Horizontal Product Differentiation in Bertrand and Cournot Duopoly: the Bertrand Paradox Revisited and Comparative Competitiveness

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Abstract: This paper provides a simple model of endogenous horizontal product differentiation that has two important implications. First, the model can explain the “empirical Bertrand paradox” – the failure to observe homogeneous product Bertrand oligopoly. If product differentiation is possible at reasonable cost, then Bertrand firms would always invest in product differentiation. Using a quadratic utility (linear demand) structure we show that Cournot firms are much less likely to engage in product differentiation than Bertrand firms. And, assuming product differentiation takes place at all, Bertrand firms would always differentiate more than Cournot firms. The second major insight of our analysis is that Bertrand competition is not necessarily more competitive than Cournot competition (in the sense of having lower prices and profits) once we allow for endogenous product differentiation. For a significant range of parameter values a Bertrand duopoly charges higher prices and earns higher profits than a corresponding Cournot duopoly.

keywords: Bertrand paradox, Cournot, horizontal product differentiation

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1. Introduction

One of the classic topics in oligopoly theory is the “Bertrand Paradox”, which dates from Bertrand’s (1883) review of Cournot (1838). Bertrand suggested a model in which symmetric price-setting duopoly firms produce a homogenous product at constant marginal cost. The resulting (Nash) equilibrium, in which price equals marginal cost, seems unreasonable. As stated by Tirole (1988, pp. 210-211): “We call this the Bertrand paradox because it is hard to believe that firms in industries with few firms never succeed in manipulating the market price to make profits.” It also seems implausible that the price should be completely unaffected by the number of firms as we go from two firms in a market to an arbitrarily large number.

Another aspect of the Bertrand paradox is that, from an empirical perspective, we rarely if ever observe an apparent Bertrand equilibrium in homogeneous product oligopolies. Slade (1995, p. 381) summarizes empirical work using Cournot and Bertrand models and reports no cases in which the Bertrand model is applied to the homogeneous product case. The Cournot model, on the other hand, is commonly applied in such cases. Recent examples of such Cournot applications include Carvajal, Deb, Fenske, & Quah (2013) for petroleum and Jansen, van Lier, van Witteloostuijn, & von Ochssée (2012) for natural gas. Thus, homogeneous product Cournot models seem to be empirically relevant while homogeneous product Bertrand models are not.

Our first objective in this paper is to provide an explanation of the empirical Bertrand paradox – an explanation of why homogeneous product oligopolies are much more likely to be well approximated by the Cournot model than by the Bertrand model. Our paper is therefore related to the substantial prior literature suggesting possible resolutions of the Bertrand paradox. Our analysis has two primary contributions relative to this prior literature. First, it provides what seems to us to be the most natural and perhaps the simplest explanation of why homogeneous
product Bertrand oligopoly is rarely if ever observed. Second, we provide a comparison with the Cournot model that explains why homogeneous product Cournot models are likely to be empirically relevant.

A widely accepted part of the conventional wisdom of oligopoly theory is that Bertrand industries are more competitive than Cournot industries in the sense of having lower prices and lower profits. However, this conventional wisdom is based on models in which product differentiation is either absent or exogenous. A second objective of this paper is to determine whether Bertrand industries are more competitive than corresponding Cournot industries once we allow for endogenous product differentiation. We obtain the striking result that Bertrand firms will charge higher prices and earn higher profits than corresponding Cournot firms if the cost of differentiation is sufficiently low.

Our approach is based on allowing firms to make horizontal product differentiation decisions prior to either quantity decisions (Cournot) or pricing decisions (Bertrand). Our basic logical structure is therefore similar to the location choice model of d’Aspremont, Gabszewicz, & Thisse (1979). This location choice paper shows that if firms first choose their locations on a Hotelling line, then choose prices, they will not locate beside each other – violating the principle of minimum differentiation established by Hotelling (1929). Thus, if we interpret location as a type of product differentiation, Bertrand firms will choose to differentiate their products in the sense that they choose different locations. Location choice may have either a horizontal or vertical interpretation. If all locations are equivalent to each other, then location choice corresponds to horizontal differentiation. However, if some locations are better than others then location choice can have a vertical interpretation.
One important difference between our analysis and the location choice literature is that we require firms to make costly investments in order to differentiate their products rather than simply choosing where to locate on a line or circle. The role of investment in product differentiation seems fundamental in many cases – making the analogy with location choice potentially misleading. Our approach also allows for a straightforward comparison of Bertrand with Cournot competition, whereas the location choice literature naturally focusses on price competition. More generally, our consideration of a different form of differentiation extends our understanding of product differentiation to a wider class of situations.

A classic example of investment in product differentiation is provided by the much-studied rivalry between Coca-Cola and Pepsi, which has been estimated using a Bertrand specification in Gasmi et al. (1992). As has been well-established in the marketing literature, many people cannot tell Coke and Pepsi apart in blind taste tests, yet high levels of advertising and other marketing activities create strong perceived product differences from the point of view of consumers. A classic study of this type is Woolfolk et al. (1983) which demonstrates that stated preferences by Pepsi and Coke “loyalists” depend strongly on whether they are given a Coke bottle or a Pepsi bottle, regardless of whether Coke or Pepsi is actually in the bottle.

These advertising investments are economically important and both Coca-Cola and Pepsi are among the world’s largest advertisers. It is the investment in advertising, packaging, and other marketing activities that “creates” product differentiation in this case. In an interesting recent paper using MRI brain imaging, McClure et al. (2004) shows that the brain responds very differently to Coke and Pepsi when brand cues are available. As stated in the article: “In the

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1 See, for example, Adbrands.net at [www.adbrands.net/top_global_advertisers.htm](http://www.adbrands.net/top_global_advertisers.htm) which has Coke and Pepsi at 7th and 11th respectively for total global advertising expenditure in 2012.
brand-cued experiment, brand knowledge for one of the drinks had a dramatic effect on expressed behavioral preferences and on the measured brain responses.” Thus the enormous advertising investments undertaken by Coke and Pepsi apparently do have a significant measurable effect on the brain’s responses to perceptual stimuli.

Another interesting example of product differentiation arises with automobiles. For the 2014 model year Lexus introduced a number of changes for its popular IS model. Perhaps the most discussed change was a new grill that gave the IS model a distinctive look that differentiated it more obviously from competing models produced by BMW, Mercedes, and Acura. The new grill had no effect on performance but required significant costs – both in development and in restructuring production lines. Why undertake such expenditures? Our analysis provides an explanation of such investments.

We consider a two-stage duopoly model in which firms make simultaneous investments that determine the level of product differentiation in the first stage and then make either simultaneous price decisions (Bertrand) or simultaneous quantity decisions (Cournot) in the second stage. The extent of possible product differentiation runs the full range from homogenous products (no differentiation) to unrelated goods, but differentiation requires costly investment.

In accordance with the empirical evidence that products tend to be differentiated when price competition is important, we show that firms have a greater incentive to differentiate their products under Bertrand competition than Cournot competition. There is a very wide range of parameter values for which Bertrand firms will make the investment to differentiate their products, but Cournot firms will not. Thus our model can explain why we might commonly observe homogeneous product industries that are consistent with the Cournot model but rarely if ever do so in the Bertrand case.
Furthermore, because Bertrand firms differentiate their products more than corresponding Cournot firms, it is possible for them to charge higher prices and earn higher profits in equilibrium. Our paper also provides what we view as valuable insights regarding the welfare comparison of Bertrand and Cournot models. Specifically we show that, in the Bertrand case, increasing differentiation is always associated with reduced consumer surplus, despite the fact that consumers like variety. The reason for the reduction in consumer surplus is that product differentiation allows for significantly higher prices and these higher prices more than offset the benefits of increased variety from the consumer point of view. Product differentiation in the Cournot case has the opposite effect, as increased product differentiation is always associated with increased consumer surplus: the benefits of increased variety are larger than the loss due to higher prices.

Section 2 is devoted a brief literature review in which we discuss other major approaches to the Bertrand paradox. Section 3 contains the basic model structure, Section 4 describes the Bertrand case, Section 5 deals with the Cournot case, and Section 6 provides comparative results regarding the two models. Section 7 contains concluding remarks.

2. Literature Review

In addition to the location choice literature already discussed, another major approach to the Bertrand paradox is based on vertical product differentiation, as pioneered by Shaked & Sutton (1982, 1983) and Motta (1993). In this literature, firms anticipate price competition in a second stage interaction and therefore choose different quality levels in a first stage decision so as to avoid cutthroat Bertrand competition in the second stage. Interestingly, however, Boccard & Wauthy (2010) show that introducing a capacity choice prior to the quality choice decision can eliminate quality differentiation.
In the context of the vertical product differentiation literature, a main contribution of our paper is to show that vertical product differentiation is not necessary to avoid the Bertrand paradox. Horizontal product differentiation is sufficient. Horizontal product differentiation is in many ways simpler and is supported by major empirical examples as with the soft drink industry, automobiles, or breakfast cereals.\(^2\) In addition, horizontal product differentiation has the advantage that it does not require asymmetry at the solution.

The other major approach to the Bertrand paradox focuses on the possibility that the mode of conduct itself (Bertrand or Cournot) might be an endogenous choice variable as in the classic treatment of Singh & Vives (1984). If so, depending on the details of model specification, the combination of homogeneous products and Bertrand conduct might be ruled out as a possibility. The basic principle is that, if firms are going to produce homogeneous products, they would opt for Cournot rivalry rather than Bertrand rivalry. This literature relies primarily on an assumed ability of firms to sign binding contracts to create mode of conduct endogeneity. Singh & Vives (1984) assume that firms can sign binding price contracts or binding quantity contracts. Breitmoser (2012) assumes that firms have access to forward contracts than can serve as binding commitments.

Instead of taking the mode of conduct as exogenous and the extent of product differentiation as endogenous, this mode of conduct choice literature does the reverse, taking the degree of differentiation as exogenous and the mode of conduct as endogenous. However, the work of Kreps & Scheinkman (1983), Friedman (1983, p. 47), Shapiro (1989, pp. 350-351), and others suggests that the mode of conduct may be largely determined by the nature of technology in an

\(^2\) As with cola soft drinks, the packaged breakfast cereal market is dominated by two large firms – General Mills and Kellogg’s, with a couple of other significant firms (Post and Quaker) and a number of small firms.
industry and therefore properly viewed as exogenous with respect to the product differentiation decision. Specifically, if quantity is hard to change – as when it is determined by capacity constraints – and price adjusts to clear the market, then the Cournot model is appropriate. If, on the other hand, quantity can be readily changed to clear the market while prices are hard to change or can be pre-committed, then the Bertrand model is suitable. Thus the mode of conduct would be exogenous – or at least hard to change relative to product differentiation decisions.

We do not take a position here on the relative empirical significance of endogenous mode of conduct models. They do seem to require the existence of contracts that, while not uncommon, are far from the norm and that can be rendered undesirable by realistic transaction costs or by uncertainties of various types. We simply argue that in many markets changes in technology are slow-moving relative to product differentiation induced through advertising (as with soft drinks) or minor changes in product specification (as with automobiles). If so then treating the mode of conduct as exogenous would often be appropriate. In our paper we consider the case in which the mode of conduct is exogenous. However, our model would allow for an endogenous mode of conduct decision to be combined with horizontal product differentiation.

Another potential resolution of the Bertrand paradox is based on the possibility of implicit collusion in repeated price-setting games as noted in Tirole (1988) and extensively studied in the subsequent literature, including, for example, Miklós-Thal (2011) and Argentona & Müller (2012). Also, experimental work on the Bertrand model, including Bruttel (2009), suggests the possibility that the Bertrand paradox may be avoided for behavioral reasons or for reasons related to bounded rationality. The Bertrand paradox can also be avoided if cost is uncertain and firms are risk averse (Wamback 1999), resulting in prices above marginal cost.
The comparative competitiveness properties of Bertrand and Cournot models have been addressed by several authors. The basic finding, provided by Cheng (1985), Singh & Vives (1984), Vives (1985), and others is that if Bertrand and Cournot duopolies face the same demand and cost conditions, then the Bertrand industry would generate lower profits, lower prices, and more consumer surplus.

Qiu (1997) provides an important extension in which cost is made endogenous through the introduction of endogenous process R&D. The paper finds that the conventional ranking of the two models may be reversed as Cournot firms will often have a stronger incentive to invest in R&D, causing costs to fall and possibly providing more consumer surplus (and more total surplus after including profits) than in the Bertrand case. Symeondis (2003) undertakes the further extension of allowing for product R&D that can improve product quality and, like Qiu (1997), finds that the Cournot model can generate more total surplus if there are strong R&D spillovers and products are not too strongly differentiated in equilibrium. Our model differs from Qiu (1997) because investment causes product differentiation not cost reduction and differs from both Qiu (1997) and Symeondis (2003) in that it is not based on R&D spillovers. In our setting, investment in product differentiation reduces, but never eliminates, the higher consumer surplus generated by Bertrand competition. Bertrand competition can also be more profitable than Cournot competition.

3. A Model of Horizontal Product Differentiation

We assume a duopoly model in which firm 1 produces quantity $x_1$ and firm 2 produces quantity $x_2$. Goods $x_1$ and $x_2$ can range between being perfect substitutes (homogeneous) to being totally unrelated. The aggregate or representative utility function is taken to be

$$U = a(x_1 + x_2) - (b/2)(x_1^2 + x_2^2) - sx_1x_2 + M$$  

(1)
where $M$ is consumption of a numeraire good. This quasi-linear utility function rules out income effects of demand. The parameter $s$ represents the degree of substitutability between the products $x_1$ and $x_2$.

Without loss of generality we undertake an algebraically convenient normalization and rescaling of variables such that $b = 1$. If $b = 1$ then the feasible range for $s$ is between 0 and 1.\(^3\) If $s = 0$, then demand for each good is independent. Since products are unrelated, each firm has a monopoly with respect to its good. If $s = 1$ (or, more generally, if $s = b$), goods are perfect substitutes and are, in effect, identical or homogenous. It is convenient for some purposes to define a parameter $v$ (for “variety”) as the extent of differentiation such that $v = 1 - s$, where $0 \leq v \leq 1$. Initially, however, we use $s$ in the specification of the demand structure, yielding the following inverse demand functions:

\[
\begin{align*}
    p_1 &= \frac{\partial U}{\partial x_1} = a - x_1 - sx_2. \\
    p_2 &= \frac{\partial U}{\partial x_2} = a - x_2 - sx_1.
\end{align*}
\]

(2)

Firms can choose to reduce the degree of substitutability between products by incurring a differentiation investment. The combined effect of the differentiation investments of both firms determines the value of $s$. One possibility is to interpret the differentiation investment as an advertising cost aimed at making the product more distinct from the other product in the eyes of consumers.

Another possibility is to interpret the investment as the cost of changing some physical characteristic of the product that differentiates it from the other product, as when breakfast cereal

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\(^3\) We assume that starting from homogenous products, firms can invest to reduce the degree of substitutability between products. We do not examine the possibility considered in Singh and Vives (1984) that the products could be complements ($s < 0$). Such an extension addresses what, in our view, is an essentially different issue – coordination of complementary products rather than competition between substitutes.
companies come up with additional variations in taste, texture, and packaging or when car manufacturers adopt new colors and or new body shapes for cars or undertake other differentiation activities of a costly but essentially horizontal nature. Since we are interested in exploring incentives with respect to the degree of substitutability, we make the simplifying assumption that the differentiation investment affects only $s$ and no other parameters of the utility function.

One characteristic of the utility function is that, other things equal (i.e. holding quantities $x_1$, $x_2$, and $M$ constant), utility is strictly decreasing in $s$ (i.e. increasing in variety, $v$) assuming output is positive: $\partial U / \partial s = -x_1x_2 < 0$. As increases in $s$ reduce product differentiation, this implies that utility is increasing in the extent of product differentiation. This property could reflect a taste for variety at the individual level or some distribution of tastes captured in aggregate utility function $U$.

The effect of differentiation investments, denoted $k_x$ and $k_y$, on effective differentiation experienced by consumers is captured by the following convenient functional form:

$$s = \frac{1}{e^{\beta K}} = e^{-\beta K}$$

(4)

where $K \equiv k_x + k_y$ and $\beta > 0$. If neither firm invests in differentiation, then $K = 0$ and $s = 1$, so the products are effectively identical (variety, $v$, is 0). If either firm invests in differentiation (such as by advertising or by superficial product adjustment) then $s < 1$ and the products are differentiated. An increase in differentiation investment by either firm reduces the degree of substitutability. The monopoly outcome with complete separation of products represents the outcome in the limit as the combined differentiation investment approaches infinity. If we use $v = 1 - s$ to represent the extent of differentiation, we can rewrite the effect of differentiation investments on the level of differentiation (variety) as
\[ v = 1 - 1/e^{\beta K} = 1 - e^{-\beta K}. \] (4')

Thus the degree of differentiation is 0 if \( K = 0 \) and approaches the limit of 1 as \( K \) approaches infinity. Strictly speaking \( v \) can never equal 1 as that would require an infinite investment in differentiation, so the admissible range for \( v \) is the half-closed interval \([0,1)\).

There are two stages in the game played by the two firms. Levels of investment in product differentiation are simultaneously determined in stage 1, and prices (Bertrand) or outputs (Cournot) are simultaneously determined in stage 2. We focus on the subgame perfect Nash equilibrium to this two-stage game in which each firm correctly anticipates the outcome of the product market decision (price or quantity) made in stage 2 when choosing its investment in stage 1.

4. Bertrand Competition

Suppose that the firms act as Bertrand competitors, simultaneously choosing prices in the second stage. We first solve for the second stage equilibrium conditional on \( s \) and show how the equilibrium changes as \( s \) changes. We subsequently consider the first stage in which \( s \) is determined by the simultaneous choices of \( k_1 \) and \( k_2 \).

4.1 Second Stage – Pricing Decisions

In the second stage, each firm maximizes variable profit with respect to its own price treating the other firm’s price as exogenous and treating \( k_1, k_2 \), and therefore \( s \) as predetermined. Variable profit for firm \( i \) for \( i = 1,2 \), denoted \( V_i \), excludes the differentiation investments, \( k_i \), sunk at stage 1:

\[ V_i = (p_i - c)x_i \] (5)
To express outputs as functions of prices and \( s \), we rewrite the inverse demand functions (2) as

\[ x_1 + sx_2 = a - p_1 \text{ and } sx_1 + x_2 = a - p_2 \]

and solve to obtain:

\[
x_1 = \frac{[(a - p_1) - (a - p_2)s]}{(1 - s^2)};
\]

\[
x_2 = \frac{[(a - p_2) - (a - p_1)s]}{(1 - s^2)}
\]

(6)

The demand equations (6) require \( s < 1 \), which applies if the products are not homogenous. If products are homogenous (\( s = 1 \)), consumers will buy from only one firm if that firm has a strictly lower price. If the firms charge the same price, we adopt the standard convention that they share the quantity demanded equally.

We next maximize variable profit (5) using (6) to solve for the Bertrand equilibrium prices and quantities. Conditional on an exogenous (or pre-determined) level of product differentiation, the properties of this model are known, but we report and prove the specific results for our setting. At the stage 2 Bertrand equilibrium, each firm has the same price, output, and variable profit. As shown in Appendix 1, these common values, denoted \( p, x, \) and \( V \) respectively, are as follows:

\[
p = (a-c)(1-s)/(2-s) + c = (a-c)v/(1+v) + c
\]

(7)

\[
x = (a-c)/(2-s)(1+s) = (a-c)/(1 + v)(2 -v)
\]

(8)

\[
V = (1-s^2)x^2 = v(2-v)x^2
\]

(9)

Equation (8) implies that in order for output to be positive the maximum willingness to pay, \( a \), must exceed marginal cost, \( c \). We impose \( a > c \) as a regularity condition for all subsequent analysis. If \( s = 1 \) (homogeneous products), then (7) reduces to \( p = c \) and (8) to \( x = (a-c)/2 \). This is the standard Bertrand solution with homogeneous products. If \( s = 0 \) (separate monopolies for goods 1 and 2), then the equilibrium prices are higher: \( p = p_1 = p_2 = (a + c)/2 \), which exceeds \( c \)
due to the requirement that \( a > c \). Interestingly, however, the quantities are the same in this dual monopoly case as in the homogeneous product case: \( x = x_1 = x_2 = (a-c)/2 \). Each firm produces the same amount but consumers are willing to pay more because the products are differentiated.

As shown in Proposition 1, increases in product differentiation cause the Bertrand price to rise for the admissible range of \( v \) (from 0 to 1). Correspondingly, reductions in \( v \) – less variety – cause price to fall. Also, starting with homogeneous products \( (v = 0) \), quantities initially fall as differentiation increases, reach a minimum at \( v = \frac{1}{2} \), then increase as differentiation increases further. Greater product differentiation always increases profits.

**Proposition 1**: Under Bertrand competition, an increase in product differentiation, \( v \) for \( v \in [0,1) \), causes

i) prices to rise,

ii) outputs to fall if \( 0 \leq v < \frac{1}{2} \), reach a minimum at \( v = 1/2 \), and then rise for \( v > 1/2 \),

iii) variable profits to rise.

**Proof:**

i) Differentiating (7) with respect to \( s \) yields

\[
\frac{dp}{ds} = - \frac{(a-c)/(2-s)^2}{0} < 0
\]  
(11)

Since \( s = v - 1 \), it follows that \( \frac{dp}{dv} = (\frac{dp}{ds})(\frac{ds}{dv}) = - \frac{dp}{ds} > 0 \).

ii) Differentiating (8) with respect to \( s \) yields

\[
\frac{dx}{ds} = \frac{x(2s-1)/(2-s)(1+s)}
\]  
(12)

It can be seen from (12) that \( \frac{dx}{ds} \) is positive if \( s > \frac{1}{2} \), zero at \( s = \frac{1}{2} \) and negative if \( s < \frac{1}{2} \). Since \( v = 1 - s \), it follows that \( \frac{dx}{dv} \) is negative if \( v < \frac{1}{2} \), zero at \( v = \frac{1}{2} \) and positive if \( v > \frac{1}{2} \).
iii) From (9) and (12), we obtain

$$\frac{dV}{ds} = -2s^2(1 - s + s^2)(2-s) < 0$$  \hspace{1cm} (13)

As $dV/dv = -dV/ds$ it follows from (13) that $dV/dv > 0$. ***

4.2 First Stage – Investments in Product Differentiation

In stage 1, firm 1 chooses $k_1$ and firm 2 chooses $k_2$. Firms are assumed to anticipate the second stage equilibrium that will emerge from their first stage decisions in accordance with sequential rationality as implied by the subgame perfect Nash equilibrium. Thus firms understand the dependence of outputs and prices on first stage differentiation decisions. They understand that a failure to differentiate their products will yield a second stage outcome in which profits are zero. This understanding provides a strong incentive to undertake positive differentiation investments in the first stage.

These (simultaneous) decisions regarding differentiation investments $k_1$ and $k_2$ jointly determine $K = k_1 + k_2$, which determines the degree of differentiation, $v$. As $v = 1 - e^{-\beta K}$ from (4’), we obtain

$$\frac{dv}{dk_1} = \frac{dv}{dk_2} = \frac{dv}{dK} = \beta e^{-\beta K} = \beta(1 - v)$$  \hspace{1cm} (14)

Thus, for any given value of $v$, a larger value of $\beta$ is associated with greater effectiveness of the investment $K$ in generating product differentiation. As $K$ is the total differentiation investment, it is useful to rewrite (14) as $dK/dv = 1/(\beta(1-v))$, which implies that the marginal cost of additional differentiation is decreasing in $\beta$ for any given value of $v$. Furthermore, generating additional differentiation gets harder – the marginal cost rises – as the extent of differentiation increases. As $v$ approaches 1 (complete differentiation) the rate at which additional differentiation
investments generate further differentiation also approaches 0 and marginal cost of further differentiation approaches infinity.

A main objective of this paper is to provide an explanation for why we rarely if ever observe the Bertrand homogeneous product equilibrium. Our answer is that firms engaged in Bertrand competition will normally differentiate their products. However, as a logical point, it is important to allow for the possibility that the cost of product differentiation might be prohibitively high. Our specification allows for this possibility. The parameter $\beta$ reflects the effectiveness of differentiation. If $\beta = 0$, then no differentiation is possible no matter how much firms invest in differentiation as $v = 1 - e^{-\beta K} = 1 - e^{0} = 0$. In effect, product differentiation is infinitely costly in this case. Positive but very low values of $\beta$ imply that differentiation is physically possible, but it may still be prohibitively costly.

The first stage profit function for firm $i$ can be written as:

$$\pi_i = V_i - k_i = (p_i - c)x_i - k_i$$

(15)

where $p_i$ and $x_i$ are both functions of $s$ (or $v$) and therefore of $k_1$ and $k_2$. In setting $k_i$, each firm $i$ correctly anticipates the effect on second stage variables, $s$, $x$, and $p$, but takes $k_j$ for $j \neq i$ as fixed. Using (13) and $\partial s/\partial k_i = ds/dK = -\beta s$, the first order condition for an interior solution ($k_i > 0$) to firm $i$’s profit maximization problem is therefore

$$\partial \pi_i/\partial k_i = -\beta s(dV_i/ds) - 1 = 2\beta sx^2(1 - s + s^2)/(2-s) - 1 = 0$$

(16)

A corner solution in which there is no investment in differentiation arises if $\partial \pi_i/\partial k_i \leq 0$ at $k_i = 0$. In order for Equation (16) to characterize a maximum rather than a minimum it is necessary that second order be conditions be satisfied, which is shown in Appendix 2.
Since \( x \) is a function of \( s \), equation (16) is a complicated function of \( s \), making a closed form solution for \( s \) and for the common level of investment, \( k = k_1 = k_2 \), difficult to obtain.

However, we are able to determine important characteristics of the solution. In particular, Proposition 2 indicates the threshold level of \( \beta \) below which product differentiation is prohibitively costly.

**Proposition 2:** Under Bertrand competition, both firms choose to differentiate their products at stage 1 if and only if \( \beta > 2/(a-c)^2 \). If \( \beta \leq 2/(a-c)^2 \) then no differentiation investment takes place and products are homogeneous at stage 2.

**Proof:** No differentiation \((s = 1)\) takes place if and only if \( k_1 = k_2 = 0 \), which occurs if and only if \( \partial \pi_i/\partial k_i \leq 0 \) at \( k_i = 0 \) and \( s = 1 \) (i.e. at \( v = 0 \)). Substituting \( s = 1 \) into (16) and using \( x = (a-c)/2 \) from (8) shows that \( \partial \pi_i/\partial k_i \leq 0 \) if and only if \( \beta \leq 2/(a-c)^2 \).

In the case of prohibitively expensive differentiation costs, it is still feasible for both firms to enter, so we cannot rule out homogeneous product Bertrand oligopoly. However, in that equilibrium each firm is indifferent about whether to produce or whether to withdraw from the industry. As a result, any positive entry cost or fixed cost would prevent the Bertrand outcome. As long as \( \beta \) exceeds the threshold level, Bertrand firms will necessarily differentiate their products.

### 4.3 Differentiation Externalities

Proposition 2 deals with the range of differentiation effectiveness that would give rise to positive differentiation in the subgame perfect Nash equilibrium. It is important to emphasize that these Nash equilibrium differentiation levels are not the levels that would maximize joint profits. There is an “underinvestment” in differentiation due to a positive externality associated with
differentiation expenditures. If firm 1 undertakes additional differentiation expenditures, this provides benefits to both firms. In our structure, it provides equal benefits to both firms. But firm 1 cares only about its own profit and hence will invest too little to maximize industry profit.

While closed form solutions for differentiation expenditures are difficult to obtain, it is straightforward to obtain solutions numerically for any specific parameter values. In Figure 1 we use the values $a = 14$, $c = 2$, and $\beta = 0.1$. For these values, the Nash equilibrium investment level is 5.0 for each firm. The diagram illustrates the effect of changes in the differentiation investment undertaken by firm 1, holding the differentiation investment of firm 2 constant at its Nash equilibrium level of 5.0.

Figure 1: Effects of Differentiation Investment by Firm 1 on Profits – Bertrand Model

As shown in Figure 1, in the Bertrand case, if firm 2 chooses a differentiation investment of 5.0, then the differentiation investment that maximizes the profit of firm 1 is also 5.0. This combination of choices is a Nash equilibrium. However, the investment by firm 1 that would
maximize joint profits is much higher: 12.7, as firm 2 also benefits from differentiation investments made by firm 1. In fact, the combined investment in differentiation that maximizes profit is $5.0 + 12.7 = 17.7$, which is the same combined investment that maximizes joint profit when the firms make symmetric investments of 8.85 each. As $k_1$ and $k_2$ are equivalent in their effect on $K$ and hence on $s$ or $v = 1 - s$, total profit is not affected by which firm makes the investment. The implied level of differentiation at the Nash equilibrium is $v = 0.63$, whereas the level of differentiation that maximizes joint profits is $v = 0.83$, which is considerably higher.

5. The Cournot Model

Suppose now that the firms are Cournot competitors – simultaneously choosing output instead of price in the second stage. As in the Bertrand case, investments in product differentiation take place in stage 1.

5.1 Second Stage – Quantity Decisions

In the second stage we take $k_i$ as predetermined (and therefore as a constant). Setting $x_1$ to maximize variable profit, $V_1$, as given by (5), holding $x_2$ fixed and setting $x_2$ to maximize $V_2$ holding $x_1$ fixed and using the demand functions given by (2), we obtain the first order conditions:

$$\frac{\partial V_1}{\partial x_1} = a - c - 2x_1 - sx_2 = 0$$

$$\frac{\partial V_2}{\partial x_2} = a - c - sx_1 - 2x_2 = 0$$

Equations (17) define the Cournot equilibrium values of output, which are unique and symmetric. Equilibrium prices and variable profits then follow from (2) and (9). Using $v = 1-s$, we express output, price and profit at the stage 2 Cournot equilibrium as follows:

$$x = x_1 = x_2 = \frac{(a-c)}{(2 + s)} = \frac{(a-c)}{(3 - v)}$$

(18)
\[ p = p_1 = p_2 = \frac{(a-c)(2+s)}{2} + c = \frac{(a-c)(3-v)}{3} + c \quad (19) \]

\[ V = V_1 = V_2 = x^2 \quad (20) \]

The effects of variation in product differentiation are set out in Proposition 3.

**Proposition 3**: An increase in product differentiation \( v \), causes outputs to rise, prices to rise, and variable profits to rise.

**Proof**: From (18), (19) and (20), we obtain

\[ \frac{dx}{ds} = -\frac{(a-c)}{2+s} = -\frac{x}{2+s}, \]

\[ \frac{dp}{ds} = -\frac{(a-c)}{2+s} \] and

\[ \frac{dV}{ds} = -\frac{2x^2}{2+s} = -\frac{2(a-c)^2}{2+s} \quad (21) \]

Since \( \frac{dv}{ds} = 1 \), the result follows. ***

In the limit as \( v \) approaches 1, each firm is effectively a monopolist over an independent good and produces output level \( \frac{(a-c)}{2} \) and the common price, \( p = \frac{(a + 2c)}{3} \), as in the Bertrand case. If \( v = 0 \), then products are homogenous and output and price are at the standard homogenous product Cournot levels: \( x = \frac{(a-c)}{3} \) and \( p = \frac{(a + 2c)}{3} \). Consequently, each firm produces a higher output when each firm has a monopoly (products are independent) than if the firms compete as Cournot competitors with homogeneous products.

**5.2 First Stage – Investments in Product Differentiation**

As with the Bertrand model, in the first stage each firm \( i \) seeks to maximize its profit \( \pi_i = V_i - k_i \) by independently choosing its differentiation investment \( k_i \), taking as exogenous the differentiation investment of the other firm and taking into account how differentiation decisions
made in the first stage affect second stage equilibrium levels of $x$ and $p$. Using (21), the associated first order condition for an interior solution for firm $i$ is given by

$$\frac{\partial \pi_i}{\partial k_i} = -\beta s(\frac{dV}{ds}) - 1 = 2\beta s x^2/(2+s) - 1 = 0$$  \hspace{1cm} (22)$$

where $x = (a-c)/(2+s)$ from (18). (See Appendix 2 for a proof that second order conditions are satisfied.) The corner solution in which $k_i = 0$ occurs if $d\pi_i/dk_i \leq 0$ at $k_i = 0$. Proposition 4 identifies the threshold level of $\beta$ below which product differentiation would not occur.

**Proposition 4:** With Cournot competition, firms choose to differentiate their products at stage 1 if and only if $\beta > 13.5/(a-c)^2$. If $\beta \leq 13.5/(a-c)^2$ then no investment takes place and products are homogeneous at stage 2.

**Proof:** No differentiation ($s = 1$) takes place if and only if and only if $\frac{\partial \pi_i}{\partial k_i} \leq 0$ at $k_i = 0$ and $s = 1$ (i.e. at $v = 0$) for $i = 1,2$. Substituting $s = 1$ into (22) and using $x = (a-c)/3$ from (18) shows that $\frac{\partial \pi_i}{\partial k_i} \leq 0$ if and only if $\beta \leq 13.5/(a-c)^2$. ***

As with the Bertrand case, firms undertake less product differentiation than would be needed to maximize joint profits.

6. **Comparing the Cournot and Bertrand Models**

Proposition 5 provides a characterization of the range of differentiation effectiveness, $\beta$, for which differentiation occurs or does not occur in the Bertrand and Cournot models. The proposition also establishes that if Bertrand firms have an incentive to differentiate, they will always invest more than their Cournot counterparts leading to greater product differentiation under Bertrand than Cournot competition. To facilitate the proof, we use superscripts $B$ and $C$ to represent Bertrand and Cournot outcomes respectively. Thus $p^B$ and $x^B$ represent the (common)
price and (common) level of output and \( v^B = 1 - s^B \) denotes the level of product differentiation under Bertrand competition.

\[ \text{6.1 The Effectiveness of Differentiation Investments and the Extent of Differentiation} \]

**Proposition 5:**

(i) If \( \beta \leq \frac{2}{(a-c)^2} \), then products are homogeneous under both Bertrand and Cournot competition.

(ii) If \( \frac{2}{(a-c)^2} < \beta \leq \frac{13.5}{(a-c)^2} \), then products are differentiated under Bertrand competition and are homogenous under Cournot competition: \( v^B > v^C = 0 \).

(iii) If \( \beta > \frac{13.5}{(a-c)^2} \), then products are differentiated under both Bertrand and Cournot competition, but are more differentiated under Bertrand than Cournot competition: \( v^B > v^C > 0 \).

**Proof:** Parts (i) and (ii) follow directly from Propositions 2 and 4. (iii) If \( \beta > \frac{13.5}{(a-c)^2} \), then \( k^C_i > 0 \) from Proposition 4. From \( k^C_i > 0 \) and (21), we have \( d\pi^C_i/dk_i = 0 \) which implies \( 2\beta s^C(x^C)^2 = 2+s^C \), where \( s^C = 1/e^{\beta K} > 0 \) for \( K = k^C_1 + k^C_2 \). Setting \( K = k^C_1 + k^C_2 \) and \( s = s^C \) in (16), we obtain

\[
\frac{\partial \pi^B_i}{\partial k_i} = 2\beta s^C(x^B)^2 - \frac{(2+s^C)(1-s^C + (s^C)^2)}{(2-s^C)} - 1. \]

For the same value of \( s > 0 \), it follows from (8) and (18) that \( x^B = \frac{(2+s)x^C(2-s)(1+s)}{1} > x^C \) and hence \( 2\beta s^C(x^B)^2 > 2\beta s^C(x^C)^2 = 2+s^C \) and \( d\pi^B_i/dk_i > \frac{(2+s^C)(1-s^C + (s^C)^2)}{(2-s^C)} - 1 \) at \( K = k^C_1 + k^C_2 \). Since \( (2+s^C)(1-s^C + (s^C)^2) - (2-s^C) = (s^C)^2(1+s^C) > 0 \), we obtain \( \frac{\partial \pi^B_i}{\partial k_i} > 0 \) for \( K = k^C_1 + k^C_2 \), which implies \( k^B_i > k^C_1 \) and \( s^B > s^C \) for \( \beta > \frac{13.5}{(a-c)^2} \) ***

Proposition 5 indicates that for both Bertrand and Cournot competition, whether the firms undertake differentiation expenditures depends on the effectiveness of differentiation investments, measured by \( \beta \), relative to the difference, \( (a-c) \) between the demand intercept and marginal cost – a reflection of the strength of demand. The stronger is demand, the less effective
differentiation investments need to be to justify differentiation. Table 1 shows the critical values of differentiation effectiveness for both the Bertrand and Cournot models.

Table 1: Critical Values of Differentiation Effectiveness, $\beta$

<table>
<thead>
<tr>
<th>Relative Demand $(a - c)$</th>
<th>Bertrand ($\beta^B$)</th>
<th>Cournot ($\beta^C$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.50</td>
<td>3.38</td>
</tr>
<tr>
<td>4</td>
<td>0.13</td>
<td>0.84</td>
</tr>
<tr>
<td>8</td>
<td>0.031</td>
<td>0.21</td>
</tr>
<tr>
<td>12</td>
<td>0.014</td>
<td>0.094</td>
</tr>
<tr>
<td>16</td>
<td>0.0078</td>
<td>0.053</td>
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<tr>
<td>20</td>
<td>0.0050</td>
<td>0.034</td>
</tr>
<tr>
<td>30</td>
<td>0.0022</td>
<td>0.015</td>
</tr>
<tr>
<td>50</td>
<td>0.00080</td>
<td>0.0054</td>
</tr>
</tbody>
</table>

Table 1 demonstrates a key point of the paper. In each cell the critical values are shown to two significant digits. For every level of relative demand the critical value of differentiation effectiveness required to induce differentiation under the Bertrand model is about an order of magnitude less than for the Cournot model. We define $\beta^B$ as the critical level of differentiation effectiveness needed for Bertrand firms to undertake differentiation and $\beta^C$ as the corresponding critical value for the Cournot case. The ratio of critical values is approximately 6.8, subject to rounding. Differentiation is much more likely under Bertrand competition than under Cournot competition in the sense that dramatically less effectiveness of differentiation investment is required.

6.2 Endogenous Product Differentiation and Comparative Competitiveness

As is well-known, in the homogeneous product case Bertrand competition is more intense than Cournot competition. As shown by Singh and Vives (1984), this insight generalizes to any common level of product differentiation short of being completely unrelated. Using a demand structure similar to ours, Singh and Vives (1984) demonstrate that, for any (exogenous) common
level of differentiation less than unrelated products ($v < 1$), the Bertrand model generates higher output, lower prices and lower profits than the Cournot model. If products are unrelated ($v = 1$), then prices and outputs are at the monopoly level in both models. However, when we allow for the important and highly relevant possibility that product differentiation is endogenous, the comparison of Cournot and Bertrand is more difficult and very different in its implications. If any differentiation at all occurs, Bertrand firms choose a higher level of product differentiation than Cournot firms. This consideration tends to raise prices and lower output in the Bertrand case to an extent that can offset the inherently greater competitiveness of Bertrand behaviour conditional on a given common level of differentiation.

Proposition 6 establishes that, even with endogenous product differentiation, the Bertrand model generates higher output than the Cournot model. However, prices and profits are not necessarily lower in the Bertrand case. Therefore, allowing for endogenous product differentiation is an important limitation on the general presumption that Bertrand industries are “more competitive” than corresponding Cournot industries.

**Proposition 6:**

i) Independent of whether product differentiation is the same across modes of competition or is chosen endogenously, output is higher under Bertrand than Cournot competition: $x^B > x^C$.

ii) For the same level of product differentiation, both price and profit are strictly lower under Bertrand than Cournot competition: $p^B < p^C$ and $\pi^B < \pi^C$.

iii) With endogenous product differentiation it is possible for Bertrand firms to charge higher prices and earn more profit than corresponding Cournot firms. For example, for an effectiveness of investment parameter, $\beta = 13.5/(a-c)^2$, Bertrand firm undertake more differentiation ($v^B = \ldots$
0.60823 > ν^C = 0), charge higher prices and earn more profit than corresponding Cournot firms.

**Proof:** i) From \( x^B = (a-c)/(2-s^B)(1+s^B) \) and \( x^C = (a-c)/(2 + s^C) \) (see (8) and (18)), we obtain

\[
x^B = (2+s^C)x^C/(2-s^B)(1+s^B),
\]
which implies \( x^B > x^C \) if and only if \( 2+s^C - (2-s^B)(1+s^B) = s^C - s^B + (s^B)^2 > 0 \). Since \( s^C \geq s^B > 0 \) from Proposition 5, the condition holds and \( x^B > x^C \).

ii) It follows immediately from downward sloping demand and \( x^B > x^C \) from part i) that the Bertrand price must be less than the Cournot price for the same \( ν \) for \( ν = 1 - s < 1 \). Now examining profits, if \( k = 0 \) (homogeneous products), then \( s = 1 \) and \( ν = 1 - s = 0 \) and from (9) and (20), we obtain \( V^C > V^B = 0 \) and hence \( π^C > π^B = 0 \). If \( k > 0 \), then \( s < 1 \), \( ν > 0 \) and, for any given \( s \), we obtain

\[
V^C = V^B(2 - s)^2(1+s)/(1-s)(2+s)^2 \quad \text{from (9) and (20)}
\]
and hence, for any given \( k \), \( π^C > π^B \) if and only if \( (2 - s)^2(1+s) > (1-s)(2+s)^2 \), which holds for all \( s, ν \in (0,1) \). If \( s = 0 \) (or \( ν = 1 \)), then each firm has a monopoly with profit, \( π^B = π^C = (a-c)^2/4 - k \).

iii) If \( β = 13.5/(a-c)^2 \), then from proposition 5(ii), products are differentiated under Bertrand Competition (\( ν^B > 0 \) and \( k^B > 0 \)) and homogeneous under Cournot competition(\( ν^C = 0 \) and \( k^C = 0 \)). Since \( k^B > 0 \), it follows from (16) that \( dπ^B/dk_i = 2βs(x^B)^2(1 - s + s^2)/(2-s) - 1 = 0 \) where \( x^B = (a-c)/(2-s)(1+s) \) from (8). Setting \( β = 13.5/(a-c)^2 \) and solving for \( s \) we obtain \( s^B = 0.39177 \) and \( ν^B = 1 - s^B = 0.60823 \). We have \( p^B > p^C \) since substituting \( ν^B = 0.60823 \) into (7) and \( ν^C = 0 \) into (19), yields \( p^B = 0.378(a-c) + c \) and \( p^C = (a-c)/3 + c \). Now examining profits, for \( β = 13.5/(a-c)^2 \), we obtain \( x^B = 0.44677 (a-c) \) and \( x^C = (a-c)/3 \) from (8) and (18). Since \( k^B = - ln(s^B)/2β \) (from \( s = e^{-βK} \)), we further obtain \( k^B = 0.0347(a-c) \). From (9) and (20) that \( π^B = 0.13426(a-c)^2 > π^C = (a-c)^2/9 = 0.1111(a-c)^2 \). ***

Figure 2 illustrates the possibility set out in part iii) of Proposition 6 that price can be higher under Bertrand than Cournot competition. For Figure 2, we assume \( β = 13.5/(a-c)^2 \), \( a = \)

24
14 and \( c = 2 \), which implies that \( \beta = 0.09375 \). In this case, Cournot firms will not invest in differentiation \((v^C = 0)\) while Bertrand firms will select a degree of differentiation given by \( v = 0.608 \). As Figure 2 shows, Cournot firms charge \$6\) when products are homogeneous \((v = 0)\), whereas Bertrand firms charge over \$6\) at \( v = 0.608 \). If Bertrand firms had produced homogeneous products, they would have set the much lower price of \$2\) as given by the marginal cost of production.

**Figure 2: Product Differentiation and Price: Cournot vs. Bertrand**

As the degree of differentiation, \( v \), rises, prices rise under both Bertrand and Cournot competition and eventually approach the monopoly level of \$8\) (unrelated goods) as \( v \) approaches 1. However, the Bertrand price is much more sensitive to product differentiation than the Cournot price. Changing the differentiation parameter from 0 (homogeneous products) to 0.4 raises the Cournot price from \$6.00\) to \$6.62\) – a modest increase of about 10%. For Bertrand firms, an increase in product differentiation from 0 to 0.4 causes price to rise dramatically – from \$2\) to \$5.43\) – an increase of over 150%! And most significantly, if \( \beta \leq 0.09375 \) the Cournot
firms will undertake no differentiation, while Bertrand firms will differentiate as long as $\beta > 0.014$.

The comparison of quantities is also interesting. Again assuming $a = 14$ and $c = 2$, Figure 3 shows how quantity per firm varies with the degree of differentiation.

Figure 3: Product Differentiation and Output per firm: Cournot vs. Bertrand

As Figure 3 shows, the Cournot equilibrium output is strictly increasing in the degree of product differentiation, despite the fact that prices are also rising. In effect, Cournot firms exploit the increased demand arising from greater product variety to raise both price and quantity.

Bertrand firms, on the other hand, initially realize the benefits of product differentiation in higher prices – to the extent that quantity sold actually falls until a fairly significant level of differentiation is reached ($v = 0.5$). If $\beta = 13.5/(a-c)^2$, then $v = 0$ for Cournot firms, which implies an output of $x^C = (a-c)/3 = 4$ when $a = 14$ and $c = 2$. Bertrand firms choose to differentiate their products resulting in $v = 0.608$ and an output of $x^B = 0.44677(a-c) = 5.36$. As shown in
Proposition 6, part i, output is always higher under Bertrand than Cournot competition despite endogenous product differentiation.

Figure 4 compares industry profit for Bertrand and Cournot competition as the degree of product differentiation varies. Again, we assume $a = 14$ and $c = 2$. (Due to differences in the incentives to invest, the value of $\beta$ required to induce the level of investment used to calculate profit for a given value of $v$ differs across the two modes of competition).

Figure 4: Effects of Differentiation on Industry Profit: Cournot vs. Bertrand

As Figure 4 shows, Cournot profits exceed Bertrand profits for any common level of differentiation. The difference in profitability is very large with homogeneous products, but becomes small for high levels of differentiation investments. As $v$ becomes close to 1, the products become nearly unrelated and profits become very close to the monopoly level, which does not depend on whether firms choose price or quantity. As emphasized earlier neither Cournot firms nor Bertrand firms maximize joint profits due to the differentiation externality.
Figure 4: Bertrand and Cournot profits at the same level of product differentiation. However, the level of product differentiation will not be the same unless the effectiveness of investment parameter, $\beta$, is sufficiently small that products are homogeneous under both modes of conduct ($v = 0$). Figure 5 illustrates the relationship between $\beta$ and profit allowing for different values of the degree of product differentiation, $v$, under the two modes of conduct. As before the parameter values used in the diagram are $a = 14$ and $c = 2$. However, the diagram has the same qualitative form for any values of $a$ and $c$ that are sufficient to allow for differentiation.

Figure 5: The effectiveness, $\beta$, of investment and profits: Cournot vs. Bertrand

If $\beta \leq 2/(a-c)^2 = 0.014$ (for $a = 14$ and $c = 2$) then products are homogeneous ($v = 0$) and as shown in Figure 5, profit is zero under Bertrand competition. Each Cournot firm earns $16, with a combined profit of $32$ for the industry. If $0.014 < \beta \leq 0.09375$, Cournot firms do not invest in product differentiation and profit is unchanged. But profit under Bertrand competition increases as more product differentiation takes place. At $\beta = 0.09375$, Bertrand firms earn a combined profit of $38.6$, which exceeds the $32$ in industry profit earned under Cournot competition. For
\( \beta > 0.0975 \), Cournot firms choose to invest, but in this region profit under Bertrand competition always exceeds profit under Cournot competition. The reason that Bertrand firms choose to differentiate more than Cournot firms is that variable profits under price competition are more sensitive to a change in the degree of product differentiation than are variable profits under quantity competition. The cutthroat nature of price competition when products are homogeneous rapidly diminishes as products become differentiated.

6.3 Endogenous Product Differentiation and Consumer Surplus.

There is a significant literature comparing the effects of Bertrand and Cournot competition on consumers. Differences in cost due to process R&D have been considered, but the analysis does not allow for the effects of different investments in product differentiation.

Letting \( G \equiv U - (p_1x_1 - p_2x_2 - M) \) denote consumer surplus (or “gains”), then from (1) for \( b = 1 \), (2) and the equality of outputs and prices across firms, we obtain

\[
G = 2(a-p)x - (1+s)x^2 = (1+s)x^2
\]

(23)

Taking into account the different relationships between product differentiation and output levels under different modes of competition (see (8) and (18)), we define \( G^B = G^B(s) = (1+s)(x^B(s))^2 \) and \( G^C = G^C(s) = (1+s)(x^C(s))^2 \) to examine consumer surplus as a function of product differentiation, \( v \), where \( v = 1 - \beta \). Proposition 7 follows.

**Proposition 7:**

i) For any given level of product differentiation, \( v \), an increase in \( v \):

(a) reduces consumer surplus under Bertrand competition.

(b) increases consumer surplus under Cournot competition.
ii) Whatever the levels of product differentiation, $v^B$ and $v^C$, other than unrelated products ($v = v^B = v^C = 1$), consumer surplus is always higher under Bertrand than Cournot competition.

Proof: i) From (23), $dG^B/ds = 3s(x^B(s))^2/(2 - s) > 0$

$\frac{dG^C}{ds} = -s(x^C(s))^2/(2+s) < 0$ (24)

Since $v = 1 - s < 1$ and $dG/dv = -dG/ds$, it follows from (23) that $dG^B/dv < 0$ proving (a) and $dG^C/dv > 0$ proving (b).

ii) If $v < 1$, then $s = 1 - v > 0$. Since $dG^B/ds > 0$ and $dG^C/ds < 0$ for $s > 0$ (see (24)), we have $G^B > G^C$ for any $v^B, v^C \in [0,1]$ provided $v^B < 1$ or $v^C < 1$. If $v = 1$(products are unrelated), then $x^B = x^C = (a-c)/2$ (see (12) and (18)) and we obtain $G^B = G^C$ from (23) ***

Proposition 7 makes the striking point that when Bertrand firms produce differentiated products rather than homogeneous products, consumers are made worse off. Even though consumers get more utility from differentiated products at given prices, Bertrand firms take advantage of product differentiation to raise prices sufficiently that the price increase more than offsets the direct gain in utility experienced by consumers. By contrast, the price increases arising from increases in differentiation under Cournot competition are relatively modest and are not sufficient to offset the consumer gains arising from more variety.

Despite the fact that consumers are made worse off by Bertrand differentiation and better off by Cournot differentiation, it is still true that for any given level of differentiation, consumer surplus is higher under Bertrand than Cournot competition. Indeed, as shown in Proposition 7, part ii, consumer surplus is always higher under Bertrand competition, regardless of product
differentiation decisions. This last result follows from the fact that the limiting case of monopoly in which products are completely independent (variety is at \( v = 1 \)), represents the lower bound for consumer surplus under Bertrand competition and the upper bound for consumer surplus under Cournot competition.

The basic facts of Proposition 7 are illustrated in Figure 6 assuming \( a = 14 \) and \( c = 2 \) as in previous figures.

Figure 5: Effects of Differentiation on Consumer Surplus: Bertrand vs. Cournot

7. Concluding Remarks

The primary objective of this paper is to provide an explanation of the empirical Bertrand paradox based on horizontal product differentiation. By the “empirical Bertrand paradox” we mean the failure to observe homogeneous product Bertrand oligopoly in practice while homogenous product Cournot oligopoly is of empirical relevance. In our model, even though
product differentiation is costly, we show that Bertrand firms have a strong incentive to undertake product differentiation – much stronger than Cournot firms.

Our model of differentiation is sufficiently general that prohibitive differentiation costs are possible, even in the Bertrand case. The critical value of differentiation effectiveness required for differentiation under Bertrand is reflected by a differentiation effectiveness parameter that we label $\beta^B$. At this level of differentiation effectiveness, Cournot firms will not undertake differentiation. Even twice or three times this level of differentiation effectiveness is not sufficient to induce Cournot firms to differentiate. The level of differentiation effectiveness must rise to approximately 6.8 times $\beta^B$ before Cournot firms will undertake differentiation. Therefore, there is a very wide range of differentiation effectiveness (and hence differentiation costs) over which Bertrand firms would differentiate but Cournot firms would not.

While there are other reasons why we might rarely if ever observe homogeneous product Bertrand oligopoly, we suggest that endogenous horizontal product differentiation is a natural explanation with significant empirical relevance. Examples include cases such as soft drinks – where differentiation is achieved through advertising – and automobiles, where much of the year to year changes that are made in model specification are small styling differences that have little to do with performance and much to do with creating perceived differentiation. In the case that differentiation investments are very ineffective in creating differentiation of relevance to consumers, Bertrand firms would earn zero profits and would be indifferent between producing and staying out of the market. Since even the slightest fixed costs or entry costs would generate negative profits, it is unlikely we would observe homogeneous product Bertrand oligopoly as an equilibrium outcome.
In addition to offering an explanation of the empirical Bertrand paradox, our analysis investigates the relative level of competitiveness of otherwise equivalent Bertrand and Cournot industries. Strikingly, we find that for sufficiently high values of differentiation effectiveness (i.e. low differentiation costs) Bertrand firms charge higher prices and earn larger profits than Cournot firms. For any given level of product differentiation short of completely unrelated products, variable profits are lower under price competition than quantity competition. However, the cutthroat nature of price competition when products are homogeneous is rapidly tempered by product differentiation. Indeed, the enhanced market power enjoyed by Bertrand firms as differentiation increases more than offsets the benefits to consumers of greater variety, with the result that consumer surplus falls. Cournot competition, however, exhibits the opposite result, as greater product differentiation generates increased consumer surplus. Nevertheless, regardless of differences in the degree of product differentiation across the two modes of competition, consumer surplus is always higher under Bertrand competition than Cournot competition.
Appendix 1: Bertrand price, output, and profit as functions of differentiation, \( v \)

We first derive (7), (8) and (9) as functions of \( s \) and then use \( s = 1 - v \) to express these conditions in terms of \( v \). Each firm \( i \) for \( i = 1,2 \) sets its price to maximize variable profit taking the price of the other firm as given. From (5) using (6), the first order conditions are

\[
\begin{align*}
\frac{dV_1}{dp_1} &= x_1 - (p_1 - c)/(1 - s^2) = [(a + c - 2p_1) - (a - p_2)s]/(1 - s^2) = 0 \\
\frac{dV_2}{dp_2} &= x_2 - (p_2 - c)/(1 - s^2) = [(a + c - 2p_2) - (a - p_1)s]/(1 - s^2) = 0
\end{align*}
\]

These first order conditions simplify to \( 2p_1 - sp_2 = (a + c) - as \) and \( 2p_2 - sp_1 = (a + c) - as \), which imply a common price, denoted, \( p \), where

\[
p = p_1 = p_2 = (a + c - as)/(2-s) = (a-c)(1-s)/(2-s) + c
\]  

(A1.1)

Substituting (A1.1) into the demand functions (6) yields the common equilibrium quantity:

\[
x = x_1 = x_2 = (a - p)/(1 + s) = (a-c)/(2-s)(1 + s)
\]  

(A1.2)

Equations (A1.1) and (A1.2) apply for \( s < 1 \) (products are differentiated) and for \( s = 1 \) (homogeneous products). Substituting (A1.1) and (A1.2) into the variable profit function for each firm (given by (5)) yields variable profits at the second stage equilibrium, denoted \( V \), where:

\[
V = V_1 = V_2 = (1 - s^2)x^2.
\]  

(A1.3)

***
Appendix 2: Second order conditions for differentiation decisions.

Show \( \partial^2 \pi_i / (\partial k_i)^2 < 0 \) for all \( s \in (0,1] \) or \( v = 1 - s \in [0,1) \)

For both Bertrand and Cournot competition (see (16) and (22)), each firm \( i \) sets \( k_i \) taking \( k_j \) for \( j \neq i \) fixed, which implies \( \partial s / \partial k_i = ds / dK = -\beta s \) and \( \partial \pi_i / \partial k_i = -\beta s (dV / ds) - 1 = 0 \). It then follows that

\[
\partial^2 \pi_i / (\partial k_i)^2 = \beta^2 s[(dV / ds) + s(d^2 V / (ds)^2)]
\]  
(A2.1)

For Bertrand competition, using \( dV^B / ds = -2(x^B)^2(1 - s + s^2)/(2-s) \) from (13) and \( dx^B / ds = -x^B(1-2s)/(2-s)(1+s) \) from (12) it can be shown that

\[
d^2 V^B / (ds)^2 = -6(x^B)^2[3s - 1 - s^2(1 - s)]/(2 - s)^2(1 + s)
\]  
(A2.2)

It can then be shown from (13) and (A2.2), that

\[
dV^B / ds + s(d^2 V^B / (ds)^2) = -2(x^B)^2\Psi^B/(2-s)^2(1+s)
\]  
(A2.3)

where \( \Psi^B = (1-s+s^2)(2-s)(1+s) + 3s[3s - 1 - s^2(1-s)] \) can be expressed as

\[
\Psi^B = 2(1-s)(1-s+s^2) + s^2(5+s+2s^2) > 0
\]  
(A2.4)

From (A2.1), (A2.3) and (A2.4), we obtain \( \partial^2 \pi^B_i / (\partial k_i)^2 < 0 \) for all \( s \in [0,1] \).

For Cournot competition, using \( dV^C / ds = -2(x^C)^2/(2+s) \) from (22) and \( x^C = (a-c)/(2+s) \) from (18), we obtain

\[
d^2 V^C / (ds)^2 = 6(x^C)^2/(2+s)^2
\]  
(A2.5)

It then follows from (22) and (A2.5), that

\[
dV^C / ds + s(d^2 V^C / (ds)^2) = -4(1-s)(x^C)^2/(2+s)^2 < 0
\]  
(A2.6)

and hence, using (A1), that \( \partial^2 \pi^C_i / (\partial k_i)^2 < 0 \) for all \( s \in [0,1] \). ***

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References


