Mitigating Losses from Climate Change through Insurance

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Abstract

Climate change potentially amplifies catastrophe risks by raising the magnitude and frequency of certain extreme events in the long term. Its crucial feature is irreversibility. The availability and affordability of insurance will also be diminished in the presence of climate change. Weather losses can be mitigated by actions of the policyholder; however, the private returns from many mitigation investments may not be sufficient to warrant such investments on a short time scale. Insurance price could thus keep rising and catastrophe insurance could become less available in the near future. This study analyzes the potential implications of climate change for catastrophic risks and examines the appropriateness of longer term insurance contracts to protect insurers against catastrophic losses and changes in risk estimates over time. The major findings are as follows. Climate change essentially plays an important role in modeling catastrophic risks, especially in the tail of the loss distribution and for longer time scales. Mitigations can completely offset the impact of climate change. Longer term insurance contract may stimulate the incentive to invest on mitigation; however, risk capital required and annual premiums could increase due to the additional premium risk faced by the insurers.

Keywords: climate change, mitigation, catastrophic risk, insurance
1 Introduction

Natural disasters have caused more severe insured losses to property in recent years than in the past. The losses caused by great natural disasters have increased dramatically in recent years, especially after 1990. Based on the recent book\(^1\), catastrophes also have a more devastating impact on insurers over the past 15 years than in the entire history. Before 1988, the worldwide insured losses from natural disasters are rarely greater than $10 billion dollars. Nonetheless, after 1990, there is a radical increase in insured losses. The representative catastrophe events lead to those losses include Hurricane Ike in 2008, which lead to insured losses of $16 billion, Hurricane Andrew in 1992, which cost insurers $23.2 billion, and Hurricane Katrina in 2005, which caused up to $46.3 billion in damage. Higher population density along the coast and increasing development in hazard-prone areas further exacerbate the situation\(^2\). These facts manifest a radial change in the scale and rhythm of catastrophes, which in turn forecast that the future catastrophic losses could rise dramatically.

Climate change also potentially amplifies catastrophe risks and challenge current risk management strategies by raising the magnitude and frequency of certain extreme events. This phenomenon is pronounced in the highly exposed areas, such as US Gulf Coast and the Caribbean, rising hazards are likely to reduce the effectiveness of current risk mitigation and threaten insurability. Based on the conclusion of Intergovernmental Panel on Climate Change (IPCC (2007)), increasing concentrations of greenhouse gases, primarily human-induced, are the major source to warm the atmosphere and oceans\(^3\). The elevated temperatures raise sea level by expanding ocean water, increasing the rate at

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1 For more detailed figures, refer to Kunreuther and Michel-Kerjan (2009)
2 Based on Changnon (2003), Muir-Wood et al. (2006), Miller et al. (2008), and Crompton and McAneney (2008), the major source of rising damages caused by natural disasters comes from the concentration of human settlements and wealth in hazard-prone areas.
3 Human behaviors, including the burning of fossil fuels, deforestation, and other land use changes, contribute to the emission of carbon dioxide and other greenhouse gasses, such as methane, which have accumulated in the atmosphere since late 19th century. Greenhouse gasses trap heat more easily, resulting in higher surface air temperature. IPCC predicts that global average surface temperatures will increase 1.1°C~2.9°C under a low emission scenario and 2.4°C ~6.4°C under a high emission scenario. Stern (2007) also suggests that positive feedback mechanisms of climate change, such as releases of methane resulting from melting of permafrost and a reduced uptake of carbon that caused by shrinking Amazon forest, may amplify greenhouse gas concentrations and lead to global warming that is more severe than anticipated by climate models.
which glaciers and ice sheets melt ice into the oceans. The number, track, rainfall quantity, and intensity of tropical cyclones might also change with global warming, driving more intense and frequent natural disasters. Sea level rise and potential stronger storms pose a more intensive threat to the economy, particularly in the coastal areas. The recent report on coastal flooding management suggests that over the next 100 years, higher sea level provides an elevated base for storm surges to build upon and diminishes the rate at which low-lying areas drain, thereby extending coastal inundation from rainstorms. Greater flood damages are also driven by increases in shore erosion, removing protective dunes, beaches, and wetlands and thus leaving previous protected properties closer to the water’s edge.

Climate change alters the catastrophic risks by increasing existing risk over time. A crucial feature of climate change lies in its irreversibility: once the climate regime in an area has change, it is unlikely to restore to the original state. Failure to take the potential of climate change into account would lead insurers to underestimate insurance premiums and reserves for catastrophic risk exposures, causing higher possibility of bankruptcy or financial distress, which in turn affect insurability. The best way to handling effects of climate change for insurers is incorporating possible changes in weather extremes to assess and manage future catastrophic risks. Even through the nature of low-probability and uncertainties of these events leads to difficulties in estimating the impact of climate change on natural disasters and their resulting damage, it is critical to establish a reasonable catastrophic risk model considering potential climate change and the associated uncertainties in order to help insurers enhance the accuracy of risk assessment, improve risk management, and further strengthen long-term sustainability against the rising trend of natural disasters.

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4 IPCC (2007) predicts that sea level may rise 0.2~0.6 meter by 2010. Although the procedures may take several centuries, an irreversible melting of Greenland ice or collapse of the West-Antarctic Ice Sheet gives rise to a substantial increase in global sea level rise, about 5 to 12 meters, as indicated in Rapley (2006) and Wood et al. (2006).

5 U.S. Climate Change Science Program “Coastal Sensitivity to Sea-Level Rise: A Focus on the Mid-Atlantic Region”

6 The irreversibility feature of climate change is further analyzed in Heal and Kristrom (2002).

7 IPCC (2007) and Botzen (2009)
Implementing mitigations can effectively reduce the potential risk and maintain insurability, protecting lives and properties\textsuperscript{8}. These measures prevent or limit damage against future natural disasters. For example, houses can be retrofitted or reinforced to withstand hurricanes, rising floods, or storm surges. However, these measures can have high up-front costs and the probability of a catastrophe seems to be remote and is often underestimated by many house owners. Investment on mitigations will only be worthwhile if the cost, which incurred in the short term, is less than the net expected benefits, which accrue over the long term. Thus, mitigations are more likely to be worthwhile when there is a risk in the near future, when people are concerned more about the future, or when the returns continue over longer terms. Due to uncertainty in the timing and magnitude of impacts, and difficulties in quantifying projected benefits and costs, it is often difficult to decide whether one should implement a specific adaptation to prepare for climate change. These uncertainties are incorporated into our model to justify the result of the benefit-cost analysis. The discount rate also matters when one would like to assess the value of implementing adaptations because it reflects people’s attitudes toward the future and the degree of risk aversion. Individuals who have higher risk aversion or are more concern about the future tend to set a lower discount rate. As indicated by Kunreuther et al. (2009), extensive experimental evidence has shown that ‘hyperbolic’ temporal discounting has been applied by humans, meaning that even the next year has a very high discount rate. The implication lies in that homeowners tend to undervalue a mitigation investment, which requires high upfront cost but delayed expected benefits over time.

Mitigation measures can reduce the loss distribution and maintain insurability in the long term. Insurance can also effectively share and transfer the risk among a risk pool and thus help individuals to manage the residual risk that can not be eliminated by installing

\textsuperscript{8} According to IFRC (2001), worldwide investments of $40 billions in disaster preparedness, prevention, and mitigation have reduced global economic losses for $280 billion during the 1990s. Kreibich et al. (2005) analyzed the impact of building precautionary measures for the Elbe flood of Germany in 2002. They found that use of buildings and interior fitting adapted to flooding reduced damage to building by 46% and 53%, and damage to contents by 48% and 53%, respectively. Moreover, Kunreuther et al. (2009) model hurricane damage in New York, Texas, South Carolina, and Florida in situations with and without mitigations according to recent building code standards, The results for a 100-year hurricane indicate that mitigation could reduce potential losses by 61% in Florida, 44% in South Carolina, 39% in New York, and 34% in Texas. Saving in Florida alone due to mitigation would result in $51 billion for a 100-year and $83 billion for a 500-year event.
mitigations. With insurance, individuals can reduce financial exposures to catastrophes by speeding the recovery, maintaining business continuity, and reducing individual suffering. For example, Luechinger and Raschky (2009) estimated the utility losses caused by flood disasters in 16 European countries between 1973 and 1998 and empirically proved that insurance indeed enhance people’s welfare. Based on this study, the presence of flood insurance almost fully mitigates the decline in the levels of life satisfaction resulting from flood disasters. In order to encourage mitigations, setting premiums at a level that reflects the underlying risk is very important. This principle allows insurers to provide lower premiums to homeowners who implement mitigations in their properties. In return, insurers can reduce the frequency and severity of claims. While some researches support the annually renewal contract\(^9\), long term insurance has been proposed to be a way to stabilize insurance premiums for house owners in coastal areas. With potential climate change, it is vital to explore this issue more in depth and see how the price of multi-year policies will be affected by climate change and its associated uncertainties.

Scientific evidence shows that weather patterns are changing over the last century, causing sea level rise and intensifying the frequency and severity of future catastrophes\(^10\). The consequences for many coastal areas could be devastating. Homeowners in these areas can purchase property insurance to prepare for and recover from natural disasters. However, the availability and affordability of insurance will be diminished in the presence of climate change. Mitigation measures help to maintain the availability of affordable insurance for coastal properties, and thus are essential to provide better protection against losses in a long time scale. Due to myopia, underestimation of the risk, failure to learn from experience, interdependencies and budget constraints, homeowners

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\(^9\) Vellinga et al. (2001) and Bouwer and Vellinga (2005) contend that allowing insurers to sign short term contracts and to adjust premiums and coverage over time ensures the solvency of the insurers against impacts of climate change.

\(^10\) Stern Review (2006) and IPCC (2007) provide more details on the scientific evidence of climate change and its impacts. According to IPCC (2007), global temperatures have increased approximately 0.76°C and sea level has risen about 20 centimeters since 1900. In addition, heat waves, heavy precipitations, and intensified tropical cyclones have emerged more frequently during late 20\(^{th}\) century worldwide. Emanuel (2005), Webster et al. (2005), and Hoyos et al. (2006) suggest that an increased intensity and frequency of hurricanes are more likely to be induced by climate change through rising sea surface temperatures. Saunders and Lea (2008) further specify that 0.5°C increase in sea temperature is associated with 40% increase in hurricane frequency and activity.
are usually hesitate to pay the high cost of adaptations and receive the relative lower premium discount that reflect the reduction in expected annual losses\textsuperscript{11}. If adaptations are not installed, insurance price will keep rising and become less available in the next decades. Policymakers must consider these problems and incentivize homeowners to implement mitigations. These measures not only reduce the levels of potential damages but also help to maintain insurability in the long term. As pointed out in Mill (2007), insurers can regard climate change as a threat as well as a new business opportunity by developing innovative insurance products to stimulate adaptations. Confronting the rising trend of catastrophic risks associated with potential climate change, public and private sectors are suggested to cooperate to cope with the problem.

This study intends to analyze the implications of climate change for catastrophic risk and to examine the appropriateness of longer term insurance contracts to protect insurers against future catastrophic losses and changes in risk estimates over time due to climate change. Specifically, three major research questions have been raised and possible answers to them would be proposed in this study.

- Insurance markets functions well when loss distributions have certain statistical properties; well defined, low correlation both cross-sectional and inter-temporal, with thinner tail, etc. Are these properties preserved in a regime of climate change? How can we model the evolutions of losses under climate change?

- Weather losses can be mitigated by actions of the policyholder; however, the private returns from many mitigation investments may not be sufficient to warrant such investments on a short time scale. Climate change threatens to increase the level of risk and possibly also the returns from mitigation. How do we choose the optimal mitigation under conditions of climate change on a longer time scale?

- Given that climate change can impact both the demand for, and supply of, insurance, we may need to re-think the design of policies. In particular, it has been suggested that long term contracts might be appropriate both for the ability to facilitate mitigation and because they provide risk protection against future insurance availability and premium volatility. What is the difference between

\textsuperscript{11} Behavioral reasons why homeowners tend to delay or deny to install adaptation measures are explained in section 12.5 in Kunreuther and Michel-Kerjan (2009)
short and long term insurance contracts under climate change? What is the likely impact of cost of capital and Bayesian-updated serial correlation on risk capital and insurance premiums for short and long term policies?

The rest of this chapter is organized as follows. In section 2, I will investigate the statistical properties of the loss distribution in the regime of climate change and examine whether insurance market still function well given this properties. In section 3 and section 4, benefits and costs of mitigation measures on catastrophic losses for short and long time scales will be analyzed both by simulations and by using loss data in St Lucia, respectively. In particular, benefit-cost analysis on simulated losses and empirical losses in St Lucia will be conducted both in the presence and in the absence of climate change for different time scales and discount rates. In section 5, historical storm activity will be used to measure the statistics of climate change factor in the near future. I will fit these statistics into two models to estimate annual insurance premiums for a longer contract and then compare the insurance prices between these two models. Finally, cost of capital and Bayesian-updated serial correlation will be taken into account in section 6 to estimate annual premiums for short term and long term catastrophe insurance contracts. Section 7 concludes. Section 8 proposes future research on long term insurance.

2 Statistical Properties in the Regime of Climate Change

The insurance market functions well in a stable world, where loss distributions are well-define (predictable), with low cross-sectional and inter-temporal correlations, and with thinner tail. In the presence of climate change, will these statistical properties still be preserved? In this section, I will explore this issue by developing a simple catastrophic risk model with potential climate change and illustrate the evolution of risk in a representative property for different features of climate change. These features include a climate change factor, the average level of risk changes over time, and the associated uncertainties regarding the factor.

2.1 A Simple Catastrophic Risk Model with Potential Climate Change

As indicated in section 1, climate change could play a critical role in estimating catastrophic risk. In order to highlight the impact of climate change on the severity of the
catastrophic loss rather than that on the likelihood of a catastrophe, the model I construct is based on the settings where a catastrophe\textsuperscript{12} occurs with an annual probability $p$ for a total $T$ years and climate change influences only the catastrophic losses but not the frequency of the catastrophe. Timing of climate change is set to be a discrete uniform distribution\textsuperscript{13} during the $T$ years. That is, for each year, climate change may occur with possibility $1/T$. The occurrence of a catastrophe and the occurrence of climate change are mutually independent. Without climate change effect, the loss resulting from a catastrophe is assumed to be a constant, $L$\textsuperscript{14}. As climate change occurs in year $\tau$ ($1<=\tau<=T$), the potential catastrophic loss increases gradually with an annual growth rate, “$a$”, until year $T$. The annual growth rate, or the climate change factor, “$a$”, can be directly associated with an external index, such as the rise in the sea level. The higher the sea level, the greater the loss caused by a flood should a storm with heavy rain strike the hazard-prone area. The impact of the uncertainty of climate change on the potential loss works as follows, if climate change occurs in year $\tau$ and a storm hits the area after climate change in year $t$ ($>\tau$), the loss caused by the storm becomes $(1+a)^{t-\tau}L$. In contrast, the loss remains $L$ if climate change occurs later than a storm. The simulation is executed by three steps: First, assume that climate change occurs in year $\tau$, $\tau$ could be $1,\ldots, T$. The potential loss caused by a catastrophe is then specified as $L(s)$, $s=1,\ldots,T$, where $L(s)=L$ for $s<=\tau$ and $L(s)=(1+a)^{s-\tau}L$ for $s>\tau$. Next, the occurrence of a catastrophe is randomly simulated with an annual probability $p$ during the $T$ years for $n$ times. $n$ denotes the number of simulations. Given that climate change occurs in year $\tau$, if a catastrophe occurs in a year $s$, a catastrophic loss $L(s)$ specified above is assigned; otherwise, no loss is incurred. Finally, $\tau$ is supposed to follow discrete uniform distribution $[1,T]$. The statistics of the simulated losses will be collected. The parameters of the benchmark case of this simulation are as follows: $a=0.05$, $p=0.01$, $L=1$, $T=20$, $n=10^5$.

\textsuperscript{12} This simplest model assumes that catastrophes are only likely to occur once each year.
\textsuperscript{13} Based on the results of our simulation, if the climate change time follows exponential distribution with the parameter $1/T$, the tail of the simulated distribution is very close to the worst case scenario, where climate change occurs in the first year. In addition, the mean losses are greater if climate change time is an uniform distribution than if it is an exponential distribution for all scenarios. Thus, the setting of uniform climate change time is more practical and more close to the real scenario.
\textsuperscript{14} This is a simplified model. In section 3, this assumption will be relaxed and use the real data/empirical distribution of losses to substitute $L$. 

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2.2 Exact Distribution of the Catastrophic Loss

In the case of no climate change, i.e., $a=0$, assuming other parameters are the same in the benchmark case, the aggregate loss is simply the sum of the annual loss for 20 years.

$$\tilde{L} = \sum_{i=1}^{20} L_i,$$

where $P(L_i = 1) = 0.01, \ P(L_i = 0) = 0.99, \ i = 1, \ldots, 20$. It is easy to deduce that the aggregate loss follows a Binomial distribution with parameters $n=20, \ p=0.01$. The probability density function of the aggregate loss is given below.

$$P(\tilde{L} = k) = \binom{20}{k}(0.01)^k(0.99)^{20-k}, \ k = 0, 1, \ldots, 20.$$ 

The statistics of the aggregate loss, such as the expected value, standard deviation, skewness, kurtosis, and VaR (Value at Risk) at different confidence levels of the aggregate loss can be expressed as follows.

- **Expected Value**
  $$E(\tilde{L}) = np = 0.2$$

- **Standard Deviation**
  $$\text{s.d.}(\tilde{L}) = \sqrt{np(1-p)} = 0.445$$

- **Skewness**
  $$\text{skewness}(\tilde{L}) = \frac{1-2p}{\sqrt{np(1-p)}} = 2.2024$$

- **Kurtosis**
  $$\text{kurtosis}(\tilde{L}) = 3 - \frac{6}{n} + \frac{1}{np(1-p)} = 7.7505$$

- **VaR at 95%**
  $$\text{VaR}_{0.95} = \text{VaR}_{0.975} = 1, \ \text{VaR}_{0.99} = 2$$

In the benchmark case, where $a=0.05$, the aggregate loss can be separated into two components: the aggregate loss prior to the occurrence of climate change and the aggregate loss following the occurrence of climate change, i.e.,

$$\tilde{L} = \sum_{i=1}^{\tau} \tilde{L}_{i,1} + \sum_{j=\tau+1}^{20} \tilde{L}_{j,2},$$

where $P(\tilde{L}_{i,1} = 1) = 0.01, \ P(\tilde{L}_{i,1} = 0) = 0.99, \ i = 1, \ldots, \tau$,

$$P(\tilde{L}_{j,2} = (1.05)^{i-\tau}) = 0.01, \ P(\tilde{L}_{j,2} = 0) = 0.99, \ j = \tau + 1, \ldots, 20.$$ 

Given the time that climate change occurs, $\tau$, the conditional aggregate loss prior to climate change follows a Binomial distribution with parameters $n=\tau, \ p=0.01$, i.e.,

$$\sum_{i=1}^{\tau} \tilde{L}_{i,1} | \tau \sim \text{Bin}(\tau,0.01);$$

however, the conditional aggregate loss posterior to climate
change, \( \sum_{i=r+1}^{20} \tilde{L}_{i,2} | \tau \), can not be identified as a well-known distribution due to the cumulative effect of climate change on the potential losses. Nevertheless, given that the climate change could occur uniformly during 20 years, the expected value of the aggregate loss can be calculated. The general form of the expected aggregate loss with potential climate change can be presented as (1). The detailed derivations are available in section 3.8.

\[
E(\tilde{L}) = \frac{p \cdot (T+1)}{2} + \frac{p \cdot (1+a)}{a} \cdot \left[ \frac{(1+a)^T - 1}{a \cdot T} - 1 \right]
\] (1)

In the benchmark case, where \( a=0.05, p=0.01, L=1, T=20 \), the expected value of the aggregate loss is equal to 0.2422. Due to the complication of the aggregate loss, higher moments are difficult to be attained. The moment generating function of the aggregate loss with climate change is given as (11) in section 8. Although higher central moment of the distribution can generally be obtained by several times differentiating the log of the moment generating function of the aggregate loss with respect to the parameter \( \theta \) and setting the value with \( \theta=0 \), the derivation in this case involves differentiation of a product with up to \( T \) terms and complicates the procedures. The derivation of the variance of the loss distribution by double expectation theorem is also available in section 8. The variance can be expressed in (18), but it is not an explicit formula since the term \( E(\tilde{L}_2^2 | \tau) \) can not be derived. Thus, we have to resort to simulations to explore the statistical properties of the aggregate loss in the presence of climate change.

2.3 The Impact of Climate Change on Catastrophic Losses with a Certain Climate Change Factor

In this subsection, I will apply the catastrophic risk model constructed in section 2.1 to quantify the impact of climate change on catastrophic losses by setting a certain value to the climate change factor, “\( a \)”. Table 1 reports the statistics of the simulated losses for the case of no climate change factor (\( a=0 \)) and the benchmark case (\( a=0.05 \)) to compare the effect of climate change. These statistics include the expected value, the standard deviation, the skewness, the kurtosis of the losses, the VaR (Value at Risk), and the ES (Expected Shortfall). For VaR and ES, the values with confidence level of 95%, 97.5%, and 99% are presented. VaR and ES are indicators of the tail of loss distributions. In
comparison of two distinct distributions, the greater the value of VaR or ES for the same confidence level, the fatter tail the loss distribution. Value at Risk with confidence level \( \alpha \) (VaR\(_\alpha\)) simply denotes the \( \alpha \)-percentile of the loss distribution. The proportion of the loss greater than VaR\(_\alpha\) is at most \((1-\alpha)\). The formal definition of VaR is shown in (2).

\[
\text{VaR}_\alpha = \inf \{ l : P(L > l) \leq 1 - \alpha \} \quad (2)
\]

Expected Shortfall (ES\(_\alpha\)) is defined as (3), which is the expected value of the tail of the loss distribution with the loss threshold VaR\(_\alpha\).

\[
\text{ES}_\alpha = E(L | L \geq \text{VaR}_\alpha) \quad (3)
\]

The advantage of ES over VaR lies in that it is a coherent risk measure. The axiom of coherence was proposed by Artzner et al. (1997, 1999). VaR violates the axiom of subadditivity, thus is not a coherent risk measure. In the case of no climate change, the statistics from simulations are quite close to the corresponding values derived from the exact loss distribution. The expected loss from simulations is 0.2018 while the expected loss from exact distribution is 0.2. For the exact distribution of the aggregate loss with climate change, almost all statistics, except for the expected value, are not available to be compared with. The exceedance probability (EP) with a specific threshold is bounded by the Chernoff bounds\(^{15}\) which are based on the probability theory.

As shown in Table 1, in the presence of climate change (“\( a \)” increases from 0 to 0.05), the simulated expected loss increases only 20.32% while the tail statistics, such as VaRs and ESs, increase at least 26.60%. (The only exception is VaR\(_{99\%}\), which increases only 16.57%. This may be due to too few samples in the extreme tail.) This result means that the climate change factor generally has more impact on the tail of the loss distribution rather than on the expected loss.

Figure 1 depicts the EPs for different climate change factors (\( a = 1, 0.05, \) and 0.1). It indicates that climate change and its associated uncertainties are critical in modeling catastrophic risks and cannot be ignored, especially when the tail of the loss distribution is the focus of insurers or reinsurers. Climate change brings in higher inter-temporal

\(^{15}\) These bounds give exponentially decreasing bounds on tail distributions of the sum of independent random variables. It is better than the first or second moment based tail bounds such as Markov's inequality or Chebyshev inequality, which only yield power-law bounds on tail decay. The Chernoff bound of the distribution of the aggregate loss with climate change is presented in section 3.9. The lowest Chernoff bound is obtained by choosing the optimal parameter \( \theta \) such that the bound is minimized. Figure A-1 in section 3.9 shows the plot of Chernoff bounds versus various thresholds for different time horizons.
correlations, which in turn lead to a fatter-tail loss distribution over time. The impact of climate change on the loss cannot be measured accurately, leading to unstable loss distributions, which create challenges to estimate the loss distribution with reliable parameters. These statistical properties could increase risk capital required and depress the supply or even the availability of insurance.

2.4 Uncertainty of Climate Change Factor

In section 2.3, the impact of climate change is certain (with a constant annual growth rate “a”). However, the impact of uncertain climate change should be of more concern for the insurance industry because the uncertainty is directly associated with the pricing and risk management of insurance contracts and insurance-linked instruments. This section further introduces the uncertainty of climate change factor into the simple catastrophic risk model. In particular, climate change factor, “a”, follows a discrete uniform distribution.

Table 2 reports the statistics of simulated losses for different settings on the uncertainty of the climate change factor. Column (1) is the case of a certain climate change, where “a”=0.05. Column (2) shows the statistics in the case of an uncertain climate change, where “a” takes three possible values: “a”=0.025, 0.05, or 0.075, each with probability of 1/3. Column (3) exhibits the statistics in the case of a more uncertain climate change, where “a” takes five possible values: “a”=0, 0.025, 0.05, 0.075, or 0.10, each with probability 1/5. Column (4) ((5)) shows the impact of the uncertainty on the statistics in term of percentage: percentage changes from column (1) to column (2) ((3)). Among all statistics, the tail probabilities are affected by the uncertainty of climate change factor to the greatest extent. As can be seen in Table 2, the contribution of uncertainty of climate change on the statistics of simulated losses most ranges from 0.62% to 4.62%. Nevertheless, the tail probabilities with threshold greater than the value of the house will increase substantially to over 9%.

Figure 2 exhibits the EP curves for different settings on the uncertainty of the climate change factor. The EP curve of “a=0.025~0.075” is quite close to the EP curve of “a=0.05”. The EP curve of “a=0.00~0.10” demonstrates a slightly fatter tail than the EP curve of “a=0.05”. However, a certain change of “a”, from 0.05 to either 0 or to 0.1, will
cause a large shift of the EP curve, compared with the “uncertain” change 
(a=0.025~0.075 or 0~0.1). As demonstrated in Figure 3, the EP curve of “a=0.00~0.10” 
matches well with the EP curve of “a=0.06”. That is to say, the impact of the uncertainty 
(from “a=0.05” to “a=0~0.1”) is equivalent to the impact of a small certain change (from 
“a=0.05” to “a=0.06”). To sum up, the uncertainty of climate change factor leads to a 
fatter-tail loss distribution, but its impact is small but positive, equivalent to a small 
certain increase in climate change factor.

3 Climate Change, Optimal Mitigations, and Time Scales

Catastrophic losses can be mitigated by investment on mitigation measures, which 
reduce both the expected value and the tail of the loss. The benefit comes from the 
reduction of potential losses while the cost is reflected in the price of the mitigation 
measures. For a short time scale, policyholders may hesitate to make the investment since 
short term benefit can not cover the cost. However, the threat of potential climate change 
stimulates us to consider risk from a longer term perspective as well as enhances the 
returns of mitigations. In this section, the optimal mitigation under conditions of climate 
change on a longer time scale will be explored.

3.1 Model Setting for Benefit-Cost Analysis

Without relevant scientific and engineering data and professional knowledge\textsuperscript{16}, an 
empirical evaluation of mitigation costs and benefits is a daunting task. In this section, we 
conduct benefit-cost analysis by simulations rather than by empirical evaluations. The 
mitigation level is defined as the reduction of losses when a catastrophe destroys a 
property. For example, if a hurricane hits a house with a reinforced roof, the damage to 
the house will reduce 10% compared to the damage to the same house with no reinforced 
roof, the mitigation level is set to be 0.1. Mitigation cost should be an increasing and 
convex function of the mitigation level because homeowners have to pay higher price for 
a higher level of protection and the marginal cost of mitigations should increase with 

\textsuperscript{16} With these data at hand, the most extensively used approach to determine the appropriate mitigation 
measure is by applying benefit-cost analysis. Smyth et al. (2009) employ benefit-cost analysis on various 
seismic retrofitting measures to mitigate losses arise from earthquakes in Istanbul area.
mitigation levels. In particular, mitigation cost is set to be $0.05 \cdot m + 0.1 \cdot m^2 + 0.05 \cdot m^3$, where $m$ stands for the mitigation level. Once a catastrophe strikes an area, it will give rise to catastrophic cost, which includes the direct cost and the indirect cost. The direct cost is simply the expected loss caused by the catastrophe. The indirect cost, such as suspension of normal operations, interruption of public transportation, and costs of human lives, is linked to the tail of the catastrophic loss. When many homes are mitigated there is an added benefit to the insurer in the form of lower catastrophic losses and lower capital costs in addition to the reduction in claims from each individual policy. This additional benefit will be considered the reduction of the indirect cost. Suppose the indirect cost is proportional to tail probabilities and is set to be the weighted sum of the EPs with different thresholds. Specifically, the cost of catastrophe $= \text{E}[L] + \text{Tail factor}(L)$.

$$\text{Tail factor}(L) = \sum_{i=1}^{9} \text{weight}(i) \cdot P(L > \overline{L}_i)$$

(weight$(i) = (0.5) \cdot (1 - (0.1) \cdot (i - 1))$, i=1,…,9, and $\overline{L}_i = 1 + 0.5 \cdot (i - 1)$).

This setting allows the tail factor to assign more weights on extreme losses rather than moderate losses. For example, the weight on $P(L > 5)$ is 2.7 while the weight on $P(1.5 > L > 1.0)$ is only 0.5. The rationale of assigning more weight on a more extreme loss lies in that insurers will encounter a great deal of claims after an extreme disaster, causing them more likely to default on claims or even declare bankruptcy. In order to avoid the consequences of a larger loss, insurers would reserve more capital in advance. The more capital reserved, the higher the cost of capital, thus the more weight to place in the extreme loss.

### 3.2 Issues on Discount Rates

The conclusions of benefit-cost analysis on a specific project are quite sensitive to the discount rates, especially when the project involves long time scales. The most popular debate which drew great attentions among economists on the global warming policy is the Stern Review on the Economics of Climate change, published in 2006 by the U.K. government. The Stern Review urges that prompt and sharp actions should be undertaken to abate potential catastrophic damages caused by greenhouse gas emissions. The major controversy on the results of Stern Review stems from the choice of very low discount rates.
rate. A large collection of comments on the Stern Review focuses on the discount rate issues, including Gollier (2006), Nordhaus (2007), Weitzman (2007), Dasgupta (2007), Heal (2009), and Mendelsohn (2008). The determination of the proper discount rate is associated with the pure rate of time preference, the measure of relative risk aversion, and the consumption growth over time, as established in the Ramsey equation. Based on Ramsey F. (1928), the equilibrium of real return on capital is governed by (5).

\[ r = \delta + \eta \cdot g \] (5)

\( r \) denotes the interest rate; \( \delta \) is the pure rate of time preference; \( \eta \) stands for the elasticity of the marginal utility of consumption, or a measure of relative risk aversion; \( g \) represents the consumption growth rate. The choice of the appropriate discount rate involves ethical judgments on preference for intergenerational utility, preference for equality within the whole society, and concern about the balance between economic development and environmental protection. Our simulations do not make a subjective judgment and thus specify various possible discount rates for analyzing the cost and benefit with optimal mitigation for different time scales.

3.3 Benefit-Cost Analysis by Simulations

The optimal mitigation level is determined by two opposite forces: A higher mitigation level costs more, but it reduces the expected losses and tail probabilities by abating the damage caused by a catastrophe. Thus, the optimal mitigation level can be achieved by minimizing the total cost, the sum of the price of mitigation measures and the damage. Figure 4 demonstrates the simulation results in the case of 20-years and 10% discount rate. The optimal mitigation level is 25%.

In order to analyze the benefit and cost of mitigations, I define optimal mitigation cost, benefit from optimal mitigation, net benefit from the optimal mitigation, and relative net benefit (RNB) from optimal mitigation as follows. Optimal mitigation cost is simply the price of mitigation measures at the optimal level of mitigation. Benefit from optimal mitigation reflects the reduction of losses resulting from the optimal mitigation. Net benefit from the optimal mitigation is the difference between benefit from optimal mitigation and optimal mitigation cost. Relative net benefit (RNB) from optimal mitigation is the ratio of net benefit from optimal mitigation to the total cost of no
mitigation. The purpose to derive this quantity is to compare the relative magnitude of net benefit from mitigations among different time scales. Table 3.3 reports the summary of the cost/benefit of optimal mitigation for different time scales with different discount rates (0% and 5%). No matter which discount rate is specified, the optimal mitigation cost increases with time scales. For instance, if discount rate=0%, when the time scale increases from 1 to 20, the optimal mitigation cost increase from 0 to 0.00563. This is because the longer time scale we are considering, the higher mitigation level will optimize the welfare (benefits minus costs). Cost reduction from optimal mitigation also magnifies with the time scale and with a higher speed, causing the net benefit (and the RNB) from optimal mitigation increases with time scales. In the case of 0% discount rate, RNB monotonically increases with the time scale from 0% to 45.44%. Based on the analysis, for a longer time scale, net benefit for all parties involved in catastrophic risk is more enhanced by investing in the mitigation measures.

Moreover, by comparing the optimal mitigation levels and net benefits from the optimal mitigation with various discount rates, there are two findings. First, the minimal time horizon that makes the investment on mitigation worthwhile increases with the discount rate. When the discount rate is 0%, the minimal time horizon that generates a positive optimal mitigation level is 2 years. As the discount rate rises to 5%, the minimal time horizon increases to 6 years. Second, for a given time scale, RNB from optimal mitigation always declines with discount rate. For instance, given time scale of 10 years, as the discount rate rises from 0% to 5%, RNB decreases from 32.49% to 4.73%. These findings suggest that a higher discount rate impedes the incentive to invest on mitigation measures, consistent with the first argument on the relationship between equity and climate change in Heal (2009): “We are less future-oriented—the CDR (consumption discount rate) is higher—and so we place less value on stopping climate change.” In sum, as human beings care more about the future, no matter in terms of setting a low discount rate or a longer time scale in modeling catastrophic risk, investing more on mitigation facilities will boost their utilities.

Climate change and mitigations have opposite effect on the total cost in the analysis. Climate change will increase total cost while mitigations will reduce total cost. Which effect dominates the other and how large is the aggregate effect? In order to answer these
questions, Table 4 reports the total cost in four scenarios: (1) no climate change, no mitigation (2) no climate change, with mitigation (3) climate change, no mitigation (4) climate change, with mitigation. Climate change effect measures the increase of total cost due to climate change and is reflected in the discrepancy between scenario (3) and scenario (1) (or scenario (4) and scenario (2)). Mitigation effect gauges the decline of total cost due to mitigation measures and can be captured by the difference between scenario (2) and scenario (1) (or between scenario (4) and scenario (3)). Aggregate effect combines both effects and is measured in terms of the difference between scenario (4) and scenario (1). Table 5 shows climate change effect, mitigation effect, and aggregate effect by calculating the percentage change described above.

As expected, the total cost magnifies with climate change factor but declines with mitigations. In Table 4, if discount rate=0%, T=5, total cost of no mitigation no climate change is 0.0729, which is less than 0.0733, total cost of no mitigation with climate change but greater than 0.0509, total cost of optimal mitigation no climate change. In some cases, especially those with a long time scale and low discount rate, mitigation effect dominates climate change effect. This shows that the implementation of mitigation measures plays a very important role to reduce catastrophic losses even in the presence of climate change. Based on the observation in Table 5, climate change effect is generally more significant for discount rate=5%, but aggregate effect is more substantial for discount rate=0%. For example, in the case of T=15, if 5% discount rate is specified, climate change increases total cost for 13.71% while mitigations decrease total cost for 11.82%; however, if 0% discount rate is specified, climate change increases total cost for 6.17% while mitigations decrease total cost for 38.23%. In addition, mitigation effect dominates for a longer time scale. In the case of 0% discount rate, if we increase the time scale from 10 years to 20 years, mitigation effect rises from 32.48% to 45.43%. These results justifies the fact that returns of mitigation measures will be sufficient to warrant such investment for longer time scales and for individuals who care more about the future.
4 Simulations on Catastrophic Risk using Empirical Loss Data:

Hurricane Risk in St. Lucia

The World Bank recently issues a report, which is a work collaborated with IIASA, RMA (Risk Management Solutions), and Wharton Risk Center, aiming to analyze the impact of cost-effective mitigation measures on the reduction of losses caused by natural disasters. This report contains several case studies, which quantitatively estimate the potential losses and implement cost-benefit analysis on various mitigation measures across different areas, including hurricane risk in St Lucia, flood risk in Jakarta, earthquake risk in Istanbul, and flood risk in Uttar Pradesh. In this section, we focus on the case study in St Lucia. EPs, losses, mitigation costs and the impact of four different mitigation measures (no mitigation, roof upgrade, opening protection, and roof & opening mitigations) on losses for different building types in various areas are provided by RMS. Because wood frame buildings in Canaries are proven to be the most effective one from mitigations, we use this case as an example. The empirical data are incorporated into our simple catastrophic risk model to simulate the situation where climate change could occur in the next 20 years. The impact of climate change and mitigation measures will also be explored for various time scales.

4.1 Impact of Climate Change and Mitigation Measures

The first step is using Monte Carlo Simulation and interpolation to randomly generate EP curve that matches data points of the loss in St. Lucia in the World Bank Report. Next, the EP curve is incorporated into our simple model described in section 2.1 to estimate the impact of climate change on hurricane risk. Figure 5-8 depict the EP curves for the wood frame building in Canaries in 20 years for four different mitigation measures and different climate change factors. A comparison of these four mitigation measures indicates that the impact of climate change declines with more or stronger mitigation measures. As can be observed, the discrepancy between “a=0.1” and “a=0” narrows down to a great extent in the presence of mitigation measures. In addition, the tail of the EP curve with the same climate change factor becomes significantly thinner with roof & opening mitigations (AB) (Figure 8) than with roof mitigation (A) (Figure 6) or opening mitigation (B) (Figure 7). These findings can also be confirmed by comparing the
quantities in Table 6 and Table 7. Table 6 (corresponding to Figure 5) reports the statistics of simulated losses for three climate change factors (a=0, 0.05, and 0.1) and no mitigation measure in 20 years, whereas Table 7 (corresponding to Figure 6) exhibits the same statistics with roof reinforcement (A). All statistics are reduced due to the roof mitigation, especially the tail statistics. For example, in the case of “a=0.05”, expected loss reduces from 1.01 to 0.74, and VaR_{95\%} also declines from 2.18 to 1.71 if roof mitigation is installed. Tail probability with threshold 2 also decreases from 0.07 to 0.02.

What is the effect of these mitigation measures in the St. Lucia case for different time horizons? Figure 9 (10) shows the EP curves for the wood frame building in Canaries with different mitigation measures in the absence of climate change, “a=0” (in the presence of climate change, “a=0.05”) when time horizon is 10 years. Figure 11 (12) depicts similar EP curves but with T=20 years. By comparing these EP curves, there are three findings:

1. The relative performance among the four mitigation measures is as follows. Roof & opening mitigation (AB) outperforms opening mitigation (B), which in turn outperforms roof mitigation (A), which follows by no mitigation.

2. As anticipated, the longer time scale, the greater the risk exposure, thus the fatter tail the loss.

3. The mitigation measures are more effective in the presence of climate change than in the absence of climate change. This suggests that the awareness of climate change will stimulate the investment on mitigations.

4.2 Benefit-Cost Analysis on Mitigations

Based on the statistics and tail probabilities of simulated loss obtained by inputting losses of St. Lucia for four different mitigation measures for the wood frame building in Canaries in the absence of climate change or in the presence of climate change, benefit-cost analysis can be conducted. Consistent with the World Bank Report, benefit from mitigation is defined as the reduction in the expected loss with a specific mitigation measure compared with the expected loss with no mitigation. Benefit-cost ratio (B/C Ratio) is simply the benefit from mitigation divided by the cost of mitigation. According to the World Bank Report, a standard wood frame building in Canaries of St. Lucia has
the value of $100,000. Roof upgrade (A) costs $9,200, opening protection (B) costs $6,720, and roof & opening mitigation (AB) costs $15,920. With this information and our simulation results for different time horizons, B/C Ratios for different mitigation measures can be easily derived.

Table 8 and Table 9 summarize benefits, costs and the B/C ratios of three different mitigation measures and various time scales for the wood frame building in Canaries of St. Lucia in the absence of climate change and in the presence of climate change, respectively. Based on numerical results in these two tables, the findings regarding to time scales are listed in the following.

1. Benefit from mitigation rises with the time scale and benefit from roof & opening mitigation (AB) is greater than benefit from opening mitigation (B), which in turn is greater than benefit from roof mitigation (A). When “a”=0 and T=10 years, benefit from roof & opening mitigation is 0.2014, benefit from opening mitigation is 0.1284, and benefit from roof mitigation is 0.1071.

2. As anticipated, B/C Ratio also rises with the time scale.

3. B/C Ratio of opening mitigation (B) is greater than B/C Ratio of roof & opening mitigation (AB), which in turn is greater than benefit from roof mitigation (A). The reason why B/C Ratio of opening mitigation (B) is with the highest value lies in that its cost, $6,720, is relative low compared with it benefit (the reduction in expected loss). In addition, B/C Ratios in the absence of climate change are consistent with the B/C Ratios in the World Bank Report.

4. The minimal time scale that makes the investment in mitigation worthwhile (the minimal time scale such that B/C Ratio >1) reduces one year for roof mitigation (A) and opening mitigation (B) if climate change is present while it stays the same for roof & opening mitigation (AB). Specifically, in the absence of climate change, roof mitigation (A) is worthwhile to be implemented for at least 9 years; opening mitigation (B) is worthwhile for at least 6 years; roof & opening mitigation (AB) is worthwhile for at least 8 years. In contrast, in the presence of climate change, the minimal time horizon for roof mitigation (A) is 8 years; for opening mitigation (B) is 5 years; for opening mitigation (AB) is still 8 years.
This implies that the presence of climate change will increase the return from mitigations.

Impact of climate change on the B/C Ratios can be observed by comparing Figure 13 and Figure 14. These two figures show the change of B/C Ratios for different mitigation measures over different time scales in the absence of climate change and in the presence of climate change, respectively, when discount rate=0%. As one would expected, B/C Ratios increase with the time horizon in both scenarios. However, they grow faster in the presence of climate change than in the absence of climate change. This phenomenon is more significant for longer time scales. For the time scale=20 years, B/C Ratio is above 4.5 in the presence of climate change while it is slightly below 4 in the absence of climate change. This indicates that the potential climate change and a longer time scale enhance the relative benefits of cost-effective mitigation measures and thus encourage human to invest in mitigations.

Figure 15 summarizes the aggregate effects of climate change and mitigation. The most upper line is the EP curve with no climate change and no mitigation while the other three lines are EP curves with climate change and different mitigation measures (A, B, and AB) when time horizon is 20 years and discount rate=0%. This figure shows that the mitigation effect dominates the climate change effect. The impact of climate change on the EPs can be completely offset by implementing mitigation measures. This phenomenon is robust for all cases, no matter which discount rate is specified and how long time scales we are considering.

4.3 Uncertainty of Climate Change Factor in the Case of St. Lucia

Section 2.4 explored the impact of uncertainty of climate change factor on the tail statistics and on the tail of the simulated loss in the simple model. Here, we conduct similar sensitivity analyses of uncertain climate change factors. In order to introducing uncertainty of climate change factor, “a” is set to be a discrete uniform distribution and has five possible values: 0, 0.025, 0.05, 0.075, and 0.1. For each value of “a”, it occurs with an equal probability, 1/5. Due to the fact that the expected value of these possible climate change factor is 0.05, the case of “a=0.05” is taken as the benchmark case. Introducing uncertainty of climate change factor is anticipated to result in a fatter tail of
the loss distribution since a greater value of climate change factor will have more significant impact on the tail of the loss.

The EP curves for the wood frame house in Canaries of St Lucia for T=20 years are presented in Figure 16. As expected, the uncertain climate change factor (“a=0–0.1”) has fatter tail than the certain climate change factor (“a=0.05”). However, the impact of the uncertainty is minor compared to the certain shift of the climate change factor. Based on the tail statistics and tail probabilities in Table 10, the uncertain climate change factor (“a=0–0.1”) is equivalent to a slightly higher climate change factor “a=0.06”. This empirical result is consistent with the simulating one in section 2.4. Moreover, Figure 17 shows the EP curves of certain (“a=0.05”) and uncertain climate change factor (“a=0–0.1”) for T=10 years and T=20 years. For T=10 years, EP curves of certain and uncertain climate change factor almost match with each other, whereas for T=20 year, EP curve with the uncertain climate change factor exhibits a slightly fatter tail than EP curve with the certain climate change factor. This indicates that uncertainty in climate change factor has more impact for a long time scale. For a shorter time horizon, such as less than 10 years, the effect of uncertainty could be negligible. If one is thinking of 5 or even 10 year guaranteed renewability insurance polities the impact of uncertainty on “a” will not play a key factor in setting premiums.

4.4 Sensitivity Analysis of Annual Premiums

In this section, we use data from the case of St. Lucia and the simple catastrophic risk model to estimate insurance premiums and conduct sensitivity analysis on the patterns of the premiums across time scales.

\[
\text{Annual Premium} = \frac{(1+\lambda)E(L_T)+k \sigma_{L,T}}{T}
\]  

Annual insurance premiums are estimated based on (6), where \(E(L_T)\) is the expected loss for T years; \(\sigma_{L,T}\) denotes the standard deviation of the loss for T years; k is the indicator of hard/soft market for insurance industry. In hard market (k=0.7), premiums are relatively more responsive to the volatility of losses than premiums in soft market (k=0.4); \(\lambda\) reflects cost of capital for insurers; and T is the term of the insurance contract. Since insurers aggregate a risk pool to allocate the risk among policyholders, the cost of capital of insurers should be related to the tail probability of losses. Specifically, we
suppose \( \lambda = m \cdot \mathbb{P}(L > L_T) \), where \( m \) denotes a factor that reflects the financial vulnerability of the insurer. If the insurer is more financially fragile and has to charge a higher premium loading to avoid potential bankruptcy, \( m \) is set to be higher. Because risk exposure for a longer time horizon should be higher, causing a fatter tail of the loss distribution, a higher threshold should be set to the loss of longer time horizons. In this section, \( m \) is set to be either 1 (a financially sound insurer) or 5 (a financially fragile insurer). In addition, \( L_1 = 0.25, L_5 = 0.5, L_{10} = 0.75, L_{15} = 1, L_{20} = 1.25 \). In the subsequent subsections, a series of sensitivity analysis will be conducted regarding the insurance premium. The parameters of the benchmark case are as follows: “a”=0.05, d=0 (discount rate=0), \( m=5 \), \( k=0.4 \), and no mitigation. A wood frame building in Canaries of St. Lucia is assumed to be worth $100,000.

The loss distribution is assumed to be the same in each year. In the case of no climate change, no mitigation, and a fixed cost of capital (cost of capital is not associated with the tail probability), the expected loss is proportional to the corresponding time horizon while standard deviation of the loss distribution is proportional to the square root of the corresponding time horizon. As a result, the annual premiums are predicted to be decreasing with time horizons since the standard deviation of the loss grows slower than the time horizon. This is the rationale for proposing long term insurance. However, based on the settings, cost of capital for different time horizons rises with tail probabilities. Climate change will have a greater impact on the tail probabilities for longer time horizons, which will push up annual premiums. Thus, it is anticipated to observe a slightly declining or even a rising pattern of annual premiums for higher climate change factors.

### 4.4.1 Adaptations

Figure 18 demonstrates the patterns of insurance premiums across time horizons for different adaptations (no mitigation, roof upgrade (A), opening protection (B), and roof & opening mitigation (AB)). As can be observed, with no adaptation, annual insurance premiums generally rise with time horizons. The rising trend of the premium will be compromised if adaptations are installed. In the case of roof & opening mitigations (AB), premiums turn to decline with time horizons. Moreover, the adaptations significantly
reduces annual premium, especially for longer time horizons. When roof & opening mitigations are installed, annual premiums decrease more than half of the original price. For instance, the annual premium for T=5 becomes one-third of the original price ($3,851 with adaptations compared to $11,308 without adaptation), and it plummets to one-fourth of the original price for T=20 (from $15,859 to $3,748). These findings support the proposition that long term insurance should be coupled with adaptations to incentivized policyholders’ willingness to invest in mitigation measures.

Based on the pricing formula, insurance premiums are determined by three factors that are related to loss distributions: Expected losses per year, standard deviations of the loss distribution per year, and tail probabilities. Further analysis on the contribution of these factors to the pattern of annual premiums indicates that, standard deviation per year, as predicted, declines with time horizons; however, due to the impact of climate change, expected losses per year and tail probabilities increase with time horizons. The level of increase in the tail probabilities is substantially reduced with more mitigation measures, causing the patterns of annual premiums shift from rising to slightly declining with time horizons.

4.4.2 Climate Change Factors

Figure 19 exhibits the patterns of annual premiums across time horizons for different climate change factors. As expected, in the absence of climate change (“a”=0), premiums slightly decline with time horizons. In contrast, in the presence of climate change, premiums show a rising trend. The rising trend is more substantial for a higher climate change factor. For the highest climate change factor (“a”=0.1) in Figure 19, the annual premium more than doubles from $12,096 for T=5 years to $26,226 for T=20 years. For the same increase in time horizon, the premium slightly decreases from $10,387 to $10,152 with no climate change. Thus, the impact of climate change on annual premiums is more significant for longer time horizons. The same as predicted in the beginning of the section. Considering climate change makes insurers raise the insurance price, especially for longer time horizons. This finding also highlights the importance of diversifying risk to reduce the likelihood of catastrophic losses.
Further analysis has been conducted on the contribution of expected losses per year, standard deviations per year, and tail probabilities to the pattern of annual premiums, respectively. In the case of no climate change, expected losses are stable, standard deviations decline with time horizons, and tail probabilities concavely rise with time horizon. Consequently, annual premiums show a slightly declining pattern. In contrast, in the presence of climate change, expected losses exhibit an increasing trend. Although the patterns of standard deviations and tail probabilities are similar to the case of no climate change, introducing climate change alleviates the decreasing pattern of standard deviations as well as exacerbates the rising pattern of tail probabilities (from concavity to a little bit convexity). All the combination of these effects substantially pushes up the insurance price, especially for longer terms.

4.4.3 Discount Rates, Hard/Soft Market, and Financial Vulnerability of Insurers

The sensitivity analysis of annual premiums with respect to discount rates, hard/soft market, and financial vulnerability of insurers can be conducted in the same manner. I will only summarize principal results as follows.

A higher discount rate applied by insurers implies that the insurers do not care about future losses, giving rise to a large reduction of annual premiums. This phenomenon is more pronounced for a longer time horizons. For T=20 years, the annual premium decreases significantly to less than one-third of the original price (from $15,859 to $4,857) when the discount rate increases from 0% to 5%.

Annual premiums should be higher in hard market than in soft market. This effect is more substantial for short time horizons than for high time horizons. For instance, the price climbs 25% for T=5 (from $11,308 to $14,108) but it only rises 15% for T=20 (from 15,859 to $18,231). The rational is associated with standard deviation since standard deviation decreases with the time horizon, which in turn reduce the impact of hard/soft market.

Financially sound insurers will charge much less premiums because the cost of capital and risk capital reserved are much less than financially vulnerable insurers, especially for longer time horizons. For T=20 years, the annual premium can be saved for almost half of the original price (from $15,859 to $8165). However, for T=5 years, policyholder only
save 35% of the annual premium (from $11,308 to $7,428). Thus, the benefit of long
term insurance is greater if more financially sound insurers are willing to sell long term
contracts.

4.5 Impact of Adaptations on Sensitivities of Annual Insurance Premium

Since implementing adaptations can reduce annual insurance premiums for a great
amount, we will further explore the change in patterns of premiums from the case of no
adaptation to the case of with adaptations. Roof and opening mitigations (AB) in St.
Lucia is taken as the representative case of adaptations because it has the greatest impact
on annual premiums. Figure 20 (21) shows the pattern of annual premiums across time
horizons when adaptations and climate change factor (financial vulnerability of insurers)
change simultaneously. A comparison of these cases indicates these findings:

1. The reduction in premiums due to adaptations dominates the reduction in premiums
due to other factors. This result indicates that adaptations are the key factor for
insurers to offer low annual premiums.

2. With no adaptation, premiums across time horizons mostly show a rising pattern,
except for the case of no climate change. Nevertheless, with adaptations, premiums
across time horizons generally exhibit a flat or a declining pattern unless a extreme
high climate change factor is specified (“a=0.1” in Figure 20). This implies that
with adaptations, insurers are more willing to provide lower or flat premiums for
longer term contracts.

3. With adaptations, the sensitivities of annual premiums across time horizons with
respect to other parameters decrease substantially. The most obvious example is the
sensitivity of premium with respect to the financil vulnerability of insurers in Figure
21. As adaptations are installed, the difference of annual premiums between a
financially sound insurer and a financially fragile insurer is very small. Thus, the
implementation of adaptations will reduce total risk exposures, thus causing
insurance premium less responsive to other risk-related factors.
5 Calculating Insurance Premiums Using Estimated Losses from Historical Storm Activities for Hurricane Risks in St. Lucia

In this section, storm activity statistics\(^{17}\) are available for us to estimate climate change factor, “a”. Alternatively, we can use lognormal loss model to fit the loss statistics derived by those storm activity statistics. The aim is to calculate insurance premiums for different timescales with and without mitigation.

5.1 Data Descriptions\(^{18}\)

The storm activity statistics are estimated by developing scenarios of storm activity over 5-year time scales using a simple model and estimating the probability and level of storm activity rate based on historical storm activity rate. Figure 22 provides the basic concept of the simple model. The next 5-year rate is predicted based on the mean over the past 5 years and the historical time series of the number of named storms and Cat 3-5 storms. ‘Upper’, ‘Middle’, and ‘Lower’ simply reflect different estimates of percentiles. For example, ‘Upper’ is 95% percentile, ‘Middle’ is the Median, and ‘Lower’ is the 5% percentile. Figure 23 and Figure 24 presents the annual number of named storms and Cat 3-5 storms in the Atlantic basin from 1950 to 2005. Based on these historical data and the simple model, the statistics of variability in storm activity over successive 5-year period can be shows as in Table 11. This tells us, for example, that on average, there is a 12% increase in the number of Cat 3-5 storms between any two successive 5-year periods; however, 35% of the time we will see a decrease in activity of at least 12%, and 35% of the time we will see an increase in activity of at least 22%.

These activity rates are used to adjust the frequencies of individual events in the RMS Caribbean Hurricane model and are converted into expected 5-year losses and standard deviation of losses, as exhibited in Table 12. These quantities are losses for 5 years. By assuming a simple constant growth rate, the annual growth rate can be derived. The climate change factor, “a”, is equivalent to the annual growth rate we obtained here.

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\(^{17}\) Centre for Climate Change Economics and Policy, London School of Economics and Political Science (LSE) provided the estimates for future hurricane-associated loss distributions of St. Lucia.

\(^{18}\) The descriptions are based on “Note on calculation of 5-year growth rates in hurricane wind-related losses and extension to longer time periods” by Nicola Ranger 2009.
5.2 Models and Methods

With the estimated annual growth rate of losses induced by hurricane risk in St. Lucia and EPs that are provided by RMS, two broadly defined models are candidates for calculating insurance premiums: the potential growth model and the lognormal loss model. The simple model we constructed in section 2.1 is the form of the potential growth model with exponential growth rate. However, since EPs of the present-day losses are available, we simply use these EPs along with the growth rate (or the climate change factor), “a”, obtained in section 5.1. The lognormal loss model assumes that the conditional loss follows lognormal distribution when a hurricane occurs.

5.2.1 Potential Growth Model

Hazard rate of climate change occurrence is defined as: 
\[ P_r(t) = 1 - (1 - P(0))^t = \frac{P(\tau = t)}{P(\tau > t - 1)} \]

After iterations and normalization, the occurrence of climate change is summarized in Table 13.

\( L_t \) are randomly generated by Monte Carlo simulation based on the EPs. The distribution of ‘a’ is transformed from the 5 climate scenarios and are summarized in Table 14. For example, -0.085 and -0.0178 are point estimates of VaR\(_{5\%}\) and VaR\(_{35\%}\). Because the middle percentile is 20%, we assign 20% to the value of -0.085. By the same token, the middle percentile of VaR\(_{35\%}\) and VaR\(_{50\%}\) is 42.5%, thus we assign 22.5\% (=42.5\%-20\%) to the value of -0.0178.

Different growth patterns, potential growth model can be divided into the following three models.

1. Step Model

\[ \sum_{t=1}^{T} L_t + \sum_{t=T+1}^{T} L_t (1 + \bar{a}) \]

2. Linear Model

\[ \sum_{t=1}^{T} L_t + \sum_{t=T+1}^{T} L_t (1 + (t - \tau) \cdot a) \]

3. Exponential Model
5.2.2. Lognormal Loss Model

In this model, if a hurricane occurs, losses are assumed to be governed by a lognormal distribution, i.e., \( \sum_{t=1}^{T} L_t + \sum_{t=\tau+1}^{T} L_t (1 + a)^{-\tau} \sim \text{lognormal}(\mu, \sigma) \). Based on the EPs, \( \text{prob}(L_t=0)=0.5 \).

Transformed conditional EPs can be obtained from the original EP curve. Since the estimates of \( \mu \) and \( \sigma \) of the lognormal distribution can be derived by fitting EPs or the expected losses and standard deviation of losses, this model is also separated into two approaches: fit EPs and fit expected losses and standard deviation of losses.

1) Fit EPs

   \( \mu \) and \( \sigma \) of the lognormal distribution are estimated by conditional EPs.

   \[
   L = e^{\mu + \sigma Z} \sim \text{lognormal}(\mu, \sigma) 
   \]

   \[
   F_L(\bar{L}) = P(e^{\mu + \sigma Z} \leq \bar{L}) = P \left( Z \leq \frac{\ln(\bar{L}) - \mu}{\sigma} \right) = \Phi \left( \frac{\ln(\bar{L}) - \mu}{\sigma} \right) 
   \]

   \[
   \Rightarrow \sigma \cdot \Phi^{-1} \left( F_L(\bar{L}) \right) + \mu = \ln(\bar{L}) 
   \]

   \( \Phi^{-1} \left( F_L(\bar{L}) \right) \) can be derived from the EPs. By minimizing the sum of the distances between RHS and LHS, these two parameters are estimated to be \( \mu = 8.1867, \sigma = 1.077 \).

2) Fit expected losses and standard deviation of losses

Data supply unconditional means and standard deviations. Let \( L \) be the unconditional loss, and \( l \) be the conditional loss. Given \( \text{prob}(L=0)=0.5 \), the transformation between \( L \) and \( l \) is summarized as follows: \( E(L) = 0.5E(l), \text{Var}(L) = 0.5\text{Var}(l) + 0.25(E(l))^2 \). Then,

   \[
   \mu \text{ and } \sigma \text{ can be estimated by } \sigma = \sqrt{\ln \left[ 1 + \frac{\text{Var}(l)}{(E(l))^2} \right]}, \mu = \ln(E(l)) - \frac{\sigma^2}{2}. 
   \]

5-year catastrophic losses are simulated based on (i) EPs of present-day losses (ii) expected loss and standard deviation of present-day losses (iii) expected losses and
standard deviations of losses of 5 potential climate scenarios. Annual insurance premiums are calculated by the formula, \( \text{Annual Premium} = \left( (1 + \lambda) \cdot \frac{E(L_T)}{\sigma_{L_T}} + k \cdot \sigma_{L_T} \right)/T \), where \( \lambda = m \cdot P(L > 0.5) \).

5.3 Simulation Results

Table 15 reports statistics, tail probabilities, and annual premiums for 5-year losses in different models. As can be seen, different assumptions on potential growth models with empirical losses do not have great impacts on annual premiums. The step model produces the lowest premium while the exponential model estimates the highest premium. If the size of loss is set to be lognormal-distributed when a catastrophe occurs, the lognormal loss model with potential climate scenarios predicts a higher insurance price than the potential growth models. However, as shown in Table 16, the lognormal loss model does not fit empirical data well. This phenomenon becomes more pronounced in the extreme tail of losses (Percentile greater than 99.9%). Another limitation of lognormal loss model lies in that we can estimate the premiums only when the statistics of losses for the time horizon are available. If we only have expected losses and standard deviation of losses for 5 years, insurance premiums for 20 years cannot be calculated.

Table 17 and Table 18 exhibit statistics, tail probabilities, and annual premiums under four different scenarios in the potential growth model and in the lognormal loss model, respectively: BCS represents Best Case Scenario, MLS denotes Most Likely Scenario, WCS corresponds to Worst Case Scenario, and UCS is Uncertain Climate Scenarios. Since UCS calculates the expected losses by assigning a distribution to five potential climate scenarios, it contains the most information on the empirical data compared to other scenarios and thus is the best estimate based on the data at hand. Lognormal loss model captures standard deviations of losses for each climate scenario, which are not used by potential growth model. Thus, lognormal loss model will estimate a higher insurance price than potential growth model. This phenomenon will be highlighted in the presence of different scenarios. Even though insurance prices are very close to each other for both models under MLS (potential growth model predicts $5,438 while lognormal loss model estimates $5,427), lognormal loss model shows greater discrepancy between BCS and WCS (a higher premium in WCS and a lower premium in BCS). Under
BCS, the annual premium estimated by lognormal loss model is $3,515, much lower than that predicted by the potential growth model, $5,091; however, under WCS, the annual premium of lognormal loss model is projected to be $8,975, much higher than that simulated using potential growth model, $6,002. Thus, it is not surprising that, under UCS, lognormal loss model, which incorporates the variance of ‘a’, estimates a higher annual premium. It simply reflects more uncertainties that will be encountered by the insurer in the future.

The choice between lognormal loss model and potential growth model lies in the availability of data. It would be more appropriate to assume the loss follows a lognormal distribution if one has sufficient data. For example, we have five different climate scenarios (ranging from 5% to 95%) as well as average annual losses and standard deviations of losses. The potential growth model uses only average annual losses but not standard deviations of losses. In order to extract more information from the data, fitting loss statistics to lognormal loss model is a better way to conduct the analysis. It is also worthy to note that lognormal loss model does not fit empirical data well in the extreme tail of losses, especially for percentiles greater than 99.9%. This is a disadvantage of all parametric distributions with few parameters. However, if one uses a generalized parametric distribution with more parameters, he will have higher variances of parameters, which could lead to imprecise estimates and higher type II errors.

6 Impact on Insurance Premiums in the Presence of Correlated Catastrophic Losses and Cost of Capital

In the previous section, we calculate the insurance premiums based on (6) by assuming that catastrophic losses are independent and that cost of capital is simply proportional to standard deviation of losses divided by the time horizon. In this section, the assumption on independence of losses over time will be relaxed and cost of capital will be reexamined based on Modigliani-Miller theorem. In addition, the expected loss for the next periods could adjust upward or downward based on the difference between the prior anticipated loss and the realized loss. This adjustment leads the losses to be Bayesian updated serial correlated. These impacts on insurance premiums will be illustrated quantitatively. Furthermore, the concept of risk capital proposed by Merton and Perold
(1993) will be introduced to compare risk capitals required in two one-period contracts and a two-period contract.

6.1 Losses Are Correlated over Time

Assume $L_t$ follows a $T$-variate unspecified distribution with parameters $(\mu, \sigma, \rho)$. Annual premiums will be associated with the variance of the aggregate loss. Based on the derivation of section 10, $Var\left(\sum_{t=1}^{T} L_t\right) = T \cdot (1 + \rho(T - 1)) \cdot \sigma^2$. The pattern of annual premium across time scales will depend on the correlation coefficient, $\rho$. Thus, except for the case of perfect correlation ($\rho=1$), annual premium will be decreasing across time scales. If $L_t$ is assumed to be autoregressive (AR(1)) and stationary, i.e., $L_t = c + \phi L_{t-1} + \epsilon_t$, $\epsilon_t \sim N(0, \sigma^2)$, $|\phi|<1$, the variance of the aggregate loss will be

$$Var\left(\sum_{t=1}^{T} L_t\right) = \sigma^2 \cdot \left[ T + 2 \cdot \sum_{i=1}^{T-1} (T-i)\phi^i \right],$$

as calculated in section 10. We have a similar result: Unless losses over time are perfect correlated ($\phi=1$), annual premium will be decreasing across time scales. Moreover, if we assume losses are independent while they are actually positively correlated, annual premiums will be underestimated because the variance is underestimated.

6.2 Cost of Capital

6.2.1 Modigliani-Miller theorem

The Modigliani-Miller theorem, proposed by Franco Modigliani and Merton Miller, forms the basis for modern thinking on capital structure (how much capital is allocated between debt and equity), though it is generally viewed as a purely theoretical result since it assumes away many important factors in the capital structure decision. These other reasons include bankruptcy costs, agency costs, taxes, information asymmetry.

Assume a perfect capital market (no transaction or bankruptcy costs; perfect information); firms and individuals can borrow at the same interest rate; no taxes; and investment decisions are not affected by financing decisions. The value of a company is independent of its capital structure (how a firm is financed is irrelevant to its value).
The analysis can be extended to take the effect of taxes into account. Under a classical
tax system, the tax deductibility of interest makes debt financing valuable; that is, the cost
of capital decreases as the proportion of debt in the capital structure increases. The
optimal structure, then, would be to have virtually no equity at all.

Furthermore, bankruptcy cost is allowed to exist. In this case, there is an advantage to
financing with debt (namely, the tax benefit of debts) and that there is a cost of financing
with debt (the bankruptcy costs of debt). The marginal benefit declines as debt increases,
while the marginal cost increases, so that a firm can optimize its value by choosing the
optimal ratio of debt and equity to use for financing.

### 6.2.2 Estimating Cost of Capital

Catastrophe insurance premiums should also consider cost of capital to insurers. Dollar
cost of capital to insurers in our catastrophe risk model can be measured by multiplying
cost of capital and expected total capital. In the benchmark case, we propose that cost of
capital=2%. Bankruptcy cost is reflected in the dollar cost of capital that is required to
maintain credit rating for insurers.

The procedures to estimate expected total capital ($K_{T}$) at initial for issuing T-year
catastrophe insurance are shown as follows:

Set $K_{r,T}$ as the total capital that is required in year $\tau$ to maintain its credit rating for
issuing T-year catastrophe insurance.

$$K_{r,T} = \left(\frac{1}{1+r}\right)^{r-1} \cdot \left(\text{VaR}_{C\%}\left(\sum_{t=1}^{T} \frac{L_{t}}{(1+r)^{t-1}}\right) - E\left(\sum_{t=1}^{T} \frac{L_{t}}{(1+r)^{t-1}}\right)\right),$$

$C$ is the critical percentile

of Value at Risk ($\text{VaR}_{C\%}$). For example, $C=99.9$ represents the case that the capital is
sufficient for 1-in-1000 years event. $r$ denotes the discount rate.

or

$$K_{r,T} = \left(\frac{1}{1+r}\right)^{r-1} \cdot \left(n \cdot \sigma \left(\sum_{t=1}^{T} \frac{L_{t}}{(1+r)^{t-1}}\right)\right),$$

$n$ is the number of standard deviation such

that $n\sigma(L)$ is equal to the distance between $\text{VaR}_{C\%}(L)$ and $E(L)$.

The expected total capital that is required at initial of the contract to maintain its credit
rating for issuing T-year catastrophe insurance is denoted as $K_{T}$.

$$K_{T} = \frac{1}{T} \sum_{r=1}^{T} K_{r,T} = \frac{1}{T} \sum_{r=1}^{T} \left(\frac{1}{1+r}\right)^{r-1} \cdot \left(\text{VaR}_{C\%}\left(\sum_{t=1}^{T} \frac{L_{t}}{(1+r)^{t-1}}\right) - E\left(\sum_{t=1}^{T} \frac{L_{t}}{(1+r)^{t-1}}\right)\right)$$
or \( K_T = \frac{1}{T} \sum_{t=1}^{T} K_{r,t} = \frac{1}{T} \sum_{t=1}^{T} \left( \frac{1}{1+r} \right)^{t-1} \left( n \cdot \sigma \left( \sum_{t=1}^{T} L_t \right) \right) \)

Cost of capital is simply \( (2\%) \cdot K_T \)

### 6.2.3 Annual Premium for T-year Catastrophe Insurance

Total premium = \( \sum_{t=1}^{T} \frac{E(L_t) + X_1}{(1+r)} + (2\%) \cdot T \cdot K_T + X_2 \)

, where \( X_1 \) is the administrative cost of marketing a policy, \( X_2 \) is the upfront cost to insurer of marketing a policy.

Inputting \( K_T \) derived in section 6.2.2, total premium

\[
= \sum_{t=1}^{T} \frac{E(L_t) + X_1}{(1+r)} + (2\%) \cdot \sum_{t=1}^{T} \left( \frac{1}{1+r} \right)^{t-1} \left( V_{\text{VaR}_{C,T}} \left( \sum_{t=1}^{T} \frac{L_t}{(1+r)^{t-1}} \right) - E \left( \sum_{t=1}^{T} \frac{L_t}{(1+r)^{t-1}} \right) \right) + X_2
\]

or

\[
= \sum_{t=1}^{T} \frac{E(L_t) + X_1}{(1+r)} + (2\%) \cdot \sum_{t=1}^{T} \left( \frac{1}{1+r} \right)^{t-1} \left( n \cdot \sigma \left( \sum_{t=1}^{T} \frac{L_t}{(1+r)^{t-1}} \right) \right) + X_2
\]

Annual premium is simply the total premium divided by \( T \).

Thus, annual premium

\[
= \frac{1}{T} \left[ \sum_{t=1}^{T} \frac{E(L_t) + X_1}{(1+r)} + (2\%) \cdot \sum_{t=1}^{T} \left( \frac{1}{1+r} \right)^{t-1} \left( V_{\text{VaR}_{C,T}} \left( \sum_{t=1}^{T} \frac{L_t}{(1+r)^{t-1}} \right) - E \left( \sum_{t=1}^{T} \frac{L_t}{(1+r)^{t-1}} \right) \right) + X_2 \right]
\]

or

\[
= \frac{1}{T} \left[ \sum_{t=1}^{T} \frac{E(L_t) + X_1}{(1+r)} + (2\%) \cdot \sum_{t=1}^{T} \left( \frac{1}{1+r} \right)^{t-1} \left( n \cdot \sigma \left( \sum_{t=1}^{T} \frac{L_t}{(1+r)^{t-1}} \right) \right) + X_2 \right]
\]

If we further set \( r=0, X_1=0, \) and \( X_2=0 \), the annual premium derived here can be compared with that estimated by (6’):

\[
\text{Annual premium} = \frac{1}{T} \left[ E \left( \sum_{t=1}^{T} L_t \right) + (2\%) \cdot n \cdot \sum_{t=1}^{T} \left( \sigma \left( \sum_{t=1}^{T} L_t \right) \right) \right]
\]

, where \( \sum_{t=1}^{T} \left( \sigma \left( \sum_{t=1}^{T} L_t \right) \right) \) stands for the sum of the standard deviation of the aggregate losses.
Annual premium = \( \frac{1}{T} \cdot \left[ E\left( \sum_{t=1}^{T} L_t \right) + (0.4) \cdot \sigma\left( \sum_{t=1}^{T} L_t \right) \right] \) \hspace{1cm} (6')

\[ \sum_{t=1}^{T} \left( \sigma\left( \sum_{t=1}^{T} L_t \right) \right) > \sigma\left( \sum_{t=1}^{T} L_t \right) \] = standard deviation of aggregate losses

Thus, using (6') to price insurance will underestimate annual premiums.

6.3 Annual Premiums with Correlations over Time and Cost of Capital

Taking climate change into account will introduce correlation of losses over time into the catastrophe model through the cumulative and uncertain effects of climate change. In addition, the annual premium of catastrophe insurance would be increased when considering cost of capital by using (7) compared with using (6'). Table 19 (20) and Figure 25 (26) depict annual premiums of catastrophe insurance with no cost of capital and with cost of capital for a house with the value of $1 million with no climate change (with climate change). As can be seen, the pattern of annual premiums changes because of cost of capital: with no cost of capital, annual premiums decline with the term of contracts; however, with cost of capital, annual premiums rise with the term of contracts. In addition, the presence of climate change increases the insurance premium. For example, with no climate change, if the term of the contract increases from 1 year to 5 years, the annual premium with no cost of capital declines from $48,300 to $44,500 while the annual premium with cost of capital rises to $52,900. In contrast, in the presence of climate change, annual premiums for a 5-year contract are $45,200 with no cost of capital and $53,700 with cost of capital, respectively.

Table 19 and 20 also provide the sensitivity analysis on the cost of capital. The equivalent cost of capital shows the cost of capital such that a potential policyholder is indifferent between purchasing a multiple-year contract or sequential 1-period contracts. A cost of capital of 2% for the one-year contract is set as the benchmark. As can be seen, the cost of capital declines with the terms of contract indicating that a lower cost of capital is required to ensure that the annual premium will not increase for longer term contracts. For example, in Table 19, the equivalent cost of capital such that the annual premium is the same as an annual renewal contract decreases from 1.56% for a 2 year contract to 1.18% for a 5-year contract.
6.4 Bayesian-Updated Serial Correlation

Suppose that the realized catastrophic loss in the past year is underestimated compared with the anticipated loss one year ago, one would adjust the expected loss of the next year upward based on the realized loss. Through this yearly updated Bayesian process, the expected loss of the next year depends on both the long term trend of catastrophic risk and the realized losses of prior years. If it is the case, annual premiums could further rise due to the Bayesian-updated correlation because more capitals are required to be reserved for the uncertainty of updated expected losses in the subsequent periods.

The volatilities of no Bayesian-updated correlated process and Bayesian-updated correlated process in the presence and in the absence of growing trend of catastrophic losses have been derived in section 11. The results show that in either case, with Bayesian-updated correlation, the volatility will be higher. In addition, the annual premiums and capital reserves for insurers will also be greater compared with the case predicted by (6’), where Bayesian-updated correlations are considered. In particular, the annual premiums show a rising trend with respect to the time scale. As can be seen in the variance of Bayesian-updated process, the increment volatility of Bayesian-updated correlation ((25)-(24) in section 11),

\[
\left\{w^2 \cdot \sum_{i=1}^{T-1} (1 + a)^{2i} \cdot (T - i) + 2 \cdot \sum_{i=1}^{T-2} (1 + a)^{2i} \cdot \frac{(T - i)(T - 1 - i)}{2}\right\} + 2w \cdot \left[\sum_{i=1}^{T-1} (1 + a)^{i} \cdot (T - i)\right]\cdot \sigma^2(\epsilon)
\]

is positively proportional to the weight put on the realized loss in each period (w), the growth rate of catastrophic losses (a), and the time horizon (T). Thus, Insurers’ updating their estimates of future losses based more weight on the revealed losses, potential climate change, and a longer term contract, will raise the annual premiums and the capital reserves that are required for insurers to support a given credit rating.

6.5 Comparison of Risk Capital in a 1-period Contract and a 2-period Contract

Based on Merton and Perold (1993), risk capital is defined as the smallest amount that can be invested to insure the value of the firm’s net assets against a loss in value relative to the risk-free investment of those net assets. This concept of risk capital can be applied to the financing, capital budgeting, and risk management decisions of financial firms. In this study, we applied this concept to estimate risk capital that is required for insurers
who wrote catastrophe insurance policies and to compare the amount of risk capital each period for two 1-period contracts and a 2-period contract. The major advantage of using this concept of risk capital to estimate risk capital lies in that we can avoid determining the credit risk appropriate for comparing of insurance policies with different time scales.

For insurers who issues catastrophe insurance, net assets are simply premiums collected minus the potential loss. Premiums are determined at the inception of the contract based on the expected loss if the price is actuarially fair, whereas the potential loss is set to follow a distribution with mean and variance. In this case, the risk capital can be expressed as the value of a European call option on the potential loss with exercise price equal to the premium.

6.5.1 Assumptions

1. $L_1$ and $L_2$ are prior belief about the loss distributions of period 1 and period 2. Specifically, they follow a normal distribution with parameters $(\mu_1, \sigma)$ and $(\mu_2, \sigma)$, respectively.

2. There exists potential default risk for insurers when realized losses exceed premiums

3. The 2-period contract binds insurers to provide and policyholders to purchase catastrophe insurance in the beginning of period 2 at a pre-determined price

4. In the second 1-period contract, the price of catastrophe insurance can change based on the posterior belief about the loss distribution of period 2, which is affected by both the realized loss in period 1 and the prior belief about the losses

5. Posterior loss distribution of period 2 conditional on the realized loss of period 1, $L_2^N | L_1^R \sim N(\mu_2^N, \sigma)$, will be derived based on an updated process. If the realized loss of period 1 turns out to be greater (less) than the mean of the prior loss distribution of period 1, the mean of the posterior loss in period 2 will be adjusted upward (downward) for the amount proportional to the discrepancy between the realized loss and the mean of the prior loss. More specifically, 

$$\mu_2^N = \mu_2 + w \cdot (L_1^R - \mu_1), \quad w > 0.$$ 

For simplicity, the updated process only changes the expected loss but not the standard deviation.
Insurance premiums are actuarially fair and are determined prior to the realization of losses in each period. Thus, \( P_1 = \mu_1 \), \( P_2 = \mu_2 \), \( P_2^N = \mu_2^N \).

Risk capital is reflected in the price of the call option, which is available in capital markets for the insurer to hedge against the potential risk.

6.5.2 Nature of Risk for Two 1-period Contracts and a 2-period Contract

In two 1-period catastrophe insurance policies, the premium of period 2 can be adjusted upward or downward based on the realized losses of period 1. The only risk is default risk when losses surpass premiums in each period. In contrast, in a 2-period catastrophe insurance policy, the premium is fixed in the beginning of period 1. The risk stems both from the catastrophe losses exceeding the premiums (default risk) and from the market-adjusted premiums exceeding the pre-determined premium (premium risk). The discrepancy of the nature of risk between two 1-period contracts and a 2-period contract leads to different amount of risk capital being reserved for different terms of insurance contracts.

6.5.3 Risk Capital for Two 1-period Contracts

In order to hedge the default risk, the insurer can purchase an option that payoff when the realized loss exceeds the premium they collected. When this situation occurs, the insurer will be indemnified for the difference between the realized loss and the premium and avoid default on the claims. Risk capital to cover losses in period 1 is determined by the value of the call option with payoff \( (L_1 - P_1)^+ \). \( S_1 = E^Q[(L_1 - P_1)^+] = \frac{\sigma}{\sqrt{2\pi}} \), where \( E^Q[.] \) represents the expected value under risk-neutral probability, whereas risk capital to cover losses in period 2 given the realized loss in period 1 is determined by the value of the call option with payoff \( (L_2^N | L_1^R - P_2)^+ \). \( S_2^N = E^Q[(L_2^N | L_1^R - P_2)^+] = \frac{\sigma}{\sqrt{2\pi}} \), where \( P_2 = \mu_2^N + \mu_2 \cdot (L_1^R - \mu_1) \). Please refer to section 12 for the derivations.
6.5.4 Risk Capital for a 2-period Contract

Assume the fixed premium predetermined at the beginning of period 1 is $P_{LT}$. The premium should be equal to the average of the expected losses in period 1 and period 2, i.e., $P_{LT} = \frac{\mu_1 + \mu_2}{2}$.

If the realized loss of period 1 turns out to be greater than the prior expectation, market premiums in period 2 will adjust upward. Insurers who wrote a 2-period contract can not change the premium at period 2, creating higher risk for the existing policyholders. Consequently, the insurers have to provide additional risk capital in period 1 to protect this possible incremental loss. The additional risk capital for insurers is equivalent to the option price with payoff $(P_2 - P_{LT})^+$, which option buyer pays to obtain the right to exercise the option if it turns out to be in-the-money $(P_2 > P_{LT})$. The option value can be derived as $w$ units of call option with underlying $L_1^R$ and strike price $P_1$ as follows.

$$E^Q \left[(P_2 - P_{LT})^+\right] = w \cdot E^Q \left[(L_1^R - P_1)^+\right] = w \cdot \frac{\sigma}{\sqrt{2\pi}}$$

In period 1, policyholders care about not only the default risk but also the premium risk, both of which will be triggered by the realized loss higher than anticipated in period 1. Risk capital to cover losses in period 1 is thus determined by $S_{1,T}^L = S_1 + E^Q \left[(P_2 - P_{LT})^+\right] = (1 + w) \cdot \frac{\sigma}{\sqrt{2\pi}}$.

If the realized loss does not exceed the anticipated loss, the premium in period 2 should be less than the fixed premium. In this case, risk capital to cover the loss in period 2 is determined solely by the value of the call option on default risk in period 2. The call option has payoff $(L_2^N | L_1^R - P_2)^+$. $S_2^N = E^Q \left[(L_2^N | L_1^R - P_2)^+\right] = -\frac{\sigma}{\sqrt{2\pi}}$, where

$$P_2 = \mu_2^N = \mu_2 + w \cdot (L_1^R - \mu_1)$$

6.5.5 Comparison of Risk Capital

A comparison of risk capital in period 1 for two 1-period contracts and a 2-period contract indicates that a 2-period contract requires more capital than a 1-period contract. The additional risk capital comes from the fact that policyholders in a 2-period contract face premium risk, which does not exist in a 1-period contract. In an annual-renewal
contract, premiums can be adjusted based on the incoming realized losses in each period; however, in a long term contract, premiums are fixed in the inception of the contract and can not adjust upward even in the presence of rising trend of catastrophic losses. If the insurers purchase an option with payoff \((P_2 - P_{LT})^+\) in a 2-period contract, when the premiums in period 2 increases due to the high realized loss in period 1, insurers will be compensated for \((P_2 - P_{LT})\) for the option of hedging premium risk. Consequently, the insurers will hold \(P_{LT} + (P_2 - P_{LT}) = P_2\) in the beginning of period 2. By purchasing this option, insurers who wrote a 2-period contract will have the same payoff structure as those who wrote two 1-period contracts in the beginning the period 2, which makes 1-period and 2-period contracts comparable.

### 6.5.6 Example

An insurer provides periodically-renewal coverage for 2 periods:

\[L_1\sim N(\mu_1, \sigma_1) \quad L_2\sim N(\mu_2, \sigma_2) \quad \mu_1= 1,000 \quad \mu_2=1,000 \quad \sigma_1= \sigma_2= 500\]

Assuming insurance is priced at an actuarially fair rate.

\[P_1= 1,000 \quad P_2=1,000\]

Risk capital in period 1 = \((L_1-P_1)^+\) = 0.4 \(\sigma_1\) = 200

Risk capital in period 2 = \((L_2-P_2)^+\) = 0.4 \(\sigma_2\) = 200

Now suppose the insurer provides long term coverage for 2 periods and wants to determine risk capital in each period.

Updated premium in period 2 is determined by \(P'_2= wL_1+(1-w) \mu_2\)

Suppose \(w=0.3\),

\[P'_2= 0.3 L_1+(0.7) 1,000 = 0.3L_1+700\]

Option for hedging premium risk of coverage in period 2

\[= (P'_2-P_{LT})^+ = w*(L_1-P_1)^+ = (0.3) (0.4) \sigma_1 = 60\]

Risk capital in period 2 = \((L_2-P_2)^+\) = 0.4 \(\sigma_2\) = 200

Risk capital in period 1 = \((L_1-P_1)^+ + (P'_2-P_{LT})^+\) = 0.4 \(\sigma_1\) + (0.3) (0.4) \(\sigma_1\) = 200+60 = 260

The principal difference between risk capital of a 1-period contract and a 2-period contract lies in the option for hedging premium risk of coverage in period 2. This example explicitly illustrates the additional premium risk faced by policyholders of a 2-period contract compared to two 1-period contracts.
There exists offsetting effects for insurers to extend the term of contracts. On one hand, a longer term insurance contract encourages policyholders to invest on mitigations. Loss distributions will diminish to some extent after mitigations being implemented. On the other hand, a longer term contract increases risk capital required to be reserved in each period. In the above example, assume that a 2-period contract induces policyholders to implement mitigation measures, which reduce $m$ proportion of the expected loss while an annual-renewal contract does not incentivize mitigation measures. The optimal level of mitigation such that 1-period and 2-period contracts are comparable can be determined by the proportion of expected loss being reduced, $m^*$. The risk capital in period 1 for a 2-period contract should be $260*(1-m^*)$ while the risk capital in period 1 for a 1-period contract should be 200. Thus, $m^*=23.08\%$. If a 2-period contract can induce mitigation such that the expected loss are reduced for more than 23.08\%, insurers would like to promote 2-period contract rather than 1-period contract due to the less risk capital required. Based on the loss data for the wood-frame building in St. Lucia, expected loss will reduce at least 27.74\% in the presence of mitigation measures. In this case, insurers are willing to issue a 2-period contract. In addition, there are other savings to the insurer from a 2-period contract—marketing costs and transaction costs associated with renewing the 1-year policy.

7 Conclusions

The impact of climate change on catastrophic risk has recently been the focus of researchers with a variety of backgrounds. In this study, I construct a simple catastrophic risk model with potential climate change, quantify the impact of potential climate change and associated uncertainties on catastrophic risk for longer timescales, conduct benefit-cost analysis of mitigation in reducing catastrophic losses with and without climate change, and estimate annual premiums for short-term and long-term catastrophe insurance under different scenarios. In particular, three research questions have been raised and potential answers to them have been proposed as follows.

First, will insurance market still function well given the distribution changes implied by climate change? In a regime of climate change, the catastrophic loss distributions are
not well-defined since the impact of climate change on catastrophic losses is changing over time along with a great deal of uncertainty and ambiguity. Specifically, uncertainties with respect to the timing of climate change and the impact of climate change on potential losses have to be taken into account in the catastrophic risk model. In addition, climate change also leads to higher correlations and cumulative effect on inter-temporal losses, which in turn create a fatter-tail loss distribution. These climate-induced statistical properties could jeopardize the originally well-functioned insurance market, giving rise to higher premiums, increasing risk capital required, and depress the supply or even the availability of insurance.

Second, how does optimal mitigation change with climate change for a longer time scale? Benefit-cost analyses on the implementation of mitigation measures have been conducted both by simulations and by using hurricane losses in St Lucia, respectively. The simulation results indicate that the optimal mitigation level will rise with people’s concern about the future (in terms of a lower discount rate or a longer time scale) while the empirical results suggest that homeowners will have more incentive to invest on mitigation in the presence of climate change and for longer time scales. Moreover, for the hurricane risk in St. Lucia, mitigation is empirically found to completely offset the impact of climate change.

Finally, what can we say about long term versus short term insurance contracts regarding the impact of cost of capital and Bayesian-updated serial correlations on risk capitals and premiums? Based on theoretical model I constructed, as cost of capital is taken into account, annual premiums increase with terms of the contract, whereas, after considering Bayesian-updated serial correlation on catastrophic losses, the volatility of aggregate losses rises compared to the case of no serial correlation. If the posterior expected loss will be adjusted based on the realized losses in the past periods, the risk capital in the sense of Merton and Perold (1993) has been proven to be greater for a 2-period contract than a 1-period contract due to the additional premium risk borne by the insurer. However, longer term contracts incentivize mitigation measures and reduce marketing and renewal transaction costs. The appropriate terms of insurance contract coupled with other risk transfer instruments can be determined by balancing the trade-off.
8 Future Research on Long Term Insurance

Potential climate change will create challenges in catastrophic risk management. The consequences can be limited if we implement effective mitigation measures and design appropriate risk sharing arrangements in advance. Mitigation measures have been shown to effectively reduce losses for all parties involved even with potential climate change. Longer term insurance contracts can incentivize mitigation measures but will increase risk capital required and premiums due to the increased uncertainty regarding future losses for longer time scales. Future research would compare long term insurance contracts for various periods with an annual renewal contract with and without mitigation with different scenarios regarding climate change include the case of no climate change.

The case of no climate change will be deemed as the benchmark, the trade-off of long term contracts and annual renewal ones will involve the benefits of mitigation measures and the extra costs associated with holding reserve capital. A longer term insurance contract encourages policyholders to implement mitigation measures, but it raises risk capital and the cost of capital. How can one determine the appropriate terms of insurance contract to balance the trade-off between short term and long term contracts? Annual insurance premiums can be used as the criteria. Mitigation reduces the catastrophic loss and also the premium while higher cost of capital pushes up the premium. In order to conduct this analysis, we need empirical estimates of cost of capital by insurers as well as amount of risk capital reserved by insurers as a function of catastrophic losses and contract length; we also need to estimate the extent to which catastrophic losses can be reduced by different mitigation measures.

With potential climate change, similar trade-offs between a longer term contract and an annual renewal contract need to be analyzed. Climate change may reinforce both the benefit and the cost of longer term contracts by raising the returns from mitigation, and by increasing the amount of reserve capital. How will the reduction (increase) of annual premiums due to mitigations (cost of capital) change with potential climate change for different time scales? In addition to applying loss data to create EP curves with and

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without mitigation for the risk-prone regions, we will also estimate the impact of climate change on catastrophic losses (such as storm activities regarding hurricane risk or sea level rise regarding flood risk). This impact can be measured based on the catastrophic loss event table provided by Willis Re, which provides the event type, the rate of occurrence, the expected loss, the independent volatility, the correlated volatility, and risk exposures.

In this study, a simple two-period model was constructed to illustrate the additional risk capital required for a two-period contract compared with a one-period contract. I also calculated the minimal level of mitigation stimulated by a two-period contract such that two one-period contracts are comparable to a two-period contact in terms of risk capital. If we would like to apply this approach to evaluate the trade-off between mitigation measures and the length of contracts, more challenges will appear. First, it will be difficult to estimate catastrophic loss distributions with reliable parameters for each period, especially with potential climate change. Second, even if we can approximately estimate the loss distribution each period, it is unlikely that losses over time are independent. With correlated losses, risk capital cannot be measured by the value of a call options with a simply form of payoff. For example, the payoff of a call option to hedge against premium risk in period one for a three-period model may depend on the correlated realized losses in the first two periods. More advanced option pricing techniques may be required to solve the problem. Third, if the losses over time are assumed to be independent and stable, the weights placed on the realized loss in the previous periods are unlikely to be constant. People tend to over-adjust large losses and under-adjust small losses. This behavior will further raise the risk capital for hedging against premium risk. In order to capture the effect of this behavior on the weights, an experimental study would be more suitable than from the empirical data.
Appendix

9 Statistics of Exact Losses with Potential Climate Change

The aggregate loss can be presented as two components, the aggregate loss prior to climate change and the aggregate loss posterior to climate change, i.e., $\tilde{L} = \tilde{L}_1 + \tilde{L}_2$.

$\tilde{L}_1$ denotes the aggregate loss prior to climate change and is the sum of annual losses before the occurrence of climate change, i.e., $\tilde{L}_1 = \sum_{i=1}^{\tau} \tilde{L}_{i,1}$, where

$$\tilde{L}_{i,1} = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1 - p \end{cases} \text{ for } i = 1, \ldots, \tau$$

$\tilde{L}_2$ represents the aggregate loss posterior to climate change and is the sum of annual losses after the occurrence of climate change, i.e., $\tilde{L}_2 = \sum_{j=\tau+1}^{T} \tilde{L}_{j,2}$, where

$$\tilde{L}_{j,2} = \begin{cases} (1+a)^{j-\tau} & \text{with probability } p \\ 0 & \text{with probability } 1 - p \end{cases} \text{ for } j = \tau+1, \ldots, T$$

With the above settings, the conditional aggregate loss prior to climate change follows Binominal distribution, i.e., $\sum_{i=1}^{\tau} \tilde{L}_{i,1} \mid \tau \sim \text{Bin}(\tau, p)$, whereas the conditional aggregate loss after climate change can not be identified. Nonetheless, the conditional expected loss is derived as follows.

$$E(\sum_{j=\tau+1}^{T} \tilde{L}_{j,2} \mid \tau) = E(\sum_{j=\tau+1}^{T} \tilde{L}_{j,2} \mid \tau) = \sum_{j=\tau+1}^{T} p \cdot (1+a)^{j-\tau} = \frac{p(1+a)}{a} \cdot [(1+a)^{\tau-1} - 1]$$

Thus, $E(\tilde{L}) = E(\sum_{i=1}^{\tau} \tilde{L}_{i,1} + \sum_{j=\tau+1}^{T} \tilde{L}_{j,2}) = E(\sum_{i=1}^{\tau} \tilde{L}_{i,1} \mid \tau) + E(\sum_{j=\tau+1}^{T} \tilde{L}_{j,2} \mid \tau)$

$$\frac{p(1+a)}{a} \cdot [E\left(\frac{(1+a)^{\tau-\tau} - 1}{a}\right) - 1]$$

$$= p \cdot E(\tau) + \frac{p(1+a)}{a} \cdot [E((1+a)^{\tau-\tau}) - 1]$$
\[ \left\{ \begin{align*} \tau & \sim \text{discrete } U(1, T) \Rightarrow E(\tau) = \frac{T + 1}{2}, \quad E\left( (1 + a)^{T-\tau} \right) = \sum_{\tau=1}^{T} (1 + a)^{T-\tau} \cdot \frac{1}{T} = \frac{(1 + a)^T - 1}{a \cdot T} \\
= & \frac{p \cdot (T + 1)}{2} + \frac{p \cdot (1 + a)}{a} \left[ \frac{(1 + a)^T - 1}{a \cdot T} - 1 \right] \end{align*} \]

By L'Hospital Rule, as \( a \to 0 \), \( \frac{p \cdot (1 + a)}{a} \left[ \frac{(1 + a)^T - 1}{a \cdot T} - 1 \right] \to p \cdot \frac{T-1}{2} \). In equation (3.8), the second term approaches \( p \cdot \frac{T-1}{2} \) while the first term does not depend on \( a \) and equates \( p \cdot \frac{T+1}{2} \), thus the expected aggregate loss approaches \( T \cdot p \), which verifies the fact that if no climate change, \( a=0 \), \( \bar{L} \sim \text{Bin}(T, p) \Rightarrow E(\bar{L}) = T \cdot p \).

Moreover, the conditional moment generating functions of the two components of the aggregate losses are given by these two equations:

\[ M_{\bar{L}_1 | \tau}(\theta) = E\left[ e^{\theta \bar{L}_1} | \tau \right] = (pe^{\theta} + 1 - p)^\tau \] \hfill (9)

\[ M_{\bar{L}_2 | \tau}(\theta) = E\left[ e^{\theta \bar{L}_2} | \tau \right] = \prod_{k=1}^{T-\tau} \left( pe^{\theta(1+a)^k} + 1 - p \right) \] \hfill (10)

Thus, the moment generating function of the aggregate loss can be written as follows:

\[ M_{\bar{L}}(\theta) = E \left[ e^{\theta \bar{L}} \right] = E_{\tau} \left[ E \left[ e^{\theta \bar{L}_1} | \tau \right] \cdot E \left[ e^{\theta \bar{L}_2} | \tau \right] \right] \]

\[ = \frac{1}{T} \sum_{\tau=1}^{T} \left( pe^{\theta} + 1 - p \right)^\tau \cdot \prod_{k=1}^{T-\tau} \left( pe^{\theta(1+a)^k} + 1 - p \right) \] \hfill (11)

We can verify that the expected loss can be derived by differentiating the log of the moment generating function of the aggregate loss with respect to the parameter \( \theta \) at the value with \( \theta=0 \).

\[ E(\bar{L}) = \frac{\partial \log(M_{\bar{L}}(\theta))}{\partial \theta} \bigg|_{\theta=0} \]

\[ = \partial \log \left\{ \sum_{\tau=1}^{T} \left( pe^{\theta} + 1 - p \right)^\tau \cdot \prod_{k=1}^{T-\tau} \left( pe^{\theta(1+a)^k} + 1 - p \right) \right\} \bigg/ \partial \theta \bigg|_{\theta=0} \]

\[ = \frac{1}{T} \sum_{\tau=1}^{T} \left( \tau \cdot p + p \cdot \sum_{k=1}^{T-\tau} (1 + a)^k \right) = \frac{p \cdot (T + 1)}{2} + \frac{p \cdot (1 + a)}{a} \left[ \frac{(1 + a)^T - 1}{a \cdot T} - 1 \right] \]
Nevertheless, higher moments are much harder to be obtained compared with the expected value since the procedures involve differentiation of a product with up to $T$ terms.

By Double Expectation Theorem, the variance of the aggregate loss can be expressed as this formula: 

$$Var(\tilde{L}) = E[Var(\tilde{L} \mid \tau)] + Var[E(\tilde{L} \mid \tau)]$$

$(13)$

$E(\tilde{L} \mid \tau)$ has been obtained in equation (8). Therefore, the second term in equation (13) can be derived as follows.

$$Var[E(\tilde{L} \mid \tau)] = Var(\varphi + \frac{p(1+a)}{a}[1 + (1+a)^{T-\tau} - 1])$$

$$= p^2 Var(\tau) + \frac{p^2(1+a)^2}{a^2} \cdot Var[(1+a)^{T-\tau}]$$

$$= \begin{cases} 
\tau \sim \text{discrete } U[1, T]: Var(\tau) = \frac{T(T+1)(2T+1)}{6} - \frac{(T+1)^2}{4} \\
Var[(1+a)^{T-\tau}] = E[(1+a)^{2(T-\tau)}] - E[(1+a)^{T-\tau}]^2 = \frac{1}{T} \sum_{\tau=1}^{T} (1+a)^{2(T-\tau)} - \left(\frac{1}{T} \sum_{\tau=1}^{T} (1+a)^{T-\tau}\right)^2 \\
= \left(\frac{1+a}{T \cdot a}\right)^{2T-1} - \left(\frac{1+a}{T \cdot a}\right)^2 \\
\end{cases}$$

$$= \left(1 + a\right)^{2T-1} \cdot \left(\frac{1+a}{T \cdot a}\right)^2$$

$$(14)$$

In order to derive the first term of equation (13), having the value of $Var(\tilde{L} \mid \tau)$ is the requirement. Since the independence between $\tilde{L}_1$ and $\tilde{L}_2$ given the timing of climate change $\tau$, $Var(\tilde{L} \mid \tau) = Var(\tilde{L}_1 \mid \tau) + Var(\tilde{L}_2 \mid \tau)$. $(15)$

$$Var(\tilde{L}_1 \mid \tau) = \varphi(1 - p)$$

However, the functional form of the first term of equation (15) is difficult to obtain. Since $E(\tilde{L}_2 \mid \tau)$ has been known in equation (8), we decompose the variance into two components: 

$$Var(\tilde{L}_2 \mid \tau) = E(\tilde{L}_2^2 \mid \tau) - [E(\tilde{L}_2 \mid \tau)]^2$$

$(16)$

, where $E(\tilde{L}_2 \mid \tau) = \frac{p(1+a)}{a \cdot T}[1 + (1+a)^{T-\tau} - 1]$. 
Thus, \( E[\text{Var}(\tilde{L} \mid \tau)] = p(1-p)E(\tau) + E\left[E\left(\tilde{L}_2^2 \mid \tau\right)\right] - \frac{p^2(1+a)^2}{a^2 \cdot T^2} E\left[\left(\frac{1+a}{a}\right)^{T-1} \right] - 1 \)

\[
= p(1-p)\frac{T+1}{2} + E\left[E\left(\tilde{L}_2^2 \mid \tau\right)\right] - \frac{p^2(1+a)^2}{a^2 \cdot T^2} \left[ \frac{(1+a)^{T-1}}{T\left(\frac{1+a}{a}\right)^{T-1}} - 2 \left(\frac{1+a}{a}\right)^{T-1} + 1 \right]
\]

(17)

We have no clue to derive the second term of RHS in equation (17) and set it to be A.

In sum, variance of loss is given as follows.

\[
\text{Var}(\tilde{L}) = E[\text{Var}(\tilde{L} \mid \tau)] + \text{Var}[E(\tilde{L} \mid \tau)]
\]

\[
= p(1-p)\frac{T+1}{2} + E\left[E\left(\tilde{L}_2^2 \mid \tau\right)\right] - \frac{p^2(1+a)^2}{a^2 \cdot T^2} \left[ \frac{(1+a)^{T-1}}{T\left(\frac{1+a}{a}\right)^{T-1}} - 2 \left(\frac{1+a}{a}\right)^{T-1} + 1 \right] + p^2 \left[ \frac{T(T+1)}{6} - \frac{(T+1)^2}{4} \right] + \frac{p^2(1+a)^2}{a^2} \left[ \frac{(1+a)^{T-1}}{T\cdot a} - \left(\frac{1+a}{a}\right)^{T-1} \right]^{2}
\]

(18)

3.10 Chernoff Bounds

By section 3.8, \( E\left[e^{\theta \tilde{L}}\right] = \frac{1}{T} \sum_{\tau=1}^{T} \left( p e^{\theta} + 1 - p \right)^{\tau} \cdot \prod_{k=1}^{T-\tau} \left( p e^{\theta(1+a)^{k}} + 1 - p \right) \)

\[
P(\tilde{L} \geq x) \leq \frac{E\left[e^{\theta \tilde{L}}\right]}{e^{\theta x}} = e^{-\theta x} \cdot \frac{1}{T} \sum_{\tau=1}^{T} \left( p e^{\theta} + 1 - p \right)^{\tau} \cdot \prod_{j=1}^{T-\tau} \left( p e^{\theta(1+a)^{j}} + 1 - p \right)
\]

(19)

where \( \theta > 0 \)

The Chernoff bound equates the minimal value of the RHS in equation (19), which are derived by finding the optimal positive \( \theta \). For example, the Chernoff bound (0.6207) of the exceedance probability with threshold of 1 in Table 1 is determined by setting \( T=20 \), \( p=0.01 \), \( a=0.05 \), and searching for the optimal \( \theta=0.9911 \).

11 Pattern of Annual Insurance Premium when Losses Are Correlated over Time

1. Assume \( L_t \) follows a T-variate unspecified distribution with parameters \( (\mu, \sigma, \rho) \)
\[ Var\left(\sum_{t=1}^{T} L_t \right) = Cov\left(\sum_{t_{j1} \leq t_{j2}}^{T} L_{t_{j1}}, \sum_{t_{j1} \leq t_{j2}}^{T} L_{t_{j2}} \right) = \sum_{t_{j1} \leq t_{j2}}^{T} \sum_{t_{j1} \leq t_{j2}}^{T} \text{cov}\left( L_{t_{j1}}, L_{t_{j2}} \right) \]

\[ = T \cdot Var\left( L_t \right) + \sum_{t_{j1} \neq t_{j2}} \text{cov}\left( L_{t_{j1}}, L_{t_{j2}} \right) \]

\[ = T \cdot \sigma^2 + \left( T^2 - T \right) \cdot \rho \cdot \sigma^2 \]

\[ = T \cdot \left( 1 + \rho(T - 1) \right) \cdot \sigma^2 \]

(20)

If \( \rho = 0 \), \( \text{Var}\left( \sum_{t=1}^{T} L_t \right) = T \cdot \sigma^2 \)

If \( \rho = 1 \), \( \text{Var}\left( \sum_{t=1}^{T} L_t \right) = T^2 \cdot \sigma^2 \Rightarrow \text{Annual premium will be constant} \)

If \( 0 < \rho < 1 \), \( \text{Var}\left( \sum_{t=1}^{T} L_t \right) = T \cdot \left( 1 + \rho(T - 1) \right) \cdot \sigma^2 \Rightarrow \text{Annual premium will be decreasing over time} \)

2. \( L_t \) is autoregressive (AR(1)) and stationary

\[ L_t = c + \phi L_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma^2), \ |\phi| < 1 \]

\[ E(L_t) = \mu = \frac{c}{1 - \phi}, \text{Var}(L_t) = \frac{\sigma^2}{1 - \phi}, \text{Cov}(L_{t+n}, L_t) = \frac{\sigma^2}{1 - \phi} \cdot \phi^n \]

\[ \text{Var}\left( \sum_{t=1}^{T} L_t \right) = Cov\left(\sum_{t_{j1} \leq t_{j2}}^{T} L_{t_{j1}}, \sum_{t_{j1} \leq t_{j2}}^{T} L_{t_{j2}} \right) = \sum_{t_{j1} \leq t_{j2}}^{T} \sum_{t_{j1} \leq t_{j2}}^{T} \text{cov}\left( L_{t_{j1}}, L_{t_{j2}} \right) \]

\[ = \frac{\sigma^2}{1 - \phi} \cdot \left[ \begin{array}{cccc}
1 & \phi & \cdots & \phi^{T-2} \\
\phi & 1 & \cdots & \phi^{T-3} \\
\vdots & \ddots & \ddots & \vdots \\
\phi^{T-2} & \phi^{T-3} & \cdots & 1 \\
\phi^{T-1} & \phi^{T-2} & \cdots & \phi \\
\end{array} \right] \]

\[ = \frac{\sigma^2}{1 - \phi} \cdot \left[ T + 2 \cdot \sum_{t=1}^{T-1} (T - i)\phi^i \right] \]

(21)

\[ T + 2 \cdot \sum_{i=1}^{T-1} (T - i)\phi^i < T^2 \Leftrightarrow \text{annual premium} \downarrow \text{with} T \]

If \( \phi = 0 \), \( T + 2 \cdot \sum_{i=1}^{T-1} (T - i)\phi^i = T \)

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If $\varphi=1$, $T + 2 \cdot \sum_{i=1}^{T-1} (T - i)\varphi^i = T^2 \Rightarrow$ Annual premium will be constant

If $-1<\varphi<1$, $T + 2 \cdot \sum_{i=1}^{T-1} (T - i)\varphi^i < T^2 \Rightarrow$ Annual premium will be decreasing over time

12 The Comparison of Volatilities with no Bayesian-updated Correlated Process and with Bayesian-updated Correlated Process

1. No growing trend for catastrophic losses

   (1) No Bayesian-updated serial correlation

   \[ L_t = L_0 + \varepsilon_t, \quad t = 1, 2, \ldots, T \]

   \[ \varepsilon_t \sim (0, \sigma^2(\varepsilon)) \]

   \[ \sum_{t=1}^{T} L_t = \sum_{t=1}^{T} (L_0 + \varepsilon_t) = T \cdot L_0 + T \cdot \sum_{t=1}^{T} \varepsilon_t \]

   \[ \sigma^2 \left( \sum_{t=1}^{T} L_t \right) = T \cdot \sigma^2(\varepsilon) \quad (22) \]

   (2) With Bayesian-updated serial correlation

   \[ L_t = E_{t-1}(L_t) + \varepsilon_t, \quad \varepsilon_t \sim (0, \sigma^2(\varepsilon)) \]

   \[ E_{t-1}(L_t) = w \cdot L_{t-1} + (1 - w) \cdot E_{t-2}(L_{t-1}) \]

   \[ t = 1, 2, \ldots, T \]

   \[ \Rightarrow L_t = L_0 + \varepsilon_t = w \cdot \sum_{i=1}^{t-1} \varepsilon_i + \varepsilon_t \]

   This result can be proved by mathematical induction.

   \[ L_1 = L_0 + \varepsilon_1 \]

   Assume that $L_k = L_0 + w \cdot \sum_{i=1}^{k-1} \varepsilon_i + \varepsilon_k$ is true.
\[ L_{k+1} = E_k \left( L_{k+1} \right) + \epsilon_{k+1} \]
\[ = w \cdot L_k + (1 - w) \cdot E_{k-1} \left( L_k \right) + \epsilon_{k+1} \]
\[ = w \cdot \left( L_0 + w \cdot \sum_{i=1}^{k-1} \epsilon_i + \epsilon_k \right) + (1 - w) \cdot \left( L_k - \epsilon_k \right) + \epsilon_{k+1} \]
\[ = L_0 + w \cdot \sum_{i=1}^{k-1} \epsilon_i + w \cdot \epsilon_k + \epsilon_{k+1} \]
\[ = L_0 + w \cdot \sum_{i=1}^{k} \epsilon_i + \epsilon_{k+1} \]

Thus, by mathematical induction, \( L_t = L_0 + w \cdot \sum_{i=1}^{t-1} \epsilon_i + \epsilon_t \) are true for \( t = 1, 2, \ldots, T \).

Based on the value of \( L_t \), we can derive the value of \( \sum_{t=1}^{T} L_t \) and \( \sigma^2_{Bayes} \left( \sum_{t=1}^{T} L_t \right) \)
\[ \sum_{t=1}^{T} L_t \]
\[ = \sum_{t=1}^{T} \left( L_0 + w \cdot \sum_{i=1}^{t-1} \epsilon_i + \epsilon_t \right) \]
\[ = T \cdot L_0 + w \cdot \left[ \sum_{t=1}^{T} \left( \sum_{i=1}^{t-1} \epsilon_i \right) \right] + \sum_{t=1}^{T} \epsilon_t \]
\[ \sigma^2_{Bayes} \left( \sum_{t=1}^{T} L_t \right) \]
\[ = \sigma^2 \left( \sum_{i=1}^{T} \epsilon_i \right) + w^2 \cdot \sigma^2 \left( \sum_{i=1}^{T} \left( \sum_{i=1}^{t-1} \epsilon_i \right) \right) + 2w \cdot \text{cov} \left( \sum_{i=1}^{T} \sum_{i=1}^{t-1} \epsilon_i, \sum_{i=1}^{T} \epsilon_t \right) \]
\[ = T \cdot \sigma^2 (\epsilon) + w^2 \cdot \left[ \sum_{i=1}^{T} i^2 \cdot \sum_{i=1}^{T} (T - 1 - i) \right] \cdot \sigma^2 (\epsilon) + 2w \cdot \left[ \frac{T(T - 1)}{2} \right] \cdot \sigma^2 (\epsilon) \]
\[ + w \cdot (T(T - 1)) \cdot \sigma^2 (\epsilon) \]
\[ = T \cdot \sigma^2 (\epsilon) + w^2 \cdot \frac{T - 1}{6} \left[ 4 \left( T - \frac{11}{8} \right)^2 + \frac{71}{16} \right] \cdot \sigma^2 (\epsilon) + w \cdot (T(T - 1)) \cdot \sigma^2 (\epsilon) \quad (23) \]

Comparing the values of (3.22) and (3.23) indicates that \( \sigma^2_{Bayes} \left( \sum_{t=1}^{T} L_t \right) \geq \sigma^2 \left( \sum_{t=1}^{T} L_t \right) \), with equality only when \( T=1 \).
As we apply variance in the presence of Bayesian-updated serial correlation to price insurance premium, annual premiums will increase with longer timescales. This result can be verified by obtaining the derivative of the volatility with respect to the timescale.

\[
\frac{\sigma_{\text{Bayes}}(\sum_{t=1}^{T} L_t)}{\partial T} = \frac{1 + w^2 \left[ 2 \left( \frac{T - 5}{4} \right)^2 + \frac{17}{24} \right] + w(2T-1)}{2 \sqrt{T + \frac{T-1}{6} \cdot \left[ 4 \left( \frac{T - 11}{8} \right)^2 + \frac{71}{16} \right]} \cdot \sigma(\varepsilon) > 0.
\]

Taking Bayesian-updated serial correlation into account leads the annual catastrophic insurance premiums show an increasing pattern even if we do not aggregate the capital reserves for each year.

2. With growing trend for catastrophic losses

(1) No Bayesian-updated serial correlation

\[ L_t = L_0 \cdot (1 + a)^t + \varepsilon_t, t = 1, 2, \ldots, T \]

\[ \varepsilon_t \sim \left( 0, \sigma^2(\varepsilon) \right) \]

\[ \sum_{t=1}^{T} L_t = \sum_{t=1}^{T} \left( L_0 (1 + a)^t + \varepsilon_t \right) = L_0 \cdot \sum_{t=1}^{T} (1 + a)^t + \sum_{t=1}^{T} \varepsilon_t \]

\[ \sigma^2 \left( \sum_{t=1}^{T} L_t \right) = T \cdot \sigma^2(\varepsilon) \] \hspace{1cm} (24)

=> The same with the case of no growing trend for catastrophic losses

(2) With Bayesian-updated serial correlation

\[ L_t = E_{t-1}(L_t) + \varepsilon_t, \varepsilon_t \sim \left( 0, \sigma^2(\varepsilon) \right) \]

\[ E_{t-1}(L_t) = E_{t-1}(L_{t-1})(1 + a) \]

\[ E_t(L_t) = w \cdot L_t + (1 - w) \cdot E_{t-1}(L_t) \]

\[ t = 1, 2, \ldots, T \]

\[ \Rightarrow L_t = L_0 \cdot (1 + a)^t + \left[ \sum_{t=1}^{T} (1 + a)^t \cdot w \cdot \varepsilon_t \right] + \varepsilon_t \]

This result can also be proved by mathematical induction.

\[ L_1 = L_0 \cdot (1 + a) + \varepsilon_1 \]

Assume that \( L_k = L_0 \cdot (1 + a)^k + \left[ \sum_{t=1}^{k-1} (1 + a)^t \cdot w \cdot \varepsilon_t \right] + \varepsilon_k \) is true.
\[ L_{k+1} = E_k(L_{k+1}) + \varepsilon_{k+1} \]
\[ = E_k(L_k) \cdot (1 + a) + \varepsilon_{k+1} \]
\[ = \left[ w \cdot L_k + (1 - w) \cdot E_{k-1}(L_k) \right] \cdot (1 + a) + \varepsilon_{k+1} \]
\[ = \left[ w \cdot L_k + (1 - w) \cdot (L_k - \varepsilon_k) \right] \cdot (1 + a) + \varepsilon_{k+1} \]
\[ = (L_k - \varepsilon_k) \cdot (1 + a) + w \cdot \varepsilon_k \cdot (1 + a) + \varepsilon_{k+1} \]
\[ = L_0 \cdot (1 + a)^{k+1} + \left[ \sum_{\tau=1}^{k} (1 + a)^{\tau} \cdot w \cdot \varepsilon_\tau \right] + \varepsilon_{k+1} \]

Thus, by mathematical induction, \( L_t = L_0 \cdot (1 + a)^t + \left[ \sum_{\tau=1}^{t-1} (1 + a)^{\tau} \cdot w \cdot \varepsilon_\tau \right] + \varepsilon_t \) are true for \( t = 1, 2, \ldots, T \)

Based on the value of \( L_t \), we can derive the value of \( \sum_{t=1}^{T} L_t \) and \( \sigma^2_{\text{Bayes}} \left( \sum_{t=1}^{T} L_t \right) \)

\[ \sum_{t=1}^{T} L_t \]
\[ = \sum_{t=1}^{T} \left( L_0 \cdot (1 + a)^t + \left[ \sum_{\tau=1}^{t-1} (1 + a)^{\tau} \cdot w \cdot \varepsilon_\tau \right] + \varepsilon_t \right) \]
\[ = L_0 \sum_{t=1}^{T} (1 + a)^t + \left[ \sum_{t=1}^{T} \left( \sum_{\tau=1}^{t-1} (1 + a)^{\tau} \cdot w \cdot \varepsilon_\tau \right) \right] \cdot w + \sum_{t=1}^{T} \varepsilon_t \]

\[ \sigma^2_{\text{Bayes}} \left( \sum_{t=1}^{T} L_t \right) \]
\[ = \sigma^2 \left( \sum_{t=1}^{T} \varepsilon_t \right) + w^2 \cdot \sigma^2 \left( \sum_{t=1}^{T} \left( \sum_{\tau=1}^{t-1} (1 + a)^{\tau} \cdot w \cdot \varepsilon_\tau \right) \right) + 2w \cdot \text{cov} \left[ \sum_{t=1}^{T} \left( \sum_{\tau=1}^{t-1} (1 + a)^{\tau} \cdot w \cdot \varepsilon_\tau \right), \sum_{t=1}^{T} \varepsilon_t \right] \]
\[ = T \cdot \sigma^2(\varepsilon) + w^2 \cdot \left[ \sum_{t=1}^{T} (1 + a)^{2t} \cdot (T - i) + 2 \cdot \sum_{i=1}^{T-2} (1 + a)^{2i} \cdot \frac{(T - i) \cdot (T - 1 - i)}{2} \right] \cdot \sigma^2(\varepsilon) \]
\[ + 2w \cdot \left[ \sum_{t=1}^{T-1} (1 + a)^i \cdot (T - i) \right] \cdot \sigma^2(\varepsilon) \]  

(25)

A Comparison of the values between (24) and (25) indicates that

\[ \sigma^2_{\text{Bayes}} \left( \sum_{t=1}^{T} L_t \right) \geq \sigma^2 \left( \sum_{t=1}^{T} L_t \right) \]

, with equality only when \( T = 1 \).
13 Derivation of Risk Capital for Two One-period Contracts

\[
E^O \left[ (L_1 - P_1^t)^+ \right] = E^O \left[ L_1 I_{(L_1 > \mu_1)} \right] - \mu_1 E^O \left[ I_{(L_1 > \mu_1)} \right] L_1 \sim N(\mu, \sigma) \\
= \int_0^\infty \left( \mu_1 + \sigma Z^O \right) \phi(Z^O) dZ^O - \frac{\mu_1}{2} \\
= \frac{\sigma}{\sqrt{2\pi}} 
\]  
(26)

\[
E^O \left[ \left( L_2^N \mid L_1^R - P_2^N \right)^+ \right] = E^O \left[ L_2^N I_{(L_2^N \mid L_1^R - P_2^N > \mu_2^N)} \right] - \mu_2^N E^O \left[ I_{(L_2^N \mid L_1^R - P_2^N > \mu_2^N)} \right] L_2^N \mid L_1 \sim N(\mu_2^N, \sigma) \\
= \int_0^\infty \left( \mu_2^N + \sigma Z^O \right) \phi(Z^O) dZ^O - \frac{\mu_2^N}{2} \\
= \frac{\sigma}{\sqrt{2\pi}} 
\]  
(27)

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---

Table 1: Climate Change Effect on the Statistics of the Simulated Losses

<table>
<thead>
<tr>
<th>Climate change factor (a)</th>
<th>Simulated</th>
<th>Exact</th>
<th>Simulated</th>
<th>Exact</th>
<th>%change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected value</td>
<td>0.2018</td>
<td>0.2</td>
<td>0.2428</td>
<td>0.2422</td>
<td>20.32%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.4482</td>
<td>0.445</td>
<td>0.5465</td>
<td></td>
<td>21.93%</td>
</tr>
<tr>
<td>Skewness</td>
<td>2.2029</td>
<td>2.2024</td>
<td>2.2903</td>
<td></td>
<td>3.97%</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>7.7498</td>
<td>7.7505</td>
<td>8.2858</td>
<td></td>
<td>6.92%</td>
</tr>
<tr>
<td>VaR (95%)</td>
<td>1</td>
<td>1</td>
<td>1.3829</td>
<td></td>
<td>38.29%</td>
</tr>
<tr>
<td>VaR (97.5%)</td>
<td>1</td>
<td>1</td>
<td>1.6533</td>
<td></td>
<td>65.33%</td>
</tr>
<tr>
<td>VaR (99%)</td>
<td>2</td>
<td>2</td>
<td>2.3314</td>
<td></td>
<td>16.57%</td>
</tr>
<tr>
<td>ES (95%)</td>
<td>1.3754</td>
<td>1.3581</td>
<td>1.8622</td>
<td></td>
<td>35.39%</td>
</tr>
<tr>
<td>ES (97.5%)</td>
<td>1.7508</td>
<td>1.7163</td>
<td>2.2166</td>
<td></td>
<td>26.60%</td>
</tr>
<tr>
<td>ES (99%)</td>
<td>2.107</td>
<td>2.1048</td>
<td>2.7279</td>
<td></td>
<td>29.47%</td>
</tr>
<tr>
<td>Prob(Loss&gt;1.0)</td>
<td>0.0177</td>
<td>0.0169</td>
<td>0.1663</td>
<td>0.6207</td>
<td>839.55%</td>
</tr>
<tr>
<td>Prob(Loss&gt;1.5)</td>
<td>0.0177</td>
<td>0.0169</td>
<td>0.0416</td>
<td>0.3554</td>
<td>135.03%</td>
</tr>
<tr>
<td>Prob(Loss&gt;2.0)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.0165</td>
<td>0.1859</td>
<td>1550.00%</td>
</tr>
<tr>
<td>Prob(Loss&gt;2.5)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.0072</td>
<td>0.0915</td>
<td>620.00%</td>
</tr>
<tr>
<td>Prob(Loss&gt;3.0)</td>
<td>0.0001</td>
<td>0</td>
<td>0.0014</td>
<td>0.043</td>
<td>1300.00%</td>
</tr>
</tbody>
</table>

The exceedance probabilities of the loss with climate change can not be derived directly; thus the Chernoff bounds are derived for the thresholds indicated in the table.

---

Table 2: Impact of Climate Change Uncertainty on the Statistics of the Simulated Losses

<table>
<thead>
<tr>
<th>Climate change uncertainty</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected value</td>
<td>0.2428</td>
<td>0.2443</td>
<td>0.2483</td>
<td>0.62%</td>
<td>2.27%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.5465</td>
<td>0.5523</td>
<td>0.562</td>
<td>1.06%</td>
<td>2.84%</td>
</tr>
</tbody>
</table>
Table 3: Time Scale and Cost/Benefit of Optimal Mitigation

<table>
<thead>
<tr>
<th>Discount Rate=0%</th>
<th>Time Scale</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>optimal mitigation levels</td>
<td>0</td>
<td>0.05</td>
<td>0.15</td>
<td>0.25</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>TC with optimal mitigation</td>
<td>0.0102</td>
<td>0.0228</td>
<td>0.0557</td>
<td>0.105</td>
<td>0.1467</td>
<td>0.1846</td>
<td></td>
</tr>
<tr>
<td>benefit from optimal mitigation</td>
<td>0</td>
<td>0.0095</td>
<td>0.0275</td>
<td>0.0701</td>
<td>0.147</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>optimal mitigation cost</td>
<td>0</td>
<td>0.0028</td>
<td>0.0099</td>
<td>0.0195</td>
<td>0.0563</td>
<td>0.0563</td>
<td></td>
</tr>
<tr>
<td>net benefit from optimal mitigation</td>
<td>0</td>
<td>0.0067</td>
<td>0.0176</td>
<td>0.0505</td>
<td>0.0908</td>
<td>0.1537</td>
<td></td>
</tr>
<tr>
<td>TC with no mitigation</td>
<td>0.0102</td>
<td>0.0296</td>
<td>0.0733</td>
<td>0.1555</td>
<td>0.2375</td>
<td>0.3383</td>
<td></td>
</tr>
<tr>
<td>RNB from optimal mitigation</td>
<td>0.00%</td>
<td>22.71%</td>
<td>23.96%</td>
<td>32.49%</td>
<td>38.21%</td>
<td>45.44%</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Total Cost with No Mitigation No Climate Change, with Optimal Mitigation No Climate Change, with No Mitigation with Climate Change, with Optimal Mitigation and Climate Change for Different Discount Rates

<table>
<thead>
<tr>
<th>Discount Rate=0%</th>
<th>Time Scale</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC with no mitigation no CC</td>
<td>0.0102</td>
<td>0.0729</td>
<td>0.1523</td>
<td>0.2237</td>
<td>0.2991</td>
</tr>
<tr>
<td>TC with optimal mitigation no CC</td>
<td>0.0102</td>
<td>0.0509</td>
<td>0.0966</td>
<td>0.1316</td>
<td>0.1573</td>
</tr>
<tr>
<td>TC with no mitigation with CC</td>
<td>0.0102</td>
<td>0.0733</td>
<td>0.1555</td>
<td>0.2375</td>
<td>0.3383</td>
</tr>
<tr>
<td>TC with optimal mitigation with CC</td>
<td>0.0102</td>
<td>0.0557</td>
<td>0.1050</td>
<td>0.1467</td>
<td>0.1846</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Discount Rate=5%</th>
<th>Time Scale</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC with no mitigation no CC</td>
<td>0.0104</td>
<td>0.0443</td>
<td>0.0793</td>
<td>0.1094</td>
<td>0.1329</td>
</tr>
<tr>
<td>TC with optimal mitigation no CC</td>
<td>0.0104</td>
<td>0.0443</td>
<td>0.0770</td>
<td>0.0996</td>
<td>0.1141</td>
</tr>
<tr>
<td>TC with no mitigation with CC</td>
<td>0.0104</td>
<td>0.0462</td>
<td>0.0869</td>
<td>0.1244</td>
<td>0.1569</td>
</tr>
<tr>
<td>TC with optimal mitigation with CC</td>
<td>0.0104</td>
<td>0.0462</td>
<td>0.0828</td>
<td>0.1097</td>
<td>0.1268</td>
</tr>
</tbody>
</table>
Table 5: Climate Change Effect, Mitigation Effect, and Aggregate Effect for Different Discount Rates

<table>
<thead>
<tr>
<th>Time Scale</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Discount Rate=0%</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Climate Change Effect</td>
<td>0.00%</td>
<td>0.55%</td>
<td>2.10%</td>
<td>6.17%</td>
<td>13.11%</td>
</tr>
<tr>
<td>Mitigation Effect</td>
<td>0.00%</td>
<td>-24.01%</td>
<td>-32.48%</td>
<td>-38.23%</td>
<td>-45.43%</td>
</tr>
<tr>
<td>Aggregate Effect</td>
<td>0.00%</td>
<td>-23.59%</td>
<td>-31.06%</td>
<td>-34.42%</td>
<td>-38.28%</td>
</tr>
<tr>
<td><strong>Discount Rate=5%</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Climate Change Effect</td>
<td>3.19%</td>
<td>4.29%</td>
<td>9.58%</td>
<td>13.71%</td>
<td>18.06%</td>
</tr>
<tr>
<td>Mitigation Effect</td>
<td>0.00%</td>
<td>0.00%</td>
<td>-4.72%</td>
<td>-11.82%</td>
<td>-19.18%</td>
</tr>
<tr>
<td>Aggregate Effect</td>
<td>3.19%</td>
<td>4.29%</td>
<td>4.41%</td>
<td>0.27%</td>
<td>-4.59%</td>
</tr>
</tbody>
</table>

Table 6: Statistics of Simulated Losses for Different Climate Change Factors (a) with Roof Mitigation, T=20

| a | Expected Value | Std | Skewness | Kurtosis | VaR (95%) | VaR (97.5%) | VaR (99%) | ES (95%) | ES (97.5%) | ES (99%) | Prob(Loss>0.5) | Prob(Loss>1) | Prob(Loss>1.5) | Prob(Loss>2) | Prob(Loss>2.5) | Prob(Loss>3) |
|---|----------------|-----|----------|----------|-----------|------------|-----------|----------|------------|----------|---------------|---------------|----------------|--------------|----------------|---------------|----------------|---------------|
| 0 | 0.83           | 0.51| 0.86     | 3.81     | 1.78      | 2.01       | 2.29      | 2.10     | 2.31       | 2.59     | 0.69          | 0.32          | 0.10           | 0.02         | 0.01          | 0.00          |
| 0.05| 1.01          | 0.63| 0.92     | 3.98     | 2.18      | 2.48       | 2.84      | 2.59     | 2.86       | 3.21     | 0.77          | 0.44          | 0.20           | 0.07         | 0.02          | 0.01          |
| 0.1 | 1.28          | 0.84| 1.05     | 4.31     | 2.89      | 3.31       | 3.81      | 3.47     | 3.85       | 4.33     | 0.82          | 0.56          | 0.35           | 0.18         | 0.09          | 0.04          |
| **Table 7: Statistics of Simulated Losses for Different Climate Change Factors (a) with Roof Mitigation (A), T=20** |     |     |     |     |     |     |     |     |     |     |               |               |                |              |               |               |
| a | Expected Value | Std | Skewness | Kurtosis | VaR (95%) | VaR (97.5%) | VaR (99%) | ES (95%) | ES (97.5%) | ES (99%) | Prob(Loss>0.5) | Prob(Loss>1) | Prob(Loss>1.5) | Prob(Loss>2) | Prob(Loss>2.5) | Prob(Loss>3) |
|---|----------------|-----|----------|----------|-----------|------------|-----------|----------|------------|----------|---------------|---------------|----------------|--------------|----------------|---------------|----------------|---------------|
| 0 | 0.61           | 0.41| 1.05     | 4.28     | 1.39      | 1.59       | 1.84      | 1.67     | 1.86       | 2.09     | 0.52          | 0.16          | 0.03           | 0.01         | 0.00          | 0.00          |
| 0.05| 0.74          | 0.50| 1.12     | 4.54     | 1.71      | 1.97       | 2.27      | 2.06     | 2.30       | 2.59     | 0.61          | 0.26          | 0.08           | 0.02         | 0.01          | 0.00          |
| 0.1 | 0.95          | 0.67| 1.25     | 4.95     | 2.26      | 2.63       | 3.09      | 2.76     | 3.10       | 3.51     | 0.72          | 0.39          | 0.18           | 0.08         | 0.03          | 0.01          |

Table 8: Benefit/Cost of Different Mitigation Measures and Different Time Horizons for Wood Frame Building in Canaries in the Absence of Climate Change (a=0.00)

<table>
<thead>
<tr>
<th>Time Scale</th>
<th>1</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benefit from Mitigation</strong> (Reduction in Expected Loss)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roof Mitigation (A)</td>
<td>0.0103</td>
<td>0.0544</td>
<td>0.0649</td>
<td>0.0768</td>
<td>0.0857</td>
<td>0.0986</td>
<td>0.1071</td>
</tr>
<tr>
<td>Opening Mitigation (B)</td>
<td>0.0119</td>
<td>0.0638</td>
<td>0.0786</td>
<td>0.0900</td>
<td>0.1005</td>
<td>0.1144</td>
<td>0.1284</td>
</tr>
<tr>
<td>Roof &amp; Opening (AB)</td>
<td>0.0198</td>
<td>0.1001</td>
<td>0.1211</td>
<td>0.1409</td>
<td>0.1607</td>
<td>0.1815</td>
<td>0.2014</td>
</tr>
<tr>
<td><strong>Mitigation Cost</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roof Mitigation (A)</td>
<td>0.0920</td>
<td>0.0920</td>
<td>0.0920</td>
<td>0.0920</td>
<td>0.0920</td>
<td>0.0920</td>
<td>0.0920</td>
</tr>
<tr>
<td>Opening Mitigation (B)</td>
<td>0.0672</td>
<td>0.0672</td>
<td>0.0672</td>
<td>0.0672</td>
<td>0.0672</td>
<td>0.0672</td>
<td>0.0672</td>
</tr>
<tr>
<td>Roof &amp; Opening (AB)</td>
<td>0.1592</td>
<td>0.1592</td>
<td>0.1592</td>
<td>0.1592</td>
<td>0.1592</td>
<td>0.1592</td>
<td>0.1592</td>
</tr>
<tr>
<td><strong>Benefit-Cost Ratio</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roof Mitigation (A)</td>
<td>0.1120</td>
<td>0.5913</td>
<td>0.7054</td>
<td>0.8348</td>
<td>0.9315</td>
<td><strong>1.0717</strong></td>
<td>1.1641</td>
</tr>
<tr>
<td>Opening Mitigation (B)</td>
<td>0.1771</td>
<td>0.9494</td>
<td><strong>1.1696</strong></td>
<td>1.3393</td>
<td>1.4955</td>
<td>1.7024</td>
<td>1.9107</td>
</tr>
<tr>
<td>Roof &amp; Opening (AB)</td>
<td>0.1244</td>
<td>0.6288</td>
<td>0.7607</td>
<td>0.8851</td>
<td><strong>1.0094</strong></td>
<td>1.1401</td>
<td>1.2651</td>
</tr>
</tbody>
</table>
Table 9: Benefit/Cost of Different Mitigation Measures and Different Time Horizons for Wood Frame Building in Canaries in the Absence of Climate Change (“a”=0.05)

<table>
<thead>
<tr>
<th>Time scale</th>
<th>Benefit from Mitigation (Reduction in Expected Loss)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Roof Mitigation (A)</td>
</tr>
<tr>
<td></td>
<td>Opening Mitigation (B)</td>
</tr>
<tr>
<td></td>
<td>Roof &amp; Opening (AB)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mitigation Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Roof Mitigation (A)</td>
</tr>
<tr>
<td>Opening Mitigation (B)</td>
</tr>
<tr>
<td>Roof &amp; Opening (AB)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Benefit-Cost Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof Mitigation (A)</td>
</tr>
<tr>
<td>Opening Mitigation (B)</td>
</tr>
<tr>
<td>Roof &amp; Opening (AB)</td>
</tr>
</tbody>
</table>

Table 10: Statistics for Wood Frame Building in Canaries in St Lucia for Different Settings of “a”

<table>
<thead>
<tr>
<th>a</th>
<th>Expected value</th>
<th>Std</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>VaR (95%)</th>
<th>VaR (97.5%)</th>
<th>VaR (99%)</th>
<th>ES (95%)</th>
<th>ES (97.5%)</th>
<th>ES (99%)</th>
</tr>
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<tbody>
<tr>
<td>0.05</td>
<td>1.0055</td>
<td>0.6254</td>
<td>0.8905</td>
<td>3.8287</td>
<td>2.1823</td>
<td>2.5791</td>
<td>2.4815</td>
<td>2.8437</td>
<td>2.8225</td>
<td>3.165</td>
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<tr>
<td>0–0.1</td>
<td>1.0291</td>
<td>0.6488</td>
<td>0.9386</td>
<td>4.0259</td>
<td>2.2521</td>
<td>2.559</td>
<td>2.938</td>
<td>2.6757</td>
<td>2.9614</td>
<td>3.3229</td>
</tr>
<tr>
<td>0.06</td>
<td>1.0506</td>
<td>0.6534</td>
<td>0.8964</td>
<td>3.8889</td>
<td>2.2768</td>
<td>2.6949</td>
<td>2.5813</td>
<td>2.9788</td>
<td>2.9626</td>
<td>3.3348</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>a</th>
<th>Prob(Loss&gt;1.5)</th>
<th>Prob(Loss&gt;2)</th>
<th>Prob(Loss&gt;2.25)</th>
<th>Prob(Loss&gt;2.5)</th>
<th>Prob(Loss&gt;2.75)</th>
<th>Prob(Loss&gt;3)</th>
<th>Prob(Loss&gt;4)</th>
<th>Prob(Loss&gt;3.5)</th>
<th>Prob(Loss&gt;3.75)</th>
<th>Prob(Loss&gt;4.25)</th>
<th>Prob(Loss&gt;4.75)</th>
<th>Prob(Loss&gt;5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.2008</td>
<td>0.1214</td>
<td>0.0416</td>
<td>0.0243</td>
<td>0.0121</td>
<td>0.0063</td>
<td>0.0002</td>
<td>0.0013</td>
<td>0.0007</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0</td>
</tr>
<tr>
<td>0–0.1</td>
<td>0.2141</td>
<td>0.1398</td>
<td>0.0534</td>
<td>0.0316</td>
<td>0.0177</td>
<td>0.0073</td>
<td>0.0006</td>
<td>0.0021</td>
<td>0.0011</td>
<td>0.0003</td>
<td>0.0002</td>
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<tr>
<td>0.06</td>
<td>0.226</td>
<td>0.1426</td>
<td>0.0541</td>
<td>0.0303</td>
<td>0.0164</td>
<td>0.0088</td>
<td>0.0005</td>
<td>0.0025</td>
<td>0.001</td>
<td>0.0003</td>
<td>0.001</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 11: Statistics of Variability in Storm Activities over Successive 5-year Period

<table>
<thead>
<tr>
<th>Statistic</th>
<th>All Named Storms</th>
<th>Cat 3 – 5 hurricanes</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>6%</td>
<td>12%</td>
</tr>
<tr>
<td>95th percentile</td>
<td>38%</td>
<td>85%</td>
</tr>
<tr>
<td>65th percentile</td>
<td>16%</td>
<td>22%</td>
</tr>
<tr>
<td>50th percentile</td>
<td>5%</td>
<td>3%</td>
</tr>
<tr>
<td>35th percentile</td>
<td>-2%</td>
<td>-12%</td>
</tr>
<tr>
<td>5th percentile</td>
<td>-24%</td>
<td>-42%</td>
</tr>
</tbody>
</table>

Table 11: Statistics of Variability in Storm Activities over Successive 5-year Period

<table>
<thead>
<tr>
<th>Statistic</th>
<th>All Named Storms</th>
<th>Cat 3 – 5 hurricanes</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>6%</td>
<td>12%</td>
</tr>
<tr>
<td>95th percentile</td>
<td>38%</td>
<td>85%</td>
</tr>
<tr>
<td>65th percentile</td>
<td>16%</td>
<td>22%</td>
</tr>
<tr>
<td>50th percentile</td>
<td>5%</td>
<td>3%</td>
</tr>
<tr>
<td>35th percentile</td>
<td>-2%</td>
<td>-12%</td>
</tr>
<tr>
<td>5th percentile</td>
<td>-24%</td>
<td>-42%</td>
</tr>
</tbody>
</table>

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Table 12: Percentiles of Frequency of Storm Activities, Expected Loss, Volatility of Loss, and Estimates of Climate Change Factor

<table>
<thead>
<tr>
<th></th>
<th>Present-Day</th>
<th>Values at (s-tau)=5 5%</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency of all named storms relative to present-day</td>
<td>1</td>
<td>0.76</td>
<td>0.98</td>
<td>1.05</td>
<td>1.16</td>
<td>1.38</td>
</tr>
<tr>
<td>Frequency of intense storms relative to present-day</td>
<td>1</td>
<td>0.58</td>
<td>0.88</td>
<td>1.03</td>
<td>1.22</td>
<td>1.85</td>
</tr>
<tr>
<td>expected loss over next 5 years standard deviation of loss</td>
<td>3376.64</td>
<td>2165.14</td>
<td>3086.28</td>
<td>3500.9</td>
<td>4050.61</td>
<td>5707.1</td>
</tr>
<tr>
<td></td>
<td>9297.99</td>
<td>7325.63</td>
<td>8794.68</td>
<td>9412.44</td>
<td>10116.8</td>
<td>12026.9</td>
</tr>
<tr>
<td>Growth rate of losses (over 5 years) standard deviation of loss</td>
<td>0</td>
<td>-8.50%</td>
<td>-1.78%</td>
<td>0.73%</td>
<td>3.71%</td>
<td>11.07%</td>
</tr>
<tr>
<td>Growth rate of standard deviation (over 5 years)</td>
<td>0</td>
<td>-4.66%</td>
<td>-1.11%</td>
<td>0.24%</td>
<td>1.70%</td>
<td>5.28%</td>
</tr>
</tbody>
</table>

Table 13: Probabilities of the Occurrence of Climate Change

<table>
<thead>
<tr>
<th>P(τ)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ=1</td>
<td>0.1848</td>
</tr>
<tr>
<td>τ=2</td>
<td>0.1479</td>
</tr>
<tr>
<td>τ=3</td>
<td>0.2129</td>
</tr>
<tr>
<td>τ=4</td>
<td>0.2309</td>
</tr>
<tr>
<td>τ=5</td>
<td>0.2235</td>
</tr>
</tbody>
</table>

Table 14: The Distribution of “a” transformed from Five Climate Scenarios

<table>
<thead>
<tr>
<th>a</th>
<th>Prob(a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.085</td>
<td>0.2</td>
</tr>
<tr>
<td>-0.0178</td>
<td>0.225</td>
</tr>
<tr>
<td>0.0073</td>
<td>0.15</td>
</tr>
<tr>
<td>0.0371</td>
<td>0.225</td>
</tr>
<tr>
<td>0.1107</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 15: Statistics, Tail Probabilities, and Annual Insurance Premiums for 5-Year Losses in Different Models

<table>
<thead>
<tr>
<th>Models</th>
<th>Potential Growth Model</th>
<th>Lognormal Loss Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>present-day loss</td>
<td>present-day loss</td>
</tr>
<tr>
<td>a_step</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a_linear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a_expox</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fit_EPs</td>
<td>Fit_E(L)&amp;σ(L)</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Expected losses</td>
<td>16,570</td>
<td>16,620</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>21,360</td>
<td>21,360</td>
</tr>
<tr>
<td>Skewness</td>
<td>2.1675</td>
<td>2.1456</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>8.4176</td>
<td>8.2913</td>
</tr>
<tr>
<td>VaR (95%)</td>
<td>64,210</td>
<td>64,180</td>
</tr>
<tr>
<td>VaR (97.5%)</td>
<td>77,850</td>
<td>77,850</td>
</tr>
<tr>
<td>VaR (99%)</td>
<td>93,780</td>
<td>93,120</td>
</tr>
<tr>
<td></td>
<td>Present-Day Loss</td>
<td>Potential Climate Scenarios</td>
</tr>
<tr>
<td>----------</td>
<td>-----------------</td>
<td>----------------------------</td>
</tr>
<tr>
<td>Fit EPs</td>
<td>Fit ( E(L) ) &amp; ( \sigma(L) )</td>
<td>Fit ( E(L) ) &amp; ( \sigma(L) )</td>
</tr>
<tr>
<td>Time Horizon</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Expected Losses</td>
<td>3,377</td>
<td>3,200</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>9,298</td>
<td>7,300</td>
</tr>
<tr>
<td>Skewness</td>
<td>7.66</td>
<td>11.83</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>133.93</td>
<td>334.85</td>
</tr>
<tr>
<td>Percentile (50%)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Percentile (80%)</td>
<td>2,090</td>
<td>4,710</td>
</tr>
<tr>
<td>Percentile (90%)</td>
<td>6,910</td>
<td>8,970</td>
</tr>
<tr>
<td>Percentile (95%)</td>
<td>20,900</td>
<td>14,390</td>
</tr>
<tr>
<td>Percentile (98%)</td>
<td>41,230</td>
<td>23,840</td>
</tr>
<tr>
<td>Percentile (99%)</td>
<td>53,150</td>
<td>32,990</td>
</tr>
<tr>
<td>Percentile (99.5%)</td>
<td>62,780</td>
<td>43,670</td>
</tr>
<tr>
<td>Percentile (99.6%)</td>
<td>65,140</td>
<td>47,510</td>
</tr>
<tr>
<td>Percentile (99.8%)</td>
<td>72,290</td>
<td>60,540</td>
</tr>
<tr>
<td>Percentile (99.9%)</td>
<td>77,110</td>
<td>74,150</td>
</tr>
<tr>
<td>Percentile (99.98%)</td>
<td>80,480</td>
<td>138,130</td>
</tr>
<tr>
<td>Percentile (99.99%)</td>
<td>87,520</td>
<td>175,230</td>
</tr>
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</table>

Table 16: Statistics and Percentiles for 1-Year Lognormal Losses under Different Assumptions
### Table 17: Statistics, Tail Probabilities, and Annual Insurance Premiums for 5-Year Losses for Different Scenarios in Potential Growth Model

<table>
<thead>
<tr>
<th></th>
<th>UCS</th>
<th>BCS</th>
<th>MLS</th>
<th>WCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time horizon</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Expected losses</td>
<td>16,630</td>
<td>15,590</td>
<td>16,460</td>
<td>17,940</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>21,370</td>
<td>20,040</td>
<td>21,210</td>
<td>22,920</td>
</tr>
<tr>
<td>Skewness</td>
<td>2.1454</td>
<td>2.1744</td>
<td>2.1714</td>
<td>2.1227</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>8.2396</td>
<td>8.5168</td>
<td>8.4269</td>
<td>8.2139</td>
</tr>
<tr>
<td>VaR (95%)</td>
<td>64,190</td>
<td>59,770</td>
<td>63,610</td>
<td>68,950</td>
</tr>
<tr>
<td>VaR (97.5%)</td>
<td>77,980</td>
<td>73,670</td>
<td>77,580</td>
<td>83,230</td>
</tr>
<tr>
<td>ES(95%)</td>
<td>93,750</td>
<td>88,930</td>
<td>93,110</td>
<td>98,710</td>
</tr>
<tr>
<td>ES(97.5%)</td>
<td>82,950</td>
<td>78,160</td>
<td>82,510</td>
<td>88,450</td>
</tr>
<tr>
<td>VaR (99%)</td>
<td>111,520</td>
<td>105,150</td>
<td>111,170</td>
<td>118,830</td>
</tr>
<tr>
<td>ES(99%)</td>
<td>111,520</td>
<td>105,150</td>
<td>111,170</td>
<td>118,830</td>
</tr>
<tr>
<td>Prob(Loss&gt;0.25)</td>
<td>0.2115</td>
<td>0.1978</td>
<td>0.2079</td>
<td>0.2271</td>
</tr>
<tr>
<td>Prob(Loss&gt;0.5)</td>
<td>0.0923</td>
<td>0.0783</td>
<td>0.0901</td>
<td>0.107</td>
</tr>
<tr>
<td>Prob(Loss&gt;0.75)</td>
<td>0.0287</td>
<td>0.0229</td>
<td>0.0281</td>
<td>0.0372</td>
</tr>
<tr>
<td>Prob(Loss&gt;1)</td>
<td>0.0071</td>
<td>0.0049</td>
<td>0.0066</td>
<td>0.0094</td>
</tr>
<tr>
<td>Prob(Loss&gt;1.25)</td>
<td>0.0018</td>
<td>0.0012</td>
<td>0.0019</td>
<td>0.0028</td>
</tr>
<tr>
<td>Prob(Loss&gt;1.5)</td>
<td>0.0004</td>
<td>0.0003</td>
<td>0.0004</td>
<td>0.0008</td>
</tr>
<tr>
<td>Prob(Loss&gt;1.75)</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0002</td>
</tr>
<tr>
<td>Prob(Loss&gt;2)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.0001</td>
</tr>
<tr>
<td>Annual Premium</td>
<td>$5,500.39</td>
<td>$5,090.87</td>
<td>$5,438.29</td>
<td>$6,001.71</td>
</tr>
</tbody>
</table>

### Table 18: Statistics, Tail Probabilities, and Annual Insurance Premiums for 5-Year Losses for Different Scenarios in Lognormal Loss Model

<table>
<thead>
<tr>
<th></th>
<th>UCS</th>
<th>BCS</th>
<th>MLS</th>
<th>WCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time horizon</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Expected losses</td>
<td>18,150</td>
<td>10,770</td>
<td>17,420</td>
<td>28,420</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>21,200</td>
<td>16,010</td>
<td>20,620</td>
<td>26,760</td>
</tr>
<tr>
<td>Skewness</td>
<td>5.0327</td>
<td>7.2403</td>
<td>4.6747</td>
<td>3.0625</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>67.9487</td>
<td>119.0596</td>
<td>52.2188</td>
<td>26.1296</td>
</tr>
<tr>
<td>VaR (95%)</td>
<td>53,830</td>
<td>35,090</td>
<td>52,310</td>
<td>76,900</td>
</tr>
<tr>
<td>VaR (97.5%)</td>
<td>70,450</td>
<td>48,750</td>
<td>68,910</td>
<td>96,190</td>
</tr>
<tr>
<td>VaR (99%)</td>
<td>98,600</td>
<td>71,900</td>
<td>96,370</td>
<td>125,940</td>
</tr>
<tr>
<td>ES(95%)</td>
<td>83,850</td>
<td>60,830</td>
<td>81,840</td>
<td>108,990</td>
</tr>
<tr>
<td>ES(97.5%)</td>
<td>106,760</td>
<td>80,620</td>
<td>104,260</td>
<td>132,750</td>
</tr>
<tr>
<td>ES(99%)</td>
<td>144,090</td>
<td>114,450</td>
<td>140,640</td>
<td>169,180</td>
</tr>
<tr>
<td>Prob(Loss&gt;0.25)</td>
<td>0.2271</td>
<td>0.0946</td>
<td>0.2045</td>
<td>0.4268</td>
</tr>
<tr>
<td>Prob(Loss&gt;0.5)</td>
<td>0.0593</td>
<td>0.0233</td>
<td>0.0571</td>
<td>0.147</td>
</tr>
<tr>
<td>Prob(Loss&gt;0.75)</td>
<td>0.021</td>
<td>0.0092</td>
<td>0.0192</td>
<td>0.0542</td>
</tr>
<tr>
<td>Prob(Loss&gt;1)</td>
<td>0.0097</td>
<td>0.00</td>
<td>0.0089</td>
<td>0.0224</td>
</tr>
<tr>
<td>Prob(Loss&gt;1.25)</td>
<td>0.0049</td>
<td>0.0026</td>
<td>0.0048</td>
<td>0.0101</td>
</tr>
<tr>
<td>Prob(Loss&gt;1.5)</td>
<td>0.0028</td>
<td>0.0014</td>
<td>0.0025</td>
<td>0.0053</td>
</tr>
<tr>
<td>Prob(Loss&gt;1.75)</td>
<td>0.0018</td>
<td>0.0008</td>
<td>0.0015</td>
<td>0.003</td>
</tr>
<tr>
<td>Prob(Loss&gt;2)</td>
<td>0.0012</td>
<td>0.0006</td>
<td>0.0011</td>
<td>0.0017</td>
</tr>
<tr>
<td>Annual Premium</td>
<td>$5,641.83</td>
<td>$3,514.83</td>
<td>$5,426.73</td>
<td>$8,975.05</td>
</tr>
</tbody>
</table>
Table 19 Annual Premiums of Catastrophe Insurance with and without Cost of Capital and Equivalent Cost of Capital in the Absence of Climate Change (a=0)

<table>
<thead>
<tr>
<th>Terms of contracts</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual premium (no CoC)</td>
<td>$48,300</td>
<td>$46,600</td>
<td>$44,900</td>
<td>$44,700</td>
<td>$44,500</td>
<td>$44,300</td>
<td>$44,200</td>
<td>$43,600</td>
<td>$43,800</td>
<td>$43,600</td>
</tr>
<tr>
<td>Annual premium (with CoC)</td>
<td>$48,300</td>
<td>$50,100</td>
<td>$50,400</td>
<td>$51,800</td>
<td>$53,900</td>
<td>$54,800</td>
<td>$55,000</td>
<td>$56,100</td>
<td>$56,700</td>
<td></td>
</tr>
<tr>
<td>Equivalent cost of capital</td>
<td>2.00%</td>
<td>1.56%</td>
<td>1.54%</td>
<td>1.32%</td>
<td>1.18%</td>
<td>1.09%</td>
<td>1.01%</td>
<td>1.02%</td>
<td>0.92%</td>
<td>0.93%</td>
</tr>
</tbody>
</table>

Table 20 Annual Premiums of Catastrophe Insurance with and without Cost of Capital and Equivalent Cost of Capital in the Presence of Climate Change (a=0.019)

<table>
<thead>
<tr>
<th>Terms of contracts</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual premium (no CoC)</td>
<td>$48,100</td>
<td>$46,500</td>
<td>$45,400</td>
<td>$45,600</td>
<td>$45,200</td>
<td>$44,900</td>
<td>$45,000</td>
<td>$44,900</td>
<td>$44,900</td>
<td>$45,100</td>
</tr>
<tr>
<td>Annual premium (with CoC)</td>
<td>$48,100</td>
<td>$50,000</td>
<td>$50,900</td>
<td>$52,700</td>
<td>$53,700</td>
<td>$54,500</td>
<td>$55,700</td>
<td>$56,500</td>
<td>$57,500</td>
<td>$58,500</td>
</tr>
<tr>
<td>Equivalent cost of capital</td>
<td>2.00%</td>
<td>1.56%</td>
<td>1.42%</td>
<td>1.14%</td>
<td>1.04%</td>
<td>0.98%</td>
<td>0.86%</td>
<td>0.81%</td>
<td>0.74%</td>
<td>0.67%</td>
</tr>
</tbody>
</table>
Figure 1: EPs for Different Climate Change Factors

Figure 2: EP Curves for Different Settings on the Uncertainty of Climate Change Factor

Figure 3: EP Curves for Uncertain Climate Change Factor and Certain CCFs

Figure 4: Total Costs Caused by Catastrophe v.s. Mitigation Levels

Total Cost of Catastrophe versus Mitigation Level; $L_0=1$, $p=0.01$, $a=0.05$, $T=20$, discount rate=0.1, Net Benefit=0.0089192
EP Curves for Wood Frame Building in Canaries with Different Mitigations

- EP Curves for Wood Frame Building in Canaries with Different Mitigations, $a=0.00, T=20, \text{ discount rate}=0$

- EP Curves for Wood Frame Building in Canaries with Different Mitigations, $a=0.05, T=20, \text{ discount rate}=0$

Benefit-Cost Ratio for Wood Frame Building in Canaries

- Benefit-Cost Ratio for Wood Frame Building in Canaries ($a=0$, discount rate=0)

- Benefit-Cost Ratio for Wood Frame Building in Canaries ($a=0.05$, discount rate=0)

EP Curves for Wood Frame Building in Canaries with No Climate Change and Different Mitigations

- EP Curves for Wood Frame Building in Canaries with No Climate Change and Different Mitigations, $T=20, \text{ discount rate}=0$
Figure 16

EP Curves for Wood Frame Building in Canaries, T=20

Figure 17

EP Curves for Wood Frame Building in Canaries

Figure 18

Premium Prices over Varying Adaptations, d=0, m=5, k=0.4
Figure 22: A Simple Model that Estimates the Probability and Level of Storm Activity Rate Based on Historical Storm Activity Rate

Figure 23: 5-yr Analysis of the Number of Named Storms

Figure 24: 5-yr Analysis of the Number of Cat 3 - 5 Storms
Figure 25: Annual Premiums of Cat Insurance for a House with Value of $1,000,000 (a=0%)  

Figure 26: Annual Premiums of Cat Insurance for a House with Value of $1,000,000 (a=1.9%)  

Figure 27: Chernoff Bounds versus Thresholds for Various Time Horizons