Game Theory Using Genetic Algorithms

I.A.Ismail , N.A.El_Ramly , M.M.El_Kafrawy , and M.M.Nasef

Abstract—**In this paper we used genetic algorithms to find the solution of game theory. We proposed new method foe solving game theory and find the optimal strategy for player A or player B. We can benefit from the relationship between game theory and the linear programming to find the fitness function and tested this fitness function at different examples .**

Index Terms— **Game theory , genetic algorithm , and optimal strategy.**

I. INTRODUCTION

Game theory is the formal study of decision–making where several players must make choices that potentially affect the interest of the other players. In other words, game theory gives mathematical expressions to the strategies of opposing players and offers techniques for choosing the best possible strategy. In most parlor games, it is relatively easy to define winning and losing and, on this basis, to quantify the best strategy for each player. However, game theory is not merely a tool for the gambler so that the person can take advantage of the odds; nor it is merely a method for winning games like tic-tac-toe or matching pennies. Games theory can be genealised to handle politics. If two people are vying for the same political office, each has open to him various comparing strategies. If it is possible to determine the impact of alternate strategies on the voters, the theory of games can be used to find the best strategy. Thus, game theory can be used in certain situations of conflict to indicate how people should behave to achieve certain goals [1,3,5]. The internal consistency and mathematical foundations of game theory make it a prime tool for modeling and designing automated decision-making processes in interactive environments. As a mathematical tool for the decision-maker the strength of game theory is the methodology which provides structuring and analyzing problems of strategic choice. The process of formally modeling a situation as a game requires the decision-maker to enumerate explicitly the players and their strategic options, and to consider their preferences and reactions. A game is essentially a set of rules describing the formal structure of a competitive situation. These rules specify (1) the alternatives among which the "players" must choose at each stage of "play", (2) the information available to each player when making such a choice, and (3) the "payoff" to each player after any particular contest. A strategy for a player is a set of directions for playing the game from beginning to end, which is "complete" in the sense that it includes instructions on what to do in every situation that might possibly arise during play [6,7,8]. The game is finite if the number of strategies available to each player is finite; otherwise it is an infinite game. Any conflict or competition between two persons (or teams) is called a two-person game. Notice that no matter what outcome occurs, whatever is lost (or gained) by player1 is gained (or lost) by player2. Such games are called two-person zero-sum games, can be denoted the gains of player1 by positive entries and his losses by negative entries. In this game each player has two strategies, which yield the following 2x2 game matrix expressed in terms of the payoff to1. Generally, any m x n matrix $A=[a_{ij}]$ can be regarded as the game matrix for a two-person zero-sum game in which player 1 chooses any one of the m rows of A and simultaneously player 2 chooses any one of the n columns of A. The entry in the row and column chosen is the payoff [9,10,11].

Professor, Dean, College of Computers and informatics , Zagazig University, Egypt
Professor, Department of Mathematics, Faculty of Science, Menofia University , Egypt
Professor of O.R. Consultant ,Institute of National Planning, Cairo , Egypt
Demonstrator, Department of Mathematics, Faculty of Science, Menofia University, Egypt **e-mail : mnasef81@yahoo.com**

II. METHODOLOGY

GA mimics all the processes based on the concept of natural evolution to find the optimum solution to the given problem residing in the search space. The GA pool contains a number of individuals called chromosomes. Each chromosome encoded from the parameters holds the potential solution. According to the evolutionary theories, the chromosomes which only have a good fitness are likely to survive and to generate the offsprings and pass its strength to them by the genetic operator. The fitness of chromosome is the way that is linked to the predefined problem or objective function, [2,4].

GA cycle can be decomposed into five steps described as follows:-

1.) Randomly initialize the population in the pool with more population, the coverage in search space is good but traded off by the calculation time in each generation. In the simplest way, the real-value parameter is binary coded to give a bit string. The bit strings for several parameters are concatenated to form a single string or chromosome.

2.) Evaluate the chromosomes by objective function. After the evaluation, all the chromosomes are ranked for the fitness values in a descending or ascending order depending on the purpose of the objective function .

ISBN:978-988-98671-5-7

3.) Select the parents from the chromosomes with the biased chances. The higher-fitness chromosome is prone to survive.

4.) Generate the offspring using genetic operators consisting of crossover and mutation. Crossover is a recombination operator that swaps the parts of two parents . Two random decisions are made prior to this operation, whether to do it or not and where the crossover point is. Mutation gives a good chance to explore the uncovered search space. It mutates, or complements some genes in the chromosome of the offspring, so that the new parameter value takes place.

5.) Entirely replace the elder generation in the pool with the newer one and return to step 2. In some cases, the few best elders may be kept away from replacement. This is known as elitist strategy. The criteria for stopping the revaluation loops are met when a) the loop number is over some predefined point or b) the steady state lasts for predetermined times.

A. Objective function

Player A’s optimum mixed strategies satisfy

$$\underset{x_i}{\text{Max}}\{\min(\sum_{i=1}^{m} a_{i1}x_i, \sum_{i=1}^{m} a_{i2}x_i, ..., \sum_{i=1}^{m} a_{in}x_i)\}$$

Subject to the constraints $x_i \geq 0$, i=1,…,m and $\sum_{i=1}^{m} x_i = 1$

This problem can be put in the linear programming form as follows.

Let

$$V = \min(\sum_{i=1}^{m} a_{i1}x_i, \sum_{i=1}^{m} a_{i2}x_i, ..., \sum_{i=1}^{m} a_{in}x_i)$$

Then the problem becomes maximize Z=V

Subject to

$$\sum_{i=1}^{m} a_{ij}x_i \geq V \quad , j=1,2,\ldots,n$$

$$x_i \geq 0 , i=1,\ldots,m \text{ and } \sum_{i=1}^{m} x_i = 1$$

V represents the value of the game in this case.

If player B wants to adopt B1, then A’s strategy must be such that

$a_{11}x_1+a_{21}x_2+a_{31}x_3+\ldots+a_{m1}x_m \geq V$

similarly if player B uses B2, then to guarantee V, A must have

$a_{12}x_1+a_{22}x_2+a_{32}x_3+\ldots+a_{m2}x_m \geq V$

A similar condition holds for any strategy B may play.

Hence the linear programming problem for A is

Maximize V

Subject to

$$\begin{aligned}
&a_{11}x_1+a_{21}x_2+a_{31}x_3+\ldots+a_{m1}x_m \geq V\\
&a_{12}x_1+a_{22}x_2+a_{32}x_3+\ldots+a_{m2}x_m \geq V\\
&.\\
&. \qquad\qquad\qquad\qquad (1)\\
&.\\
&a_{1n}x_1+a_{2n}x_2+a_{3n}x_3+\ldots+a_{mn}x_m \geq V\\
&x_1+x_2+x_3+\ldots+x_m = 1\\
&\text{all } x_i \geq 0
\end{aligned}$$

The solution of this problem gives the equilibrium mixed strategy $(x_1,x_2,\ldots,x_m)$ for player A and the value of the game V.

From formula (4) assuming that V>0, the constraints of the linear program becomes

$$\begin{aligned}
&a_{11}\frac{x_1}{V}+a_{21}\frac{x_2}{V}+\ldots+a_{m1}\frac{x_m}{V} \geq 1\\
&a_{12}\frac{x_1}{V}+a_{22}\frac{x_2}{V}+\ldots+a_{m2}\frac{x_m}{V} \geq 1\\
&.\\
&. \qquad\qquad\qquad\qquad (2)\\
&.\\
&a_{1n}\frac{x_1}{V}+a_{2n}\frac{x_2}{V}+\ldots+a_{mn}\frac{x_m}{V} \geq 1\\
&\frac{x_1}{V}+\frac{x_2}{V}+\ldots+\frac{x_m}{V} = \frac{1}{V}\\
&\text{all } x_i \geq 0
\end{aligned}$$

Let $X_i = \frac{x_i}{V}$, i=1,2,…,m since

$$\text{Max } V = \min \frac{1}{V} = \min\{X_1 + X_2+\ldots+X_m\}$$

The problem becomes

Minimize $Z= X_1 + X_2+\ldots+X_m$

Subject to

$$\begin{aligned}
&a_{11}X_1+a_{21}X_2+\ldots+a_{m1}X_m \geq 1\\
&a_{12}X_1+a_{22}X_2+\ldots+a_{m2}X_m \geq 1\\
&.\\
&. \qquad\qquad\qquad\qquad (3)\\
&.\\
&a_{1n}X_1+a_{2n}X_2+\ldots+a_{mn}X_m \geq 1\\
&\text{all } X_i \geq 0 \text{ for i } =1,2,\ldots,m
\end{aligned}$$

where $Z=\frac{1}{V}$, $X_i=\frac{x_i}{V}$, i=1,2,…,m

Player B’s problem is given by

$$\underset{y_j}{\text{Min}}\{\max(\sum_{j=1}^{n} a_{1j}y_j, \sum_{j=1}^{n} a_{2j}y_j, ..., \sum_{j=1}^{n} a_{mj}y_j)\}$$

Subject to the constraints $y_j \geq 0$, j=1,…,n and $\sum_{j=1}^{n} y_j = 1$

This can also be expressed as a linear program as follows

Maximize $W= Y_1 + Y_2+\ldots+Y_n$

Subject to

$$\begin{aligned}
&a_{11}Y_1+a_{12}Y_2+\ldots+a_{1n}Y_n \leq 1\\
&a_{21}Y_1+a_{22}Y_2+\ldots+a_{2n}Y_n \leq 1\\
&.\\
&. \qquad\qquad\qquad\qquad (4)\\
&.\\
&a_{m1}Y_1+a_{m2}Y_2+\ldots+a_{mn}Y_n \leq 1\\
&\text{all } Y_i \geq 0 \text{ for i } =1,2,\ldots,n
\end{aligned}$$

where $W=\frac{1}{V}$, $Y_j=\frac{y_j}{V}$, j=1,2,…,n

Noticing that B’s problem is actually the dual of A’s problem, thus the optimal solution of one problem automatically yields the optimal solution to the other. Player B’s problem can be solved by the regular simplex

ISBN:978-988-98671-5-7

method, and player A's problem is solved by the dual simplex method, [7,9,10,11].

We can benefite from the relationship between the game theory and the linear programming to find the fitness function respective the genetic algorithm. The fitness function to find player A's strategies is the formula (3) to get the minimum value of the (Z) and the expected values of X_i , hence calculate the value of the game and the value of x_i . If we want to find player B's strategies using the formula (4) to get the maximize value of (W) and the expected values of Y_j , hence calculate the value of the game and the value of y_j, then The fitness function in this case is the formula (4).

B. Coding Scheme

The coding scheme for this problem can be accomplished using a relatively standard coding scheme. The parameters of the search problem are represented as bit strings. The equation is represented collectively as equation (3) and the unknowns are x_i , for i = 1,2,..,n and $0 \le x_i \le 1$, then each parameter is coded as binary strings according to the formula

$$P = P_{\min} + \frac{b}{2^i - 1}(P_{\max} - P_{\min})$$

where P is the parameter to be coded ,P_{min} is the minimum value of the parameter (in the case of x_i, the minimum value is 0). P_{max} is the maximum value of the parameter (in the case of x_i, the maximum value is 1). B is the decimal value associated with a binary string of length.

III. RESULTS

We tested the genetic algorithm (GA) with game theory by different examples.

A. *Example1*

Consider the following (3x3) game

Player B

$$\text{Player A}\quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 3 & -1 & -3 \\ -3 & 3 & -1 \\ -4 & -3 & 3 \end{bmatrix} \end{array}$$

The number of generation = 50
The number of population = 24
The length of the string of each parameter = 8
The crossover probability = 0.04
The mutation probability = 0.007

Table of results :-

GEN_NUM	OPTIMAL STRATEGY	THE VALUE OF X_1	THE VALUE OF X_2	THE VALUE OF X_3
8	0.670588	0.185841	0.419469	0.394690
15	0.537255	0.193841	0.347826	0.458333
35	0.262745	0.189655	0.633621	0.176724
50	0.223539	0.418919	0.322072	0.259009

The exact solution by the linear programming

OPTIMAL STRATEGY	THE VALUE OF X_1	THE VALUE OF X_2	THE VALUE OF X_3
0.229591	0.4444	0.31111	0.24444

B. *Example2*

Consider the following (3x3) game

Player B

$$\text{Player A}\quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 2 \\ 3 & 0 & 0 \\ 0 & 2 & 1 \end{bmatrix} \end{array}$$

The number of generation = 90
The number of population = 8
The length of the string of each parameter = 4
The crossover probability = 0.07
The mutation probability = 0.07

Table of results :-

GEN_NUM	OPTIMAL STRATEGY	THE VALUE OF X_1	THE VALUE OF X_2	THE VALUE OF X_3
40	1.533333	0.500000	0.433333	0.066667
50	1.266667	0.240000	0.280000	0.480000
71	1.066667	0.120000	0.360000	0.520000
90	1.000000	0.240000	0.240000	0.520000

The exact solution by the linear programming

OPTIMAL STRATEGY	THE VALUE OF X_1	THE VALUE OF X_2	THE VALUE OF X_3
1.000000	0.250000	0.250000	0.500000

C. *Example3*

Consider the following (3x4) game

Player B

$$\text{Player A}\quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 3 & -1 & 1 & 2 \\ -2 & 3 & 2 & 3 \\ 2 & -2 & -1 & 1 \end{bmatrix} \end{array}$$

The number of generation = 500
The number of population = 4
The length of the string of each parameter = 3
The crossover probability = 0.09
The mutation probability = 0.1

Table of results :-

GEN_NUM	OPTIMAL STRATEGY	THE VALUE OF X_1	THE VALUE OF X_2	THE VALUE OF X_3
100	1.714286	0.333333	0.277778	0.388889
260	1.571429	0.000000	0.363636	0.636364
470	1.428571	0.500000	0.200000	0.300000
500	1.285714	0.555556	0.444444	0.000000

The exact solution by the linear programming

OPTIMAL STRATEGY	THE VALUE OF X_1	THE VALUE OF X_2	THE VALUE OF X_3
1.285714	0.5555556	0.44444445	0.0000000

IV. CONCLUSIONS

In this work we have developed a new method to find the solution of game theory . The method utilizes a genetic algorithm to find the optimal strategy of game theory for player A or player B.

V. REFERENCE

[1] Bierman, H. , Jr. , C. P. Bonini , and W. H. Hausman, " Quantitative Analysis For Business Decisions " , RichardD. Irwin, Inc. Homewood, Illinois, 1973.
[2] Coley, D. A. , " An Introduction To Genetic Algorithms For Scientists And Engineers " , World Scientific publishing Co. Pte. Ltd, 1999.
[3] Cullom, C. , and D. J. Rowley , " Design Considerations For Computewr Simulation Games In The Management Discipline " , Journal of Intelligent information management , Vol. 6 , pp. 41 – 48 , 1997.
[4] Goldberg, D.E.. Genetic Algorithms in Search,Optimization and Machine Learning. Addison-Wesley, 1989.
[5] Gordon, G. , and I. Ressman , " Quantitative Decision-Making For Business " , Prentice Hall International , Inc. , London , 1978.
[6] Kwak, N. K. , and S. A. Delurgio , " Quantitative Models For Business Decisions ", Duxbury Press North Scituate , Massachusetts , 1980.
[7] Mizrahi, A. , and M. Sullivan , " Finite Mathematics With Application " , John Wiley & sons , 1988.
[8] Nelder, J. A. , and R. Mead , " A Simplex Method For Function Minimization", Computer Journal , Vol. 7 , pp. 308 – 321 , 1965.
[9] Özkan, B. , H. V. Akcaöz , " Game Theory And Its Application To Field Crops In Antalya Province " , Turk J Agric for 26 , pp. 303 – 309 , 2002.
[10] Saaty, T. L. , " Lectures On Methods Of Optimization" , Memo. No. 391, pp.183 – 216 , INP,Cairo,1964.
[11] Taha, H. A. , " Operations Research An Introduction" , Macmillan publishing co., Inc. United States , 1982.