Stable running with segmented legs

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Abstract

In human and animal running spring-like leg behavior is found. In a spring-mass model, running proves to be self-stable in terms of external perturbations or variations in leg properties (e.g. landing angle). However, biological limbs are not made of springs but, rather, consist of segments where spring-like behavior can be localized at joint level. Here, we use a two-segment leg model to investigate the effects of leg compliance originating from joint level on running stability.

Due to leg geometry a nonlinear relationship between leg force and leg compression is found. In contrast to the linear leg spring, the segmented leg is capable of reducing the minimum speed for self-stable running from 3.5 m/s in the spring-mass model to 1.5 m/s for almost straight joint configurations which is below the preferred transition speed from human walking to running (≈ 2 m/s).

At moderate speeds the tolerated range of landing angle is largely increased (17 degrees at 5 m/s) compared to the linear leg spring model (2 degrees). However, for fast running an increase of joint stiffness is required to compensate the mechanical disadvantage of larger leg compression. This could be achieved by nonlinear springs enhancing joint stiffness in fast running.

Keywords: self-stability; running; two-segment leg; spring-mass model; SLIP;
knee joint stiffness; nonlinear stiffness.

1 Introduction

The structure of biological limbs appears to be very complex. It comprises bones, cartilage, muscles, tendons, ligaments, connective tissues. However, in bouncing gaits such as running, trotting or galloping, the dynamics of the center of mass can be described by the action of a simple mechanical leg spring acting during stance. Based on reduced models representing this spring-like action of biological limbs, an appropriate description of the mechanics of hopping and running at different speeds is provided. Such spring-mass models (Blickhan, 1989; McMahon and Cheng, 1990) are also known as Spring Loaded Inverted Pendulum (SLIP) (Schwind and Koditschek, 1997).

The use of compliance in legged systems appears to have several advantages. As the leg is flexing and extending during stance phase, elastic structures can temporarily store mechanical energy (Cavagna et al., 1977; Dickinson et al., 2000). Being part of the muscle-tendon complex, the compliant structures (e.g. tendons (Ker, 1981), aponeuroses (Alexander, 2002), titin (Tskhovrebova et al., 1997)) not only allow energy reuse but do also support the reduction of the limb’s effective mass during landing impact (Gruber et al., 1998; Guenther and Blickhan, 2002). Furthermore, the arrangement of muscles (spanning two or more joints) parallel to bones additionally increases the bending stiffness of the segments (Moehl, 2003), depending on the activity of these muscles. Only a minor part of the leg segments consists of comparatively stiff bones whereas soft tissue, including muscle, dominate the overall mass.

Another advantage of compliant leg behavior might be to shape and potentially simplify the control of highly dynamic movements. The muscle-skeletal system of humans or animals is characterized by a high number of mechanical degrees of freedom as well as a large number of actuators spanning one or more joints. This leads to a challenging control task (e.g. the kinematic and motor redundancy problem (Bernstein, 1967)). Here, spring-like behavior of the
muscle-tendon complex could provide an adequate solution to the control task. In a segmented leg, a proper adjustment of joint stiffness (nonlinear torque-angle characteristics) can guarantee homogeneous and stable joint flexion when the leg is dynamically loaded (Seyfarth et al., 2001). This is required in fast movements such as hopping or running. Furthermore, the synchronized unloading of the leg (e.g. in vertical jumping) is largely facilitated by biarticular muscles (e.g. the gastrocnemius spanning knee and ankle joint (Van Ingen Schenau, 1989)). Thus, a spring-like function of muscles spanning single or multiple joints can support the intersegmental stability and efficiency of fast movements.

It is important to note that on the neuro-muscular level, spring-like behavior could even be generated without compliant structures connected to the muscle fibers. A simple, monosynaptic feedback circuit based on proprioceptive force signals (e.g. delivered by Golgi organs) could provide an appropriate muscle stimulation pattern resulting in a stable hopping pattern (Geyer et al., 2003). The dynamics of this muscle-reflex system allows a decentralization of the motor control whereby a central drive (descending pathway) controls the energy (e.g. hopping height) and the local feedback generates the required muscle activation pattern. By having compliant structures in series (tendons, aponeuroses) to the muscle fibers, the efficiency of spring-like muscle behavior is further enhanced. At the same time, the muscle’s ability to act as stiff actuator is lost.

This is in agreement with a recent simulation study in which spring-like leg behavior was identified to allow both self-stable running and walking (Geyer et al., 2006). This idea is supported by bouncing robots (Raibert, 1986) and Scout II (De Lasas and Buehler, 2001) and JenaWalker (Iida et al., 2007; Seyfarth et al., 2006b) compliant walking robots which demonstrate that stable gaits can be achieved by employing simple control approaches which take advantage of the dynamics of compliant legs.

After Raibert’s pioneering work, further research on robots with telescoping, springy legs was carried out, resulting in a variety of stable gaits based on the chosen system configuration and control (Poulakakis et al., 2003). Here, linear springs were introduced into the telescoping legs, representing the intrinsically
compliant leg behavior found in animals and humans. However, biological limbs are not telescopic but, rather, consist of an arrangement of leg segments where elasticity is localized at joint level (Guenther and Blickhan, 2002; Kuitunen et al., 2002; Hansen et al., 2004; Seyfarth et al., 2001).

This concept was implemented in a number of robots by using segmented legs with elastic structures spanning the joints (Schmiedeler, 2001; Iida and Pfeifer, 2004; Seyfarth et al., 2006b). In these robots stable hopping or running was implemented with little sensory effort or by simply applying feed-forward control approaches. Based on articulated compliant legs, the fundamental characteristics of the SLIP template could be inherited (Altendorfer et al., 2001). This is supported by Zhang et al. (2004) who found stability in a running model employing a two-segment leg. Similar two-segment leg models were used in biomechanical studies aiming at identifying appropriate muscle designs or optimum leg strategies for several bouncing tasks (e.g. hopping (Geyer et al., 2003), vertical swinging (Wagner and Blickhan, 1999), long jump (Alexander, 1990; Seyfarth et al., 2000)).

So far, the potential impact of leg segmentation on running stability remains an unresolved issue. Therefore, this paper aims to gain better insights into the advantages and disadvantages of segmented legs with compliant joints in running. Similar to previous studies, leg segmentation is represented by only two massless segments with a rotational spring located at the intersegmental joint generating spring-like leg behavior. The focus will be on contributions of leg geometry to leg force generation and the resultant running dynamics. The results will be compared to the findings of the spring-mass model.

We expect that the two-segment leg model will inherit fundamental properties of the SLIP system leading to similar or shifted regions for self-stable running with a fixed angle of attack leg policy. Secondly, we hope that previously unstable or even completely new solutions can be stabilized using segmented legs. Finally, we expect that in some cases running stability might be threatened by the dynamics of the segmented leg. These findings will allow us to identify potential strategies of employing leg segmentation for stable running.
Figure 1: Simple spring-mass model (a) and two-segment leg model (b). Running of a two-segment model with a fixed angle of attack $\alpha_0$ leg policy is shown in (c). The direction of the intersegmental joint has no influence on running dynamics. For explanations see text.

The main focus of this paper is on stabilizing mechanisms based on passive dynamics similar to McGeer’s pioneering work on passive dynamic walking (McGeer, 1990). Within this concept even complex movements can originate purely from the design of the mechanical system, which allows for simple and energy efficient control to further enhance dynamic stability (Collins et al., 2005). Such control strategies also include online adjustments of the mechanical system parameters and will be addressed with respect to legged robots in the discussion.
2 Methods

2.1 Simple spring-mass model

Within the original spring-mass model (Fig. 1(a), Blickhan (1989); McMahon and Cheng (1990); Seyfarth et al. (2002)) the action of the stance leg is represented by a linear spring of rest length $l_0$ and leg stiffness $k$. As a consequence, the leg can only generate forces directed from a fixed contact point at the ground to the center of mass. Furthermore, the amount of leg force is assumed to depend on leg compression $\Delta l = l_0 - l(t)$ but not on leg orientation or compression velocity. Finally, a linear relationship between leg compression and leg force as defined by a constant leg stiffness $k$ is assumed.

2.2 Two-segmented leg model

Here we address the potential role of leg segmentation on the dynamics of running with spring-like legs. In order to keep the analysis simple we decided to take advantage of the above mentioned assumptions made in the spring-mass model. Again, we describe the leg function by the action of leg force depending on leg compression. The segmented leg is defined by two massless segments (upper and lower leg) of length $\lambda_1$ and $\lambda_2$ connected by the intersegmental leg joint with an inner angle $\beta$ (Fig. 1(b)).

In order to generate spring-like forces in a segmented leg we introduce a torsional spring of stiffness $c$ at the intersegmental joint with joint torque

$$\tau(\Delta \beta) = c \Delta \beta.$$  \hspace{1cm} (1)

Here $\Delta \beta$ denotes the amount of joint flexion $\beta_0 - \beta$ with the rest angle $\beta_0$. The instantaneous joint angle $\beta$ is a function of leg length $l$ with

$$\beta(l) = \arccos \frac{\lambda_1^2 + \lambda_2^2 - l^2}{2 \lambda_1 \lambda_2}.$$  \hspace{1cm} (2)

To calculate the amount of joint flexion $\Delta \beta$ we need to specify a rest angle $\beta_0$ which corresponds to a rest length of the leg.
\[ l_0(\beta_0) = \sqrt{\lambda_1^2 + \lambda_2^2 - 2\lambda_1\lambda_2 \cos(\beta_0)}. \]  

In consequence, any amount of joint flexion \( \Delta \beta \) translates into a corresponding amount of leg compression \( \Delta l \) depending on the selected rest angle \( \beta_0 \). The joint torque (1) results in a leg force with

\[ F_{\text{leg}}(\tau) = \frac{l}{\lambda_1\lambda_2} \frac{\tau}{\sin\beta}. \]

Thus, for any two-segment leg with a rotational spring \( \tau(\Delta \beta) \) the corresponding leg force-leg compression dependency \( F_{\text{leg}}(\Delta l) \) can be calculated. Compared to the spring-mass model, only a few new parameters are required: the segment lengths \( \lambda_1 \) and \( \lambda_2 \), the rotational stiffness \( c \) and the rest angle \( \beta_0 \). The rest length \( l_0 \) and leg force \( F_{\text{leg}} \) are now functions of these parameters.

The two-segment leg is not unique but the most reduced model we could find to address the question of how leg segmentation and joint stiffness influence running stability. The highly reduced complexity of the model still suffices to identify the fundamental dependencies without too many interfering effects.

2.3 Reference stiffness and effective stiffness

As demonstrated in the last section, for a given two-segment leg (parameters \( \lambda_1, \lambda_2, c, \) and \( \beta_0 \)) the leg force can be calculated for any given leg length \( l \). To facilitate the comparison with the linear leg spring model (stiffness \( k \), rest length \( l_0 \)) we define a reference leg compression at 10% of leg length \( \Delta l_{10\%} = 0.1\ l_0 \). This is a typical value for running in humans (Farley and Gonzalez, 1996). Based on the corresponding leg force \( F_{10\%}(\Delta l_{10\%}) \) of the two-segment model, a reference stiffness \( k_{10\%} = F_{10\%}/\Delta l_{10\%} \) can be defined (see Appendix for calculating the corresponding joint stiffness). This stiffness can be compared to the stiffness of a linear leg spring model.

However, for largely varying leg compressions the concept of reference stiffness must be adapted. Here, we can take the maximum leg compression \( \Delta l_{\text{max}} \) as new reference condition resulting in an effective stiffness \( k_{\text{max}} = F_{\text{max}}/\Delta l_{\text{max}} \).
In contrast to the reference stiffness (which is constant for a given leg configuration) the effective stiffness represents the actual leg dynamics. Hence, the effective stiffness may vary for a given two-segment leg and describes its adaptation to different loading conditions.

2.4 Running mechanics

The following section introduces a model describing the center of mass dynamics during running. The body is represented by a point mass \( m \) which is supported by the leg force \( F_{\text{leg}} \) during stance phase to counteract the effect of gravity. During flight phase the leg does not effect the system dynamics \( (F_{\text{leg}} = 0) \). The equation of motion is given by

\[
m \ddot{r} = F_{\text{leg}} + mg,
\]

where \( \dot{r} = [x, y]^T \) is the position of the point-mass and \( g = [0, -g]^T \) is the gravitational acceleration vector. Throughout the stance phase, the foot is fixed on the ground. The force vector \( F_{\text{leg}} = [F_x, F_y]^T \) directs from the foot point to the point mass at \( r \). We consider transitions between flight and stance phases using the following conditions:

\[
\text{flight} \rightarrow \text{stance} : \quad y \leq l_0 \cdot \sin \alpha_0, \\
\text{stance} \rightarrow \text{flight} : \quad l > l_0,
\]

where \( \alpha_0 \) is the predefined touch-down angle and \( l_0 \) is the rest length of the leg.

The state of the running model is defined by the position \( r \) and velocity \( v \) of the center of mass. During stance phase, the leg parameters are \( k \) and \( l_0 \) in the linear leg spring (section 2.1) and \( \lambda_1, \lambda_2, c \) and \( \beta_0 \) in the segmented leg (section 2.2). Furthermore, the angle of attack \( \alpha_0 \) defines the instant of touch-down (6).

During flight phase, the forward velocity \( v_x \) is a direct function of the total system energy \( E_{\text{sys}} \) and the apex height \( y_{\text{apex}} \):

\[
v_x(E_{\text{sys}}, y_{\text{apex}}) = \sqrt{2 \left( E_{\text{sys}}/m - g y_{\text{apex}} \right)}. \tag{7}
\]
Hence, for a given system energy $E_{sys}$, the model state is completely described by $x$ and $y_{apex}$. However, on even ground, the actual horizontal position $x$ has no influence on the future system dynamics and $y_{apex}$ completely describes the system’s state.

The number of free parameters can be further reduced by using dimensionless parameters (Geyer et al., 2005). The model parameters are the dimensionless system energy $\tilde{E}_{sys} = E_{sys}/(m g l_0)$, the dimensionless leg stiffness $\tilde{k} = (k l_0)/(m g)$ and the angle of attack $\tilde{\alpha}_0 = \alpha_0$. In the two-segment model, the dimensionless segment lengths are $\tilde{\lambda}_1 = \lambda_1/l_0$ and $\tilde{\lambda}_2 = \lambda_2/l_0$; the dimensionless rotational stiffness is $\tilde{c} = c/(m g l_0)$ and the nominal angle $\tilde{\beta}_0 = \beta_0$ is given by:

$$\tilde{\beta}_0 = \arccos \frac{\tilde{\lambda}_1^2 + \tilde{\lambda}_2^2 - 1}{2 \tilde{\lambda}_1 \tilde{\lambda}_2}.$$  \hspace{1cm} (8)

In the case of equal segment lengths $\tilde{\lambda} = \tilde{\lambda}_1 = \tilde{\lambda}_2$ only two free parameters remain to describe the leg function: $\tilde{c}$ and $\tilde{\beta}_0$. The dimensionless segment length is then given by:

$$\tilde{\lambda} = \sqrt{\frac{1}{2(1 - \cos \tilde{\beta}_0)}}.$$  \hspace{1cm} (9)

Any combination of $\tilde{c}$ and $\tilde{\beta}_0$ can be expressed by a dimensionless reference stiffness $\tilde{k}_{10\%} = k_{10\%}l_0/(m g)$ (section 2.3). Compared to the linear leg spring model (parameter $\tilde{k}$), the two-segment leg (dimensionless rotational stiffness $\tilde{c}$) has two more parameters $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ for unequal segment lengths and only one additional parameter $\tilde{\beta}_0$ for equal segment lengths. In this paper we focus on the latter segment configuration. In addition, we fix the parameters for a human-like model (mass $m = 80$ kg, leg length $l_0 = 1$ m, and gravitational acceleration $g = 9.81$ m/s$^2$).

### 2.5 System analysis

In this section two methods for analyzing the running dynamics as described by the two models (see sections 2.1 and 2.2) are presented. The first approach - the
steps-to-fall analysis - examines the ability of the model to generate continuous running patterns by simply counting the number of successful steps. In a second method - the single-step analysis - the system behavior is analyzed based on a Poincaré map of two adjacent apex heights, further called the apex return map.

In the steps-to-fall analysis the number of successive steps is counted for a given initial condition specified by the apex height \( y_{\text{apex},0} \) and the horizontal velocity \( v_{x,0}(y_{\text{apex}}, E_{\text{sys}}) \) (see Eq. 7). Three possible situations may occur: (1) the systems runs continuously, (2) the body hits the ground, or (3) the forward speed crosses zero. In case (1) the number of steps is limited by a predefined value \( \text{maxsteps} = 50 \). In cases (2) and (3) the number of steps is counted. In this study an initial apex height \( y_{\text{apex},0} = l_0 \) is used. This initial condition was found to be appropriate to find stable running patterns in the spring-mass model (Seyfarth et al., 2002).

The previous method provides an intuitive way of identifying potentially stable solutions for the running models. However, in some cases the system might still fail after the predefined maximum number of steps. Even by an increased maximum number of steps this problem can not be solved. On the other hand, the system could be potentially stable but the selected initial condition is not attracted to the stable limit cycle. Here, the following method provides an adequate solution.

In the second method, the apex return map, running stability requires that the following two conditions must be fulfilled: (1) the solution must be periodic with identical apex heights and (2) perturbations in the initial apex height are reduced after one step. Here, the step is defined as the period between two subsequent apexes \( i \) and \( i + 1 \). These conditions can be conveniently verified based on a single step analysis of the apex height \( y_{\text{apex},i+1} = f(y_{\text{apex},i}) \). The conditions for periodicity and local stability are:

\[
y^* = y_{\text{apex},i+1} = y_{\text{apex},i}
\]

\[
\left| \frac{dy_{\text{apex},i+1}}{dy_{\text{apex},i}} \right|_{y^*} < 1.
\]
For a given system energy $E_{sys}$, angle of attack $\alpha_0$ and leg properties (section 2.4) the apex return map $y_{\text{apex},i+1} = f(y_{\text{apex},i})$ can be calculated for all possible initial apex heights $l_0 \sin \alpha_0 \leq y_{\text{apex},i} \leq E_{sys}/(mg)$ based on the equation of motion (5).

In this paper both methods, the step-to-fall method and the apex return map, are used to analyze the system’s stabilizing behavior. First, potentially stable running solutions are located by the steps-to-fall method (using the above described initial condition and running 50 steps). This method is also practical in testing robotic systems (Neville et al., 2006). Afterwards, we prove stability of these solutions using the Poincaré map of the apex height (apex return map). Using an iterative search, the fixed point was identified and local stability was tested using Eq. 11. It is important to note that the initial steps-to-fall method is not a proof of stability. Therefore, it is only used to search physically reasonable and potentially stable fixed points in a straightforward and intuitive way.

3 Results

3.1 Force-length relationship of the segmented leg

In order to examine the effects of leg segmentation on running stability, we first investigate the geometrical effects of the two-segment leg on the force-length relationship. This function includes a number of calculations which can be divided into three steps in Fig. 2(a). Steps 1 and 3 are purely geometric relationships and are referred to as $\Gamma_1$ and $\Gamma_2$, respectively. In the second step, the joint torque $\tau$ at the intersegmental joint is calculated depending on the joint angle $\beta$. Here, a linear rotational spring is used, as introduced in section 2.2.

The geometric function $\Gamma_1$ (Fig. 2(b)) relates leg length to joint angle with $\beta(l) = \Gamma_1(l) l$, where

$$
\Gamma_1(l) = \frac{1}{l} \arccos \left( 1 - \frac{l^2}{2\lambda^2} \right).
$$

Here, $\lambda$ denotes the length of both segments. At almost straight leg configura-
Figure 2: The calculation of leg force $F_{\text{leg}}$ as a function of leg length $l$ consists of three subsequent steps (a). First, the calculation of the joint angle $\beta$, second the estimation of the joint torque $\tau$, and finally the calculation of the resulting leg force $F_{\text{leg}}$. The geometric function $\Gamma_1$, illustrated in (b), describes the relationship between intersegmental joint angle $\beta$ and leg length $l$. The geometry function $\Gamma_2$ drawn in (c) is the ratio of leg force $F_{\text{leg}}$ to joint torque $\tau$ as a function of leg length $l$. The geometric functions $\Gamma_1$ and $\Gamma_2$ represent steps (1) and (3) in the flow chart of (a). The length of the segments $\lambda$ is 0.5 m.

Equations (i.e. for leg lengths $l$ close to $2\lambda$) the nonlinear nature of $\Gamma_1$ results in large changes in joint angle $\beta$ for small variations in leg length $l$. After calculating the joint torque $\tau$ based on joint angle $\beta$ (step 2 in Fig. 2(a)), the leg force $F_{\text{leg}}$ can be determined based on the geometric function $\Gamma_2$ (Fig. 2(c)) with $F_{\text{leg}} = \Gamma_2(l) \tau$ given by:

$$\Gamma_2(l) = \frac{l}{\lambda^2 \sin \beta(l)}.$$  \hspace{1cm} (13)
This function is almost constant at small leg lengths but changes very rapidly at almost extended leg configurations. In fact, at fully extended leg configuration (i.e. normalized leg length of 1) the function approaches infinity according to equation 13.

\[
\text{normalized leg compression } \Delta \frac{l}{l_0} \%
\]

\[
\text{normalized leg force } \frac{F}{F_{\text{leg 10}}}
\]

Figure 3: Normalized force-length relationships of elastic two-segment legs with the same reference stiffness \(k_{10\%}\) and different nominal angles \(\beta_0\) of the rotational spring. For comparison, the linear force-length curve of the spring-mass model is shown. Leg compression is given by \(\Delta l = l_0 - l\).

Hence, both geometric functions \(\Gamma_1\) and \(\Gamma_2\) contribute to the resulting force-length relationship of the segmented leg. For almost extended leg configurations, small changes in leg length result in large changes in leg force. In contrast, if the leg is bent, the change in leg force becomes about proportional to the variation of leg length. For a linear rotational spring at the intersegmental joint, the slope of the force-length relationship, and thus the leg stiffness at a given leg compression, depends further on the nominal angle \(\beta_0\) as shown for \(\beta_0 = 110, 150, 170\) degrees in Fig. 3. The larger the nominal angle, the greater the drop in leg stiffness with increasing leg compression.

### 3.2 Running stability

In this section, we will analyze the effects of the previously identified force-length relationships (Fig. 3) on running stability. First we compare the regions of stable running of the two-segment leg with the spring-mass model (SLIP
Figure 4: Regions of stable running for given reference stiffness $\tilde{k}_{10\%}$ and angle of attack $\alpha_0$ in the spring-mass model (left column) and the two-segment model (middle column $\beta_0 = 150$ deg, right column $\beta_0 = 170$ deg). Running stability was tested by analyzing the slope of the apex return map following Eq. 11 (see section 2.5). In the two-segment model, the values of the joint stiffness $c$ corresponding to the reference stiffness $\tilde{k}_{10\%}$ are shown on the left axis.
model) at different running speeds and nominal joint angles. The regions of stability are calculated for given combinations of stiffness (joint stiffness \( c \) in the two-segment model, leg stiffness \( k \) in the spring-mass model) and angle of attack \( \alpha_0 \). Both stiffness values \((c, k)\) can be easily compared using the dimensionless reference stiffness \( \tilde{k}_{10\%} \) defined at 10% leg compression (see section 2.3).

At moderate running speeds (3.5 m/s and 5 m/s), we find a substantial extension of the stability region in the segmented leg (Fig. 4) for moderate or larger values of the reference stiffness. For instance, for 5 m/s and \( \tilde{k}_{10\%} = 30 \) the tolerated range in leg angle adjustments for running stability increases from about 2 degrees in the spring-mass model to as much as 17 degrees in the two-segment leg \((\beta_0 = 170 \text{ deg})\). At low speeds (e.g. 2 m/s) only the two-segment model can predict stable running movements for a constant angle of attack leg policy. The tolerated range in angle of attack in the two-segment model at 2 m/s, \( \beta_0 = 170 \text{ deg} \) and moderate reference stiffness (e.g. \( \tilde{k}_{10\%} = 30 \)) is quite comparable to corresponding robustness of the spring-mass model at 5 m/s. In contrast, at low speeds (below 3.5 m/s) no stability region is found in the spring-mass model.

In the simple spring-mass model the tolerated range in angle of attack \( \alpha_0 \) does not change between moderate and high stiffnesses. The range of \( \alpha_0 \) is constant at two degrees for reference stiffnesses from 15 to 50 at a speed of 5 m/s. Not so in the two-segment leg, here the range becomes smaller with increased stiffness. For the straighter leg configuration \( \beta_0 = 170 \text{ deg} \) and a velocity of 3.5 m/s the range of angle of attack decreases from 9.0 to 5.4 degrees for reference stiffnesses of 30 to 50, respectively.

In all three models (three columns in Fig. 4) a minimum reference stiffness \( \tilde{k}_{10\%} \) is required to achieve stable running. However, in the case of the two-segment leg a lower joint stiffness \( c \) (denoted on the right hand side) is necessary with straighter nominal leg configurations. For instance, using a straight leg \((\beta_0 = 170 \text{ deg})\) a minimum joint stiffness of \( c = 8 \text{ Nm/deg} \) is required for stable running at 5 m/s (Fig. 4, right column, upper panel) whereas with a more bent joint configuration \((\beta_0 = 150 \text{ deg})\) a rotational stiffness of more than 10 Nm/deg
Figure 5: Regions of stable running for given angle of attack $\alpha_0$ and running speed $v_{x,0}$ predicted by (a) the spring-mass model and (b) and (c) the two-segment leg model. Running speed corresponds to the initial velocity $v_{x,0}$ at apex height $y_0 = 1$ m. The effect of selected values of reference stiffness $\tilde{k}_{10\%}$ (15, 25, 35) on the region of stable running is shown.

is necessary (Fig. 4, middle column, upper panel).

As with the spring-mass model (Fig. 5(a)), the two-segment leg demonstrates regions of stability that increase in size with increases in speed. However, at larger speeds (Fig. 5(b) and 5(c)) these regions are limited in the two-segment system. This upper boundary can be shifted to higher speeds by increasing the reference stiffness (increased joint stiffness). Furthermore, a minimum angle of attack is required for stable running with two-segment legs (Fig. 5(b) and 5(c)). For the selected stiffness $\tilde{k}_{10\%} = 35$ upper limits in velocity where found at 11.5 m/s and 8.7 m/s for resting angles $\beta_0 = 150$ and 170 degrees, respectively. In contrast to the spring-mass model, where stable running is predicted at speeds higher than 20 m/s, the two-segment leg’s forward speed is clearly constrained to more moderate speeds. Running at high velocities (e.g. 20 m/s) in the two-segment leg requires sufficient joint stiffness. Furthermore, regions of stable running also have a limit at small angles of attack. At an angle of approximately 30 degrees these regions end abruptly.

Fig. 6 shows the attraction of initial apex heights towards stable running in both models for a given angle of attack and system energy. The basin of
Figure 6: Dependency of apex heights corresponding to periodic running solutions $y_{\text{apex}}^*$ (thick solid and dashed lines) and the resulting basin of attraction (grey area) of stable fixed points (thick solid line) on dimensionless reference stiffness. For a given reference stiffness, the basin of attraction of the stable fixed point is restricted by (1) the touch-down height $y_{TD} = l_0 \sin \alpha_0$ (thin dotted lines), and (2) the apex of the second unstable fixed point (thick dashed line). Model parameters: angle of attack $\alpha_0 = 68 \, \text{deg}$, system energy $E_{sys} = 1785 \, J$ (corresponds to a velocity of $v_{x,0} = 5 \, \text{m/s}$ at an apex height of 1 m).

attraction (gray area) has one lower limit (initial apex height must be above landing height $y_{TD}$, Eq. 6) and two upper limits. These two conditions (apex of unstable fixed point and the subsequent apex compared to the landing height) depend on the reference stiffness. With segmented legs ($\beta_0 = 150$ and 170 deg), the maximum tolerated apex height is 1.8 m and 1.85 m, respectively. In contrast, in the spring-mass model (SLIP model), the maximum height is 1.4 m. With segmented legs, stable running is also found for higher apex conditions (up to about 1.4 m) compared to the SLIP model.

We further analyze the model behavior based on single apex return maps. The results are shown in Fig. 7(a) for constant system energy and angle of attack. For an easy model comparison, reference stiffnesses were chosen such that in all systems a periodic running solution at the same apex height $y_{\text{apex}}^* =$
Figure 7: Return maps of the apex height \( y_{i+1}(y_i) \) of a single step is shown in (a) for the spring-mass model and the two-segment model with two different nominal joint angles \( (\beta_0 = 150, 170 \text{ deg}) \). Normalized tracings of the ground reaction force are illustrated in (b) for stable running corresponding to the fixed point condition in (a). The graph (c) shows the adaptation of effective leg stiffness to the selected initial apex height corresponding to the maps in (a).

Model parameters: reference velocity \( v_{x,0} = 5 \text{ m/s} \) (refers to an apex height of 1 m), angle of attack \( \alpha_0 = 68 \text{ deg} \), reference stiffness of the spring-mass model \( k_{10\%} = 29.16 \) and the two-segment legs \( k_{10\%} = 29.83 \) \( (\beta_0 = 150 \text{ deg}) \) and \( \tilde{k}_{10\%} = 28.10 \) \( (\beta_0 = 170 \text{ deg}) \).

1 m exists. A periodic solution (fixed point) is defined by \( y_{i+1} = y_i \), i.e. an intersection of the mapping \( y_{i+1}(y_i) \) with the diagonal. With increasing nominal angle \( \beta_0 \), the slope of the solutions \( y_{\text{apex},i+1}(y_{\text{apex},i}) \) becomes lower around the fixed point. The more the map is aligned to the diagonal (as in the SLIP model) the more steps are needed to approach the stable limit cycle. The highest attraction of the fixed point is defined for zero slopes, as approximately found for the two-segment leg with a resting angle of \( \beta_0 = 150 \text{ deg} \). Here, a perturbation of 0.2 m \( (y_{\text{apex},i} = 1.2 \text{ m}) \) can be compensated in only about 2 running steps.
3.3 Running dynamics

The nonlinear force-length relationship (Fig. 3) of the two-segment leg effects the stance phase dynamics represented by the pattern of the ground reaction force and the effective leg stiffness (Fig. 7(b,c)). In Fig. 7(b) normalized vertical ground reaction forces are drawn for the spring-mass model and two configurations of the two-segment leg. The predicted patterns correspond to stable running solutions of the apex return map (Fig. 7(a)) with an apex height of 1 m. The more the force-length relationship becomes nonlinear the lower the maximum force at midstance. In the case of the spring-mass model (Fig. 7(b)) the vertical ground reaction force is about four times body weight. In contrast, in the two-segment leg with straight nominal leg configuration ($\beta_0 = 170$ deg) the highest force is less than three times body weight. Almost no difference in stance time between SLIP model and two-segment leg ($\beta_0 = 170$ deg) (155 ms and 157 ms, respectively) is found. The pattern of the ground reaction force changes from a sinusoidal curve in the spring-mass model to a more rectangular shape in the segmented leg with a rapid force increase at the beginning of stance phase.

In the segmented leg, the effective leg stiffness is dependent on leg compression. Due to the nonlinear force-length relationship (Fig. 3) the effective stiffness $\tilde{k}_{\text{max}}$ becomes lower at larger leg compressions. This is the case in running with larger apex heights as shown in Fig. 7(c). Hence, for longer flight phases the effective stiffness decreases and the segmented leg becomes softer. Even at the fixed point with apex height $y_{\text{apex}}^* = 1$ m (dashed line), the effective stiffness is already lower in the segmented leg compared to the SLIP model.

4 Discussion

In this paper, the potential influence of leg segmentation on the stability of running is addressed. Compared to the simple spring-mass model, the two-segment leg reveals, that (1) segmented legs provide self-stable running at an
enlarged range of running speeds (lower minimum speed), (2) for given speed and corresponding joint stiffness running with segmented legs is more robust to variations in angle of attack and perturbations in apex height, (3) for running with comparable apex heights and speed the maximum leg force is reduced resulting in a decreased effective leg stiffness, and (4) low and moderate values of the joint stiffness provide more robustness with respect to variations in angle of attack. At a given speed, joint stiffness must be higher for more bent nominal joint configurations to achieve stable running. However, with increased speed a corresponding increase in joint stiffness is required to guarantee running stability.

These novel features of segmented legs with respect to running stability can be seen as a consequence of the nonlinear relationships between leg length and joint angle (geometric function $\Gamma_1$) and between joint torque and leg force (geometric function $\Gamma_2$), resulting in a decreased effective leg stiffness with larger leg compressions. Despite the highly simplified structure of the segmented leg model compared to the function of human or animal legs during running, we believe that this reduced description (also called template or minimalistic model, (Full and Koditschek, 1999)) is appropriate to reveal fundamental consequences of leg segmentation which can not be represented by the spring-mass model. Such templates can be used to interpret legged locomotion observed in more complicated and higher dimensional systems like animals and robots.

Due to the reduced number of system parameters, the natural dynamics of such template models can be used to derive simple control strategies further enhancing stability or even stabilizing periodic solutions which are unstable without control (Schmitt and Holmes, 2000; Seyfarth et al., 2003). This includes swing-leg strategies to adjust the leg angle before touch-down (Raibert, 1986) and changes of mechanical properties (e.g. leg stiffness (Blum et al., 2007)) either by purely mechanical adjustments (Van Ham, 2006) or simulated by computed-torque control (Park and Chung, 1999). In the latter case the required torque is calculated, for instance, using an inverse model of the robot dynamics including motor properties. As an alternative approach, the concept
of a simulated leg stiffness (without mechanical springs) was implemented in a simple hopping robot (Seyfarth et al., 2007) by purely employing feedback control with leg force proportional to the amount of leg compression during stance. In order to deal with energy losses (e.g. landing impacts and internal friction), energy supply strategies (e.g. increased leg stiffness at midstance) had to be introduced. Additional control effort was required in preparation of the stance phase (leg shortening) to minimize the risk of structural damage due to high impact forces acting on the mechanical components of the robot.

Due to the highly reduced structure of the two-segment model used here, no information can be obtained about effects of segment mass distribution (e.g. swing dynamics), ground contact mechanisms (e.g. foot deformation) including energy losses or upper body dynamics such as body rotation. These important aspects are not addressed with this model but have to be considered in the design and implementation of robotic systems. Due to the segmented structure of most legged robots we expect that the mechanisms identified here will still hold. However, additional effects will shape the system dynamics and hence influence the control required for stable locomotion.

The two-segment leg has a direct relationship between intersegmental joint angle and leg length. In a three-segment leg including knee and ankle joint, however, a gait specific inter-joint coordination of both joints is observed (Seyfarth et al., 2006a). Here, the issue of internal leg stability is of importance as one joint could extend at the cost of the other (Seyfarth et al., 2001). Such issues can not be represented in the two-segment model used in this study. Equally, the influence of acting hip torques on leg function is excluded. Hence, the leg has no preferred direction of locomotion; the leg is merely described by forces acting from the foot point to center of mass similar to the spring-mass model.

4.1 Running at low speeds

Spring-mass running with fixed angle of attack provides self-stability for speeds larger than about 3.5 m/s (Fig. 4). However, the transition from walking to running occurs already at about 2 m/s (Hreljac, 1993). To stabilize low running
speeds additional control strategies like swing-leg retraction (Seyfarth et al., 2003) are required. Such control is based on sensory information, e.g. to detect the instant of apex from which the leg starts to retract. Another possibility is to replace the simple leg spring by more detailed mechanical (segmentation) or neuromechanical structures (including muscles and reflexes). Here, the transition from a linear leg spring to a two-segment leg lead to a nonlinear leg stiffness characteristic. As a result, the minimum speed for self-stable running is clearly reduced depending on the selected nominal joint angle (about 2 m/s at $\beta_0 = 150$ degrees and less than 1.3 m/s at $\beta_0 = 170$ degrees). Experiments on human running (De Wit et al., 2000) indicate an increase of knee angle at touch down from 163 degrees at 5.5 m/s to 168 degrees at 3.5 m/s. According to the prediction of the two-segment model (Fig. 4) such knee angles could support self-stable running at speeds of 2 m/s and lower.

A number of legged robots (Iida and Pfeifer, 2006; Seyfarth et al., 2006a; Palmer et al., 2003) use elastic two-segment legs in a similar configuration as in the model investigated in this study. These robots demonstrate that stable hopping can be achieved with only little or no sensory feedback, potentially taking advantage of the mechanical self-stability identified in the model. One possible criterion, whether or not the systems are operating in a self-stable region, could be the analysis of proximal leg joint torque (e.g. hip joint). In the model, no hip torque is required to stabilize the running pattern since the role of the trunk is ignored for systematic reasons. If the center of mass of the trunk is located above the hip joint with small horizontal displacements, the required hip torques to stabilize the trunk can be small compared to the knee torques. This was checked in a simulation study on self-stable running with a rigid trunk kept in an upright position. The trunk position was measured relatively to the hip angle unlike other approaches (e.g. Poulakakis and Grizzle (2007)) where the absolute trunk position is necessary. Interestingly, both control methods take advantage of the self-stabilizing behavior of compliant legs. As the hip torque was calculated based on the relative trunk position, system energy was directly affected. The resulting steady-state running pattern, however, was characterized
by small hip torques associated with minor energy fluctuations. Thus, in a real robot, we would expect that small hip torques would suffice to compensate for energy losses due to landing impacts and joint friction. According to the results of the simulations on running with segmented legs, self-stable running is predicted within a certain speed range. This is in agreement with robotic trials, where hopping at higher speeds turned out to be unstable (Rummel et al., 2006). By applying swing leg strategies, this speed range of stable locomotion can be further enhanced (Seyfarth et al., 2003; Blum et al., 2007).

4.2 Three-segment legs

Segmented legs (for instance in humans or humanoid robots) often consist of more than two leg segments. For joints with compliant structures operating in-phase (i.e. flexion and extension occurring at the same time) this allows for synchronous storage and release of elastic energy at both joints. Such leg behavior is found in running. In-phase joint operation with the same amounts of joint flexion is comparable to the function of the two-segment leg investigated here.

For out-of-phase joint operation or different joint angles a more detailed description is necessary. For instance, in hopping in place the leg function is dominated by the ankle joint (Farley and Morgenroth, 1999). Thus, depending on the selected movement pattern and the desired intensity of the movement, a proper distribution of leg function into joint operation is required. This addresses the issue of solving the redundancy problem (Gielen et al., 1995) imposed by a three-segment structure.

One advantage of having a foot as a third segment is the ability to shift the centre of pressure from the posterior to the anterior end of the foot during ground contact (Bullimore and Burn, 2006). The speed of the center of pressure results in a reduction of the effective speed of locomotion and reduced risk of foot slip (less horizontal ground reaction forces).

After heel-off the initiation of the swing-phase is supported by ankle extension and knee flexion. The leg segments move forward while the tip of the foot
(toes) remains at the ground. This clearly reduces the swing time. In most recent humanoid robots (e.g. ASIMO, QRIO and Johnnie (Sakagami et al., 2002; Geppert, 2004; Lohmeier et al., 2004)), this function is hardly represented. It remains for future research to investigate the dynamics and stability of walking and running based on a three-segmented leg geometry.

### 4.3 Adjustment of joint stiffness

Animals and robots with spring-like leg behavior can stabilize running at different speeds by using simple leg strategies, such as (1) adjustment of leg stiffness (i.e. increasing leg stiffness with increasing speed (Arampatzis et al., 1999)), (2) adjustment of angle of attack (i.e. using flatter angles at higher running speeds (Farley et al., 1993; Seyfarth et al., 2002)), (3) adjustment of leg length (e.g. shortening the leg as preparation for possible obstacles (Blickhan et al., 2007) or lengthening the leg while running unexpectedly a step down (Daley et al., 2007)), or a combination of these (e.g. (1) and (2) in Fig. 5(a)). The analysis of the spring-mass model indicates that no adjustment of the leg stiffness is required to increase speed. In the two-segment leg, however, a maximum running speed is predicted for a given joint stiffness. In addition, at given speed a rather moderate joint stiffness is preferred resulting in a large tolerance in angle of attack (right column in Fig. 4). The findings indicate that for stable running at higher speeds an increase in joint stiffness is required.

Robotic testbeds can be used to prove control and design concepts as suggested by model predictions in a real-world context. Alternatively, we also like to motivate our results based on human trials. This provides a valuable source of insights for the design of future walking and running robots as impressively demonstrated by the passive dynamic walkers (Collins et al., 2005).

In order to verify the prediction of the segmented model, i.e. that joint stiffness has to increase with running speed for adequate stability, we performed experiments with seven human subjects (for details see caption of Fig. 8) running on an instrumented treadmill (ADAL WR, Tecmachine) at three speeds (2, 3, and 4 m/s) and analyzed kinematic (joint trajectories) and kinetic data.
Figure 8: Experimental results (mean ± s.d. shown as circles and vertical lines) of effective knee joint stiffness from seven human subjects (mean ± s.d. of mass: 77 ± 8 kg, leg length: 0.99 ± 0.05 m) running on a treadmill at three speeds ($v_x = 2, 3$ and $4 \text{ m/s}$). The participants ran 10 seconds for each speed and the procedure was repeated twice. The gray region shows the predicted joint stiffness for self-stable running in a two-segment leg with $\beta_0 = 170 \text{ deg}$ and $\alpha_0 = 70 \text{ deg}$.

The experimentally observed knee joint stiffness values are closely aligned with the range of joint stiffness for stable running in the two-segment model (gray region in Fig. 8). The rather moderate increase of human joint stiffness over speed compared to model data may be explained by the chosen angles of attack (flatter angles with increased speed, (Knuesel et al., 2005)). The adaptation of landing angle to running speed was not taken into account in the calculation based on the two-segment model (fixed angle of attack).
The joint torques in humans and animals are generated by muscles-tendon complexes which consist of passive and active elements. To achieve high joint stiffness at fast running speed, the active part of the muscles - the contractile elements - must generate more force. This functional dependency required for stable running in the two-segment model is supported by EMG (electromyogram) measurements of knee extensor muscles (e.g. vastus medialis (Kuitunen et al., 2002)) where an increase in muscle activation with speed is observed. As a consequence of this increased muscle activity the elastic tendons attached to the muscle fibers are further stretched. Here, the nonlinear stress-strain relationship of the tendons (Ker, 1981) might help to increase the overall muscle-tendon stiffness at higher running speeds (Seyfarth et al., 2000).

Our results in simulation and experiments indicate that joint stiffness must be adapted to running speed. This holds not only for animals and humans but probably also for robots with segmented legs. So far, most robots are clearly restricted in running speed. The findings based on spring-mass models suggest that elastic legs could provide passive stability in running. Within a segmented leg, however, an adjustment of the joint stiffness to the amount of joint flexion would be preferable. Within the last years, a number of technical solutions to adjust joint stiffness and nominal angles were developed. These concepts include arrangements of mechanical springs (AMASC (Hurst et al., 2004), MACCEPA (Van Ham, 2006)) and controlled pneumatic actuators (pleated pneumatic muscles (Verrelst et al., 2000)). For instance, in the passive dynamic walker Veronica (Van Ham et al., 2005) the MACCEPA mechanism was implemented to adjust the leg properties during the swing phase. Another promising approach is the two-segmented BiMASC leg (Hurst et al., 2007) with adaptable leg stiffness based on the AMASC design. In the bipedal walking robot JenaWalker II (Seyfarth et al., 2006b) a set of motors is used to adjust the nominal length of springs simulating the action of biological muscles in the segmented leg. Here, a transition from heel-toe walking to toe walking could be introduced by reducing the nominal length of the biarticular calf muscle (gastrocnemius) leading to a shift of the nominal angle in the ankle joint. So far, the underlying mechanisms
required to guarantee running stability at different speeds are not well understood. The potential contribution of adjustable joint stiffness characteristics as presented in this study could be an appropriate starting point for further investigations.

4.4 Adaptive mechanics in segmented legs

Another result of this study is the adaptation of effective leg stiffness in a two-segment leg dependent on leg compression while the joint stiffness is constant. Figure 7(c) shows that for a given system energy and angle of attack the leg compression and the resultant effective stiffness depend on the previous apex height. The leg becomes softer with increasing apex height, which contributes to the lower apex height in the forthcoming step. Thus, a flatter apex return map with greater robustness against perturbations in initial apex height is observed (Fig. 7(a)). The self-adapting mechanism in the segmented leg lead to a basin of tolerated apex heights which is twice as large compared to that of the spring-mass model (Fig. 6). Furthermore, the apex height of periodic running patterns is found to be relatively high. This allows for overrunning larger obstacles due to the increased ground clearance (difference between apex height and landing height).

The adaptation of effective leg stiffness corresponding to the duration of the flight phase results in similar rotation of the apex return map, as already predicted for control strategies during swing phase, e.g. swing-leg retraction, stabilizing running on uneven ground or at low speeds (Seyfarth et al., 2003). In contrast to leg retraction control, the adaptation of the contact dynamics to different flight times, represented here, is a truly passive mechanism which does not rely on sensory information (e.g. no detection of the instant of apex, no continuous adjustment of the leg angle).

For a given nominal joint angle $\beta_0$ of the rotational spring the effect of leg segmentation could be equally described by a nonlinear leg spring with decreasing stiffness at increasing leg compression. In the segmented leg, however, the nonlinearity of leg stiffness is not constant but is the result of leg geometry and
selected nominal angle $\beta_0$. The implementation of a mechanical spring has the advantage that energy can be efficiently stored and released during stance phase. Furthermore, given the dependency of joint torque to the joint angle, mainly harmonic (sinusoidal) movement patterns will be generated. A change of the physical spring parameters may shift the frequency or amplitude while keeping the general pattern. The control of compliant operating actuators spanning the leg joint (e.g. MACCEPA (Van Ham, 2006)) could actively shift the nominal joint angle in preparation for ground contact, leading to a combination of leg rotation and leg shortening, as discussed in the next section.

4.5 Leg response to landing

The segmented leg offers an implementation of leg retraction by employing a combination of hip extension and knee flexion. In order to avoid high impacts at landing, it suffices to introduce leg retraction by rotation of the shank with a significantly lower moment of inertia compared to the thigh. This opens up the possibility of not only accounting for running stability but also to reduce the foot’s landing velocity. For instance, in human running at 4.5 m/s a reduction in horizontal heel velocity at touch-down from around 1.64 m/s in shod running to about 1.16 m/s in barefoot running is observed with a corresponding increase in knee flexion velocity from 265 deg/s to 353 deg/s (De Wit et al. (2000)). A proper combination of hip and knee rotation prior to landing could be used to control the initial impact force due to non-zero landing velocities of the foot. This was not considered here and remains for further investigations.

A constant joint stiffness (joint torque proportional to joint flexion) as assumed in this study leads to high force rates in early stance (Fig. 7(b)). Such loading rates might become critical and lead to structural damage within the body. At high running speeds, a more flexed knee angle at contact is found (Knuitunen et al., 2002) which reduces the speed of force build-up. Furthermore, a nonlinear joint stiffness behavior, whereby stiffness increases with joint flexion, could help to reduce the initial force rate after landing. This could be achieved by control schemes or by well-tuned mechanical properties. Recently, novel
concepts for implementing joints of adjustable rotational stiffness and nominal angles were introduced (Hurst et al., 2004; Van Ham, 2006). Similar to the muscles within the muscle-tendon complex, actuators can then be understood as active elements which are able to configure the mechanical properties (stiffness, damping) of joints. By adding motors in the leg segments to actuate the joints, additional masses are introduced increasing the effective mass (Gruber et al., 1998) during landing impact. This will again increase the loading rates, potentially damaging the structures at higher speed. A potential solution to that problem is to shift the motors proximally (e.g. implemented in the Spring Flamingo (Pratt, 2000) and the JenaWalker II (Seyfarth et al., 2006b)) or to decouple the motors mechanically from the rigid segments by tendons and other compliant tissues as found in animals using endoskeletons (such as vertebrates). Both strategies reduce the effective mass during landing impacts.

5 Conclusion

In this paper a simple two-segment leg model is used to analyze stability in running. Due to the elastic joint function of the two-segment model, basic properties of the more abstract spring-mass model can be inherited, namely (1) the ability to generate periodic running movements and (2) the stability of these trajectories for fixed touch-down leg angles and with sufficient speed. The segmented leg offers a number of advantages over the simpler spring-mass model: (1) it allows for stable running at lower speeds and (2), at given speeds, larger variations in the angle of attack can be tolerated. The model further predicts that an adjustment of joint stiffness is necessary to achieve stable running within a wide range of running speeds.

The two-segment model serves as a conceptual model in between the simpler spring-mass model and more detailed segmented models of human or animal bodies. With increasing number of degrees of freedom the possibility to incorporate additional movement goals is given. This can be illustrated based on the two-segment leg, which can potentially integrate issues of running stability.
and strategies of ground impact avoidance by taking advantage of the extra intersegmental joint.

Acknowledgments

This study is supported by the German Research Foundation (DFG, SE1042). The authors thank Susanne Lipfert for realization of experiments on human locomotion and James Andrew Smith for his help on the manuscript.

Appendix

In order to compare the two-segment leg model with the spring-mass model, a reference stiffness $k_{10\%}$ was introduced in section 2.3 and is used in dimensionless form $\tilde{k}_{10\%}$. For the two-segment leg, the corresponding joint stiffness $c$ is given as

$$c = \tilde{k}_{10\%} \frac{m g}{l_0} \frac{\lambda_1 \lambda_2 \Delta l_{10\%}}{l_0 - \Delta l_{10\%}} \frac{\sin \beta_{10\%}}{\beta_0 - \beta_{10\%}}$$

with the joint angle $\beta_{10\%}$ at reference leg compression $\Delta l_{10\%}$

$$\beta_{10\%} = \arccos \frac{\lambda_2^2 + \lambda_2^2 - \left(l_0 - \Delta l_{10\%}\right)^2}{2 \lambda_1 \lambda_2}.$$  

References


