On the Outage Capacity of Sensing-Enhanced Spectrum Sharing Cognitive Radio Systems in Fading Channels

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Abstract—Spectrum sharing has received an increasing amount of attention in cognitive radio over the past few years as an effective method of alleviating the spectrum scarcity problem in wireless communications by allowing unlicensed users to use the same spectrum as the licensed users under the condition of protecting the latter from harmful interference using a received interference power constraint at the licensed receivers. In this paper, we study the outage capacity and the truncated channel inversion with fixed rate (TIFR) capacity of a sensing-enhanced spectrum sharing cognitive radio system under two different scenarios, namely with and without missed-detection interference power constraints for the protection of the primary users, for both Rayleigh and Nakagami-μ fading channels. In our analysis, we consider various constraints on the capacity that include: (i) average transmit power constraints, (ii) peak interference power constraints, (iii) average interference power constraints and (iv) target detection probability constraints, and derive the power allocation strategy, as well as the TIFR and outage capacity for each scenario. Finally, we provide simulation results, which indicate that the cognitive radio system can achieve higher outage and TIFR capacity compared to the conventional non-sensing spectrum sharing cognitive radio system.

Index Terms—Spectrum sharing, outage capacity, optimal power allocation, spectrum sensing, cognitive radio.

I. INTRODUCTION

Cognitive radio [1], [2] has received a lot of interest over the past few years as a promising technology that aims to alleviate the spectrum scarcity problem by allowing unlicensed (secondary) users to access spectrum that is allocated to licensed (primary) users under the condition of preserving the quality-of-service (QoS) of the licensed networks. This is achieved by operating in a way that remains undetected by the primary users and that guarantees their uninterrupted communication in the presence of a secondary network. The research in cognitive radio has been significantly motivated by the demand for more and better wireless applications, as well as by the recent spectrum usage measurements [3], which have revealed that the spectrum under the current fixed spectrum allocation policy is severely underutilized.

Two main spectrum access approaches have been proposed for cognitive radio so far, regarding the way a secondary user accesses the licensed spectrum. The first one is called opportunistic spectrum access or interweave access scheme, according to which a secondary user is allowed to access a licensed frequency band only when the latter is detected not being used by the primary users [4]. The second spectrum access approach is called spectrum sharing or underlay access scheme, according to which the secondary users coexist with the primary users by using the frequency band at the same time as the primary users under the condition of protecting the latter from harmful interference that could be caused by their operation [5], [6]. Finally, a third hybrid spectrum access approach called sensing-based spectrum sharing was recently proposed that aims to increase the achievable throughput of the two aforementioned spectrum access schemes by combining their functions [7]. According to this last scheme, the secondary users use spectrum sensing to determine the status (active/idle) of a frequency band (as in the opportunistic spectrum access scheme) and choose their transmit power based on the decision of spectrum sensing, namely higher if it is idle and lower if the frequency band is active, in order to avoid causing harmful interference similar to the spectrum sharing scheme. The work in [7] focused mainly on increasing the throughput of opportunistic spectrum access cognitive radio networks. We will distinguish its application on improving the throughput of spectrum sharing cognitive radio networks by using the term “sensing-enhanced spectrum sharing”. This spectrum access technique leads to a sensing-throughput tradeoff problem similar to the one studied for the opportunistic spectrum access scheme in [8] and which was studied in [7] for the ergodic capacity of a sensing-based spectrum sharing cognitive radio system. The ergodic capacity is an effective metric for delay-insensitive applications such as data transmission, but not for delay-sensitive applications, such as voice and video transmission, where the outage capacity comprises a more appropriate metric for the capacity of a cognitive radio system.

The capacity of spectrum sharing systems was initially studied in [9], where the author derived the capacity for different additive white Gaussian noise (AWGN) channels under an average received-power constraint at a third party’s receiver, namely the receiver of the primary user, and showed that the received-power constraint results in a similar to the transmit power constraint capacity formula, which can be explained by the fact that for a non-varying channel the received-power
constraint is equivalent to a scaled transmit power constraint. In the following, we define the received-power constraint as interference power constraint, as it represents the interference caused to the primary user and which therefore should be limited for the protection of the QoS of the primary network. The capacity of spectrum sharing systems under various fading channel models was later studied in [5], where the authors studied the ergodic capacity of a point-to-point system under either a peak or an average interference power constraint at the primary receiver and showed that significant capacity gains can be achieved in time-varying channels due to fading, as opposed to the non-spectrum-sharing systems, namely systems where only a transmit power constraint is imposed, where the capacity is degraded under fading [10]. The ergodic, truncated channel inversion with fixed rate (TIFR) and outage capacity of a point-to-point system under joint average and peak interference power constraints was studied in [11] for Rayleigh fading channels. In the TIFR technique, the secondary transmitter uses the channel side information (CSI) to invert the channel fading and achieve a constant signal-to-noise ratio (SNR) at the secondary receiver during the periods when the channels fade above a certain “cutoff” value [10]. Finally, the ergodic and outage capacity of a point-to-point system under a combination of peak or average transmit and peak or average interference power constraints for various channel models was studied in [12].

In this paper, we consider a point-to-point sensing-enhanced spectrum sharing cognitive radio system and study the TIFR and the outage capacity under Rayleigh and Nakagami-\(m\) fading channels. More specifically, we consider several imposed constraints on the capacity that include: (i) average transmit power constraints, (ii) peak interference power constraints, (iii) average interference power constraints and (iv) target detection probability constraints, and derive the power allocation strategy, the TIFR and the outage capacity for each case. Finally, simulation results are provided for the two scenarios of the sensing-enhanced spectrum sharing system and the conventional non-sensing spectrum sharing cognitive radio system.

The rest of the paper is organized as follows. In Section II, we present the system and channel models, whereas the outage and TIFR capacity, as well as the imposed constraints are described in Section III. In Sections IV and V, we study the TIFR and the outage capacity under the sensing-enhanced spectrum sharing access scheme for a combination of various constraints that include average transmit and interference power constraints, peak interference power constraints and target detection probability constraints with and without missed-detection protection for the primary users, respectively. Finally, the simulation results are presented in Section VI and the conclusions are drawn in Section VII.

**Notations:** \(E\{\cdot\}\) denotes the expectation operation, \(\log\) represents the natural logarithm, \(\min(x, y)\) denotes the minimum of \(x\) and \(y\), \(P\) denotes power and \(\mathcal{P}\) probability.

## II. System Model

We consider the cognitive radio system presented in Fig. 1 that operates based on the sensing-enhanced spectrum sharing access scheme, according to which the secondary users perform spectrum sensing at the beginning of each frame and adjust their transmit power based on the sensing decision by using higher transmit power \(P_h\) if the frequency band is detected to be idle \((H_0)\) and lower transmit power \(P_l\) if it is detected to be active \((H_1)\). The frame structure of the cognitive radio system consists of a quiet sensing slot of duration \(\tau\) and a data transmission slot of duration \(T-\tau\), as shown in Fig. 2. Let \(g\) and \(h\) denote the instantaneous channel power gains from the secondary transmitter (SU-Tx) to the secondary receiver (SU-Rx) and the primary receiver (PU-Rx), respectively, as shown in Fig. 1. The channel power gains \(g\) and \(h\) are assumed to be independent and identically distributed (i.i.d.) ergodic and stationary random processes with probability density function (pdf) \(f_g(g)\) and \(f_h(h)\), respectively.

We consider similar to [7] the energy detection [15] in our simulation results, although it should be noted here that the following analysis does not depend on the spectrum sensing technique and therefore any spectrum sensing algorithm can be used to determine the status of the frequency band, e.g. [16]-[19]. The detection and false alarm probability under the energy detection scheme is given by

\[
\mathcal{P}_d = Q\left(\left(\frac{\epsilon}{\sigma^2} - \psi - 1\right)\sqrt{\frac{\tau f_s}{2\psi + 1}}\right),
\]

\[
\mathcal{P}_{fa} = Q\left(\left(\frac{\epsilon}{\sigma^2} - 1\right)\sqrt{\tau f_s}\right),
\]

respectively [8], where \(Q(\cdot)\) denotes the complementary cumulative distribution function that is given by \(Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{t^2}{2}\right) \, dt\), \(\tau\) represents the sensing time, \(\epsilon\) denotes the decision threshold of the energy detector, \(\psi\) is the received signal-to-noise ratio (SNR) from the primary user at the secondary detector and \(f_s\) represents the sampling function.

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**Fig. 1.** System model of the sensing-enhanced spectrum sharing cognitive radio system.

**Fig. 2.** Frame structure of the sensing-enhanced spectrum sharing cognitive radio system.
frequency. Finally, the noise is assumed to be circularly symmetric complex Gaussian (CSCG) with zero mean and variance $\sigma^2$, namely $\mathcal{CN}(0, \sigma^2)$.

III. IMPOSED CONSTRAINTS AND OUTAGE CAPACITY

The ergodic capacity studied in [7] is an effective metric for fast fading channels or delay-insensitive applications [20], where a block of information can experience all different fading states of the channel during transmission, whereas for slow fading channels or delay-sensitive applications, such as voice and video transmission, the outage capacity [20], [21] comprises a more appropriate metric for the capacity of the system due to the fact that only a cross section of the channel characteristics is experienced during the transmission period of a block of information. The outage capacity $C_{\text{out}}$ is defined as the highest transmission rate $R$ that can be achieved by the communications system, while keeping the probability of outage under a maximum value equal to $\mathcal{P}_{\text{out}}$ [21].

In the following two sections, we study the outage capacity of the sensing-enhanced spectrum sharing cognitive radio system for two different scenarios and derive the power allocation strategy for a combination of different constraints on the outage capacity that include average transmit power constraints, as well as average and peak interference power constraints. In the first scenario, we study the case of imperfect spectrum sensing where the secondary users impose an interference power constraint to protect the primary users in the case of missed-detection, namely when an active frequency band is falsely detected to be idle, whereas in the second scenario we consider a similar approach to opportunistic spectrum access, according to which the secondary user does not impose any interference power constraint if a frequency band is detected to be idle. In addition, we study the truncated channel inversion with fixed rate (TIFR) technique, where the secondary transmitter uses the channel side information (CSI) to invert the channel fading and achieve a constant signal-to-noise ratio (SNR) at the secondary receiver during the periods when the channels fade above a certain “cutoff” value [10]. This adaptive transmission scheme offers the advantages of non-zero achievable rates for a target outage probability $\mathcal{P}_{\text{out}} = \mathcal{P}_{\text{out}}$, even when the fading is extremely severe such as in Rayleigh fading cases, where a constant transmission rate cannot be achieved under all fading states of the channel.

In the following, we present the imposed power constraints on the sensing-enhanced spectrum sharing cognitive radio system’s capacity. Firstly, we consider an average (over all fading states) transmit power constraint that is imposed in order to keep the long-term power budget of the secondary users, similar to previous studies of the capacity [25], [26], that is given by

$$P_0 \geq 0, \quad P_1 \geq 0. \quad (4)$$

Finally, in order to effectively protect the primary users from harmful interference, we consider an average and a peak interference constraint for the first scenario (namely for the case of imperfect spectrum sensing where the secondary users impose an interference power constraint for the protection of the primary users in the case of missed detection), which can be expressed as follows

$$\frac{T - \tau}{T} \mathbb{E} \left\{ \mathcal{P}(H_1)(1 - \mathcal{P}_d)hP_0 + \mathcal{P}(H_1)\mathcal{P}_d h P_1 \right\} \leq Q_{\text{av}}, \quad (5)$$

$$\mathcal{P}(H_1)(1 - \mathcal{P}_d)P_0h \leq Q_{\text{peak}}, \quad \mathcal{P}(H_1)\mathcal{P}_d P_1h \leq Q_{\text{peak}}. \quad (6)$$

Here $Q_{\text{av}}$ and $Q_{\text{peak}}$ represent the maximum average and peak interference power, respectively. For the second scenario (namely when a similar approach to the opportunistic spectrum access is adopted, according to which the secondary user does not impose any interference power constraint if a frequency band is detected to be idle) the respective average and peak interference power constraints for a target detection probability $\mathcal{P}_d = \mathcal{P}_d$ can be formulated as follows

$$\frac{T - \tau}{T} \mathbb{E} \left\{ \mathcal{P}(H_1)\mathcal{P}_d h P_1 \right\} \leq Q_{\text{av}}, \quad (7)$$

$$\mathcal{P}(H_1)\mathcal{P}_d P_1h \leq Q_{\text{peak}}. \quad (8)$$

respectively. In the following sections, we consider both average and peak interference power constraints, in addition to the average transmit power constraint for the derivation of the capacity under the TIFR transmission policy and the outage capacity of the sensing-enhanced spectrum sharing cognitive radio system. However, all different combinations of the above constraints can be easily obtained under the following analysis by setting the respective parameters related to the constraint equal to zero.

IV. SENSING-ENHANCED SPECTRUM SHARING OUTAGE CAPACITY WITH MISSED-DETECTION PROTECTION CONSTRAINTS FOR THE PRIMARY USERS

In this section, we study the capacity under the TIFR transmission policy and the outage capacity of a sensing-enhanced spectrum sharing cognitive radio network that employs a missed-detection interference power constraint for the protection of the primary users. We consider both average transmit and interference power constraints, as well as peak interference power constraints on the capacity and derive the expressions of the TIFR and outage capacity under Rayleigh and Nakagami fading for both the channel between the secondary users and the channel between the secondary transmitter and the primary receiver.
As mentioned in the previous section, in the TIFR technique the transmitter inverts the channel fading, in order to achieve a constant rate at the receiver when the channel fading is higher than a “cutoff” threshold. We define here this threshold by \( \gamma_0 \) when the primary users are detected to be idle and by \( \gamma_1 \) when they are detected to be active. The respective transmit powers of the secondary user are given by

\[
P_0 (g, h) = \begin{cases} \frac{\alpha}{g}, & 0 < \frac{h}{g} \leq \frac{\gamma_0}{\tau_1} \\ 0, & \frac{h}{g} > \frac{\gamma_0}{\tau_1} \end{cases} \tag{9}
\]

\[
P_1 (g, h) = \begin{cases} \frac{\alpha}{g}, & 0 < \frac{h}{g} \leq \frac{\gamma_1}{\tau_1} \\ 0, & \frac{h}{g} > \frac{\gamma_1}{\tau_1} \end{cases} \tag{10}
\]

respectively, where the parameters \( \gamma_0 \), \( \gamma_1 \) and \( \alpha \) must be found such that the average transmit power constraint (3), the constraint (4), the average interference power constraint (5) and the peak interference power constraint (6) are met.

The transmit power in both cases is suspended (as in [11]) when the link \( g \) between the secondary transmitter and the secondary receiver is weak compared to the interference channel \( h \) from the secondary transmitter to the primary receiver. We consider here the same metric, i.e. \( \frac{h}{g} \), for the case that the primary users are detected to be idle, namely for \( P_0 \), in order to take into consideration the realistic scenario of imperfect spectrum sensing, so that to effectively protect the primary users from harmful interference when a miss-detection occurs, i.e. when an active frequency band is falsely detected to be idle, which might be unavoidable due to the limitations of the spectrum sensing techniques and the nature of wireless communications that include phenomena such as shadowing and fading.

### A. Rayleigh fading channels

In this subsection, we study the TIFR and outage capacity for the case that the channel between the secondary users and the channel between the secondary transmitter and the primary receiver follow the Rayleigh distribution. In this case, the respective channel power gains \( g \) and \( h \) would follow the exponential distribution with unit mean. In order to calculate the parameters \( \gamma_0 \), \( \gamma_1 \) and \( \alpha \), we need to derive an analytical expression of the average interference power constraint (5), the average transmit power constraint (3) and the peak interference power constraints in (6). Based on the aforementioned constraints and Appendix I, the parameter \( \alpha \) should satisfy the following constraints

\[
\alpha \leq \frac{Q_{\text{peak}} \sigma^2}{\mathcal{P} (H_1) (1 - \mathcal{P}_d) \gamma_0^2} = t_3^R (\gamma_0), \tag{13}
\]

\[
\alpha \leq \frac{Q_{\text{peak}} \sigma^2}{\mathcal{P} (H_1) \mathcal{P}_d \gamma_1} = t_4^R (\gamma_1), \tag{14}
\]

where the parameters \( K_0 \) and \( K_1 \) are given by

\[
K_0 = \mathcal{P} (H_0) (1 - \mathcal{P}_{fa}) + \mathcal{P} (H_1) (1 - \mathcal{P}_d), \tag{15}
\]

\[
K_1 = \mathcal{P} (H_0) \mathcal{P}_{fa} + \mathcal{P} (H_1) \mathcal{P}_d, \tag{16}
\]

respectively.

Furthermore, the outage probability of the sensing-enhanced spectrum sharing cognitive radio system under Rayleigh fading channels is proven in Appendix I to be equal to

\[
\mathcal{P}_{out,1} (\gamma_0, \gamma_1) = \frac{K_0 \sigma^2}{\gamma_0 + \sigma^2} + \frac{K_1 \sigma^2}{\gamma_1 + \sigma^2}. \tag{17}
\]

As a result, the channel capacity under the TIFR transmission policy for Rayleigh fading channels can be obtained as follows

\[
C^R_{TIFR,1} = \max_{\gamma_0, \gamma_1, \tau} \left\{ \frac{T - \tau}{T} \left[ (1 - \mathcal{P}_{out,1}) \log \left( 1 + \frac{1}{\sigma^2} \right) \right. \right.
\]

\[
\left. \cdot \min \left\{ t_1^R (\gamma_0, \gamma_1), t_2^R (\gamma_0, \gamma_1), t_3^R (\gamma_0, \gamma_1) \right\} \right\}. \tag{18}
\]

The maximum can be found by searching numerically for the optimal value of the sensing time \( \tau \) and the thresholds \( \gamma_0 \) and \( \gamma_1 \).

On the other hand, the outage capacity requires that a target outage probability is met. Therefore, the parameters \( t_1^R \) and \( t_2^R \) from (11) and (12) for \( \mathcal{P}_{out,1} (\gamma_0, \gamma_1) = \mathcal{P}_{out} \) take the following form

\[
t_1^R (\gamma_0) = \frac{Q_{av} T}{(T - \tau) \mathcal{P} (H_1)} \left\{ \log \left( 1 + \frac{\gamma_0}{\sigma^2} \right) - \frac{\gamma_0}{\gamma_0 + \sigma^2} \right\}
\]

\[
\cdot \left[ \frac{\mathcal{P}_d}{\mathcal{P}_{out} - \frac{K_0 \sigma^2}{\gamma_0 + \sigma^2}} + \frac{K_1}{\mathcal{P}_{out} - \frac{K_1 \sigma^2}{\gamma_1 + \sigma^2}} - 1 \right]^{-1}, \tag{19}
\]

\[
t_2^R (\gamma_0, \gamma_1) = \frac{T P_{av} (T - \tau)^{-1}}{K_0 \log (1 + \frac{\gamma_0}{\sigma^2}) + K_1 \log (K_1) - K_1 \log (K_1 - 1)
\]

\[
\cdot \log \left( \frac{\mathcal{P}_{out} - \frac{K_0 \sigma^2}{\gamma_0 + \sigma^2}}{\mathcal{P}_{out} - \frac{K_1 \sigma^2}{\gamma_1 + \sigma^2}} \right)^{-1}, \tag{20}
\]

respectively. Hence, the outage capacity of the sensing-enhanced spectrum sharing cognitive radio system under Rayleigh fading channels for a target outage probability equal to \( \mathcal{P}_{out} \) is given by

\[
C^R_{out,1} = \max_{\gamma_0, \tau} \left\{ \frac{T - \tau}{T} \log \left( 1 + \frac{1}{\sigma^2} \right) \min \left\{ t_1^R (\gamma_0), t_2^R (\gamma_0, \gamma_1) \right\} \right\}
\]

\[
\cdot \left( 1 - \mathcal{P}_{out} \right). \tag{21}
\]

The maximum can be found by searching numerically for the optimal value of \( \gamma_0 \) and \( \tau \).
B. Nakagami-m fading channels

In this subsection, we study the TIFR and outage capacity for the case that the channel between the secondary users and the channel between the secondary transmitter and the primary receiver follow the Nakagami-\( m \) distribution. The Nakagami-\( m \) distribution comprises a more versatile channel model that can better fit a wide range of empirical data by simply modifying the parameter \( m \). For a unit mean channel gain, the distribution of the channel power gain is given by [13]

\[
f_x(x) = \frac{m^m x^{m-1}}{\Gamma(m)} e^{-mx}, \quad x \geq 0, \tag{22}
\]

where \( m \) denotes the Nakagami-\( m \) fading parameter that ranges from \( \frac{1}{2} \) to \( \infty \) and measures the ratio of the line-of-sight (LOS) signal power to that of the multipath component. Finally, \( \Gamma(\cdot) \) represents the Gamma function that is given by [14]

\[
\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt. \tag{23}
\]

In order to calculate the parameters \( \gamma_0, \gamma_1 \) and \( \alpha \), we need to derive an analytical expression of the average interference power constraint (5), the average transmit power constraint (3) and the peak interference power constraints (6). Based on the aforementioned constraints and Appendix II, the parameter \( \alpha \) should satisfy the following constraints

\[
\alpha = \frac{T \rho^{m_h} (m_h + 1) B(m_g, m_h) Q_{av}}{(T - \tau) \mathcal{P}(H_1)} \left\{ \mathcal{P}_d \left( \frac{\gamma_1}{\sigma^2} \right)^{m_h+1} \right\} \cdot 2 F_1 \left( m_g + m_h, m_h + 1; m_h + 2; -\frac{\gamma_0}{\rho \sigma^2} \right) + (1 - \mathcal{P}_d) \cdot \left( \frac{\gamma_0}{\sigma^2} \right)^{m_h+1} 2 F_1 \left( m_g + m_h, m_h + 1; m_h + 2; -\frac{\gamma_0}{\rho \sigma^2} \right)^{-1} = t_1^N(\gamma_0, \gamma_1), \tag{24}
\]

Furthermore, the outage probability of the sensing-enhanced spectrum sharing cognitive radio system under Nakagami-\( m \) fading channels is proven in Appendix II to be equal to

\[
\mathcal{P}_{out,1}(\gamma_0, \gamma_1) = \frac{(\rho \sigma^2)^{m_g}}{m_g B(m_g, m_h)} K_0 \left( \frac{\gamma_0}{\gamma_1} \right)^{m_h} \cdot 2 F_1 \left( m_g + m_h, m_h; m_h + 1; -\frac{\rho \sigma^2}{\gamma_0} \right) + K_1 \left( \frac{\gamma_0}{\gamma_1} \right)^{m_h} 2 F_1 \left( m_g + m_h, m_h; m_h + 1; -\frac{\rho \sigma^2}{\gamma_1} \right). \tag{28}
\]

The maximum can be found by searching numerically for the optimal value of the sensing time \( \tau \) and the thresholds \( \gamma_0 \) and \( \gamma_1 \).

On the other hand, the outage capacity requires a target outage probability to be met, i.e. \( \mathcal{P}_{out,1}(\gamma_0, \gamma_1) = \mathcal{P}_{out} \). Based on the latter equation, the parameter \( \gamma_1 \) can be written as a function of the parameter \( \gamma_0 \)

\[
\Omega_{N,1}(\gamma_0) = 2 F_1 \left( m_g + m_h, m_g; m_h + 1; -\frac{\rho \sigma^2}{\gamma_0} \right) \cdot \bar{\gamma}_1^{m_h}, \tag{30}
\]

where \( \bar{\gamma}_1 \) denotes the parameter \( \gamma_1 \) as a function of \( \gamma_0 \), i.e. \( \bar{\gamma}_1 = \gamma_1(\gamma_0) \), and \( \Omega_{N,1}(\gamma_0) \) is given by

\[
\Omega_{N,1}(\gamma_0) = \frac{\mathcal{P}_{out} m_g B(m_g, m_h)}{K_1 (\rho \sigma^2)^{m_g}} - K_0 \left( \frac{\gamma_0}{\gamma_1} \right)^{m_h} \cdot 2 F_1 \left( m_g + m_h, m_g; m_h + 1; -\frac{\rho \sigma^2}{\gamma_1} \right). \tag{31}
\]

As a result, the outage capacity of the sensing-enhanced spectrum sharing cognitive radio system under average transmit, average and peak interference power constraints for Nakagami-\( m \) fading channels and for a target outage probability equal to \( \mathcal{P}_{out} \) is finally given by

\[
C_{TIFR,1}^N = \max_{\gamma_0, \tau} \left\{ \frac{T - \tau}{T} (1 - \mathcal{P}_{out}) \log \left( 1 + \frac{1}{\sigma^2} \right) \cdot \min \left\{ t_1^N(\gamma_0, \gamma_1), t_2^N(\gamma_0, \gamma_1), t_3^N(\gamma_0, \gamma_1) \right\} \right\} \tag{29}
\]

The maximum can be found by searching numerically for the optimal value of \( \sigma^2 \) and \( \tau \).

V. SENSING-ENHANCED SPECTRUM SHARING OUTAGE CAPACITY WITHOUT MISSED-DETECTION PROTECTION CONSTRAINTS FOR THE PRIMARY USERS

In this section, we study the capacity under the TIFR transmission policy and the outage capacity of a sensing-enhanced spectrum sharing cognitive radio network that does not employ missed-detection interference power constraints
A. Rayleigh fading channels

In this subsection, we study the TIFR and outage capacity for the case that the channel between the secondary users and the channel between the secondary transmitter and the primary receiver follow the Rayleigh distribution. In order to calculate the parameters $\gamma_0$, $\gamma_1$, and $\alpha$, we need to derive an analytical expression of the average interference power constraint (7), the average transmit power constraint (3) and the peak interference power constraints in (8). Based on the aforementioned constraints and Appendix III, the parameter $\alpha$ should satisfy the following constraints

$$\alpha = \frac{Q_{av} \cdot T}{(T - \tau) \mathcal{P}(H_1) \mathcal{P}_d \log \left(1 + \frac{\gamma_1}{\gamma_1 + \sigma^2}\right)} = q^R_1(\gamma_1),$$  
(35)

$$\alpha \leq \frac{TP_{av}}{(T - \tau) [K_0 E_1 \left(\frac{\gamma_1}{\sigma^2}\right) + K_1 \log (1 + \frac{\gamma_1}{\sigma^2})]} = q^{R}_{2}(\gamma_0, \gamma_1),$$  
(36)

$$\alpha \leq \frac{Q_{peak} \sigma^2}{\mathcal{P}(H_1) \mathcal{P}_d \gamma_1} = q^{N}_{3}(\gamma_1),$$  
(37)

where the parameters $K_0$ and $K_1$ are given from $K_0$ and $K_1$ in (15)-(16) for $\mathcal{P}_d = \mathcal{P}_d$.

Furthermore, the outage probability of the sensing-enhanced spectrum sharing cognitive radio system under Rayleigh fading channels is proven in Appendix III to be equal to

$$\mathcal{P}_{out,2}(\gamma_0, \gamma_1) = \frac{\bar{K}_0 - K_0 \exp \left(-\frac{\gamma_0}{\sigma^2}\right) + \bar{K}_1 \sigma^2}{\gamma_1 + \sigma^2}. $$  
(38)

The channel capacity under the TIFR transmission policy for Rayleigh fading channels can finally be obtained as follows

$$C^R_{out,2} = \max_{\gamma_0, \gamma_1} \left\{ \frac{T - \tau}{T} \left(1 - \mathcal{P}_{out,2}\right) \log \left(1 + \frac{1}{\sigma^2}\right) \cdot \min \left\{ q^R_1(\gamma_1), q^R_2(\gamma_0, \gamma_1), q^R_3(\gamma_0) \right\} \right\}. $$  
(39)

The maximum can be found by searching numerically for the optimal value of the sensing time $\tau$ and the thresholds $\gamma_0$ and $\gamma_1$.

On the other hand, the outage capacity requires that a target outage probability is met, i.e. $\mathcal{P}_{out,2}(\gamma_0, \gamma_1) = \mathcal{P}_{out}$. As a result, the parameter $\gamma_1$ can be written as a function of the parameter $\gamma_0$ as follows

$$\gamma_1(\gamma_0) = \frac{\bar{K}_1 \sigma^2}{\mathcal{P}_{out} - \bar{K}_0 + \bar{K}_0 \exp \left(-\frac{\gamma_0}{\sigma^2}\right)} - \frac{\sigma^2}{\gamma_1}. $$  
(40)

Therefore, the outage capacity of the sensing-enhanced spectrum sharing cognitive radio system under Rayleigh fading channels for a target outage probability equal to $\mathcal{P}_{out}$ is given by

$$C^R_{out,2} = \max_{\gamma_0, \tau} \left\{ \frac{T - \tau}{T} \left(1 - \mathcal{P}_{out}\right) \log \left(1 + \frac{1}{\sigma^2}\right) \cdot \min \left\{ q^R_1(\gamma_1), q^R_2(\gamma_0, \gamma_1), q^R_3(\gamma_0) \right\} \right\}. $$  
(41)

The maximum can be found by searching numerically for the optimal value of $\gamma_0$ and $\tau$.

B. Nakagami-\(m\) fading channels

In this subsection, we study the TIFR and outage capacity for the case that the channel between the secondary users and the channel between the secondary transmitter and the primary receiver follow the Nakagami-\(m\) distribution. In order to calculate the parameters $\gamma_0$, $\gamma_1$, and $\alpha$, we need to derive an analytical expression of the average interference power constraint (7), the average transmit power constraint (3) and the peak interference power constraints in (8). Based on the aforementioned constraints and Appendix IV, the parameter $\alpha$ should satisfy the following constraints

$$\alpha = \frac{T \rho^{m_h} (m_h + 1) B(m_g, m_h) Q_{av}}{(T - \tau) \mathcal{P}(H_1) \mathcal{P}_d \gamma_1^m \rho^{m_h} + 1^m} = q^N_1(\gamma_1),$$  
(43)

$$\alpha \leq \frac{Q_{peak} \sigma^2}{\mathcal{P}(H_1) \mathcal{P}_d \gamma_1} = q^N_3(\gamma_1),$$  
(44)
\[ \alpha \leq \frac{TP_{sv} (m_g + m_h - 1) B (m_g, m_h) \left( \frac{\gamma_1}{\sigma^2} + \rho \right)^{m_g + m_h - 1}}{T - \tau} \]

\[ \left( \frac{m_g + m_h - 1} {K_0^{-1} \left( \frac{\gamma_1}{\sigma^2} + \rho \right)} \right) \cdot \frac{1} {m_g B (m_g, m_h)} \cdot \left( \frac{\rho \sigma^2}{\gamma_1} \right)^{m_g} \cdot \left( \frac{\rho \sigma^2}{\gamma_1} \right)^{m_h} \cdot 2 F_1 \left( 1, m_g + m_h - 1; m + 1; \frac{\gamma_1}{\gamma_1 + \rho \sigma^2} \right)^{-1} \]

\[ = q_2^N (\gamma_0, \gamma_1), \]

\[ \alpha \leq \frac{Q_{peak} \sigma^2}{P (H_1) \mathcal{D}_{\gamma_1}^N} = q_3^N (\gamma_1). \] 

Furthermore, the outage probability of the sensing-enhanced spectrum sharing cognitive radio system under Nakagami-\( m \) fading channels is proven in Appendix IV to be equal to

\[ P_{out,2}^N (\gamma_0, \gamma_1) = \frac{K_0}{m_g \rho \sigma^2 B (m_g, m_h) \left( \frac{\gamma_0}{\sigma^2} \right)^{m_h}} \cdot \frac{2 F_1 \left( m_g + m_h, m_g; m_g + 1; -\frac{\gamma_0}{\rho \sigma^2} \right)} {m_g B (m_g, m_h)} \cdot \frac{K_1}{\gamma_1^{m_g}} \cdot 2 F_1 \left( m_g + m_h, m_g; m_g + 1; -\frac{\rho \sigma^2}{\gamma_1} \right). \] 

The channel capacity under the TIFR transmission policy can be obtained as follows

\[ C_{TIFR,2}^N = \max_{\gamma_0, \gamma_1, \tau} \left\{ \frac{T - \tau} {T} \left( 1 - P_{out,2}^N (\gamma_0, \gamma_1) \right) \log \left( 1 + \frac{1} {\sigma^2} \right), \right. \]

\[ \left. \cdot \min \{ q_1^N (\gamma_1), q_2^N (\gamma_0, \gamma_1), q_3^N (\gamma_1) \} \right\}. \] 

The maximum can be found by searching numerically for the optimal value of \( \gamma_0 \) and \( \tau \).

On the other hand, the target outage capacity requires that a target outage probability is met, i.e. \( P_{out,2}^{min} (\gamma_0, \gamma_1) = \mathcal{P}_{out} \), based on which, the parameter \( \gamma_1 \) can be written as a function of \( \gamma_0 \) as

\[ \Omega_{N,2} (\gamma_0) = 2 F_1 \left( m_g + m_h, m_g; m_g + 1; -\frac{\rho \sigma^2}{\gamma_1} \right) \left( \frac{\gamma_0}{\gamma_1} \right)^{-m_g}, \]

where \( \gamma_1 \) denotes the parameter \( \gamma_1 \) as a function of \( \gamma_0 \), i.e. \( \gamma_1 = \gamma_1 (\gamma_0) \) and \( \Omega_{N,2} (\gamma_0) \) is given by

\[ \Omega_{N,2} (\gamma_0) = \frac{m_g B (m_g, m_h) \mathcal{P}_{out} - \rho \tilde{K} \gamma_0^{m_h}} {K_1 (\rho \sigma^2)^{m_g} - \rho \tilde{K} \gamma_0^{m_h}} \cdot 2 F_1 \left( m_g + m_h, m_h; m_h + 1; -\frac{\gamma_0}{\rho \sigma^2} \right). \] 

As a result, the outage capacity of the sensing-enhanced spectrum sharing cognitive radio system under average transmit, average and peak interference power constraints for Nakagami-\( m \) fading channels and for a target outage probability equal to \( \mathcal{P}_{out} \) is finally given by

\[ C_{out,2}^N = \max_{\gamma_0, \tau} \left\{ \frac{T - \tau} {T} \left( 1 - \mathcal{P}_{out} \right) \log \left( 1 + \frac{1} {\sigma^2} \right), \right. \]

\[ \left. \cdot \min \{ q_1^N (\gamma_1), q_2^N (\gamma_0, \gamma_1), q_3^N (\gamma_1) \} \right\}. \]

VI. SIMULATION RESULTS

In this section, we present the simulation results for the outage capacity and the capacity under the TIFR transmission policy of the sensing-enhanced spectrum sharing cognitive radio system. We adopt the energy detection scheme [8] as a method of spectrum sensing, although any spectrum sensing technique [16]-[19] can be used under the previous analysis. The frame duration of the secondary system is set to \( T = 100 \) ms, the sampling frequency \( f_s \) is assumed to be 6 MHz, \( \mathcal{P} (H_0) = 0.6 \), the target detection probability is set to \( \mathcal{P}_{out} = 90\% \) for a received SNR from the primary user equal to \( \psi = -20 \) dB. The Nakagami-\( m \) fading parameters for the channels \( g \) and \( h \) are considered to be \( m_g = 2 \) and \( m_h = 2 \), respectively. Finally, the target outage probability is assumed to be \( \mathcal{P}_{out} = 0.1 \), whereas the maximum peak interference constraint \( Q_{peak} \) is related to the average interference constraint \( Q_{av} \) by \( \xi = Q_{peak} / Q_{av} \), as in [11]. In our simulations, we consider \( \xi = 1.5 \), \( P_{av} = 20 \) dBW, whereas the noise variance is considered to be \( \sigma^2 = 1 \).

In Fig. 3, the \( C_{TIFR} \) capacity of the cognitive radio system is presented versus the sensing time for Rayleigh and Nakagami fading channels under the scenario of Section IV and V, which are denoted in the figures and in the following by ‘1’ and ‘2’, respectively. It can be seen in Fig. 3 that the capacity under the TIFR transmission policy is a convex function of the sensing time, which yields that an optimal sensing time that maximizes the \( C_{TIFR} \) capacity exists. Furthermore, it can be seen that the \( C_{TIFR} \) capacity under the scenario of Section V, namely without missed detection protection constraints for the primary users, is significantly higher compared to the respective of the scenario with missed-detection protection constraints. This can be easily explained by the fact that the transmit power of the secondary users when the frequency band is detected to be idle is restricted by the average
interference power constraint for the scenario 1 (i.e. of Section IV) as opposed to the one of scenario 2 (i.e. of Section V), where no restriction applies on the transmit power for the secondary users besides the average transmit power constraint.

The latter remark can be also observed in Fig. 4, where the $C_{TIFR}$ capacity is presented versus the maximum average interference power $Q_{av}$. It is interesting to notice that the achievable $C_{TIFR}$ capacity under Nakagami fading channels is significantly higher compared to the respective of Rayleigh fading channels taking into consideration that not only the communication channel between the secondary users, but also the interference channel between the secondary transmitter and the primary receiver is more favorable as well. Another interesting observation that can be made is that the $C_{TIFR}$ capacity of the cognitive radio system under the scenario 1 increases significantly as the maximum interference power $Q_{av}$ receives higher values (or equivalently the distance between the secondary transmitter and the primary receiver increases), as opposed to the respective capacity under the scenario 2, which indicates that for the latter, the transmit power is mainly allocated during the periods when the frequency band is detected to be idle. Finally, it can be seen in Fig. 4 that for the scenario 2, a higher value of maximum interference power results in a bigger increase on the $C_{TIFR}$ capacity under less favorable channels (Rayleigh fading), compared to more favorable channels (Nakagami fading channels with parameters $m_g = 2$ and $m_h = 2$).

Another observation that can be made from Fig. 3 is that the optimal sensing time that maximizes the $C_{TIFR}$ capacity varies not only between the two scenarios, but also under different channel fading models. We investigate this further in Fig. 5, where the optimal sensing time that maximizes the $C_{TIFR}$ capacity is presented versus the maximum interference power $Q_{av}$. It can be seen that the optimal sensing time decreases as the maximum interference power $Q_{av}$ receives higher values, regardless of the scenario or the channel fading model. Furthermore, it can be observed that the optimal sensing time under the scenario 1 varies more with respect to the maximum average interference power $Q_{av}$ than that of the scenario 2, regardless of the channel fading model. Moreover, the optimal sensing time under more favorable channels for the scenario 2 is less susceptible to the maximum average interference power $Q_{av}$ and appears to be almost constant. The latter can be explained in conjunction with Fig. 4, where it can be seen that the power is mainly allocated during the periods that the frequency band is detected to be idle and that the contribution of the "spectrum sharing" component is more significant under higher values of maximum average interference power $Q_{av}$ and under less favorable communication channels, i.e. Rayleigh fading channels, where due to the severe fading, allocating more power during the periods when the frequency band is detected to be active results in an increased total capacity for the cognitive radio system.

In Fig. 6, the outage capacity is presented versus the sensing
radio systems and that this time varies with the channel capacity of the sensing-enhanced spectrum sharing cognitive system. The simulation results indicate there exists an optimal sensing time that maximizes the outage and the TIFR fading channels. The optimal sensing time decreases as the maximum average interference power $Q_{\text{av}}$ receives higher values or equivalently as the distance between the secondary transmitter and the primary receiver increases, regardless of the considered scenario. Finally, another interesting observation that can be made is that as both the communication and the interference channel become more favorable, the optimal sensing time decreases, whereas the outage capacity increases, as seen in Fig. 8. Different from the $C_{\text{TIFR}}$ capacity, the outage capacity increases significantly for both scenarios as the maximum average interference power $Q_{\text{av}}$ receives higher values, as well as for the two channel fading models considered. Finally, it should be noted that the $C_{\text{TIFR}}$ and the outage capacity under both scenarios and channel fading models is considerably higher compared to the conventional non-sensing spectrum sharing (SS) scheme, as seen from Fig. 4 and 8.

VII. CONCLUSIONS

In this paper, we studied the outage capacity and the truncated channel inversion with fixed rate (TIFR) capacity of a sensing-enhanced spectrum sharing cognitive radio system under two different scenarios, namely with and without missed-detection interference power constraints for the protection of the primary users, for both Rayleigh and Nakagami-$m$ fading channels. The simulation results indicate there exists an optimal sensing time that maximizes the outage and the TIFR capacity of the sensing-enhanced spectrum sharing cognitive radio systems and that this time varies with the channel fading model, the application of missed-detection interference power constraints for the protection of the primary users, as well as with the tolerable maximum average interference power or equivalently with the distance between the secondary transmitter and the primary receiver. Finally, it was shown that the sensing-enhanced cognitive radio systems can achieve significantly increased outage and TIFR capacity under both scenarios and channel fading models compared to the conventional non-sensing spectrum sharing cognitive radio systems.

APPENDIX I

ANALYTICAL EXPRESSIONS FOR THE SCENARIO OF SECTION IV UNDER RAYLEIGH FADING

A. Average interference power constraint

From the average interference power constraint (5), we have the following expression

$$\frac{T - \tau}{T} E_{g,h} \left\{ \mathcal{P} (H_1) (1 - \mathcal{P}_d) h P_0 + \mathcal{P} (H_1) \mathcal{P}_d h P_1 \right\} =$$

$$= \frac{T - \tau}{T} \left\{ \int_{\frac{h}{g} \leq \frac{\sigma}{g}} f(h) f(g) f_h dhdg + \int_{\frac{h}{g} \geq \frac{\sigma}{g}} f(h) f(g) f_h dhdg \right\} .$$

(52)

In order to calculate the above integrals, we define the new random variable $u = \frac{h}{g}$. It can be easily shown [5] that for Rayleigh fading channels $h$ and $g$, the random variable $u$ follows a log-logistic distribution given by $f_u (u) = \frac{1}{(1+u)^2}, \, u \geq 0$. This distribution is used to model the fading severity in many wireless communication systems.
0. Therefore, the equation (52) can be written as follows
\[
\frac{T - \tau}{T} \int_0^\frac{2\pi}{\alpha T} \mathcal{P}(H_1) \frac{(1 - \mathcal{P}_d)\alpha u}{(u + 1)^2} du + \frac{T - \tau}{T} \int_0^\frac{2\pi}{\alpha T} \mathcal{P}(H_1) \frac{\mathcal{P}(H_1) d\alpha u}{(u + 1)^2} du = \mathcal{P}(H_1) \frac{T - \tau}{T} \left[ (1 - \mathcal{P}_d) \left( \log \left(1 + \frac{\gamma_0}{\sigma^2}\right) - \frac{\gamma_0}{\gamma_0 + \sigma^2} \right) + \mathcal{P}_d \left( \log \left(1 + \frac{\gamma_1}{\sigma^2}\right) - \frac{\gamma_0}{\gamma_1 + \sigma^2} \right) \right].
\]

B. Average transmit power constraint

By defining the parameters \( K_0 = \mathcal{P}(H_0)(1 - \mathcal{P}_f_a) + \mathcal{P}(H_1)(1 - \mathcal{P}_d) \) and \( K_1 = \mathcal{P}(H_0)\mathcal{P}_f_a + \mathcal{P}(H_1)\mathcal{P}_d \), we can derive an analytical expression for the average transmit power constraint (3) under Rayleigh fading channels as follows
\[
\frac{T - \tau}{T} \mathbb{E}_{g,h} \left\{ K_0 P_0 + K_1 P_1 \right\} = \int_{\frac{\gamma_0}{\sigma^2}}^1 K_0 \alpha \mathbb{E}_{g,h} \left\{ f_h(h) f_g(g) dg dh \right\} + \frac{T - \tau}{T} \int_{\frac{\gamma_0}{\sigma^2}}^1 K_1 \alpha \mathbb{E}_{g,h} \left\{ f_h(h) f_g(g) dg dh \right\} = \frac{T - \tau}{T} K_0 \alpha \int_{\frac{\gamma_0}{\sigma^2}}^1 \left(1 + \frac{\gamma_0}{\sigma^2}\right) + \frac{T - \tau}{T} K_1 \alpha \int_{\frac{\gamma_0}{\sigma^2}}^1 \left(1 + \frac{\gamma_1}{\sigma^2}\right).
\]

C. Outage probability

The outage probability for the scenario of Section IV under Rayleigh fading channels is given by
\[
\mathcal{P}_{out,1}(\gamma_0, \gamma_1) = \mathcal{P} \left( \frac{h}{g} > \frac{\gamma_0}{\sigma^2} \right) H_0 \left( 1 - \mathcal{P}_f_a \right) H_0 + \mathcal{P} \left( \frac{h}{g} > \frac{\gamma_1}{\sigma^2} \right) H_1 \mathcal{P}_d + \mathcal{P} \left( \frac{h}{g} > \frac{\gamma_0}{\sigma^2} \right) H_1 \mathcal{P}_d = \int_{\frac{\gamma_0}{\sigma^2}}^1 K_0 (1 + u)^2 du + \int_{\frac{\gamma_0}{\sigma^2}}^1 K_1 (1 + u)^2 du = \frac{K_0 \sigma^2}{\gamma_0 + \sigma^2} + \frac{K_1 \sigma^2}{\gamma_1 + \sigma^2},
\]
where \( u \) is the random variable defined in Appendix I.A.

APPENDIX II

ANALYTICAL EXPRESSIONS FOR THE SCENARIO OF SECTION IV UNDER NAKAGAMI-\( m \) FADING

A. Average interference power constraint

In order to derive an analytical formula for the average interference power constraint (5), we need to calculate the distribution of the random variable \( w = \frac{h}{g} \), when \( g \) and \( h \) follow the Nakagami-\( m \) distribution. Considering the fact that the distribution of the ratio of two Gamma distributed random variables with parameters \( \alpha_1 \) and \( \alpha_2 \) is a beta prime distribution (also known as beta distribution of the second kind [27]) with parameters \( \alpha_1 \) and \( \alpha_2 \) [24], [22], the distribution of the random variable \( w \) is given by
\[
f_w(w) = \frac{m_w^{m_w} \Gamma(m_w + m_h)}{B(m_w, m_h) \Gamma(m_w + m_h + m_h)} \Gamma(m_w + m_h), \quad w > 0 \text{, where } m_g \text{ and } m_h \text{ denote the Nakagami-}\( m \) fading parameter for the channels } g \text{ and } h \text{, respectively, whereas the parameter } \rho \text{ and the beta function } B(m_g, m_h) \text{ are given by } \rho = \frac{m_g}{m_h}, \quad B(m_g, m_h) = \frac{\Gamma(m_g) \Gamma(m_h)}{\Gamma(m_g + m_h)}, \text{ respectively [14]. As a result, the constraint (5) can be written as}
\[
\int_{\frac{\gamma_0}{\sigma^2}}^1 \frac{T - \tau}{T} \mathbb{E}_{g,h} \left\{ \mathcal{P}(H_1) (1 - \mathcal{P}_d) h P_0 + \mathcal{P}(H_1) \mathcal{P}_d h P_1 \right\} = \int_{\frac{\gamma_0}{\sigma^2}}^1 \frac{T - \tau}{T} \int_{\frac{\gamma_0}{\sigma^2}}^1 \mathcal{P}(H_1) (1 - \mathcal{P}_d) \frac{\alpha w \rho_p}{w} \frac{w^{m_h - 1}}{(w + \rho)^{m_g + m_h}} \, dw + \frac{T - \tau}{T} \int_{\frac{\gamma_0}{\sigma^2}}^1 \mathcal{P}(H_1) \mathcal{P}_d \frac{w^{m_h - 1}}{(w + \rho)^{m_g + m_h}} \, dw = \int_{\frac{\gamma_0}{\sigma^2}}^1 \frac{T - \tau}{T} \mathcal{P}(H_1) \alpha \rho_m \frac{\gamma_0}{\sigma^2} \, m_h + 1 \right\} \cdot 2F1 \left( m_g + m_h, m_h + 1, m_h + 2, -\frac{\gamma_0}{\rho \sigma^2} \right) + \mathcal{P}_d \cdot \left( \frac{\gamma_1}{\sigma^2} \right)^{m_h + 1} 2F1 \left( m_g + m_h, m_h + 1, m_h + 2, -\frac{\gamma_1}{\rho \sigma^2} \right),
\]
where \( 2F1 \left( a, b; c; z \right) \) denotes the Gauss hypergeometric function [14] that is given by \( 2F1 \left( a, b; c; z \right) = \sum_{n=0}^\infty \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} \), for \( \text{Re}(c) > \text{Re}(b) > 0 \) and \( |z| < 1 \). Finally, \((\cdot)_n\) denotes the pochhammer symbol [14].

B. Average transmit power constraint

Using the parameters \( K_0 \) and \( K_1 \) given by (15) and (16), respectively, an analytical expression can be derived for the average transmit power constraint (3) under Nakagami-\( m \) fading channels as follows
\[
\int_{\frac{\gamma_0}{\sigma^2}}^1 \frac{T - \tau}{T} \mathbb{E}_{g,h} \left\{ K_0 P_0 + K_1 P_1 \right\} = \int_{\frac{\gamma_0}{\sigma^2}}^1 K_0 \alpha \frac{f_h(h) f_g(g) dg dh}{\frac{\gamma_0}{\sigma^2}} + \frac{T - \tau}{T} \int_{\frac{\gamma_0}{\sigma^2}}^1 K_1 \alpha \frac{f_h(h) f_g(g) dg dh}{\frac{\gamma_0}{\sigma^2}} = \frac{T - \tau}{T} \int_{\frac{\gamma_0}{\sigma^2}}^1 K_0 \alpha \frac{f_h(h) f_g(g) dg}{\frac{\gamma_0}{\sigma^2}} + \frac{T - \tau}{T} \int_{\frac{\gamma_0}{\sigma^2}}^1 K_1 \alpha \frac{f_h(h) f_g(g) dg}{\frac{\gamma_0}{\sigma^2}} = \frac{T - \tau}{T} \left( \frac{\gamma_0}{\sigma^2} \right)^{m_h} \cdot 2F1 \left( 1, m_g + m_h - 1, m_h + 1, -\frac{\gamma_0}{\rho \sigma^2} \right) + \mathcal{P}_d \cdot \frac{K_1 \cdot 2F1 \left( 1, m_g + m_h - 1, m_h + 1, -\frac{\gamma_1}{\rho \sigma^2} \right)}{\left( \frac{\gamma_1}{\sigma^2} + \rho \right)^{m_g + m_h - 1}}.\]
C. Outage probability

Using the random variable $w$ from Appendix II.A, the outage probability of the scenario of Section IV under Nakagami-$m$ fading channels can be derived as follows

\[ P_{\text{out},1}(\gamma_0, \gamma_1) = \mathcal{P} \left( \frac{h}{g} > \frac{\gamma_0}{\sigma^2} \right| H_0 \right) (1 - \mathcal{P}_{fa} \mathcal{P}(H_0) + \mathcal{P} \left( \frac{h}{g} > \frac{\gamma_1}{\sigma^2} \right| H_1 \right) \mathcal{P}(H_1) (1 - \mathcal{P}_d) = \]

\[ = \int_{\frac{\gamma_0}{\sigma^2}}^{\infty} K_0 \rho_{m_0} B(m_g, m_h) (w + \rho)^{m_g + m_h} dw + \int_{\frac{\gamma_1}{\sigma^2}}^{\infty} K_1 \rho_{m_0} B(m_g, m_h) (w + \rho)^{m_g + m_h} dw = \]

\[ = \frac{(\rho \sigma^2)^{m_g}}{m_g B(m_g, m_h)} \left[ 2 F_1 \left( m_g + m_h, m_g; m_g + 1; -\frac{\rho \sigma^2}{\gamma_0} \right) \right] + \frac{K_0}{\gamma_0} m_g + K_1 \cdot 2 F_1 \left( m_g + m_h, m_g + 1; -\frac{\rho \sigma^2}{\gamma_1} \right), \]

where \( E_1(\cdot) \) denotes the exponential integral of order 1 \[ [14] \] given by \( E_1(z) = \int_1^{\infty} e^{-zt} t^{-1} dt, \text{Re}\{z\} > 0 \).

C. Outage probability

The outage probability of the scenario of Section V under Rayleigh fading channels is given by

\[ P_{\text{out},2}(\gamma_0, \gamma_1) = \mathcal{P} \left( g < \frac{\gamma_0}{\sigma^2} \right| H_0 \right) (1 - \mathcal{P}_{fa} \mathcal{P}(H_0) + \mathcal{P} \left( g < \frac{\gamma_1}{\sigma^2} \right| H_1 \right) \mathcal{P}(H_1) (1 - \mathcal{P}_d) = \]

\[ = \int_0^{\frac{\gamma_0}{\sigma^2}} K_0 e^{-g} dg + \int_{\frac{\gamma_1}{\sigma^2}}^{\infty} \frac{K_1}{(1 + u)^2} du = \]

\[ = \bar{K}_0 - \bar{K}_0 \exp \left( -\frac{\gamma_0}{\sigma^2} \right) + \frac{K_1 \sigma^2}{\gamma_1 + \rho \sigma^2}, \]

where $u$ is the random variable defined in Appendix I.A.

APPENDIX IV

ANALYTICAL EXPRESSIONS FOR THE SCENARIO OF SECTION V UNDER NAKAGAMI-$m$ FADING

A. Average interference power constraint

From the average interference power constraint (7) under Nakagami-$m$ fading channels, we have

\[ \frac{T - \tau}{T} \mathbb{E}_{g,h} \left\{ \mathcal{P}(H_1) \mathcal{P}_d h P_1 \right\} = \]

\[ = \frac{T - \tau}{T} \int_{\frac{\gamma_0}{\sigma^2}}^{\infty} \mathcal{P}(H_1) \mathcal{P}_d h \frac{f_h(h) f_g(g) dhdg}{g} (53) \]

In order to calculate the above integral, we use the random variable $u = \frac{h}{g}$ of Appendix I.A, whose distribution is given by $f_u(u) = \frac{1}{(1 + u)^2}, u \geq 0$. As a result, the integral (53) can be written as follows

\[ \frac{T - \tau}{T} \mathbb{E}_{g,h} \left\{ \mathcal{P}(H_1) \mathcal{P}_d h P_1 \right\} = \int_{\frac{\gamma_0}{\sigma^2}}^{\infty} \mathcal{P}(H_1) \mathcal{P}_d \mathcal{P}_o \left[ \log \left( 1 + \frac{\gamma_1}{\sigma^2} \right) \right] \]

\[ = \frac{T - \tau}{T} \mathbb{E}_{g,h} \left\{ F_1 \left( m_g + m_h, m_g + 1; -\frac{\rho \sigma^2}{\gamma_1} \right) \right\} \]

B. Average transmit power constraint

By defining the parameters $\bar{K}_0$ and $\bar{K}_1$ from $K_0$ and $K_1$ in (15)-(16) for $\mathcal{P}_d = \mathcal{P}_d$, we can derive an analytical expression for the average transmit power constraint (3) under Rayleigh fading channels as follows

\[ \frac{T - \tau}{T} \mathbb{E}_{g,h} \left\{ K_0 P_0 + K_1 P_1 \right\} = \frac{T - \tau}{T} \int_{\frac{\gamma_0}{\sigma^2}}^{\infty} \frac{K_0}{g} f_g(g) dg + \]

\[ + \frac{T - \tau}{T} \int_{\frac{\gamma_1}{\sigma^2}}^{\infty} \frac{K_1}{g} f_h(h) f_g(g) dhdg = \]

\[ = \frac{T - \tau}{T} \bar{K}_0 \mathbb{E}_1 \left( \frac{\gamma_0}{\sigma^2} \right) + \frac{T - \tau}{T} \bar{K}_1 \mathbb{E}_1 \left( \frac{\gamma_1}{\sigma^2} \right), \]
C. Outage probability

The outage probability of the scenario of Section V under Nakagami-$m$ fading channels is given by

\[
\Phi_{\text{out},2}(\gamma_0^m, \gamma_1^m) = \Phi \left( g < \frac{\gamma_0^m}{\sigma^2} | H_0 \right) + \left( 1 - \Phi_{\text{fa}} \right) \Phi \left( H_1 \right) + \Phi \left( \frac{h}{g} > \frac{\gamma_1^m}{\sigma^2} | H_2 \right) \Phi \left( H_1 \right) \Phi \left( H_1 \right).
\]

\[
= \frac{g^m}{\gamma_1^m \bar{H}_0} \cdot 2 F_1 \left( m_g + m_h, m_h; m_h + 1; -\frac{\mu g}{\gamma_1^m} \right) + \frac{g^m \bar{K}_1 \left( \rho^2 \bar{H}_0 \right)}{m_g \bar{H}_0 \left( m_g, m_g \right)} + \frac{g^m \bar{K}_1 \left( \rho^2 \bar{H}_0 \right) 2 F_1 \left( m_g + m_h, m_h; m_h + 1; -\frac{\mu g}{\gamma_1^m} \right)}{m_g \bar{H}_0 \left( m_g, m_g \right)}.
\]

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