Single-Carrier FDMA versus Cyclic-Prefix CDMA
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Abstract—In order to meet the data rate and quality-of-service (QoS) requirements of the future cellular systems, new air interfaces are currently under development. In this paper, we compare two air interfaces of particular interest for the uplink: cyclic-prefix code-division multiple access (CP-CDMA) proposed in the literature as an evolution of direct-sequence code-division multiple access (DS-CDMA) because it enables the low complexity equalization of the multipath channel in the frequency domain, and single-carrier frequency-division multiple access (SC-FDMA), recently proposed in the long term evolution of the 3GPP standard because it enables the easy separation of the users in the frequency domain. We demonstrate analytically that SC-FDMA is a special case of CP-CDMA, in which the CDMA codes have been optimized to minimize the symbol estimation mean square error (MSE) under a constraint of received power. Numerical results show that SC-FDMA outperforms significantly CP-CDMA at high user loads. The transmit power necessary to fulfill the received power constraint is higher in case of SC-FDMA than in case of CP-CDMA when the carrier sub-sets are allocated randomly to the users, and lower when the carrier sub-sets are allocated in an optimized way.

I. INTRODUCTION

Cellular systems of the third generation (3G) are based on the direct-sequence code-division multiple access (DS-CDMA) technology [1]. Because DS-CDMA enables the reuse of the same carrier frequency in neighboring cells, the system capacity is significantly increased and interesting networking abilities are offered. Especially, soft hand-over is supported between two cells. However, the system suffers from inter-symbol interfer-
ence (ISI) and multi-user interference (MUI) caused by multi-
path propagation, leading to a significant loss of performance in typical outdoor environments. For this reason, the spectral efficiency of 3G DS-CDMA communication systems is very low.

In order to enable the design of low complexity transceivers that can cope with multipath channels while still taking benefit from the good properties of DS-CDMA, next generation cellular systems could combine the DS-CDMA accessing scheme with the single-carrier block transmission (SCBT), also known as single-carrier (SC) modulation with cyclic prefix (CP) [2]. The DS-CDMA signals are spread within one block across the SC sub-channels, leading to cyclic-prefix code-division multiple access (CP-CDMA) [3], [4]. CP-CDMA does not preserve the orthogonality among the users in the presence of multipath propagation. In the downlink, a single-user detector is usually used, that consists of a chip equalizer to estimate the streams of chips by simple channel inversion in the frequency domain, and of a code correlator to separate the user signals [4]. In the uplink, however, advanced joint detection techniques have to be considered that detect jointly the user symbol blocks transmitted over different propagation channels. The complexity of the joint detector can still be significantly reduced by using the circulant properties of the channel matrices [3].

On the other hand, single-carrier frequency-division multiple access (SC-FDMA) has recently been identified as one of the most promising air interface for the uplink of the next generation 3GPP communication system [5]. A feasibility study of the technology has been performed in the 3GPP long term evolution (LTE) study group. The SC-FDMA air interface relies on orthogonal frequency-division multiple access (OFDMA) to separate the users in the frequency domain, and hence can benefit from the multi-user diversity offered in the different frequency selective channels by allocating the carriers to the users in an optimized way. SC-FDMA is further characterized by a pre-
coding of the vector of symbols applied in front of OFDMA in order to reduce the peak-to-average power ratio (PAPR).

The goal of the present paper is to demonstrate analytically that SC-FDMA is an optimized case of CP-CDMA, in which the CDMA codes are selected to minimize the symbol estimation mean square error (MSE) under a constraint of received power. The paper is organized as follows. In Section II, we build a generalized transmitter model of CP-CDMA and demonstrate that SC-FDMA is a special case of it. A matrix model of the CP-CDMA communication system uplink is proposed in Section III, based on which the minimum mean square error (MMSE) joint detector, necessary to mitigate the interference among the users, is derived in Section IV. The last section V considers the optimization of the CP-CDMA spreading codes according to the MMSE criterion and demonstrates that SC-FDMA corresponds to an optimized case.

In the sequel, we use single- and double-underlined letters for the vectors and matrices respectively. Matrix 1N is the identity matrix of size N and matrix 0M×N is a matrix of zeros of size M × N. The operators (·)*, (·)T and (·)H denote respectively the complex conjugate, transpose and transpose conjugate of a vector or a matrix. The trace of matrix M is denoted by tr[M]. The operator ⊗ is the Kronecker product.

II. TRANSMITTER MODEL

A. CP-CDMA

The CP-CDMA transmission scheme for the m-th user (m = 1 · · · M) in the uplink is depicted in Figure 1. As we have indi-
cated in the introduction, CP-CDMA first performs classical DS-CDMA symbol spreading, followed by SCBT modulation,
such that the information symbols are spread across the different SCBT sub-channels [3], [4].

The information symbols, \(s^m[n]\), which are assumed independent and of variance equal to \(\sigma^2\), are first serial-to-parallel converted into blocks of \(B\) symbols, leading to the symbol block sequence, \(x^m[n] := [s^m[nB], \ldots, s^m[(n+1)B-1]]^T\).

With \(Q := BN\) and \(N\) the spreading code length, the \(Q \times B\) spreading matrix, \(\varphi^m\), that spreads the symbols across the subchannels, is generally defined as:

\[
\varphi^m := \mathbb{F}_B \otimes \varphi^m,
\]

with \(\varphi^m := [c^m[0], \ldots, c^m[N-1]]^T\) the \(m\)-th user’s \(N \times 1\) code vector. A generalized form of CP-CDMA is obtained when \(\varphi^m\) is a block circulant matrix constructed based on a sequence of \(B\) code vectors \(\varphi^m_b\) \((b = 0, \ldots, B-1)\) of size \(N \times 1:\n
\[
\varphi^m := \begin{bmatrix}
\varphi^m_0 & \varphi^m_1 & \cdots & \varphi^m_{B-1} \\
\varphi^m_{B-1} & \varphi^m_0 & \cdots & \varphi^m_{B-2} \\
\cdots & \cdots & \cdots & \cdots \\
\varphi^m_1 & \varphi^m_2 & \cdots & \varphi^m_{B-1}
\end{bmatrix}.
\]

It is clear that (2) reduces to (1) if the vector \(\varphi^m_{B-1}\) is composed of zeros when \(b \neq 0\). The chip blocks, \(z^m[n]\), are obtained by multiplying the blocks \(x^m[n]\) with the spreading matrix \(\varphi^m:\n
\[
z^m[n] := \varphi^m \cdot x^m[n].
\]

Finally, the \(K \times Q\) \((K \geq Q)\) transmit matrix, \(T\), adds some transmit redundancy to the time-domain chip blocks:

\[
y^m[n] := T \cdot z^m[n].
\]

With \(K = Q + L\), \(T := [I_B,\ I^T_B]\), \(I_B\) consists of the last \(L\) rows of \(\mathbb{F}_Q\), \(\geq p\) adds redundancy in the form of a length-\(L\) cyclic prefix. The resulting transmitted chip block sequence, \(y^m[n]\), is parallel-to-serial converted into the scalar sequence, \([y^m[nK], \ldots, y^m[(n+1)K - 1]]^T := y^m[n]\), and transmitted over the air at a rate \(1/T_c\).

**B. SC-FDMA**

The SC-FDMA transmission scheme for the \(m\)-th user \((m = 1, \ldots, M)\) in the uplink is depicted in Figure 2. SC-FDMA first performs a pre-coding to reduce the PAPR, followed by a separation of the users in the frequency domain based on OFDMA [5].

The information symbols, \(s^m[n]\), are serial-to-parallel converted into blocks of \(B\) symbols, \(z^m[n]\). The vector of symbols, \(s^m[n]\), is first pre-coded with a fast Fourier transform (FFT) matrix \(\mathbb{F}_B\) of size \(B\) defined as:

\[
\mathbb{F}_B := \frac{1}{\sqrt{B}} \begin{bmatrix}
e^{-j2\pi \frac{p}{B}} \end{bmatrix}_{p,q=0,\ldots,B-1},
\]

and with a diagonal matrix \(\Delta^m\) of size \(B\). The diagonal of \(\Delta^m\) is composed with the elements \(a^m[b]\) which can take any complex value in the general case (for example, the inverse of the channel coefficients in the zero-forcing pre-coding case). However the matrix \(\Delta^m\) is originally an identity matrix, so that \(a^m[b] = 1\).

The pre-coded vector is placed on the adjacent carriers \((n-1)B\) to \(nB - 1\) (localized transmission) by a multiplication with the allocation matrix \(\mathbb{E}^m_{t}\) consisting of the columns \((n-1)B + 1\) to \(nB\) of the matrix \(\mathbb{I}^Q\). Finally, a frequency-to-time domain conversion is performed by a multiplication of the result with the inverse fast Fourier transform (IFFT) matrix \(\mathbb{F}_Q^H\) of size \(Q\). We obtain:

\[
y^m[n] = \mathbb{F}_B \cdot \Delta^m \cdot \mathbb{F}_Q \cdot x^m[n].
\]

Redundancy is added in the form of a cyclic prefix, and the resulting sequence of blocks, \(y^m[n]\), is parallel-to-serial converted into the scalar sequence \(y^m[n]\).

**C. SC-FDMA versus CP-CDMA**

The SC-FDMA transmitter (6) can be written as a special case of the CP-CDMA transmitter (3). Based on (6), the element \(q\) \((q = 0, \ldots, Q - 1)\), \(x^m[n,q]\), of the transmitted block \(x^m[n]\) is given as a function of the element \(b\) \((b = 0, \ldots, B - 1)\), \(s^m[n,b]\), of the symbol block \(s^m[n]\) by:

\[
x^m[n,q] = \frac{1}{B \sqrt{N}} \sum_{b=0}^{B-1} s^m[n,b] \sum_{p=0}^{B-1} a^m[p] e^{j2\pi \frac{b q}{B}}.
\]

By defining the polyphase components \(x^m[n,i]\) of \(x^m[n,q]\) such that:

\[
x^m[n,i] := x^m[n,q = iN + \rho]
\]

with \(i = 0, \ldots, B - 1\) and \(\rho = 0, \ldots, N - 1\), we obtain:

\[
x^m[n,i] = \sum_{b=0}^{i} s^m[n,b] e^{j2\pi \frac{b(iN-1)}{Q}} f^m_{\rho}[i - b] + \sum_{b=i+1}^{B-1} s^m[n,b] e^{j2\pi \frac{b(iN-1)}{Q}} f^m_{\rho}[B + i - b]
\]

in which \(f^m_{\rho}[b]\) is the element \(b\) of the IFFT of size \(B\) of the function \(a^m[b] e^{j2\pi \rho b}/BN\), as expressed in:

\[
f^m_{\rho}[b] := \sum_{p=0}^{B-1} a^m[p] e^{j2\pi \frac{p b}{B}} e^{j2\pi \frac{\rho b}{N}}.
\]
in which:

\[
\mathbf{E}_m = \begin{bmatrix} \frac{1}{\sqrt{Q}} f_m^0[b] \\ \frac{e^{2\pi j (m/2+1)}}{\sqrt{Q}} f_m^1[b] \\ \vdots \\ \frac{e^{2\pi j (m/(Q-1)(m-1))}}{\sqrt{Q}} f_m^{n-1}[b] \end{bmatrix}.
\] (11)

In the following sections, we build a matrix model and design the MMSE joint detector for the CP-CDMA system (also applicable for SC-FDMA since it is a special case of CP-CDMA). We give a sufficient condition for optimality of the CP-CDMA codes according to the MMSE criterion and show that the codes corresponding to SC-FDMA fulfill this condition.

### III. SYSTEM MODEL

#### A. Cyclo-stationarization of the channels

After propagation through the different user channels, the signal is received at the base station. Adopting a discrete-time baseband equivalent model, the chip-sampled received signal, \( v[n] \), is the superposition of a channel-distorted version of the \( M \) transmitted user signals, which can be written as:

\[
v[n] = \sum_{m=1}^{M} L_{m} h_{m}[\tau] u_{m}[n - \ell] + w[n],
\] (12)

where \( h_{m}[\tau] \) is the chip-sampled finite impulse response (FIR) channel of order \( L_{m} \) that models the frequency-selective multipath propagation between the \( m \)-th user’s terminal and the base station, including the effect of transmit/receive filters and the remaining asynchronism of the quasi-synchronous users, and \( w[n] \) is additive white Gaussian noise (AWGN) sequence at the base station. Assuming a perfect low-pass filter at the receiver, the variance of the noise samples is given by \( \sigma_{w}^{2} = 2N_{0}/T_{c} \) in which \( N_{0} \) denotes the one-sided power spectral density of the noise. Furthermore, the maximum channel order \( L_{\max} \), that is \( L_{\max} = \max_{m} \{ L_{m} \} \), can be well approximated by \( L_{\max} \approx (\tau_{\max,a} + \tau_{\max,s})/T_{c} + 1 \), where \( \tau_{\max,a} \) is the maximum asynchronism between the nearest and the farthest user of the cell, and \( \tau_{\max,s} \) is the maximum excess delay within the given propagation environment. In the sequel, we assume that the CP length has been chosen such that \( L \geq L_{\max} \).

Assuming perfect time and frequency synchronization, the received sequence \( v[n] \) is serial-to-parallel converted into the block sequence, \( v[1:nK] = [v[nK], \ldots v[(n+1)K-1]]^{T} \). From the scalar input/output relationship in (12), we can derive the corresponding block input/output relationship:

\[
y[n] = \sum_{m=1}^{M} \left( H_{m}^{0}[0] \cdot u_{m}[n] + H_{m}^{1}[0] \cdot u_{m}[n-1] \right) + w[n],
\] (13)

where \( \sqrt{Q} \) is the corresponding noise block sequence, \( H_{m}^{0}[0] \) is a \( K \times K \) lower triangular Toeplitz matrix with entries \( H_{m}^{0}[0]_{p,q} = h_{m}(p - q) \), and \( H_{m}^{1}[1] \) is a \( K \times K \) upper triangular Toeplitz matrix with entries \( H_{m}^{1}[1]_{p,q} = h_{m}[K + p - q] \) (see e.g. [6] for a detailed derivation of the single-user case). The delay-dispersive nature of the multipath propagation gives rise to so-called inter-block interference (IBI) between successive blocks, which is modeled by the second term in (13).

The \( Q \times K \) receive matrix, \( \hat{R} \), removes the redundancy from the chip blocks, that is, \( y[n] := \hat{R} \cdot v[n] \). With \( R = \hat{R}_{p} := [0_{Q \times L_{p}}] \), \( \hat{R} \) again discards the length-\( L_{p} \) cyclic prefix. The purpose of the transmit \( T_{r} \)/receive \( T_{r} \) pair is twofold. First, it allows for simple block-by-block processing by removing the IBI, that is, \( \hat{R} \cdot H_{m}^{1}[1] \cdot T_{r} = 0_{Q \times Q} \) provided the CP length to be at least the maximum channel order \( L \). Second, it enables low-complexity frequency-domain processing by making the linear channel convolution to appear circulant to the received block. This results in a simplified block input/output relationship in the time-domain:

\[
y[n] = \sum_{m=1}^{M} \hat{H}_{m}^{m} \cdot x_{m}[n] + z[n],
\] (14)

where \( \hat{H}_{m}^{m} := \hat{R} \cdot H_{m}^{0}[0] \cdot T_{r} \) is a circulant channel matrix, \( z[n] := \hat{R} \cdot w[n] \) is the corresponding noise block sequence. Note that circulant matrices can be diagonalized by FFT operations, that is, \( \hat{H}_{m}^{m} = F_{Q} \cdot \Lambda_{m} \cdot F_{Q}^{*} \), where \( \Lambda_{m} \) is a diagonal matrix composed of the frequency-domain channel response between the \( m \)-th user’s terminal and the base station.

#### B. Matrix model

In order to design the multi-user joint detector for the CP-CDMA system, a generalized matrix model is built that links the vector of input symbols to the vector of received samples. The generalized input/output matrix model that relates the symbol vector defined as:

\[
x[n] := \begin{bmatrix} x_{1}[n]^{T} \\ \vdots \\ x_{M}[n]^{T} \end{bmatrix}
\] (15)

to the received vector \( y[n] \) and noise vector \( z[n] \), is given by:

\[
y[n] = H \cdot \theta \cdot x[n] + z[n]
\] (16)

where the channel matrix is equal to:

\[
H := \begin{bmatrix} \hat{H}_{1}^{1} & \cdots & \hat{H}_{M}^{1} \end{bmatrix}
\] (17)

and the spreading matrix is equal to:

\[
\theta := \begin{bmatrix} \theta_{1} & \cdots & \theta_{Q \times K} \\ \vdots & \ddots & \vdots \\ \theta_{Q \times K} \end{bmatrix}
\] (18)

#### C. Frequency domain representation

A factorization of the matrix \( H \cdot \theta \) serves as the basis of the MMSE linear joint detector simplification. Based on (17) and (18), we have that:

\[
H \cdot \theta = \begin{bmatrix} \hat{H}_{1}^{1} \cdot \theta_{1} & \cdots & \hat{H}_{M}^{1} \cdot \theta_{M} \end{bmatrix},
\] (19)
which can be interestingly reorganized into:

$$H \cdot \Theta = \Psi \cdot \Upsilon$$

(20)

where $\Upsilon$ is a permutation matrix of size $MB$ whose role is to reorganize the columns of the initial matrix $H \cdot \Theta$ according to the symbol and user indexes successively, and:

$$\Psi := \begin{bmatrix} \tilde{\psi}[1] & \cdots & \tilde{\psi}[B] \end{bmatrix}$$

(21)

in which the matrix $\tilde{\psi}[1]$ of size $Q \times M$ is composed of the $M$ user channel impulse responses (CIR) convolved with the user codes, and each matrix $\tilde{\psi}[b]$ of size $Q \times M$ is a $(b-1)N$ cyclic rotation of the matrix $\tilde{\psi}[1]$ ($b = 1 \cdots B$).

The block circulant matrix $\Psi$ can be decomposed according to [3] into:

$$\Psi = F^H(N) \cdot \Psi \cdot F(M)$$

(22)

where the matrices $F(N)$ and $F(M)$ are block Fourier transforms, defined as:

$$F(n) := F_B \otimes I_n$$

(23)

where $F_B$ is the orthogonal Fourier transform matrix of size $B$ and $I_n$ is the identity matrix of size $n$ ($n$ equal to $N$ or $M$). The inner matrix $\Psi$ is equal to:

$$\Psi := \begin{bmatrix} \tilde{\psi}[1] & \cdots & 0_{N \times M} \\ \vdots & \ddots & \vdots \\ 0_{N \times M} & \cdots & \tilde{\psi}[B] \end{bmatrix},$$

(24)

where the block diagonal is found by dividing the result of the product $F(N) \cdot \tilde{\psi}[1]$ in blocks $\tilde{\psi}[b]$ of size $N \times M$.

We get finally:

$$H \cdot \Theta = F^H(N) \cdot \Psi \cdot F(M) \cdot \Upsilon,$$

(25)

and the matrix model (16) becomes:

$$y[n] = F^H(N) \cdot \Psi \cdot F(M) \cdot \Upsilon \cdot z[n] + z[n].$$

(26)

IV. RECEIVER DESIGN

The optimal solution to detect the CP-CDMA transmitted signals is to estimate jointly the symbol blocks of the different users within the transmitted vector, $\hat{s}[n]$, based on the received block, $\hat{y}[n]$ (multi-user joint detection). The optimum linear joint detector according to the MMSE criterion is computed in [7]. At the output of the MMSE multiuser detector, the estimate of the transmitted vector is:

$$\hat{s}[n] = \Psi^H \cdot F(M) \left( \frac{\sigma_w^2}{\sigma_s^2} L_{MB} + \Psi^H \cdot \Psi \right)^{-1} \cdot \Psi^H \cdot F(N) \cdot \hat{y}[n]$$

(27)

which has been obtained by using the fact that the matrices $F(N)$, $F(M)$, and $\Upsilon$ are orthogonal. The MMSE linear joint detector decomposes successively into the following operations (see Figure 3): (a) the FFT to move to the frequency domain, (b) the matched filtering $\Psi^H$ in the frequency domain, (c) the mitigation of the multi-user interference by multiplication with the inner matrix, (d) the IFFT to go back to the time domain, (e) the permutation $\Upsilon$ to arrange the result according to the user and symbol indexes successively. The matrix $\Psi$ defined in (25) is block diagonal so that the computation of the MMSE joint detector involves only the inversion of $B$ square auto-correlation matrices of size $M$. The symbol estimation error auto-correlation matrix is [7]:

$$R_s := E \left[ (\hat{\alpha}[n] - \hat{\alpha}[n]) \cdot (\hat{s}[n] - \hat{s}[n])^H \right]$$

(28)

$$= \Psi^H \cdot F^H(M) \cdot \left( \frac{1}{\sigma_s^2} L_{MB} + \frac{1}{\sigma_w^2} \Psi^H \cdot \Psi \right)^{-1} \cdot F(M) \cdot \Upsilon,$$

(29)

in which the operator $E[.]$ denotes the expectation over the random symbols and the noise.

V. CODE OPTIMIZATION

A. Sufficient condition for optimality

The optimization of the CP-CDMA codes is performed under a constraint of constant received symbol energy $E_s$. The approach followed in this paper is similar to the one that we have proposed in [8] for burst transmissions.

As the symbols are independent, we minimize the sum of the symbol estimation error variances. The matched filter bound (MFB), defined as the signal-to-noise power ratio (SNR) at the output of the filter $\Psi^H \cdot F^H(M) \cdot \Psi^H \cdot F(N)$ matched to the total impulse response (25), is equal to the constant $\gamma := E_s/N_0$ for each user. The problem becomes:

$$\min_{\hat{\alpha}} \text{tr} \left[ R_s \right]$$

(30)

subject to $\frac{\sigma^2_s}{\sigma^2_w} \left( \Psi^H \cdot F^H(M) \cdot \Psi^H \cdot F(N) \cdot \Upsilon \right) = \gamma$ for $i = 1 \cdots MB$. An interesting result, due to Witsenhausen [9], and restated in [8], gives a sufficient condition for the optimality of the codes.

Proposition (Witsenhausen): Let $M$ be a square non-negative definite Hermitian matrix of size $Q$ and $M_{\alpha}$ be the diagonal matrix obtained from $M$ by setting all non-diagonal elements to zero. It can be proven, for any positive real number $k$, that:

$$\text{tr} \left( L_Q + kM_{\alpha} \right)^{-1} \leq \text{tr} \left( L_Q + kM_{\alpha} \right)^{-1}. $$

(31)

If $M = \Psi^H \cdot F^H(M) \cdot \Psi^H \cdot F(N) \cdot \Upsilon$, the term on the right side of the inequality is proportional to the trace of the error auto-correlation matrix (29). On the other hand, the diagonal matrix $M_{\alpha}$ is constant due to the received symbol energy constraint and the term on the left side of the inequality (31) is a constant. As a result of the proposition, a diagonalization of the inner matrix $\Psi^H \cdot F^H(M) \cdot \Psi^H \cdot F(N) \cdot \Upsilon$ results in the minimization of the trace of the error auto-correlation matrix $R_s$ (the term on the right side of the inequality is equal to the constant term on the left side).
Hence, the CP-CDMA codes $c_n^m$ are ideally chosen such that:

$$\hat{s}[n] = \frac{\sigma_w^2}{\sigma_w^2 + 1 + \gamma H} \cdot F_{(N)}^H \cdot \psi^H \cdot \psi \cdot F_{(M)} \cdot Y = \gamma \frac{\sigma_w^2}{\sigma_s^2} I_{MB}. \quad (32)$$

In this case, the linear MMSE joint detector reduces to:

$$\hat{s}[n] = \frac{\sigma_w^2}{\sigma_w^2 + 1 + \gamma H} \cdot F_{(N)}^H \cdot \psi^H \cdot \psi \cdot F_{(M)} \cdot y[n]. \quad (33)$$

The interference is eliminated at the output of the matched filter and each user achieves the MFB.

### B. Zero-forcing SC-FDMA

In this subsection, we demonstrate that the pre-coding matrix $A^m$ can be selected so that the codes corresponding to SC-FDMA fulfill the optimality condition.

As stated in (24), the block diagonal of the matrix $H$ is obtained by dividing the result of the FFT $F_{(N)}$ of the user total impulse responses into blocks $\psi[b]$ of size $N \times M$. In case of SC-FDMA, the column $m$ of the block $\psi[b]$, denoted by $\psi^m[b]$, is the part $b$ of the first column of $F_{(N)}^H \cdot H^m \cdot F_{(M)}^H \cdot \psi^m \cdot \psi \cdot A^m \cdot F_B$ (taking (6) into account). After computation, it reduces to:

$$\psi^m[b] = \frac{1}{\sqrt{N}} \lambda^m[(m-1)B+b] \cdot a^m[b] \cdot \left[ e^{j2\pi \frac{b}{(m-1)+\frac{B}{2}}} \right]$$

$$\psi^m[b] = \frac{1}{\sqrt{N}} \lambda^m[(m-1)B+b] \cdot a^m[b] \cdot \left[ e^{j2\pi \frac{b}{(m-1)+\frac{B}{2}}} \right]$$

$$\psi^m[b] = \frac{1}{\sqrt{N}} \lambda^m[(m-1)B+b] \cdot a^m[b] \cdot \left[ e^{j2\pi \frac{b}{(m-1)+\frac{B}{2}}} \right]$$

in which $\lambda^m[q]$ is the coefficient $q$ of the $m$-th user’s channel in the frequency domain.

$$\psi^m[b] = \frac{1}{\sqrt{N}} \lambda^m[(m-1)B+b] \cdot a^m[b] \cdot \left[ e^{j2\pi \frac{b}{(m-1)+\frac{B}{2}}} \right]$$

$$(\psi^m[b])^H \cdot \psi^F[b] = \delta[m-k] |\lambda^m[(m-1)B+b]|^2 |a^m[b]|^2,$$

the matrix $\psi^H \cdot \psi$ is always diagonal and the multi-user interference is completely canceled out at the output of the matched filter. In order to further fulfill the optimality condition (32), the pre-coding matrices $A^m$ should be chosen such that:

$$a^m[b] = \frac{1}{\sqrt{\gamma}} \frac{\sigma_w}{\sigma_s} |\lambda^m[(m-1)B+b]|. \quad (34)$$

As a result, SC-FDMA is an optimized case of CP-CDMA if a pre-coding is applied at the transmitter in the frequency domain that inverts the modulus of the channel user coefficients (zero-forcing pre-coder).

Compared to the optimized pre-coding proposed in [8] for burst transmissions, the pre-coding in the present system can be computed for each user independently of the other users.

The goal of this section is to compare numerically the performance of the CP-CDMA and SC-FDMA systems. We assume a physical bandwidth of 5 MHz. The channels are composed of 5 equi-distant paths. The delay spread is equal to the 0.8μs. A linearly decreasing power profile is considered, given by [1; 0.8; 0.6; 0.4; 0.2]. Each path has a Rayleigh distribution. The performance is averaged over $10^4$ stochastic channel realizations. A power control is assumed, such that the ratio received symbol energy $E_r$ to noise one-sided power spectral density $N_0$ is equal to 20dB. The number of carriers is fixed to 64, and the cyclic prefix is 16. In case of CP-CDMA, the user signals are spread by periodic Walsh-Hadamard codes. A spreading factor equal to 16 has been chosen. In case of SC-FDMA, each user is allocated a sub-set of 4 adjacent carriers. We compare the case where the users are allocated the carrier sub-sets randomly, to the case where the users are allocated the carrier sub-sets in a progressive order (the user with the highest fading channel coefficients selects his sub-set of carriers first to minimize his required transmit power, then the user with the second highest fading channel coefficients follows the same process but by excluding the carriers selected by the first user ...). Both SC-FDMA systems with and without zero-forcing (ZF) pre-coders are studied.

Figure 5 illustrates the necessary transmit power of the SC-FDMA systems relatively to the CP-CDMA system as a function of the number of active users. The SNR of the CP-CDMA system is highly degraded for an increasing number of users because of the remaining interference between the users at the output of the linear MMSE joint detector. Since the SC-FDMA systems are orthogonal in the users, the SNR of those systems is independent on the number of users. As predicted, the SC-FDMA systems with a zero-forcing pre-coder achieve the MFB equal to 20 dB.

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Figure 5 illustrates the necessary transmit power of the SC-FDMA systems relatively to the CP-CDMA system as a function of the number of active users. When the SC-FDMA carrier sub-sets are allocated randomly to the users, the transmit power necessary to reach the desired power level at the receiver is higher than that of the CP-CDMA system. Especially, when a zero-forcing pre-coder is assumed, the increase of power is significant. However, when the sub-sets of carriers are allocated to the users in a progressive order, the transmit power of the SC-FDMA systems is on the overall smaller than that of the CP-CDMA system, even when a zero-forcing pre-coder is used. The SC-FDMA system applying a zero-forcing pre-coder has a transmit power slightly increasing with the number of users.
VII. CONCLUSIONS

We have demonstrated analytically that the SC-FDMA communication system, applying a zero-forcing pre-coder at the transmitter, corresponds to an optimized case of the CP-CDMA communication system where the codes and the receiver have been chosen to minimize the MSE under a constraint of constant received symbol energy. Because the interference between the users and between the symbols is completely eliminated at the output of the matched filter, the SC-FDMA system achieves the MFB. When the sub-sets of carriers are allocated randomly to the users, the cost in transmit power to fulfill the received symbol energy constraint is significant with respect to the CP-CDMA system. Therefore, when the sub-sets of carriers are allocated to the users in a progressive order, the required transmit power is smaller than that of the CP-CDMA system. Therefore, the SC-FDMA system can outperform significantly the CP-CDMA system both in performance and in transmit power. It has however to be optimized based on the channel knowledge at the transmitter (allocation of the sub-sets of carriers and zero-forcing pre-coder), which is only a valid assumption under low-mobility conditions.

REFERENCES