

A Privacy-Preserving Grouping Proof Protocol Based on ECC with Untraceability for RFID

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ABSTRACT

An RFID (Radio-Frequency Identification) system provides the mechanism to identify tags to readers and then to execute specific RFID-enabled applications. In those applications, secure protocols using lightweight cryptography need to be developed and the privacy of tags must be ensured. In 2010, Batina *et al.* proposed a privacy-preserving grouping proof protocol for RFID based on ECC (Elliptic Curve Cryptography) in public-key cryptosystem. In the next year, Lv *et al.* had shown that Batina *et al.*'s protocol was insecure against the tracking attack such that the privacy of tags did not be preserved properly. Then they proposed a revised protocol based on Batina *et al.*'s work. Their revised protocol was claimed to have all security properties and resisted tracking attack. But in this paper, we prove that Lv *et al.*'s protocol cannot work properly. Then we propose a new version protocol with some nonce to satisfy the functions of Batina *et al.*'s privacy-preserving grouping proof protocol. Further we try the tracing attack made by Lv *et al.* on our protocol and prove our protocol can resist this attack to recover the untraceability.

Keywords: ECC; RFID; Grouping Proof; Privacy-Preserving

1. Introduction

An RFID system provides an identification mechanism to identify objects, having RFID tags attached, to reader by communicating over an insecure RF-channel. The basic architecture of an RFID system is combined with a tag, a reader and a backend database server. An RFID tag is a small and cheap device which consists of an IC chip and an antenna for radio communications. RFID tags provide more functionalities as barcodes. Each tag has memory to store more information than barcode. And tags can execute the communication process of answering the request of a reader. An RFID reader is used for querying, reading and writing tag data in no line-of-sight, contactlessly and bulkily. All the data between tags and reader need to send to backend database server. Therefore, RFID is considered to be a suitable replacement for barcodes to reduce the cost of store managements and goods distribution, and to increase the asset visibility.

Owing to RFID is based on radio waves, a kind of unsecured communication channel, RFID system needs secure protocol to protect tag's identify information. Especially when a tag is linked to a person, then the tracing of a tag is equivalent to the tracing of a person. In that case, tag's privacy will become a critical security issue in

the RFID system.

In 2004, Juels [1] introduced the concept of RFID yoking proof. The proof means that two RFID tags have been scanned simultaneously. The RFID yoking proof also named grouping proof which is designed for any application that requires proving two or more entities are present. These applications of grouping proof are increasing in modern life such as delivering some related medication in groups, launching some kind of weapon system after the presence of certain group entities or starting a vehicle when the owner and his driver license on the scene. Most of RFID grouping proof schemes are designed based on symmetric-key cryptography. However, the significant disadvantage of symmetric-key cryptosystem is the key distribution problem that needs all parties to have shared the same key in a secure and authenticated channel before the secure communication happening. The key management is also a great challenge to symmetric-key cryptosystem. In 1976, Diffie and Hellman [2] introduced the fundamental public-key cryptography. In the public-key cryptosystem, the key is split into a public key and a private key, many parties can encrypt message with the receiver's public key, and the encrypted message only can be decrypted by the receiver with her or his private key. In addition, one party can sign a message with her or his private key and send

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to many message signature receivers who can verify the signature with the sender's public key. Therefore, the key management of public-key cryptosystem is easier than symmetric-key cryptosystem.

In 2007, Vaudenay [3] have proven that public-key cryptography can assure the highest level of feasible privacy in RFID applications. Up to now, there are major classes to construct the public-key cryptosystem, which are all based on a mathematical problem that is hard to solve, such as RSA based on large Integer Factorization Problem (IFP), the Diffie-Hellman and ElGamal based on the Discrete Logarithm Problem (DLP), and the Elliptic Curve Cryptosystem (ECC) based on the Elliptic Curve Discrete Logarithm Problem (ECDLP). Among these hard mathematical problems, there are subexponential algorithms for IFP and DLP. In the end of 1980s, Koblitz [4] and Miller [5] independently proposed using the group of points on an elliptic curve defined over a finite field in discrete logarithm cryptosystem. The advantage of ECDLP is that there is absent a subexponential algorithm [6] that could find discrete logarithm in these groups, provided that the curve and the finite field are suitably chosen. Hence, the ECDLP can be regarded as one of the hardest mathematical problem among these public-key cryptosystems. Therefore, the key length for similar level of security in ECC is far less than those public-key cryptosystems based on the IFP and DLP. Consequently, ECC increasingly becomes one of the most popular public-key cryptosystem and is used widely in constrained environment.

Recently in [7,8], ECC was proved to be suitable for RFID applications. In 2010, Batina *et al.* [9] first proposed a privacy-preserving grouping-proof RFID protocol based on ECC. The protocol allows a pair of RFID tags to prove that they have been scanned simultaneously. But in 2011, Lv *et al.* [10] proved the protocol in [9] that failed to resist the tracking attack and lost the untraceability. In an RFID system, the untraceability of a protocol means that an attacker cannot distinguish, based on protocol messages, whether two actions were performed by the same tag or by two different tags [11]. Attacking the untraceability of an RFID system, the attacker is trying to figure out that two (or more) seemingly unrelated interactions were with the same tag [12]. In the same article, Lv *et al.* [10] proposed an intensive protocol to fix the problem. Unfortunately, we found that Lv *et al.*'s protocol [10] had a defect that caused the protocol to execute improperly. In this paper, at first we review two privacy-preserving grouping proof protocols of [9] and [10]. The vulnerability of Batina *et al.* [9] will be discussed in detail. And we demonstrate the defect that we found in Lv *et al.*'s protocol [10]. Furthermore, we propose a new protocol with some nonce to fix the impracticability of Lv *et al.*'s protocol [10]. We also prove that

our protocol can resist the Lv *et al.*'s tracking attack [10] to possess the untraceability. Therefore our new protocol can concurrently solve the defect of Lv *et al.*'s protocol [10] and the vulnerability of Batina *et al.*'s protocol [9].

The rest of this paper is organized as follows. Section 2 introduces the background of ECC. And then, the related works are particularly reviewed in Section 3. In Section 4, we analyze Lv *et al.*'s protocol and give the proof of the defect in Lv *et al.*'s protocol [10]. Section 5 gives our new protocol and proves it can resist Lv *et al.*'s tracing attack [10]. A comparison between protocols of [9,10] and ours are shown in Section 6. Finally, the conclusions and the acknowledgement are given in Section 7 and Section 8 respectively.

2. Background of ECC

This section gives some background of ECC. The addition cyclic subgroup, consisted by the points on an elliptic curve over a finite field, is described and the general form of an elliptic curve is given. Then the ECDLP, the security is relied on in ECC, is mentioned.

2.1. Cyclic Group for ECC

The ECC has a set of points, generated by a primitive point, on the elliptic curve over finite field. These points and the point at infinity, denoted \mathcal{O} , construct an addition abelian group. Point \mathcal{O} is also said on the curve as an addition identity element of the addition abelian group. Then the ECC is established by taking advantage to the difficult ECDLP in cyclic subgroups of such elliptic curve groups. In the affine plane coordinate system, the elliptic curve equation in general form can be represented as $y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6$, known as the affine long Weierstrass equation. Let q be a great prime number, and let F_q denote the finite field of integers modulo q . The equation can be rewritten as its isomorphic curve form $y^2 = x^3 + ax + b \pmod{q}$ by changing variables transforms, where $q > 3$, $a, b \in F_q$ and $4a^3 + 27b^2 \neq 0 \pmod{q}$.

2.2. ECDLP

The security of ECC is based on the intractability of ECDLP. Given an elliptic curve E , over a finite field F_q , denoted $E(F_q)$. There is a point $P \in E(F_q)$ with prime order n . Then P generates the cyclic subgroup, $\langle P \rangle = \{\mathcal{O}, P, 2P, 3P, \dots, (n-1)P\}$, of $E(F_q)$. The public domain parameters are the prime q , the elliptic curve E , the primitive point P and its order n . When given the public domain parameters and a point Q in $\langle P \rangle$, to find the integer $K \in [1, n-1]$ such that $Q = kP$ is an ECDLP. The integer k is the discrete logarithm of Q to the base P , denoted $k = \log_P Q$ [13].

3. Related Works

In this section, the notations using throughout in this paper are given. The Batina *et al.*'s protocol [9], the Lv *et al.*'s tracing attack [10] on Batina *et al.*'s work and the protocol proposed by Lv *et al.* are reviewed in detail.

3.1. Notations

- P : a primitive point of a cyclic subgroup on an elliptic curve defined over a finite field;
- $\mathcal{X}(T)$: a function that allows to input an EC point $T = (x_i, y_i)$ and to return the x -coordinate x_i of the point T ;
- v : Verifier's private key;
- V : Verifier's public key, $V = vP$;
- s_i : Tag m 's private key;
- S_i : Tag m 's public key, $S_i = s_iP$.

3.2. The Privacy-Preserving ECC-Based Grouping Proof Protocol of Batina *et al.*

In the Batina *et al.*'s protocol [9], the tag and/or the reader will abort when a timeout occurs or when they receive the EC (Elliptic Curve) point at infinity. On the basis of public-key cryptography, each tag has its own private key and the public key of verifier before executing the protocol. On the other side, verifier has all tag's public key in backend database when tags have registered. Then, the details of protocol execution steps are described as follows and shown as **Figure 1**.

Reader sends the message "start left" to Tag A for assigning the role of tags.

- 1) Tag A generates a random number r_a and computes the corresponding EC point $T_{a,1} = r_aP$. Then Tag A sends $T_{a,1}$ to Reader.
- 2) Reader generates a random number r_s . Then Reader sends "start right", $T_{a,1}$ and r_s to Tag B.
- 3) Tag B generates a random number r_b and computes

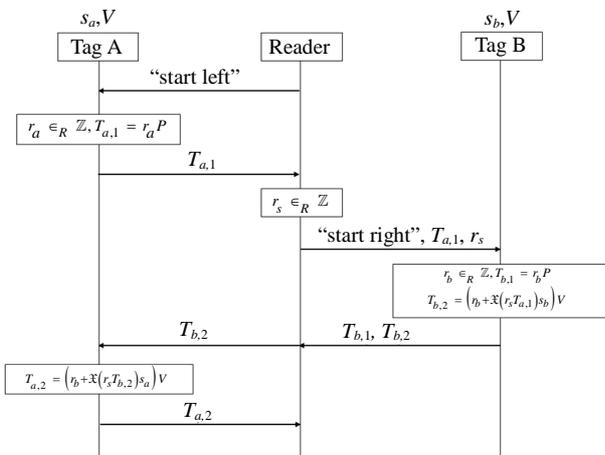


Figure 1. Batina *et al.*'s grouping-proof protocol [9].

EC points $T_{b,1} = r_bP$ and

$$T_{b,2} = (r_b + \mathcal{X}(r_s T_{a,1})s_b)V.$$

Then Tag B responds $T_{b,1}$ and $T_{b,2}$ to Reader.

4) Reader forwards $T_{b,2}$ to Tag A. Tag A computes $T_{a,2} = (r_a + \mathcal{X}(T_{b,2})s_a)V$, and sends $T_{a,2}$ to Reader.

5) Then Reader collects the grouping proof $\{T_{a,1}, T_{a,2}, r_s, T_{b,1}, T_{b,2}\}$ and forwards to Verifier.

6) At the last, Verifier verifies

$$S_a = (\mathcal{X}(T_{b,2}))^{-1} (v^{-1}T_{a,2} - T_{a,1})$$

and $S_b = (\mathcal{X}(r_s T_{a,1}))^{-1} (v^{-1}T_{b,2} - T_{b,1}).$

3.3. Vulnerability of Batina *et al.*'s Protocol

In 2011, Lv *et al.* [10] performed the tracking attack on Batina *et al.*'s grouping-proof protocol [9] in three phases to prove the vulnerability. These three phases are described as follows and shown as **Figure 2**.

Phase I:

Attacker eavesdrops on the normal messages

$T_{a,1}, T_{a,2}, r_s, T_{b,1}, T_{b,2}$ as her or his knowledge.

Phase II:

Attacker impersonates Reader to challenge Tag B with "start right", $T_{a,1}, r_s$, where $T_{a,1}, r_s$ got from Phase I. After receiving the challenge from the fake Reader, Tag B generates a new random number r'_b and computes $T'_{b,1} = r'_bP$ and

$$T'_{b,2} = (r'_b + \mathcal{X}(r_s T_{a,1})s_b)V.$$

Then, Tag B replies the message $T'_{b,1}$ and $T'_{b,2}$.

Phase III:

At the last phase, Attacker listens and waits for next normal session happening. In that case, Reader sends "start left" to Tag A. Then Tag A generates a new random number r''_a and calculates $T''_{a,1} = r''_aP$. Tag A replies $T''_{a,1}$ for Reader. Reader generates a new random number r''_s as $T''_{a,1}$ being received. Then Reader challenges Tag

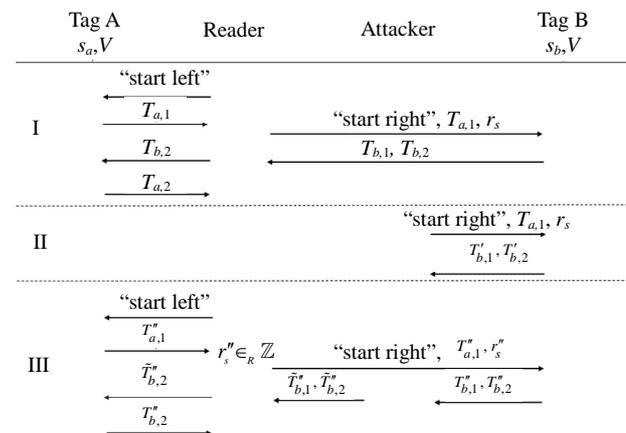


Figure 2. Lv *et al.*'s tracking attack [10].

B with $T_{a,1}''$ and r_s'' . After the challenge messages arriving, Tag B generates a random number r_b'' as well as computes $T_{b,1}'' = r_b''P$ and $T_{b,2}'' = (r_b'' + \mathcal{X}(r_s''T_{a,1}'')s_b)V$. Then Tag B replies $T_{b,1}''$ and $T_{b,2}''$. At this moment, Attacker blocks these messages and forges these messages as

$$\tilde{T}_{b,1}'' = T_{b,1}'' + T_{b,1}' - T_{b,1} = (r_b'' + r_a'' - r_b)P$$

and

$$\tilde{T}_{b,2}'' = T_{b,2}'' + T_{b,2}' - T_{b,2} = (r_b'' + r_b' - r_b + \mathcal{X}(r_s''T_{a,1}'')s_b)V.$$

Then Attacker sends $\tilde{T}_{b,1}''$ and $\tilde{T}_{b,2}''$ to Reader. And Reader forwards $\tilde{T}_{b,2}''$ to Tag A. Once Tag A received $\tilde{T}_{b,2}''$, Tag A computes $T_{a,2}'' = (r_a'' + \mathcal{X}(\tilde{T}_{b,2}'')s_a)V$ and replies it for Reader. Reader collects grouping proof messages as $\{\tilde{T}_{a,1}'', \tilde{T}_{a,2}'', r_s'', \tilde{T}_{b,1}'', \tilde{T}_{b,2}''\}$ and forwards to Verifier. Then Verifier verifies

$$S_a = (\mathcal{X}(\tilde{T}_{b,2}''))^{-1} (v^{-1}T_{a,2}'' - T_{a,1}'')$$

and

$$S_b = (\mathcal{X}(r_s''T_{a,1}''))^{-1} (v^{-1}T_{b,2}'' - T_{b,1}'')$$

successfully. Thus, Attacker can perform a tracking attack which makes the leakage of tag location in the protocol.

3.4. The Revised Protocol Proposed by Lv et al.

Lv et al.'s proposed a revised protocol [10] to resist tracking attack for Batina et al.'s protocol [9]. The revised protocol is shown in **Figure 3** and described as follows.

- 1) Reader sends the messages “start left” to Tag A.
- 2) Tag A generates a random number r_a and computes the corresponding EC point $T_{a,1} = r_aP$. Then Tag A sends $T_{a,1}$ to Reader.
- 3) Reader generates a random number r_s . Then Read-

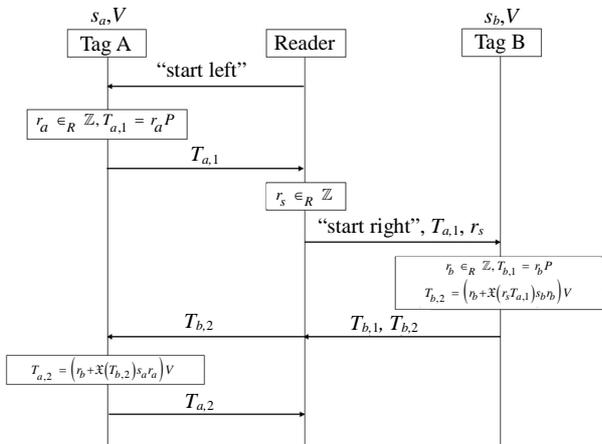


Figure 3. Lv et al.'s revised protocol [10].

er sends “start right”, $T_{a,1}$ and r_s to challenge Tag B .

- 4) Tag B generates a random number r_b and computes EC points $T_{b,1} = r_bP$ and

$$T_{b,2} = (r_b + \mathcal{X}(r_sT_{a,1})s_br_b)V.$$

Then Tag B responds $T_{b,1}$ and $T_{b,2}$ to Reader.

- 5) Reader forward $T_{b,2}$ to tag A. Tag A computes $T_{a,2} = (r_a + \mathcal{X}(T_{b,2})s_ar_a)V$ and sends $T_{a,2}$ to Reader.

- 6) Then Reader collects the grouping proof $\{T_{a,1}, T_{a,2}, r_s, T_{b,1}, T_{b,2}\}$ and forwards to Verifier.

- 7) At the last, Verifier verifies

$$s_aT_{a,1} = (\mathcal{X}(T_{b,2}))^{-1} (v^{-1}T_{a,2} - T_{a,1})$$

and

$$s_bT_{b,1} = (\mathcal{X}(r_sT_{a,1}))^{-1} (v^{-1}T_{b,2} - T_{b,1}).$$

4. The Impracticability of Lv et al.'s Revised Protocol

Batia et al.'s protocol [9] was designed on the basis of public-key cryptography, therefore public key and private key were involved. Basically, Lv et al.'s protocol [10] was revised from Batia et al.'s protocol [9]. Thus, Lv et al.'s revised protocol [10] should follow the principle of public-key cryptography. However, we find Lv et al.'s revised protocol [10] has impracticability on the basis of public-key cryptography.

In Lv et al.'s protocol [10], Reader collects the grouping proof $\{T_{a,1}, T_{a,2}, r_s, T_{b,1}, T_{b,2}\}$ and provides for Verifier to verify. Then in accordance with the step (7) in subsection 3.4, Verifier needs to compute $s_aT_{a,1}$ and $s_bT_{b,1}$. But based on public-key cryptography, Verifier cannot have tags' secret keys, s_a and s_b , to execute this verification. In the other case of $s_aT_{a,1} = s_ar_aP = r_aS_a$ and $s_bT_{b,1} = s_br_bP = r_bS_b$, Verifier can get tags' public keys, S_a and S_b , but cannot get r_a and r_b to compute r_aS_a and r_bS_b . Consequently, this verification cannot be completed. Obviously, Lv et al.'s protocol [10] is impracticable in the public-key cryptography.

5. Proposed Protocol

In this section, we propose a new protocol to satisfy the functionalities of Batia et al.'s protocol [9] and resist the Lv et al.'s attack model [10]. The new protocol is described step by step in subsection 5.1. Then we analyze the security of the protocol and use Lv et al.'s attack [10] to our protocol to show its resistibility for this kind tracing attack.

5.1. Protocol Description

The proposed protocol is described as the following steps

and shown as **Figure 4**.

- 1) Reader sends the message “start left” to Tag A.
- 2) Tag A generates a random number r_a and a nonce n_a . Then Tag A computes the corresponding EC point $T_{a,1} = r_a P$ and sends $T_{a,1}$ to reader.
- 3) Reader generates a random number r_s . Then Reader sends “start right”, r_s and $T_{a,1}$ to challenge Tag B.
- 4) Tag B generates a random number r_b and a nonce n_b . Then Tag B computes EC points $T_{b,1} = r_b P$ and $T_{b,2} = (r_b + \mathcal{X}(r_s T_{a,1})(s_b + n_b))V$. Then Tag B responds $T_{b,1}$ and $T_{b,2}$ to Reader.
- 5) Reader forwards $T_{b,2}$ to Tag A. Tag A computes $T_{a,2} = (r_a + \mathcal{X}(T_{b,2})(s_a + n_a))V$ and sends $T_{a,2}$ to Reader.
- 6) Then Reader collects the grouping proof $\{T_{a,1}, T_{a,2}, r_s, n_a, n_b, T_{b,1}, T_{b,2}\}$ and forwards to Verifier.
- 7) At the last, Verifier verifies

$$S_a + n_a P = (\mathcal{X}(T_{b,2}))^{-1} (v^{-1} T_{a,2} - T_{a,1})$$

and

$$S_b + n_b P = (\mathcal{X}(r_s T_{a,1}))^{-1} (v^{-1} T_{b,2} - T_{b,1}).$$

5.2. Analysis

In this section, we use Lv *et al.*'s attack [10] on our protocol and prove the protocol can resist this attack. As the tracking attack shown in **Figure 2**, the attacker eavesdrops on messages $T_{a,1}$, $T_{a,2}$, r_s , $T_{b,1}$, $T_{b,2}$ in Phase I. Then the attacker challenges Tag B by sending $T_{a,1}$ and

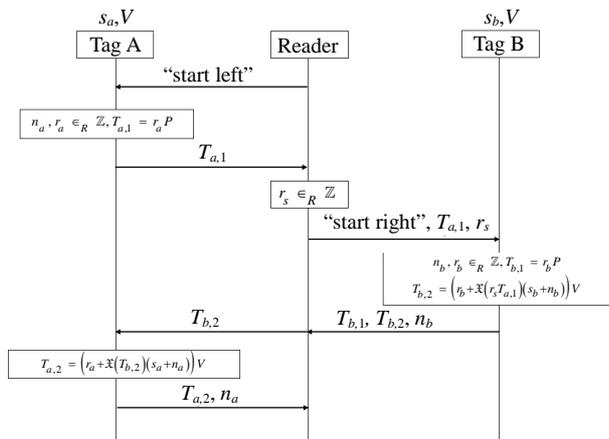


Figure 4. Proposed protocol.

Table 1. Comparison between ECC-based privacy-preserving grouping proof protocols.

Protocols	Public-key cryptosystem based	Untraceability	With nonce	Computation overhead
Batina <i>et al.</i> 's protocol [9]	Yes	No	No	$5M_{EC} + 2M_S + 2A_S$
Lv <i>et al.</i> 's protocol [10]	No	Yes	No	$5M_{EC} + 2M_S + 2A_S$
Our protocol	Yes	Yes	Yes	$5M_{EC} + 2M_S + 4A_S$

r_s . In our protocol, Tag B generates a nonce n'_b which guarantees every response include a different nonce in

$$T'_{b,2} = (r'_b + \mathcal{X}(r_s T_{a,1})(s_b + n'_b))V.$$

Then in the Phase III,

$$\tilde{T}''_{b,1} = T''_{b,1} + T'_{b,1} - T_{b,1} = (r''_b + r'_b - r_b)P$$

and

$$\begin{aligned} \tilde{T}''_{b,2} &= T''_{b,2} + T'_{b,2} - T_{b,2} \\ &= (r''_a + r'_b - r_b + \mathcal{X}(r''_s T''_{a,1})(s_b + n''_b) \\ &\quad + \mathcal{X}(r_s T_{a,1})(n'_b - n_b))V \end{aligned}$$

Verifier computes

$$(\mathcal{X}(\tilde{T}''_{b,2}))^{-1} (v^{-1} T''_{a,2} - T''_{a,1}) = S_a + n''_b P$$

and

$$\begin{aligned} &(\mathcal{X}(r''_s T''_{a,1}))^{-1} (v^{-1} \tilde{T}''_{b,2} - \tilde{T}''_{b,1}) \\ &= s_b P + n''_b P + (\mathcal{X}(r''_s T''_{a,1}))^{-1} \mathcal{X}(r_s T_{a,1})(n'_b - n_b) P \\ &\neq S_b + n''_b P. \end{aligned}$$

Therefore, the verification is failed. Thus our protocol can resist Lv *et al.*'s attack [10] and keep all secure properties of Batina *et al.*'s protocol [9].

6. Comparison with Previous Protocols

In this section we compare our protocol with previous ECC-based privacy-preserving grouping proof protocols as **Table 1**. At first, our protocol and Batina *et al.*'s protocol [9] are based on public-key cryptosystem that can avoid key management problem and support those applications which have large number of users. Both our protocol and Lv *et al.*'s protocol [10] can resist the tracking attack of [10] to possess untraceability, but our protocol is based on public-key cryptosystem that means our protocol has the practicability. To get better privacy security in our protocol, we needed additional two nonce involve in the protocol. In the last column of **Table 1**, we let M_{EC} , M_S and A_S denote the scale multiplication of elliptic curve point, scale multiplication and the scale addition separately. The protocol computation overhead is shown in this column. And our protocol is only two more scale addition operations than the other protocols.

7. Conclusion

In this paper, we have reviewed related papers those are based on ECC and provided the privacy-preserving grouping proof for RFID applications. Lv *et al.* [10] successfully attacked on Batina *et al.*'s protocol [9] in untraceability. And then they proposed revised Batina *et al.*'s protocol [9] to resist the tracing attack. However, we found that Batina *et al.*'s protocol [9] was designed on the basis of public-key cryptography, but Lv *et al.*'s revised protocol [10] cannot execute properly in public-key cryptography. During the execution of the Lv *et al.*'s protocol [10], Verifier cannot get tags' public keys to implement their verification. Besides, Verifier can get tags' public keys, but cannot solve the ECDLP from $T_{a,1}$ and $T_{b,1}$ to get r_a and r_b for the verification. Therefore, Lv *et al.*'s protocol [10] is impractical. To fix this problem, we propose a practical ECC-based privacy-preserving grouping proof protocol on the basis of public-key cryptography. We have proved that our protocol can resist the Lv *et al.*'s tracking attack [10] to complete the untraceability and inherits the security properties of Batina *et al.*'s protocol [9]. Therefore our new protocol provide the contributions to give the solutions for the defect of Lv *et al.*'s protocol [10] and the vulnerability of Batina *et al.*'s protocol [9] simultaneously.

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