cancels in Eq.(22), other advantages survive in a simpler divertor; the potential for a higher magnetic field, yielding a much smaller confinement system; and perhaps a lower fusion power (which is  $\propto \kappa A$ ) at a given  $\$/Kwe$ , allowing for modular units and less costly development. Higher field is possible because: (1) only simple poloidal coils are required, and (2) the poloidal coils are located outside where stresses are lower for a given  $B$  at the magnetic axis. The small size, simple geometry and the possibility an external divertor (as in mirrors) may allow full or partial use of liquid walls to increase the wall load, which both reduces facility cost and increases  $Q_E$ . The Hagenson-Krakowski reactor design, with  $a = 1.5$ , yields  $P_{WALL} = 20$ , which is twice the likely maximum with conventional heat flow into thin metal coolant tubes and 4 - 5 times that in ARIES design studies. Flowing liquid walls could sustain much higher wall loads [17].

Given success with profile control, the main disadvantage of spheromak reactors based on auxiliary current drive is a lower  $Q_E$  due to small bootstrap current, and possibly higher current required to achieve ignition with aspect ratio  $A = 1$  [11]. However, this can be traded against a lower confinement facility cost [1,2]. Eq. (10) gives  $P = 265$  MW to sustain a current of 47 MA ( $n = 2.26, a = 1.5, T = 20$ ). With a fusion power 2775 MW, this gives, for  $\eta_{TH} = 40\%$ ,  $Q_E = 4$  corresponding to 33% recirculation of electric power

(the same as ARIES 1 and ARIES-ST); and higher  $Q_E$  if alpha-channeling drives current [18].

## 8. Conclusions

A concerted numerical effort using the NIMROD resistive MHD code, as proposed in Refs. [1] - [3], could determine whether profile control can be successful in the  $q < 1$  regime of spheromaks and compact RFP's, and, in doing so, open the door to advanced toroidal reactors very different from those based on tokamaks or stellarators.

In this note we have attempted to show for spheromaks that a successful prognosis from computer simulations could be followed by comparatively rapid experimental demonstrations, rapid because the required devices are small in size (like SSPX for the POP) and they would only require NBI power levels already available in tokamak laboratories  $< 40$  MW. That such low power can drive the full current required for good confinement in spheromaks follows from the small size giving  $\langle n_a \rangle < 1$ , but only if beam instability is avoided as discussed in Section 4.

### Appendix. Cordey Formula, Comparison with Experiment and Theory

Results in Table 1 are intended only as an indication that future work using Corsica and NIMROD is warranted, to determine useful paths for spheromak development.

The Cordey formula yielding the power for buildup in Eq. (5) and to sustain the current in Eq. (10) is idealized in that we do not account for beam orientation ( $\cos \theta \approx 1$ ) and we have not constrained  $nR = n_a$  (in spheromaks) to prevent "shine through" of the beam. We also assume stabilization of fast ion modes, by Eqs. (11) and (12), and MHD modes by profile control, to be explored on NIMROD.

Beam orientation and other effects apparently contribute to significant differences from Eq. (10) in DIII-D experiments modeled by Corsica and other codes [12, 13]. In an invited paper at APS in 2005, Politzer reported a case in which 6.5 MW of beam power

was required to drive 0.14 MA of current at  $T = 2.1$  KeV [19]. With  $n = 0.5$  and  $R = 1.5$ , Eq. (10) gives  $P = 1.7$ , or 4 times less power. For experiments reported in Ref. [12] (cited also in Ref. [13]), at the time when non-inductive current drive and bootstrap provided all of the current,  $P = 12.5$  MW was required to drive  $I = 0.34$  MA at  $T = 4.8$  KeV, in part due to fluctuations transporting fast ions [12], while Eq. (10), with  $n = 0.4$ , gives  $P = 1.4$  or 9 times less. As noted in Section 3, Corsica calculations for SSPX give results within a factor  $\approx 2$  of that predicted by Eq. (10), despite compromises in injection angle to optimize against ion confinement and shine through ( $35^\circ$  off perpendicular to the geometric axis) [11]. Closer agreement is obtained with code calculations in Ref. [20] for tangential injection into an SSPX-like device ( $n = 1$ ,  $a = 0.25$ ), giving  $I/P = 0.06$  by Eq. (10) compared to 0.05 for the optimum in Fig. 4 of Ref. [20] and variation off-optimum within 20 % of the scaling  $TJ(x,y)$  per the discussion below (case  $E = 30$  KeV, Fig. 9). Ref. [20] also deals with shine through. In general, shine through places restrictions on  $nR\sigma > 1$  where  $\sigma$  is the effective cross-section for ionization of the neutral beam, determined by the injection energy  $E$  and electron temperature  $T$  [7]. Ref. [20] also discusses increases in the shielding factor  $F$  due to electron trapping, which we ignore for spheromaks, taking  $F = 1 - 1/Z$  as in Ohkawa's original paper.

The Cordey formula is evaluated at an optimum  $E = 80T$  (for deuterium). Scaling around the optimum can be understood from Fig. 2 of Ref. [8], and analytically by comparison with the analytical formula of Mikkelsen-Singer [7] that agrees with the Cordey formula within a factor 1.5, as noted in Section 2. The circulating current  $I_{\text{CIRC}} = A_0 j_{\text{CIRC}}$  is obtained from a beam current  $I_B$  with cross-sectional area  $A_0$ , where, by Eq. (11) of Ref. [7]:

$$j_{\text{CIRC}} = eS\tau_{\text{se}} v_0 I(x,y) = eS\tau_{\text{se}} (v_0^2/v_C)J(x,y) \quad (\text{A1})$$

$$eS = (I_B/2\pi R A_0) = (P/2\pi 10^3 E R A_0) \quad (\text{A2})$$

$$\tau_{\text{se}} = A_B(0.37/\sqrt{10})(T^{3/2}/n) = 0.012(T^{3/2}/n) \quad (\text{A3})$$

$$v_0^2 = 1.6 \times 10^{-16} (2E/A_B m_p) \quad (\text{A4})$$

$$v_C = (1.6 \times 10^{-16} 2E_C/A_B m_p)^{1/2} = 1.68 \times 10^6 (Z/A_B)^{1/3} \sqrt{T} \quad (\text{A5})$$

Here  $R = a$  is the major radius for the spheromak;  $A_B$  is the beam ion atomic number;  $E$  is the beam energy in KeV and  $v_0$  the velocity;  $T$  is the electron temperature in KeV; and  $E_C = 15T(Z/A_B)^{2/3}$  is the cutoff energy in KeV from Ref. [7] and Ref. [6], Eq. (11).

Multiplying Eq. (A1) by  $A_0$  to obtain the total circulating ion current  $I_{CIRC}$  and using Eqs. (A2) - (A5), we obtain after some algebra:

$$I_{CIRC} = [0.21J(x,y)] (A_B/Z)^{1/3} (PT/nR) \quad (\text{A6})$$

where for  $y = 1$  (deuterium,  $Z = 2$ )  $J(x,y) = x^2 / \{7 + x^3 + 2x^2\}$  as in Section 2. At the optimum  $J = 0.18$  [7],  $0.21J = 0.04$ , to be compared with 0.06 in Cordey's formula; hence a factor 1.5 greater power in Eq. (10). Results are not much different for hydrogen beams, giving  $(A_B/Z)^{1/3} = 0.8$  for  $Z = 2$ . For that case, the optimum  $J = 0.16$  -- almost the same (case  $y = 1.6$ , Table 1, Ref. [7]). For both hydrogen and deuterium, Fig. 2 of Ref. [7] shows little change in  $J$  for  $E/T$  higher than the optimum value (for deuterium,  $E/E_C = 5.3$  or  $E/T = 80$ ), but a 50% decrease at  $E/E_C = 1$  at the margin of validity.

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