Fuzzy Almost Strongly \((r,s)\)-Semicontinuous Mappings

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Abstract

In this paper, we introduce the concept of fuzzy almost strongly \((r,s)\)-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak’s sense. The relationships among fuzzy strongly \((r,s)\)-semicontinuous, fuzzy almost \((r,s)\)-semicontinuous, and fuzzy almost strongly \((r,s)\)-semicontinuous mappings are discussed. The characterization for the fuzzy almost strongly \((r,s)\)-semicontinuous mappings is obtained.

Key Words: fuzzy continuous, fuzzy topology, fuzzy almost strongly \((r,s)\)-semicontinuous

1. Introduction

The concept of fuzzy set was introduced by Zadeh [1]. Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [3], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [4], and by Ramadan [5].


In this paper, we introduce the concept of fuzzy almost strongly \((r,s)\)-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak’s sense. The relationships among fuzzy strongly \((r,s)\)-semicontinuous, fuzzy almost \((r,s)\)-continuous, fuzzy almost \((r,s)\)-semicontinuous, and fuzzy almost strongly \((r,s)\)-semicontinuous mappings are discussed. The characterization for the fuzzy almost strongly \((r,s)\)-semicontinuous mappings is obtained.

2. Preliminaries

For the nonstandard definitions and notations we refer to [10, 11, 12, 13]. Let \(I(X)\) be a family of all intuitionistic fuzzy sets in \(X\) and let \(I \otimes I\) be the set of the pair \((r,s)\) such that \(r, s \in I\) and \(r + s \leq 1\).

Definition 2.1. ([8]) Let \(X\) be a nonempty set. An intuitionistic fuzzy topology in Šostak’s sense (SoIFT for short) \(\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)\) on \(X\) is a mapping \(\mathcal{T} : I(X) \to I \otimes I\) which satisfies the following properties:

1. \(\mathcal{T}_1(\emptyset) = \mathcal{T}_1(1) = 1\) and \(\mathcal{T}_2(\emptyset) = \mathcal{T}_2(1) = 0\).
2. \(\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \cap \mathcal{T}_1(B)\) and \(\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)\).
3. \(\mathcal{T}_1(\bigcup A_i) \geq A \mathcal{T}_1(A_i)\) and \(\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)\).

The \((X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)\) is said to be an intuitionistic fuzzy topological space in Šostak’s sense (SoIFTS for short). Also, we call \(\mathcal{T}_1(A)\) a gradation of openness of \(A\) and \(\mathcal{T}_2(A)\) a gradation of nonopenness of \(A\).

Definition 2.2. ([10, 11, 13]) Let \(A\) be an intuitionistic fuzzy set in a SoIFTS \((X, \mathcal{T}_1, \mathcal{T}_2)\) and \((r,s) \in I \otimes I\). Then \(A\) is said to be

1. fuzzy \((r,s)\)-semipenopen if \(\text{cl}(\text{int}(A, r, s), r, s) \supseteq A\),
2. fuzzy \((r,s)\)-semiclosed if \(\text{int}(\text{cl}(A, r, s), r, s) \subseteq A\),
3. fuzzy \((r,s)\)-regular open if \(\text{int}(\text{cl}(A, r, s), r, s) = A\),
4. fuzzy \((r,s)\)-regular closed if \(\text{cl}(\text{int}(A, r, s), r, s) = A\),
5. fuzzy strongly \((r,s)\)-semipenopen if \(A \subseteq \text{int}(\text{cl}(A, r, s), r, s), r, s)\),

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(6) fuzzy strongly \((r,s)\)-semiclosed if 
\[ A \supseteq \text{cl}(\text{int}(A, r,s), r,s), r,s). \]

**Definition 2.3.** ([13]) Let \((X, T_1, T_2)\) be a SoIFTS. For each \((r,s) \in I \otimes I\) and for each \(A \in I(X)\), the fuzzy strongly \((r,s)\)-semiinterior is defined by
\[ \text{ssint}(A, r,s) = \bigcup \{B \in I(X) \mid B \subseteq A, \]
\[ B \text{ is fuzzy strongly } (r,s)\text{-semipositive}\} \]
and the fuzzy strongly \((r,s)\)-semiclosure is defined by
\[ \text{sscl}(A, r,s) = \bigcap \{B \in I(X) \mid A \subseteq B, \]
\[ B \text{ is fuzzy strongly } (r,s)\text{-semiclosed}\} \]

**Theorem 2.4.** ([11]) (1) The fuzzy \((r,s)\)-closure of a fuzzy \((r,s)\)-open set is fuzzy \((r,s)\)-regular closed for each \((r,s) \in I \otimes I\).
(2) The fuzzy \((r,s)\)-interior of a fuzzy \((r,s)\)-closed set is fuzzy \((r,s)\)-regular open for each \((r,s) \in I \otimes I\).

**Definition 2.5.** ([11, 12, 14]) Let \(f : (X, T_1, T_2) \to (Y, U_1, U_2)\) be a mapping from a SoIFTS \(X\) to a SoIFTS \(Y\) and \((r,s) \in I \otimes I\). Then \(f\) is called
(1) a fuzzy strongly \((r,s)\)-semicontinuous mapping if \(f^{-1}(B)\) is a fuzzy strongly \((r,s)\)-semipositive set in \(X\) for each fuzzy \((r,s)\)-open set \(B\) in \(Y\),
(2) a fuzzy almost \((r,s)\)-continuous mapping if \(f^{-1}(B)\) is a fuzzy \((r,s)\)-open set in \(X\) for each fuzzy \((r,s)\)-regular open set \(B\) in \(Y\),
(3) a fuzzy almost \((r,s)\)-semicontinuous mapping if \(f^{-1}(B)\) is a fuzzy \((r,s)\)-semipositive set in \(X\) for each fuzzy \((r,s)\)-regular open set \(B\) in \(Y\).

**Definition 2.6.** ([11]) Let \(x_{(\alpha, \beta)}\) be an intuitionistic fuzzy point in a SoIFTS \((X, T_1, T_2)\) and \((r,s) \in I \otimes I\). Then an intuitionistic fuzzy set \(A\) in \(X\) is called a fuzzy \((r,s)\)-neighborhood of \(x_{(\alpha, \beta)}\) if there is a fuzzy \((r,s)\)-open set \(B\) in \(X\) such that \(x_{(\alpha, \beta)} \in B \subseteq A\).

**Definition 2.7.** ([9]) Let \(f : (X_1, \delta_1) \to (X_2, \delta_2)\) be a mapping from a fuzzy space \(X_1\) to another fuzzy space \(X_2\). Then \(f\) is called a fuzzy almost strongly semicontinuous mapping if \(f^{-1}(B)\) is a fuzzy strongly semipositive set of \(X_1\) for each fuzzy regular open set \(B\) of \(X_2\).

### 3. Fuzzy almost strongly \((r,s)\)-semicontinuous mappings

Now, we define the notion of fuzzy almost strongly \((r,s)\)-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak’s sense, and then we investigate some of their properties.

**Definition 3.1.** Let \(f : (X, T_1, T_2) \to (Y, U_1, U_2)\) be a mapping from a SoIFTS \(X\) to a SoIFTS \(Y\) and \((r,s) \in I \otimes I\). Then \(f\) is called a fuzzy almost strongly \((r,s)\)-semicontinuous mapping if \(f^{-1}(B)\) is a fuzzy strongly \((r,s)\)-semipositive set in \(X\) for each fuzzy \((r,s)\)-regular open set \(B\) in \(Y\).

**Definition 3.2.** Let \(f : (X, T_1, T_2) \to (Y, U_1, U_2)\) be a mapping from a SoIFTS \(X\) to a SoIFTS \(Y\) and \((r,s) \in I \otimes I\). Then \(f\) is said to be fuzzy almost strongly \((r,s)\)-semicontinuous at an intuitionistic fuzzy point \(x_{(\alpha, \beta)}\) in \(X\) if for each fuzzy \((r,s)\)-regular open set \(B\) in \(Y\) with \(f(x_{(\alpha, \beta)}) \in B\), there is a fuzzy strongly \((r,s)\)-semipositive set \(A\) in \(X\) such that \(x_{(\alpha, \beta)} \in A\) and \(f(A) \subseteq B\).

**Theorem 3.3.** Let \(f : (X, T_1, T_2) \to (Y, U_1, U_2)\) be a mapping from a SoIFTS \(X\) to a SoIFTS \(Y\) and \((r,s) \in I \otimes I\). Then \(f\) is fuzzy almost strongly \((r,s)\)-semicontinuous if and only if \(f\) is fuzzy almost strongly \((r,s)\)-semicontinuous at each intuitionistic fuzzy point \(x_{(\alpha, \beta)}\) in \(X\).

**Proof.** Let \(f\) be fuzzy almost strongly \((r,s)\)-semicontinuous, \(x_{(\alpha, \beta)}\) an intuitionistic fuzzy point in \(X\), and \(B\) a fuzzy \((r,s)\)-regular open set in \(Y\) with \(f(x_{(\alpha, \beta)}) \in B\). Since \(f\) is fuzzy almost strongly \((r,s)\)-semicontinuous, \(f^{-1}(B)\) is a fuzzy strongly \((r,s)\)-semipositive set in \(X\). Putting \(A = f^{-1}(B)\). Then \(A\) is fuzzy strongly \((r,s)\)-semipositive in \(X\), \(x_{(\alpha, \beta)} \in A\), and \(f(A) = f(f^{-1}(B)) \subseteq B\). Since \(x_{(\alpha, \beta)}\) is an arbitrary intuitionistic fuzzy point in \(X\), we conclude that \(f\) is fuzzy almost strongly \((r,s)\)-semicontinuous at each intuitionistic fuzzy point \(x_{(\alpha, \beta)}\) in \(X\).

Conversely, let \(B\) be a fuzzy \((r,s)\)-regular open set in \(Y\) and \(x_{(\alpha, \beta)} \in f^{-1}(B)\). Then \(f(x_{(\alpha, \beta)}) \in B\). From the assumption, there is a fuzzy strongly \((r,s)\)-semipositive set \(A_{x_{(\alpha, \beta)}}\) in \(X\) such that \(x_{(\alpha, \beta)} \in A_{x_{(\alpha, \beta)}}\) and \(f(A_{x_{(\alpha, \beta)}}) \subseteq B\). Thus
\[ f^{-1}(B) = \bigcup \{x_{(\alpha, \beta)} \mid x_{(\alpha, \beta)} \in f^{-1}(B)\} \]
\[ \subseteq \bigcup \{A_{x_{(\alpha, \beta)}} \mid x_{(\alpha, \beta)} \in f^{-1}(B)\} \]
\[ \subseteq f^{-1}(B). \]

Hence \(f^{-1}(B) = \bigcup \{A_{x_{(\alpha, \beta)}} \mid x_{(\alpha, \beta)} \in f^{-1}(B)\}\), which is a fuzzy strongly \((r,s)\)-semipositive set in \(X\). Therefore \(f\) is fuzzy almost strongly \((r,s)\)-semicontinuous. \(\square\)
Remark 3.4. It is clear that the following implications are true:

(1) fuzzy strongly $(r, s)$-semicontinuous $\Rightarrow$
    fuzzy almost strongly $(r, s)$-semicontinuous.

(2) fuzzy almost $(r, s)$-continuous $\Rightarrow$
    fuzzy almost strongly $(r, s)$-semicontinuous.

(3) fuzzy almost strongly $(r, s)$-semicontinuous $\Rightarrow$
    fuzzy almost $(r, s)$-semicontinuous.

However, the following examples show that all of the
converses need not be true.

Example 3.5. Let $X = \{x, y, z\}$ and let $A_1$, $A_2$, $A_3$, and
$A_4$ be intuitionistic fuzzy sets in $X$ defined as

$A_1(x) = (0.1, 0.8)$, $A_1(y) = (0, 1)$, $A_1(z) = (0.2, 0.6)$;

$A_2(x) = (0.5, 0.5)$, $A_2(y) = (0.5, 0.5)$, $A_2(z) = (0.5, 0.5)$;

$A_3(x) = (0.2, 0.7)$, $A_3(y) = (0.3, 0.6)$, $A_3(z) = (0.4, 0.5)$;

and

$A_4(x) = (0.6, 0.3)$, $A_4(y) = (0.5, 0.5)$, $A_4(z) = (0.7, 0.1)$.

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

\[
\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases}
(1, 0) & \text{if } A = 0, 1, \\
\left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } A = A_1, A_2, \\
(0, 1) & \text{otherwise};
\end{cases}
\]

and

\[
\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases}
(1, 0) & \text{if } A = 0, 1, \\
\left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } A = A_3, A_4, \\
(0, 1) & \text{otherwise}.
\end{cases}
\]

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIIFTs on $X$. Consider a mapping $f : (X, T) \to (X, U)$ defined by $f(x) = x, f(y) = y,$
and $f(z) = z$. Note that

\[
\int(\text{cl}(A_3), \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_3,
\]

\[
\int(\text{cl}(A_4), \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = 1 \neq A_2 \text{ in } (X, U).
\]

Thus $A_3$ is fuzzy $(\frac{1}{2}, \frac{1}{3})$-regular open but $A_4$ is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-regular open set in $(X, U)$. Since

\[
f^{-1}(A_3) = A_3 \subseteq \int(\text{cl}(A_3), \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_2,
\]

we conclude that $f$ is fuzzy almost strongly $(\frac{1}{2}, \frac{1}{3})$-semicontinuous. However, $f$ is neither fuzzy almost
$(\frac{1}{2}, \frac{1}{3})$-continuous nor fuzzy strongly $(\frac{1}{2}, \frac{1}{3})$-semicontinuous. For $f^{-1}(A_3) = A_3$ is not fuzzy
$(\frac{1}{2}, \frac{1}{3})$-open in $(X, T)$ and

\[
f^{-1}(A_4) = A_4 \not\subseteq \int(\text{cl}(A_4), \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_2.
\]

Example 3.6. Let $X = \{x, y, z\}$ and let $A_1$ and $A_2$ be
intuitionistic fuzzy sets in $X$ defined as

$A_1(x) = (0, 0.5)$, $A_1(y) = (0.3, 0.5)$, $A_1(z) = (0.3, 0.5)$;

and

$A_2(x) = (0.7, 0.2)$, $A_2(y) = (0.2, 0.7)$, $A_2(z) = (0.2, 0.8)$.

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

\[
\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases}
(1, 0) & \text{if } A = 0, 1, \\
\left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } A = A_2, \\
(0, 1) & \text{otherwise};
\end{cases}
\]

and

\[
\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases}
(1, 0) & \text{if } A = 0, 1, \\
\left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } A = A_1, \\
(0, 1) & \text{otherwise}.
\end{cases}
\]

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIIFTs on $X$. Consider a mapping $f : (X, T) \to (X, U)$ defined by $f(x) = x, f(y) = y,$
and $f(z) = z$. Note that

\[
\int(\text{cl}(A_1), \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_1 \text{ in } (X, U),
\]

and hence $A_1$ is fuzzy $(\frac{1}{2}, \frac{1}{3})$-regular open in $(X, U)$. Since

\[
f^{-1}(A_1) = A_1 \not\subseteq \int(\text{cl}(int(A_1), \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_2.
\]

Theorem 3.7. Let $f : (X, T_1, T_2) \to (Y, U_1, U_2)$ be a mapping from a SoIIFTS $X$ to a SoIIFTS $Y$ and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) $f$ is fuzzy almost strongly $(r, s)$-semicontinuous.
\begin{enumerate}
\item $f^{-1}(B)$ is fuzzy strongly $(r,s)$-semiclosed in $X$ for each fuzzy $(r,s)$-regular closed set $B$ in $Y$.
\item For each fuzzy $(r,s)$-closed set $B$ in $Y$,
$$\text{sscl}(f^{-1}(\text{cl}(\text{int}(B, r, s), r, s)), r, s) \subseteq f^{-1}(B).$$
\item For each fuzzy $(r,s)$-open set $B$ in $Y$,
$$f^{-1}(B) \subseteq \text{ssint}(f^{-1}(\text{cl}(\text{int}(B, r, s), r, s)), r, s).$$
\item For each fuzzy $(r,s)$-semiopen set $B$ in $Y$,
$$\text{sscl}(f^{-1}(B), r, s) \subseteq f^{-1}(\text{cl}(B, r, s)).$$
\item For each fuzzy $(r,s)$-semiclosed set $B$ in $Y$,
$$f^{-1}(\text{int}(B, r, s)) \subseteq \text{ssint}(f^{-1}(B), r, s).$$
\end{enumerate}

\textbf{Proof.} It is clear that (1) $\Leftrightarrow$ (2), (3) $\Leftrightarrow$ (4), (5) $\Leftrightarrow$ (6).

(2) $\Rightarrow$ (3) Let $B$ be a fuzzy $(r,s)$-closed set in $Y$. By Theorem 2.4, $\text{cl}(\text{int}(B, r, s), r, s)$ is fuzzy $(r,s)$-regular closed in $Y$. By (2), $f^{-1}(\text{cl}(\text{int}(B, r, s), r, s))$ is a fuzzy strongly $(r,s)$-semiclosed set in $X$. Hence
$$\text{sscl}(f^{-1}(\text{cl}(\text{int}(B, r, s), r, s)), r, s) \subseteq \text{ssint}(f^{-1}(\text{cl}(B, r, s)), r, s) \subseteq f^{-1}(\text{cl}(B, r, s)) = f^{-1}(B).$$

(3) $\Rightarrow$ (5) Let $B$ be a fuzzy $(r,s)$-semiclosed set in $Y$. Then $\text{cl}(B, r, s)$ is fuzzy $(r,s)$-closed in $Y$. By (3), we have
$$\text{sscl}(f^{-1}(B), r, s) \subseteq \text{sscl}(f^{-1}(\text{cl}(B, r, s)), r, s) \subseteq f^{-1}(\text{cl}(B, r, s)).$$

(5) $\Rightarrow$ (2) Let $B$ be a fuzzy $(r,s)$-regular closed set in $Y$. Then $B$ is fuzzy $(r,s)$-closed and fuzzy $(r,s)$-semiopen in $Y$. By (5), we obtain
$$f^{-1}(B) \subseteq \text{sscl}(f^{-1}(B), r, s) \subseteq \text{ssint}(f^{-1}\text{cl}(B, r, s)) = f^{-1}(B).$$

Thus we have $f^{-1}(B) = \text{sscl}(f^{-1}(B), r, s)$, which is a fuzzy strongly $(r,s)$-semiclosed set in $X$.

\textbf{Definition 3.8.} Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in a SoIFTS $(X, T_1, T_2)$ and $(r,s) \in I \otimes I$. Then an intuitionistic fuzzy set $A$ in $X$ is called a fuzzy strongly $(r,s)$-semineighborhood of $x_{(\alpha,\beta)}$ if there is a fuzzy strongly $(r,s)$-semiopen set $B$ in $X$ such that $x_{(\alpha,\beta)} \in B \subseteq A$.

\textbf{Theorem 3.9.} Let $f : (X, T_1, T_2) \rightarrow (Y, U_1, U_2)$ be a mapping from a SoIFTS $X$ to a SoIFTS $Y$ and $(r,s) \in I \otimes I$. Then $f$ is fuzzy almost strongly $(r,s)$-semicontinuous if and only if for each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in $X$ and each fuzzy $(r,s)$-neighborhood $B$ of $f(x_{(\alpha,\beta)})$ in $Y$, there is a fuzzy strongly $(r,s)$-semineighborhood $A$ of $x_{(\alpha,\beta)}$ such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq \text{int}(\text{cl}(B, r, s), r, s)$.

\textbf{Proof.} Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in $X$ and $B$ a fuzzy $(r,s)$-neighborhood of $f(x_{(\alpha,\beta)})$. Then there is a fuzzy $(r,s)$-open set $C$ in $Y$ such that $f(x_{(\alpha,\beta)}) \subseteq C \subseteq B$. Hence $x_{(\alpha,\beta)} \in f^{-1}(C) \subseteq f^{-1}(B)$. Since $f$ is fuzzy almost strongly $(r,s)$-semicontinuous, by Theorem 3.7, we have
$$f^{-1}(C) \subseteq \text{ssint}(f^{-1}(\text{cl}(C, r, s), r, s)), r, s) \subseteq f^{-1}(\text{sscl}(f^{-1}(\text{cl}(B, r, s), r, s)), r, s).$$

Put $A = f^{-1}(\text{int}(\text{cl}(B, r, s), r, s))$. By Theorem 2.4, $\text{int}(\text{cl}(B, r, s), r, s)$ is fuzzy $(r,s)$-regular open in $Y$, and hence $A = f^{-1}(\text{int}(\text{cl}(B, r, s), r, s))$ is a fuzzy strongly $(r,s)$-semiopen set in $X$. Thus
$$x_{(\alpha,\beta)} \in f^{-1}(C) \subseteq \text{ssint}(f^{-1}(\text{cl}(B, r, s), r, s)), r, s) = \text{ssint}(A, r, s) = A.$$

Hence we conclude that $A$ is a fuzzy strongly $(r,s)$-semineighborhood of $x_{(\alpha,\beta)}$ and
$$f(A) \subseteq f(f^{-1}(\text{int}(\text{cl}(B, r, s), r, s))) \subseteq \text{int}(\text{cl}(B, r, s), r, s).$$

Conversely, let $B$ be a fuzzy $(r,s)$-regular open set in $Y$ and $x_{(\alpha,\beta)} \in f^{-1}(B)$. Then $B$ is fuzzy $(r,s)$-open in $Y$, and hence $B$ is a fuzzy $(r,s)$-neighborhood of $f(x_{(\alpha,\beta)})$. From the assumption, there is a fuzzy strongly $(r,s)$-semineighborhood $A_{x_{(\alpha,\beta)}}$ of $x_{(\alpha,\beta)}$ such that $x_{(\alpha,\beta)} \in A_{x_{(\alpha,\beta)}}$ and
$$f(A_{x_{(\alpha,\beta)}}) \subseteq \text{int}(\text{cl}(B, r, s), r, s) = B.$$

Because $A_{x_{(\alpha,\beta)}}$ is a fuzzy strongly $(r,s)$-semineighborhood of $x_{(\alpha,\beta)}$, there is a fuzzy strongly $(r,s)$-semiopen set $C_{x_{(\alpha,\beta)}}$ such that
$$x_{(\alpha,\beta)} \in C_{x_{(\alpha,\beta)}} \subseteq A_{x_{(\alpha,\beta)}} \subseteq f^{-1}(f(A_{x_{(\alpha,\beta)}})) \subseteq f^{-1}(B).$$

Hence $f^{-1}(B) = \{C_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\}$, which is a fuzzy strongly $(r,s)$-semiopen set in $X$. Therefore $f$ is fuzzy almost strongly $(r,s)$-semicontinuous.
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