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Effects of a general set of interactions on neutrino propagation in matter*

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Abstract

An analysis of the effective potential for neutrino propagation in matter, assuming a generic set of Lorentz invariant non-derivative interactions is presented. In addition to vector and axial vector couplings, also tensor interactions can give coherent effects if the medium is polarized, and the components of a tensor potential transverse to the direction of neutrino propagation can induce a neutrino spin-flip.

I. INTRODUCTION

Neutrino physics currently provides the strongest experimental evidence for physics beyond the Standard Model (SM). The atmospheric neutrino anomaly [2] and the solar neutrino problem [3] are best explained by neutrino oscillations. This require massive neutrinos that mix, and hence physics beyond the SM. When neutrinos propagate in matter, the physics of neutrino oscillations can be very different from the case of vacuum propagation. This is because coherent interactions with the background give to the neutrino an "index of refraction" that depends on the type of background and on the neutrino flavor. For example, in normal matter only electron neutrinos have SM charged current interactions, and thus the effective ν_e mass is enhanced with respect to the other flavors. This allows for the possibility of level crossing between different neutrino eigenstates in matter, and can result in a significant amplification of neutrino oscillations. This is known as the MSW effect. [4] For light sterile neutrinos also neutral current interactions are important. [5] Finally, in a polarized medium the neutrino effective mass depends also on the average polarization of the background and on the angle between the neutrino momentum and the polarization vector. [6]

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New physics models that imply massive neutrinos often predict also new neutrino interactions, that can significantly modify the SM picture. [7,8] For example, non-universal interactions may give rise to matter effects that distinguish between muon and tau neutrinos. Lepton flavor violating interactions can induce an effective mixing in matter, allowing for a resonant conversion even in the absence of vacuum mixing. The two effects combined together can induce neutrino flavor transitions even for massless neutrinos. Most of the discussions of these non standard effects assume new interactions just of vector and axial vector types. However, recently the possible effects of a much more general set of interactions have been analyzed. [1] In this talk we discuss the main results of this investigation.

II. NEUTRINO PROPAGATION IN MATTER WITH GENERAL INTERACTIONS

Our aim is to study neutrino propagation in matter in the presence of the most general pointlike and Lorentz invariant four-fermion interaction with the background fermions ($f = e, p, n, \nu$). We assume an interaction Hamiltonian of the form

$$\mathcal{H}_{\rm int} = \frac{G_F}{\sqrt{2}} \sum_a (\bar{\nu}\, \Gamma^a\, \nu) \left[\bar{\psi}_f\, \Gamma_a\, (g_a + g'_a \gamma^5)\, \psi_f\right] + \text{h.c.}\,, \tag{2.1}$$

where $\Gamma^a = \{I, \gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}\}$, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ and $a = \{S, P, V, A, T\}$. The Fermi constant G_F has been factored out so that all the couplings are dimensionless. In general, ν is a vector of the different neutrino types, and g_a, g'_a are 10 matrices in the space of neutrino flavors that describe the coupling strengths. Note that new interactions can include both flavor diagonal and off-diagonal couplings. To derive the equation of motion for the neutrino propagation in matter we first average the effective interactions over the background fermions. We select only coherent transitions, which leave the many-fermion background system in the same state, since incoherent effects become negligible after averaging. In particular, while we allow for neutrino spin-flips, we require that the background fermions do not change their spin. Accordingly, we introduce matrix elements of the fermion currents between initial and final states with the same quantum numbers

$$\mathcal{M}_a^f \equiv \langle f, \boldsymbol{p}, \boldsymbol{\lambda} | \bar{\psi}_f\, \Gamma_a\, (g_a + g'_a \gamma^5)\, \psi_f | f, \boldsymbol{p}, \boldsymbol{\lambda} \rangle\,, \tag{2.2}$$

where $\boldsymbol{p}$ and $\boldsymbol{\lambda}$ denote respectively the momentum and polarization vectors of the background fermion f. The expectation value of $\mathcal{M}_a^f$, averaged over the fermion distribution $\rho_f(\boldsymbol{p}, \boldsymbol{\lambda})$ reads

$$V_a^f = \frac{G_F}{\sqrt{2}} \sum_{\boldsymbol{\lambda}} \int \frac{d^3p}{(2\pi)^3} \rho_f(\boldsymbol{p}, \boldsymbol{\lambda}) \mathcal{M}_a^f\,. \tag{2.3}$$

The effect of the medium on the neutrino propagation in the presence of the general interactions (2.1) is then described by the interaction Lagrangian

$$-\mathcal{L}_{int} = \sum_{a,f} (\bar{\nu}\,\Gamma^a\,\nu)\,V_a^f\,. \tag{2.4}$$

The computation of the various $\mathcal{M}_a^f$ is straightforward. [1] After performing the contractions $\Gamma^a\,V_a^f$ in (2.4) we obtain

$$\Sigma^{SP} \equiv \left[V^S + V^P\,\gamma^5\right] = \frac{G_F}{\sqrt{2}}\,n_f \left\langle \frac{m_f}{E_f} \right\rangle \left(g_S + g_P'\,\gamma^5\right) \tag{2.5}$$

$$\begin{aligned}\Sigma^{VA} &\equiv \gamma^\mu\left[V_\mu^V + V_\mu^A\,\gamma^5\right] \\ &= \frac{G_F}{\sqrt{2}}\,n_f \left[\left\langle \frac{\not{p}}{E_f}\right\rangle \left(g_V + g_A'\,\gamma^5\right) \;+ m_f \left\langle \frac{\not{s}}{E_f}\right\rangle \left(g_V' + g_A\,\gamma^5\right)\right]\end{aligned} \tag{2.6}$$

$$\Sigma^T \equiv \varsigma^i\left[V_i^B + iV_i^E\,\gamma^5\right] = \frac{G_F}{\sqrt{2}}\,n_f \left\langle \frac{[\not{s},\not{p}]}{E_f}\right\rangle \left(g_T' + g_T\,\gamma^5\right)\,, \tag{2.7}$$

where $\varsigma^i \equiv \mathrm{diag}\,(\sigma^i, \sigma^i)$. The spin vector s satisfies $s^2 = -1$ and $s_\mu\,p^\mu = 0$ (the explicit expression can be found in [1]). We have also introduced

$$n_f = \sum_{\boldsymbol{\lambda}} \int \frac{d^3p}{(2\pi)^3}\rho_f(\boldsymbol{p},\boldsymbol{\lambda})\,, \qquad \langle x\rangle = \frac{1}{n_f}\sum_{\boldsymbol{\lambda}}\int \frac{d^3p}{(2\pi)^3}\rho_f(\boldsymbol{p},\boldsymbol{\lambda})\,x(\boldsymbol{p},\boldsymbol{\lambda}) \tag{2.8}$$

to denote, respectively, the number density n_f of the fermion f and the average of some function $x(\boldsymbol{p},\boldsymbol{\lambda})$ over the fermion distribution. In (2.7) we have decomposed the tensor term $V_{\mu\nu}^T$ in analogy to the electro-magnetic field tensor $F_{\mu\nu}$, as $V_i^B = \epsilon_{ijk}\,V_{jk}^T$ and $V_i^E = 2\,V_{0i}^T$. Note that the second equality in (2.7) makes apparent that the tensor interaction can contribute only in the presence of a polarized background.

The equation of motion for the neutrino propagation can be deduced from the Lagrangian

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int} = \bar{\nu}(i\not{\partial} - m - \Sigma)\nu \tag{2.9}$$

where the matrix of the potentials $\Sigma \equiv \Sigma^{SP} + \Sigma^{VA} + \Sigma^T$ depends on the background density and polarization, and in general will vary along the neutrino propagation path. In the general case both Σ and m are matrices in the space of neutrino types. In the chiral basis the interaction part in (2.9) reads

$$-\mathcal{L}_{int} = \bar{\nu}\,\Sigma\,\nu = \begin{pmatrix}\nu_L^\dagger \\ \nu_R^\dagger\end{pmatrix}^T \begin{pmatrix} V_\mu^{LL}\bar{\sigma}^\mu & V_\mu^{LR}\sigma^\mu \\ V_\mu^{RL}\sigma^\mu & V_\mu^{RR}\sigma^\mu\end{pmatrix}\begin{pmatrix}\nu_L \\ \nu_R\end{pmatrix}, \tag{2.10}$$

where $\sigma^\mu\,(\bar{\sigma}^\mu) = (\sigma^0\,, (-)\,\sigma^i)$ with $\sigma^0 = I$, and

$$V_\mu^{LL} \equiv V_\mu^V - V_\mu^A\,, \qquad V_0^{RL} \equiv V^S - V^P\,, \qquad V_i^{RL} \equiv V_i^B - i\,V_i^E\,, \tag{2.11}$$

$$V_\mu^{RR} \equiv V_\mu^V + V_\mu^A\,, \qquad V_0^{LR} \equiv V^S + V^P\,, \qquad V_i^{LR} \equiv V_i^B + i\,V_i^E\,. \tag{2.12}$$

It is apparent that the (axial)vector potentials appearing in the diagonal entries in (2.10) couple neutrinos of the same chirality, while the (off-diagonal) (pseudo)scalar and tensor potentials couple neutrinos of opposite chirality. The equations of motion for neutrinos and antineutrinos derived from (2.9) read

$$\gamma_0(\not{k} - m - \Sigma)u = 0\,, \qquad\qquad \gamma_0(\not{k} + m + \Sigma)v = 0\,. \tag{2.13}$$

Note that the signs of m and Σ are opposite for the antineutrinos. The dispersion relations for the neutrino propagation are given by the solutions of

$$\det\left[\mathcal{O}\right] = \det\left[\gamma_0(\not{k} - m - \Sigma)\right] = 0. \tag{2.14}$$

Let us take the neutrino momentum $\boldsymbol{k} = k\hat{\boldsymbol{z}}$ along the z-axis. Assuming that $V^{V,A,T}, m \ll E$ and neglecting terms of $\mathcal{O}(m/E)$ the relevant terms in (2.10) are $V^{LL}_{0,3}$, $V^{RR}_{0,3}$ and the tensor components $V^{LR}_{1,2}$, $V^{RL}_{1,2}$ transverse with respect to the neutrino propagation direction. Solving the determinant equation (2.14) yields the neutrino energies

$$E_{\pm} = k + \frac{m^2}{2k} + \tfrac{1}{2}\left[V^{LL}_{0-3} + V^{RR}_{0-3} \pm \sqrt{(V^{LL}_{0-3} - V^{RR}_{0-3})^2 + 4\,V^{LR}_{+}\,V^{RL}_{-}}\,\right], \tag{2.15}$$

where $V_{0\pm3} \equiv V_0 \pm V_3$ and $V_{\pm} \equiv V_1 \pm iV_2$. In (2.15) the plus (minus) sign refers to neutrinos that are mainly left(right)-handed states. Eliminating the two helicity suppressed states from the equations of motion we obtain a Schrödinger-like equation that governs the neutrino propagation:

$$i\frac{d}{dt}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix} = \mathcal{H}_\nu \begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix} \qquad \text{with} \qquad \mathcal{H}_\nu = k + \frac{m^2}{2k} + \begin{pmatrix} V^{LL}_{0-3} & V^{LR}_{+} \\ V^{RL}_{-} & V^{RR}_{0-3}\end{pmatrix}. \tag{2.16}$$

The two eigenvalues of the effective Hamiltonian $\mathcal{H}_\nu$ are the solutions (2.15) of the determinant equation (2.14). The results for the antineutrinos can be obtained from (2.16) and (2.15) by changing the sign of the potentials ($V \to -V$). In the case of more than one neutrino flavor (2.15) is a matrix equation in the space of the neutrino types. In the one flavor case, the energy gap between the two states is

$$\Delta E_\nu = \sqrt{(V^{LL}_{0-3} - V^{RR}_{0-3})^2 + 4\,V^{LR}_{+}\,V^{RL}_{-}}\,. \tag{2.17}$$

In the limit of vanishing tensor interaction ($V_T = 0$) ν_L decouples from ν_R. Setting also $V^{RR} = 0$ and V^{LL} equal to the SM charged current and neutral current interactions, we recover the SM case, with decoupled non-interacting right-handed states.

III. IMPLICATIONS AND DISCUSSION

Setting $g_V = -g'_A = g_A = -g'_V = 1$ in (2.6) and $\Sigma^{SP} = \Sigma^T = 0$ yields the SM result for the potential felt by an electron neutrino propagating in an electron background. Introducing a unit vector in the direction of the neutrino momentum $\hat{\boldsymbol{k}} = \boldsymbol{k}/|\boldsymbol{k}|$ and using the explicit expression for the spin vector s [1] we reproduce the result [6]

$$V^{SM}_{\nu,\bar\nu} = \pm\sqrt{2}\,G_F\,n_e\left[1-\left\langle \frac{(\boldsymbol{p}+m_e\boldsymbol{\lambda})\cdot\hat{\boldsymbol{k}}-\boldsymbol{p}\cdot\boldsymbol{\lambda}}{E_e}+\frac{(\hat{\boldsymbol{k}}\cdot\boldsymbol{p})\,(\boldsymbol{p}\cdot\boldsymbol{\lambda})}{E_e(m_e+E_e)}\right\rangle\right] \tag{3.1}$$

where the plus-sign refers to neutrinos and the minus-sign to antineutrinos.

However, our main result is that in the presence of a neutrino tensor interaction with the background fermions, the neutrino can undergo spin-flip. This effect is similar to the spin-precession induced by a transverse magnetic field $B_\perp$ that couples to the neutrino magnetic dipole moment μ_ν. In fact, if we substitute in (2.16) the off-diagonal term $V^{LR}_\pm$ by $\mu_\nu B_\perp$ we obtain the equation of motion for a neutrino that propagates in a magnetic field. [9] Of course the two scenarios originate from different physics, however, formally they can be treated in the same way.

In the simplest one generation case, a left-handed neutrino produced at $t = 0$ and propagating for a time t in a constant medium will be converted into a right-handed neutrino with a probability $P^{LR}_\nu(t) = \sin^2 2\theta\,\sin^2(\Delta E_\nu\,t/2)$ where the effective mixing angle is given by

$$\sin^2 2\theta = \frac{|2V^{LR}_+|^2}{(\Delta E_\nu)^2}\,, \tag{3.2}$$

and ΔE_ν is given in (2.17). In the case of more than one neutrino flavor, propagation in a medium with changing density can lead to resonance effects in complete analogy to the magnetic field induced resonant spin-flip. [9]

Now, let us discuss shortly the results for different types of background matter. First consider a medium where the average momentum of the background fermions vanishes: $\langle\boldsymbol{p}\rangle = 0$ (*e.g.* when the momentum distribution is isotropic). The tensor component determining the effective mixing (3.2) is given by

$$|V^{LR}_+| = \sqrt{2}\,G_F\,n_f\sqrt{|g_T|^2+|g'_T|^2}\,\left\langle\lambda_\perp\left(\sin^2\vartheta+\frac{m_f}{E_f}\cos^2\vartheta\right)\right\rangle\,, \tag{3.3}$$

where ϑ denotes the angle between the momentum and the transverse polarization of the background fermion, and $\lambda_\perp = \sqrt{\lambda_1^2+\lambda_2^2}$. Note that $|V^{LR}_+|$ vanishes if the neutrino propagates along the direction of the average background polarization ($\lambda_\perp = 0$). For a non-relativistic background ($E_f \simeq m_f \gg p_i$) this yields $|V^{LR}_+| = \sqrt{2}\,G_F\,n_f\sqrt{|g_T|^2+|g'_T|^2}\,\langle\lambda_\perp\rangle$ while in the ultra-relativistic limit we find $|V^{LR}_+| \propto \langle\lambda_\perp\,\sin^2\vartheta\rangle$. Finally, for a degenerate background in the presence of a magnetic field, only the fermions in the lowest Landau level contribute to the polarization, with the spin oriented antiparallel to the momentum. In this case the background is not isotropic, and one obtains

$$|V^{LR}_+| = \sqrt{2}\,G_F\,n_f\sqrt{|g_T|^2+|g'_T|^2}\,\left\langle\lambda_\perp\frac{m_f}{E_f}\right\rangle\,, \tag{3.4}$$

which vanishes in the ultra-relativistic limit.

It is interesting to note that tensor interactions could result from neutrino scalar couplings after Fierz rearrangement. Consider the tree level Lagrangian

$$- \mathcal{L}_{\rm tree} = \lambda_\phi \phi \, (\overline{L_L}\, e_R) + \lambda'_\phi \tilde{\phi}\, (\overline{L_L}\, \nu_R) \; + {\rm h.c.}\,, \tag{3.5}$$

involving a right-handed neutrino singlet (ν_R) and a doublet scalar field ϕ with mass m_ϕ and couplings $\lambda_\phi, \lambda'_\phi$ to the lepton fields. The resulting set of low energy effective interactions contains the following terms:

$$\mathcal{H}^{\phi}_{\rm int} = -\frac{\lambda'_\phi \lambda_\phi}{m_\phi^2} \left[\tfrac{1}{2} (\overline{\nu_R}\, \nu_L)\, (\overline{e_R}\, e_L) + \tfrac{1}{8} (\overline{\nu_R}\, \sigma_{\mu\nu}\, \nu_L)\, (\overline{e_R}\, \sigma^{\mu\nu}\, e_L) \right] \,, \tag{3.6}$$

implying $g_T \sim \lambda'_\phi \lambda_\phi / m_\phi^2$. When different scalar fields mix, operators of this kind can be generated also in supersymmetric models without R-parity.

Let us now address the issue whether the new tensor term could be relevant for real physical systems, like the sun or a galactic supernova. From eqs. (3.3)–(3.4) it follows that, with respect to the SM vector potential, the effective tensor potential is suppressed by a factor

$$\epsilon \equiv \left| \frac{V_+^{LR}}{V_0^{LL}} \right| \lesssim \sqrt{|g_T|^2 + |g'_T|^2}\, \langle \lambda_\perp \rangle \,. \tag{3.7}$$

New physics effects can be relevant to neutrino oscillations only if g_T, g'_T and $\langle \lambda_\perp \rangle$ are large enough to affect sizably the results obtained within the SM. In particular ϵ should satisfy the lower limits: [1,8] $\epsilon_{sun} \gtrsim 10^{-2}$ and $\epsilon_{SN} \gtrsim 10^{-4}$. The excellent agreement between the SM predictions and various experimental results, suggests that g_T and g'_T are small, probably not exceeding the few percent level. However, the tiny values of the average polarization is by far the most important suppression factor. In the solar interior, the magnetic field can be at most of the order of several kG. This results in a very small electron polarization [1] $\langle \lambda_e \rangle \simeq 10^{-8}$ and quite likely neutrino propagation in the sun cannot be affected by the new tensor interaction. For a proto-neutron star in the early cooling phase, soon after the supernova explosion, the magnetic field strength can reach very large values. However, the temperature is also large, thus suppressing the induced polarization. For a magnetic field $B \sim 10^{13}$ G, it was estimated [1] $\langle \lambda_e \rangle \simeq 10^{-4}$ and for the nucleon polarization $\langle \lambda_{p,n} \rangle \simeq 10^{-5}$. Thus, if $B \lesssim 10^{13}$ G the propagation of supernova neutrinos would not be affected. However, it was pointed out [1] that collision effects could increase the production of right-handed states and thus enhance the effects of the tensor interaction. Also, the value of the proto-neutron star magnetic field is poorly known. It has been proposed that at early times it could be as large as 10^{16} G. [10] This would imply an enhancement of the polarization of about three orders of magnitude, opening the possibility of observing these effects.

Finally, let us note that since the presence of right-handed neutrinos implies in general a non-vanishing magnetic moment, the effect of the tensor interaction will be accompanied by similar effects due to the neutrino magnetic moment coupled to the strong magnetic field. Clearly, in this case both effects have to be taken into account simultaneously.

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