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# Strategy Change and Wealth Accumulation: An Analysis of S&P 500 Data

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# Strategy Change and Wealth Accumulation: An Analysis of S&P 500 Data

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## Abstract

This paper studies investors' strategy change frequency and their wealth accumulation by financial investments. Artificial investors are put into a real stock market. They trade S&P 500 following common strategies in practice. Fundamental analysis generally surpasses technical analysis in all market situations except boom periods. Though investors' strategy change behavior, which is driven by the past performance of strategies, seems reasonable, a faster strategy change does not guarantee a higher final wealth. Active strategy change hurts investor' wealth in bear markets and in markets with major trend reversals. In bull markets, both fast and slow strategy change behaviors work better than a moderate speed of strategy change. A detailed decomposition of wealth accumulation via financial investment shows the dependence of wealth on investors' past transactions. This may explain the relation between investors' strategy change frequency and their wealth.

**Keywords:** Financial Investment Strategy, Strategy Change Frequency, Wealth Accumulation, Standard & Poor's 500

**JEL Classification:** C63, G19

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# 1 Introduction

This is a world full of changes and choices. Life today moves faster than ever before. To catch chances, people often need to make quick decisions. When lack of information, one tends to imitate the behavior of people who are believed having more information. Since most people only have access to limited information, this method is frequently adopted by the majority. Therefore, we see all kinds of trends and fashions in our daily lives. In financial markets, the imitation among investors causes Shiller's irrational exuberance (Shiller, 2000) which may further lead to disastrous financial crises.

In stock markets, the imitation of strategies among investors is usually called herding (Lux, 1995) or adaptive learning (Brock and Hommes, 1997, 1998). Investors observe others' payoff and adopt others' strategy if such strategy produces a payoff higher than their old strategy does. The motivation of herding behavior is to pursue a higher payoff in the short term and to accumulate more wealth in the long term. Since no strategy definitely dominate other strategies, otherwise all investors only need to use this dominant strategy and doing this would eventually invalid this strategy, investors naturally have the incentive to change their strategies. But finding the right moment of changing strategies is no less difficult than forecasting the beginning of a financial crisis. Even with the same information of historical performance of strategies, different investors may have different frequency of adjusting their strategies. Does a faster adjustment of strategies lead to a higher final wealth? We devote this chapter to these questions.

In this chapter, artificial investors are put into a real stock market to see how their strategy change behavior affects their wealth accumulation. Some popular trading strategies in practice, e.g. fundamental analysis, trend following and moving average crossovers, are formulized for investors to choose. S&P 500 is taken as the risky asset as it is more representative than individual stocks. Investors trade the index while adopting one of many strategies. When the performance of other strategies is better than an investor's current strategy, this investor has an incentive to change his strategy. However, an investor does not abandon his present strategy whenever it has been surpassed. He changes his strategy only when the performance of another strategy is higher than the performance of his old strategy by an amount over a threshold. Different investors have different thresholds. A higher threshold means that the investor changes his strategy less often. We calculate investors' final wealth under various market situations and find that in general investors with higher threshold end up with higher final wealth.

## 2 Strategies and strategy change

There are four strategies for agents to pick, fundamental or technical, all representing typical strategies widely used in practice. Agents only adopt one strategy at a time.

## 2.1 Fundamental analysis

In practice, the fundamental value of an individual stock is usually derived from its price-to-earnings ratio or price-to-dividends ratio. Since we take the index S&P 500 rather than a single stock as the risky asset, the search for its true fundamental is much more complicated. To simplify our study, we take the annual moving average of prices as the fundamental value  $F_t$ , as shown in Eq. 1. Here we take 240 working days as a year so that 5 days as a week, 20 days as a month and 60 days as a quarter. In our study, we calculate the fundamental value according to Eq. 1 and feed the result to investors taking fundamental analysis. As for those investors, they receive the exogenous information of fundamental value without knowing how it is derived. Eq. 1 is beyond their knowledge, and it is impossible for them to infer accurate future prices from the time series of fundamental. The fact that future prices are used to calculate the current fundamental makes the fundamental analysis a forward-looking strategy, though without investors' awareness.

Investors adopting fundamental analysis believe that the price will converge to the fundamental eventually. So they buy shares when the stock is undervalued compared with its fundamental and sell when it is overvalued. Investors' market orders (Day and Huang, 1990) at time  $t$   $D_t^f$  are formulized as Eq. 2.

$$F_t = \frac{P_{t-119} + P_{t-118} + \cdots + P_{t+120}}{240} \quad (1)$$

$$D_t^f = F_t - P_t \quad (2)$$

## 2.2 Technical analysis

Investors with technical analysis usually derive their anticipation and trading from past information. They read charts, count waves, look for patterns, etc. They believe that what has happened before will reoccur. Here we focus on one-period trend following and moving averages.

### 2.2.1 One-period trend following

Investors adopting one-period trend following strategy believe that the latest price change will continue. As a result, they place buy market orders  $D_t^{tf}$  when the latest price change is positive and place sell market orders under negative price change.

$$D_t^{tf} = P_t - P_{t-1} \quad (3)$$

### 2.2.2 Moving average

Moving average (Chiarella, He and Hommes, 2006) is widely used in technical analysis to filter out random price fluctuations. It usually takes two moving averages to form a

concrete strategy: one with a shorter time frame and the other with a relatively longer time frame. The shorter moving average crossing above the longer moving average, known as a “golden cross”, indicates the shifting up of a trend. It sends a buy signal. The shorter moving average crossing below the longer moving average, known as a “dead/death cross”, sends a sell signal. When the shorter moving average is above or below the longer moving average, it still sends a buy or sell signal, though may not be a chance as good as these crossovers. Essentially, the moving average strategy is following short-term trend against relative long-term trend. Here we take two popular combinations: 10/20 moving averages and 15/30 moving averages. Investors who adopt these two strategies place market orders  $D_t^{10/20}$  and  $D_t^{15/30}$  respectively in period  $t$ .

$$D_t^{10/20} = \frac{P_{t-9} + P_{t-8} + \cdots + P_t}{10} - \frac{P_{t-19} + P_{t-18} + \cdots + P_t}{20} \quad (4)$$

$$D_t^{15/30} = \frac{P_{t-14} + P_{t-13} + \cdots + P_t}{15} - \frac{P_{t-29} + P_{t-28} + \cdots + P_t}{30} \quad (5)$$

### 2.3 Fitness of strategies

An agent can only pick one strategy at a time. When other strategies work better than his current strategy, naturally this agent has an incentive to change his strategy. Agents’ strategy change behavior is driven by comparing strategies’ fitness, that is, the past performance of strategies.

Different strategies share one feature in common, that is, to buy shares when expecting higher future price and to sell when expecting lower future price. The difference among strategies is more about the different price expectations. Eq. 6 shows a linear relation between the current expectation on future price  $E_t^h(P_{t+1})$  and the market order  $D_t^h$  determined by strategy  $h$ . Substitution of Eq. 6 into Eq. 7 presents the relation between the price expectation of the strategy  $h$   $E_t^h(P_{t+1})$  and the corresponding profit  $\pi_t^h$ .

$$D_t^h = E_t^h(P_{t+1}) - P_t \quad (6)$$

$$\begin{aligned} \pi_t^h &= D_t^h \cdot (P_{t+1} - P_t) \\ &= (E_t^h(P_{t+1}) - P_t) \cdot (P_{t+1} - P_t) \end{aligned} \quad (7)$$

The profit earned by the market order determined by a strategy (Eq. 7) is widely used as the quantitative measure of a strategy’s fitness (Hommes, 2006). It is reasonable when the price fluctuates in a moderate range. But when the price changes a lot, proper adjustment is required. For example, a profit of 200 means differently when the price is 20.16 on 11/22/1950 and when the price is 2074.78 on 11/28/2014.

As the daily close price of S&P 500 increased over 100 times after more than 60 years, we should take this point into consideration. The fitness of strategy  $h$  is reformed as follows

$$\begin{aligned}\pi_t^h &= E_t^h(\rho_t) \cdot \rho_t \\ &= \frac{E_t^h(P_{t+1}) - P_t}{P_t} \cdot \frac{P_{t+1} - P_t}{P_t},\end{aligned}\tag{8}$$

where  $\rho_t$  is the return of stock in period  $t$ . The fitness is determined by the real return and the expected return of strategy  $h$ . Fig. 1 shown us the linear and logarithmic daily close prices of S&P 500. Fig. 2 shows us the one period profit (Eq. 7) and the reformed fitness (Eq. 8) of the fundamental strategy. The reformed fitness seems more reasonable than the old one.

As mentioned before, an investor does not abandon his present strategy whenever it has been surpassed by other strategies. He changes his strategy only when the fitness of another strategy is higher than the fitness of his old strategy by an amount higher than a threshold. When multiple strategies meet this criterion, this investor replaces his old strategy by the strategy with the highest fitness.

### 3 Simulations with S&P 500 historical prices

In the artificial financial market we build, agents can invest either in the risky S&P 500 or in a risk-free asset. We assume zero interest rate of risk-free asset, i.e.  $R = 1$ , as the daily interest rate of risk-free asset is negligible in practice. Then, agents' investment of risk-free asset can be interpreted as holding cash. Agents place market orders according to their current strategies. These market orders are later satisfied by a market maker under the current price. Agents' wealth is updated as follows.

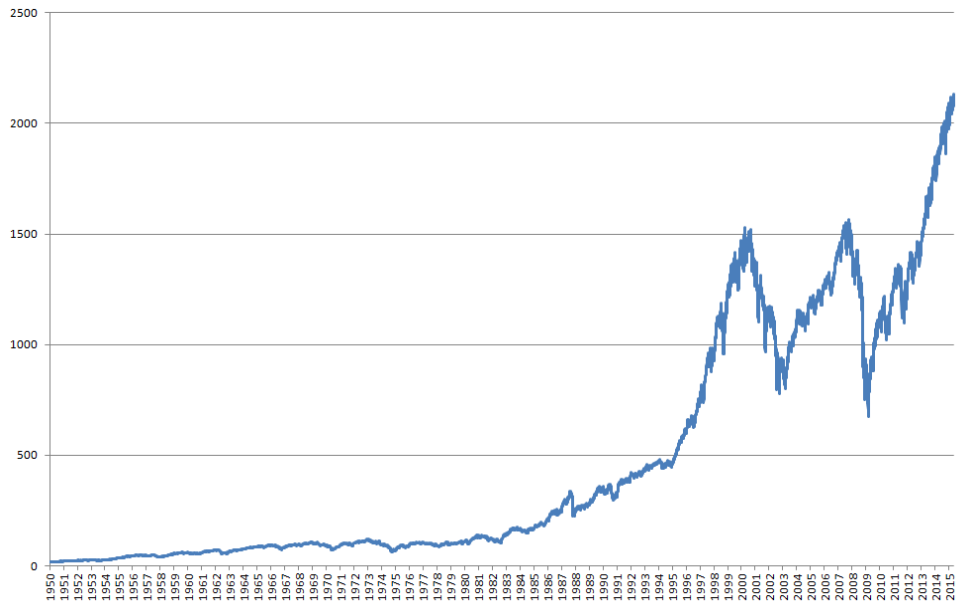
$$W_{i,t+1} = S_{i,t+1} + P_{t+1}Z_{i,t+1}\tag{9}$$

$$S_{i,t+1} = (S_{i,t} - D_{i,t}P_t)R\tag{10}$$

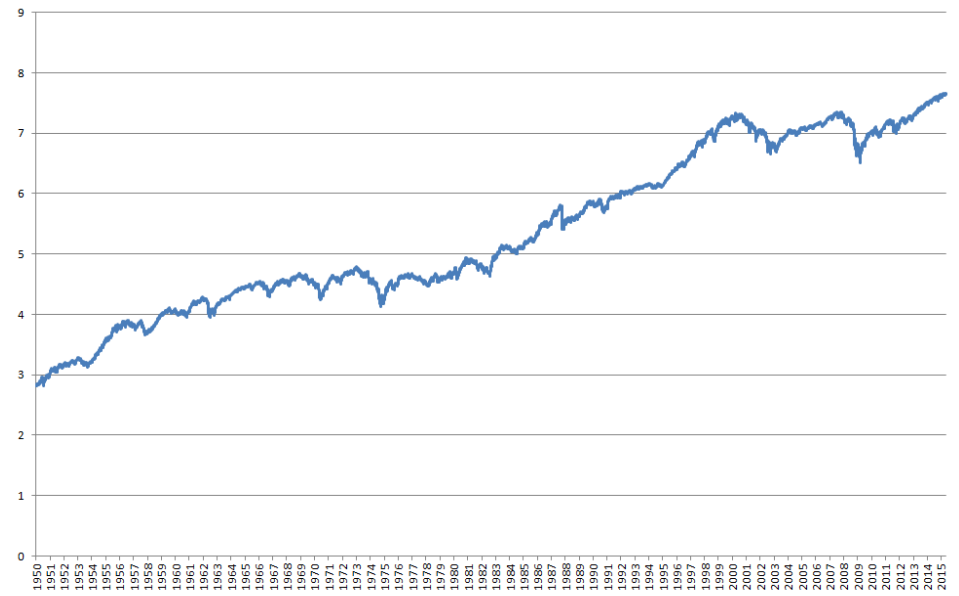
$$Z_{i,t+1} = Z_{i,t} + D_{i,t}\tag{11}$$

At the beginning of period  $t$ , agent  $i$  holds  $S_{i,t}$  risk-free asset and  $Z_{i,t}$  shares of risky asset, and his total wealth is  $W_{i,t}$ . After he places a market order  $D_{i,t}$  and the order is later executed by the market maker under the current price  $P_t$ , the amount of his risk-free asset, risky asset and total wealth are updated to  $S_{i,t+1}$ ,  $Z_{i,t+1}$  and  $W_{i,t+1}$  respectively.

In our study, agents have zero initial endowment, i.e.  $\forall i, S_{i,0} = 0$  and  $Z_{i,0} = 0$ . All agents start with the same initial condition. No budget constraint or short-sale constraint is imposed on agents. So they can borrow risk-free asset to buy index, or sell short index to gain risk-free asset.



(a) Linear price



Logarithmic price

Figure 1: Historical daily close prices of SP500 from 1/3/1950 to 5/21/2015, 16452 days in total.



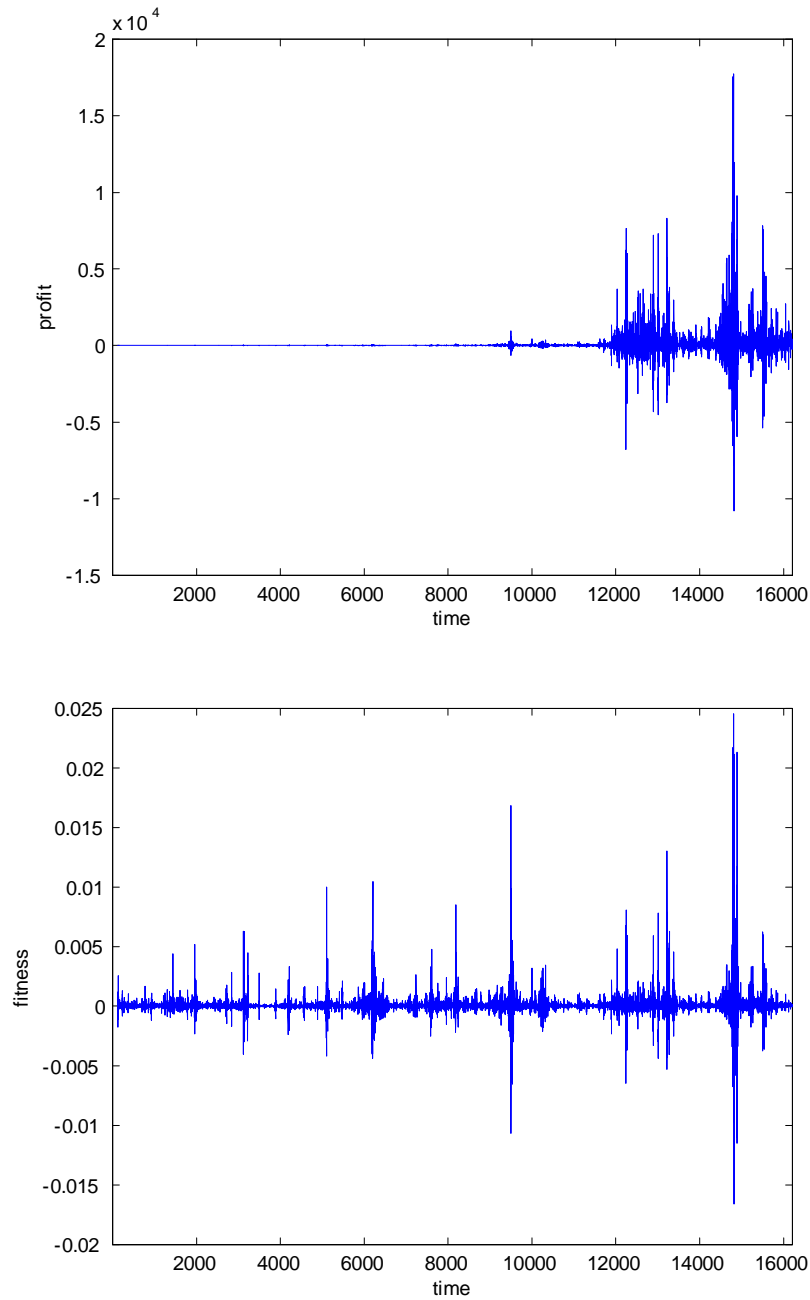


Figure 2: Comparison of one period profit and the reformed fitness of the fundamental strategy through the history of SP500. Top panel: daily profit. Bottom panel: reformed profit derived from expected and real daily returns. Because of the long run increase of price, the daily profits in the last twenty years take larger weights and overwhelm those happened before 1990s. The reformed fitness eases this situation. But still, the fitness of fundamental strategy becomes more fluctuant as time passes.

### 3.1 The whole history: 1950~2014

The daily close prices of S&P 500 from 1/3/1950 to 5/21/2015 are shown in Fig. 1.

We take the whole history except data lost during the calculation of moving averages. Fig. 3 compares the four strategies mentioned before. In general, the fundamental strategy accumulates more final wealth than others. But this does not mean fundamental analysis is always the best. During the formation of bubbles, we see these two moving average strategies accumulate wealth faster than the fundamental strategy does. While during crises, fundamental strategy is the only one that can make a large increase of wealth, and all technical strategies suffer a huge drop of wealth. The reason behind is clear. When there are bubbles, short-term trends are strong, and the index price continuously deviates from its fundamental. Trend following strategies benefit more from the strong short-term price trend than the fundamental strategy does. During crises, price trends reverse and trend following strategies fail. As the price drops, it converges to its fundamental, and fundamentalists who are holding right expectations benefit from selling short the index.

We put 100 artificial agents in our simulation. The 100 agents' strategy change thresholds are evenly distributed from 0 to a maximum threshold. The maximum strategy change threshold is properly chosen according to the fitness of strategies, as shown in the bottom panel of Fig. 2. A "too big" maximum threshold makes some agents never be able to change strategies. A "too small" maximum threshold only considers agents with fast strategy change and leaves out agents with slow strategy change. Here, the maximum threshold is 0.025. The actual minimum strategy change threshold is 0.00025, instead of 0, which makes no sense. Every agent's initial strategy is randomly picked from these four strategies. Agents have zero initial endowment. No budget constraint or short-sale constraint is imposed on agents. After ranking agents according to their strategy change thresholds, the frequency of their strategy change and their final wealth are shown in Fig. 4. Agents with higher thresholds change their strategies less often. In the bottom panel of Fig. 4, there seems a upward trend, blurred by fluctuations. These fluctuations are caused by agents' random initial strategies. This randomness affects agents with higher thresholds more, as such agents change their strategies less frequently.

We run the simulation 1000 times and average the results to cancel out the randomness of initial strategies. Fig. 5 shows the results. Again, agents with higher threshold change their strategies less frequently. Compare with Fig. 4 (b), Fig. 5 (b) shows less fluctuations. As agents' threshold increases, agents' final wealth drops at first and then increases, and at last stays at the level around the maximum.

The lowest point in Fig. 5 (b) represents the agent with threshold 0.00125 who on average has changed strategies 372.7 times throughout the whole history of S&P 500. For agents with thresholds lower than 0.00125, a higher threshold relates to a lower final wealth. For agents with thresholds higher than 0.00125, a higher threshold leads to a higher final wealth. It seems that these two parts give us contradictive results. This may be caused by the long history of data, which is over 60 years. Besides, if we

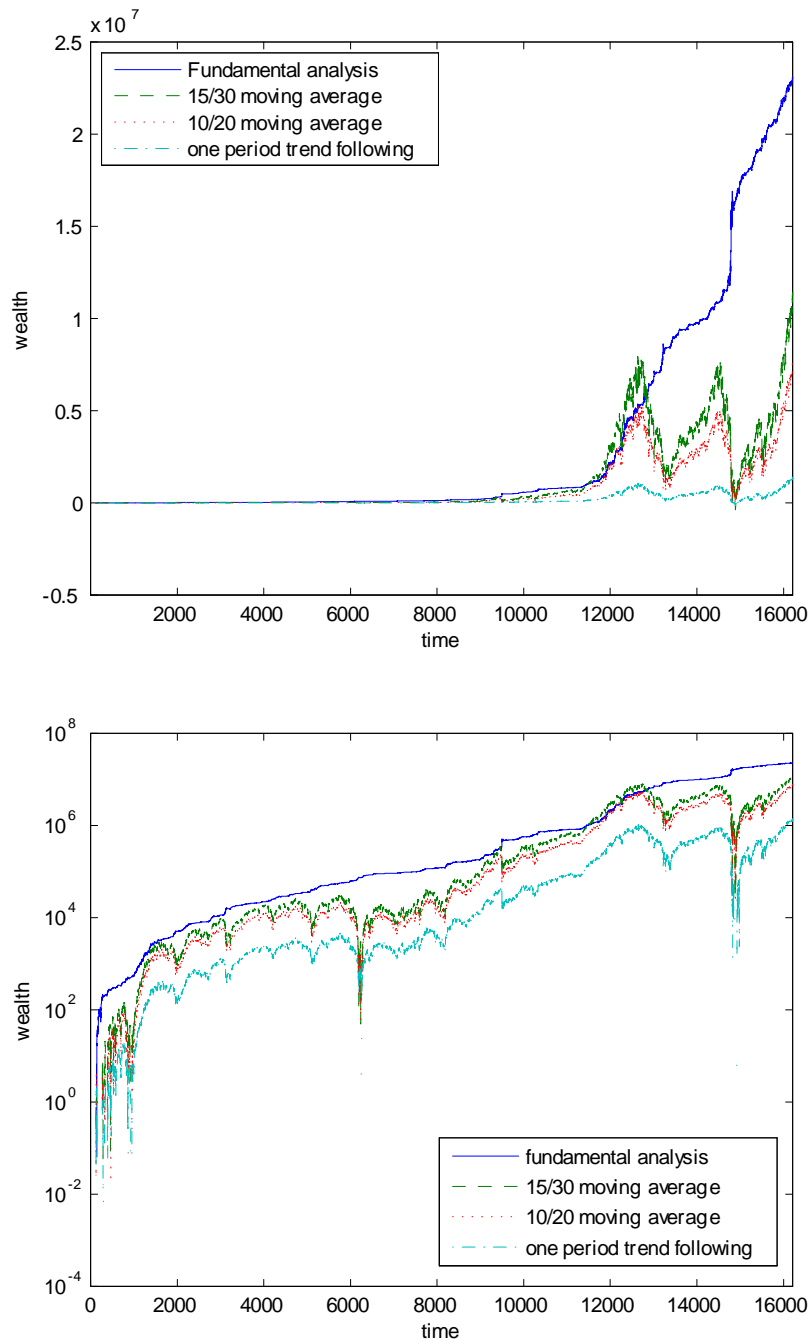


Figure 3: Wealth accumulations by four strategies: fundamental analysis, 15/30 moving average, 10/20 moving average and one period trend following. Top panel: linear scale of wealth. Bottom panel: logarithmic scale of wealth. Later we choose the linear scale to present future results on wealth as it presents the cumulation process clearer.

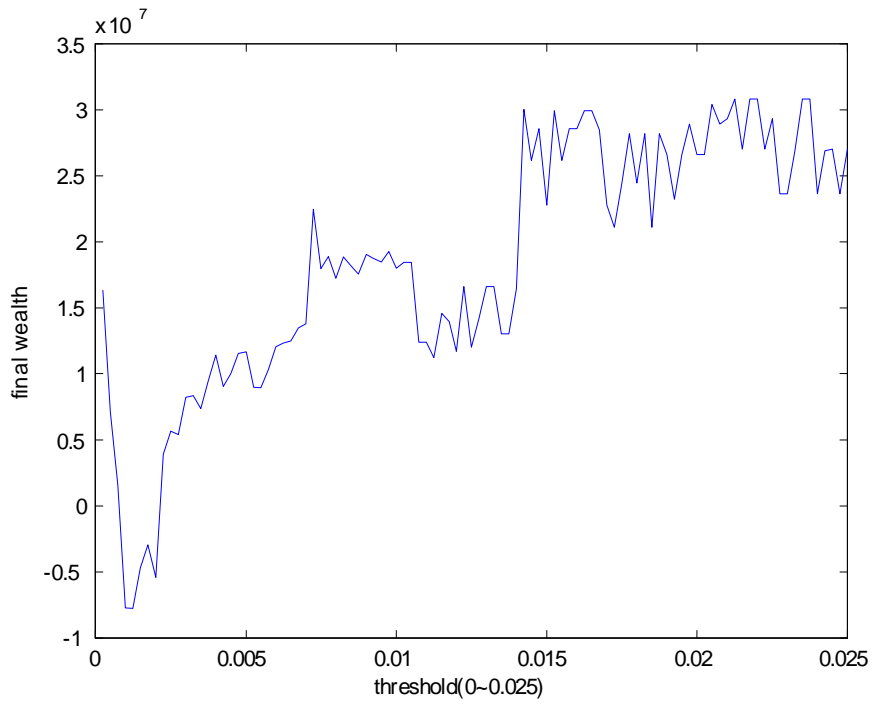
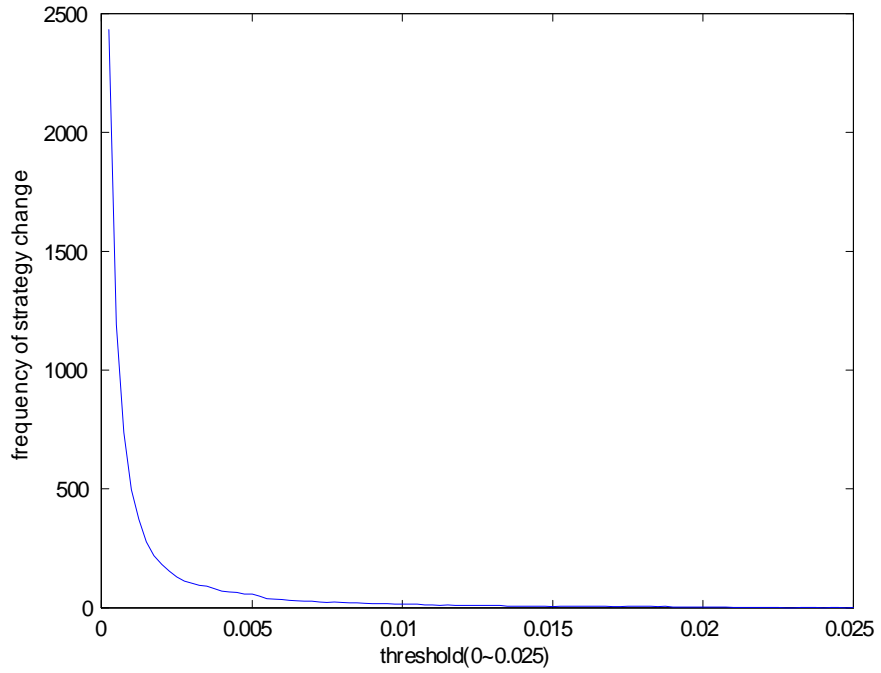


Figure 4: Agents' frequency of strategy change (a) and their final wealth (b). Note that agents are ranked by their strategy change thresholds.

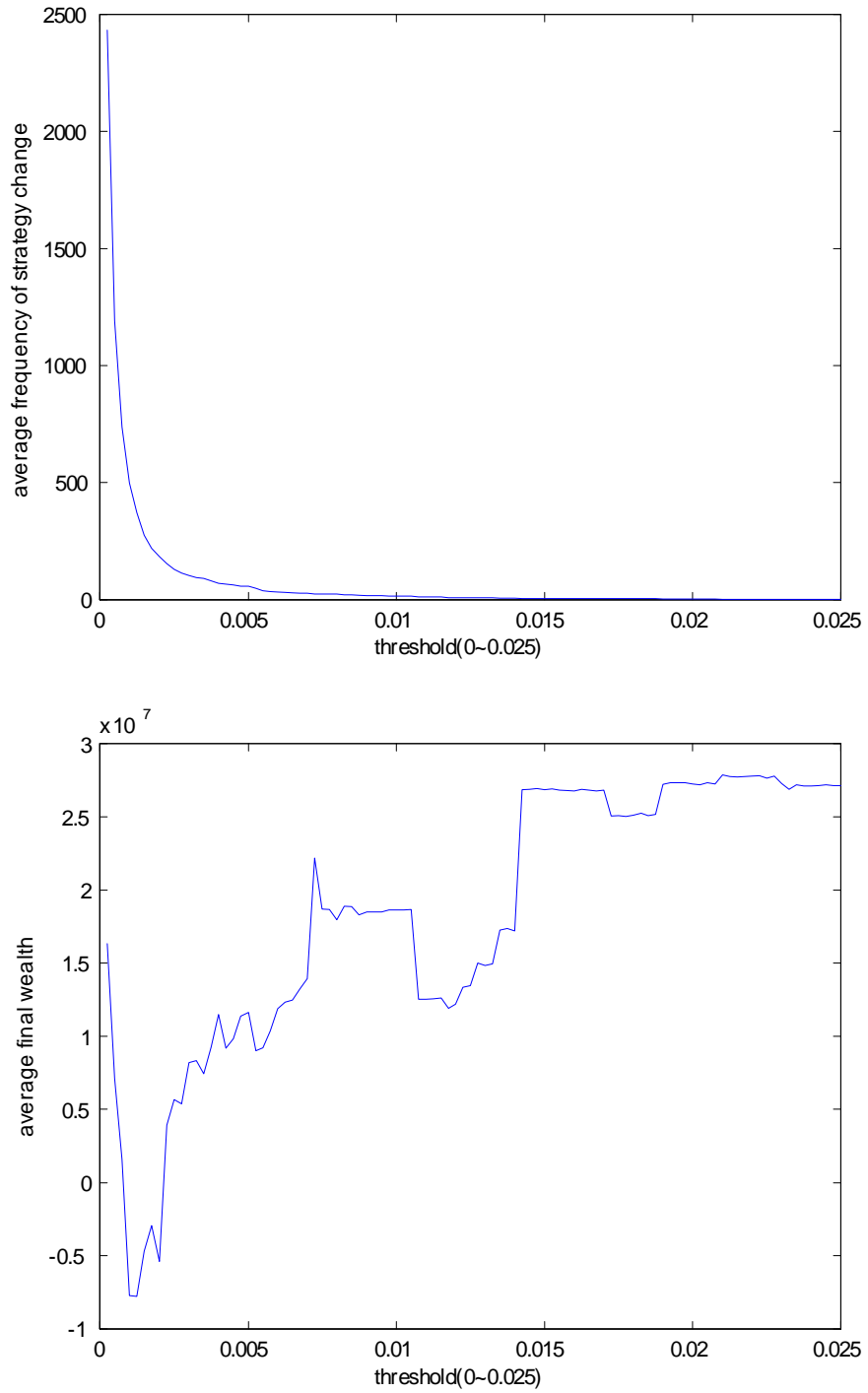


Figure 5: Agents' average frequency of strategy change and their average final wealth of 1000 simulations. Note that agents are ranked by their strategy change thresholds.

take the cost of strategy change into consideration, changing strategy too frequently, i.e. more than 372.7 times here, may cost a lot.

### 3.2 Bulls, bears and others

We studied the history of S&P 500 as a whole in the last subsection. Now, we pick typical time periods of S&P 500 to investigate how agents' strategy change behavior affects their wealth accumulation under specific market trends. The maximum strategy change threshold in each segment is adjusted according to strategies' fitness throughout each portion.

Fig. 6 shows the analysis of two bull markets: one from 12/8/1994 to 3/24/2000 (left panels) and the other from 3/9/2009 to 11/24/2014 (right panels). The former covers the dot-com bubble. The latter shows the current bull market starting from the recovery of 2007/08 financial crisis. The top six panels show the daily close prices and fundamental values, wealth accumulation of four strategies, and the fitness of four strategies through these two bull markets. In the second row, four stubborn agents, each sticks to one strategy, start with zero initial endowment. The bottom four panels show the average results of 1000 simulations. 100 artificial agents are ranked by their strategy change thresholds from 0.00005 to 0.005. In the fourth row, agents with higher thresholds change their strategies less frequently. In the last row, as agents' strategy change thresholds increase, their average final wealths drop at first and then increase. This is consistent with our finding in the whole historical data of S&P 500. It seems that agents' final wealths drop at first and then increase is a feature of bull markets, as the whole history of S&P 500 can be taken as a long lasting bull market.

Fig. 7 demonstrates the analysis of two bear markets: one from 11/9/2000 to 11/11/2002 (left panels) and the other from 10/30/2007 to 3/23/2009 (right panels). The first one presents the burst of dot-com bubble. The last one shows the 2007/08 financial crisis. The top six panels show the daily close prices and fundamental values, wealth accumulation of four strategies, and the fitness of four strategies during these two bear markets. The bottom four panels show the average results of 1000 simulations. 100 artificial agents are ranked by their strategy change thresholds from 0.00005 to 0.005. In the fourth row, again, agents with higher thresholds change their strategies less frequently. In the last row, as agents' strategy change thresholds increase, their average final wealths increase. In the bottom right panel, agents' average final wealths drop when their thresholds are higher than 0.00385. This is because when agents seldom change strategies, their initial strategies have large impact on their final wealths. Agents with better initial strategies end up with being as wealthy as the richest agent. Agents with relative poorer initial strategies have much lower final wealths. As a result of averaging agents with better and poorer initial strategies, the final wealths of agents with thresholds higher than 0.00385 drop with their thresholds.

Fig. 8 shows the analysis of markets with major trend reverses: one from 9/10/1997 to 8/13/2009 (left panels) and the other from 8/30/2000 to 1/14/2013 (right panels). The former presents two rounds of boom and bust, covering the dot-com bubble, its burst, the US housing bubble, and the 07/08 financial crisis caused by its burst. The latter shows the burst of dot-com bubble, its recovery, US housing bubble, 07/08 financial crisis, and the recovery to the peak of housing bubble. The top six panels show the daily close prices and fundamental values, wealth accumulation of four strategies, and the fitness of four strategies. The bottom four panels show the average results of 1000 simulations. 100 artificial agents are ranked by their strategy change thresholds from 0.0001 to 0.01. In the fourth row, agents with higher thresholds change their strategies less frequently. In the last row, as agents' strategy change thresholds increase, their average final wealths increase. In the bottom left panel, agents become less wealthy when their thresholds are higher than 0.0094 out of the same reason discussed in the last paragraph.

From the study of the whole history and typical segments of S&P 500, we find that the fundamental analysis performs better than technical analysis when sharp price drops happen, and fundamental and technical analyses are comparable in boom periods. It is natural that agents with higher strategy change thresholds change their strategies less frequently. Though no simple clear relation of agents' final wealth and their strategy change frequency is revealed, our study is able to provide suggestions to investors under specific price trends. In bull markets, both fast and slow strategy change behaviors work better than a moderate speed of strategy change. Investors either change their strategies frequently to beat the market, or just stick to one strategy, for example the buy-and-hold strategy. In bear markets and markets with major trend reversals, active strategy change hurts investor' wealth. When large price drops happen, fundamental analysis performs much better than technical strategies do. So there is no need for active strategy change.

## 4 Path dependent wealth accumulation

Investors' wealth accumulation is path dependent on their past transactions.

Assume investor  $i$ 's portfolio at the beginning of period  $t$  is  $(S_{i,t}, Z_{i,t})$ , which means he holds  $S_{i,t}$  amount of risk-free asset and  $Z_{i,t}$  shares of risky asset. Under the present price<sup>1</sup> of risky asset  $P_t$ , this investor's wealth is  $W_{i,t} = S_{i,t} + P_t \cdot Z_{i,t}$ . Every period, this investor places a market order  $D_{i,t}$ , which is executed under the present price  $P_t$ . Then the rest of his risk-free asset grow at the rate  $R$ . At the beginning of next period, the value of his portfolio is  $W_{i,t+1}$ .

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<sup>1</sup>Note that the formation of price is not mentioned here. This means our discussion does not depend on the generation process of price. Our discussion can be apply to empirical data, theoretical analysis or numerical simulation.

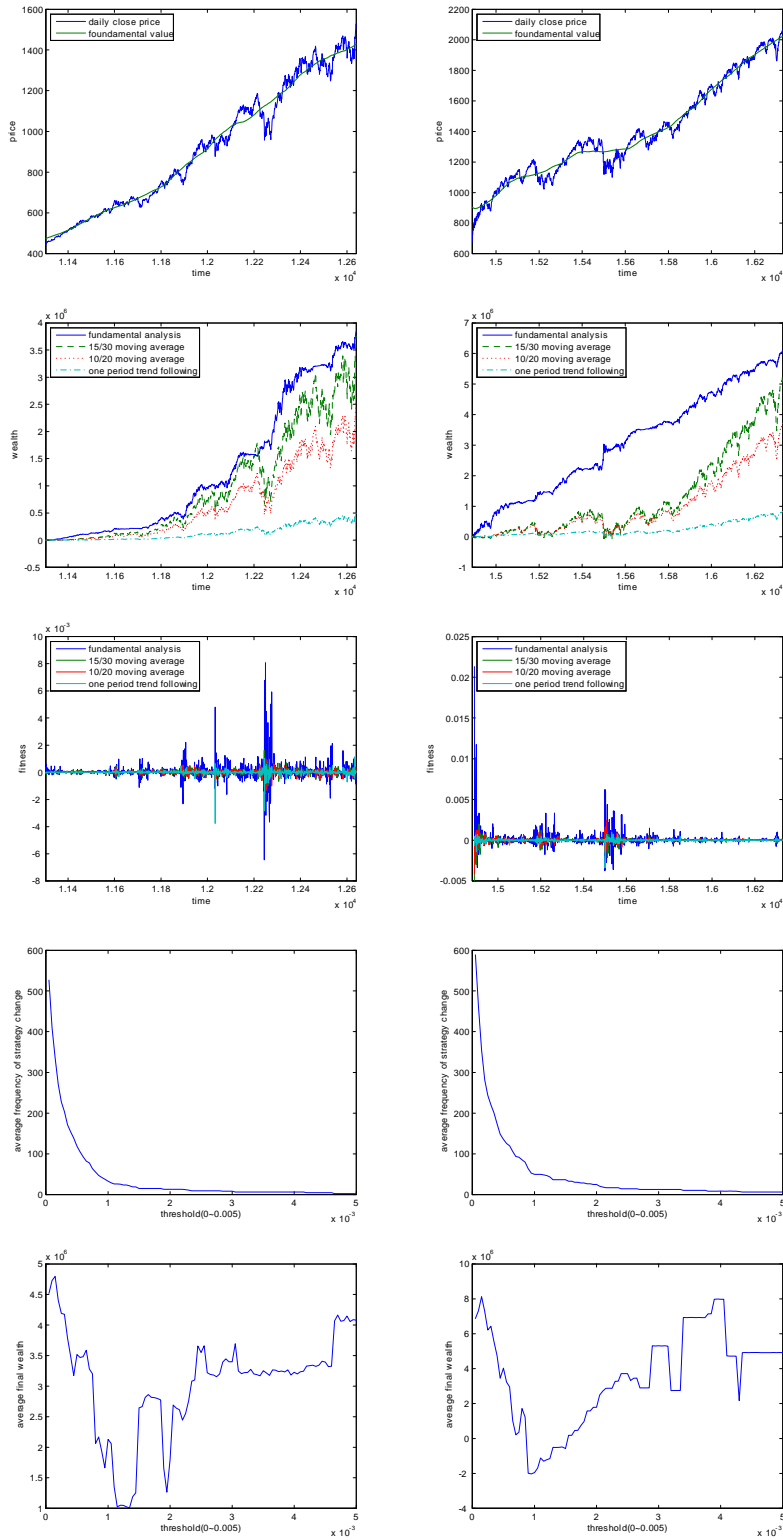


Figure 6: Analysis of bull markets. Left panels: from 12/8/1994 to 3/24/2000. Right panels: from 3/9/2009 to 11/24/2014.



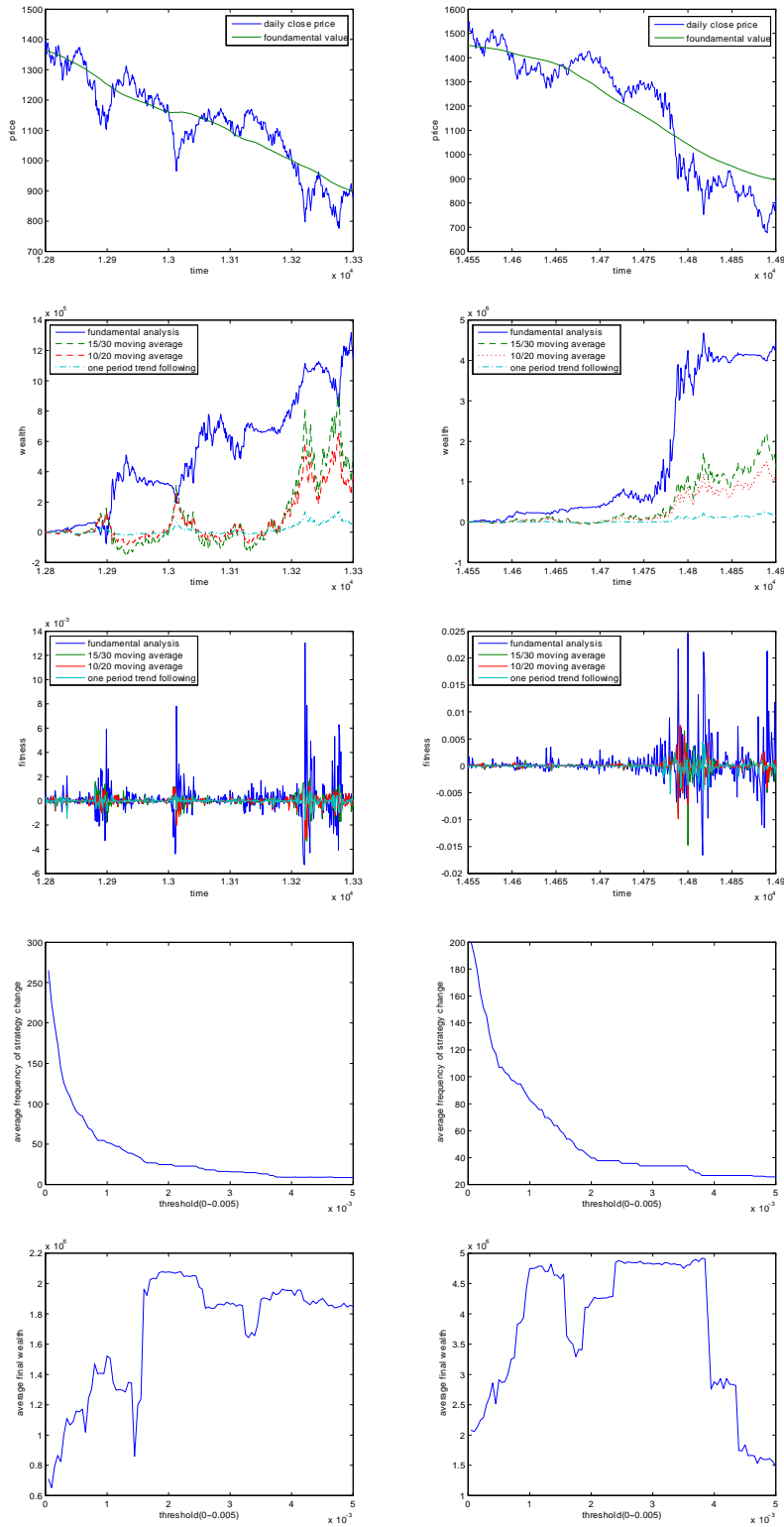


Figure 7: Analysis of bear markets. Left panels: from 11/9/2000 to 11/11/2002. Right panels: from 10/30/2007 to 3/23/2009.

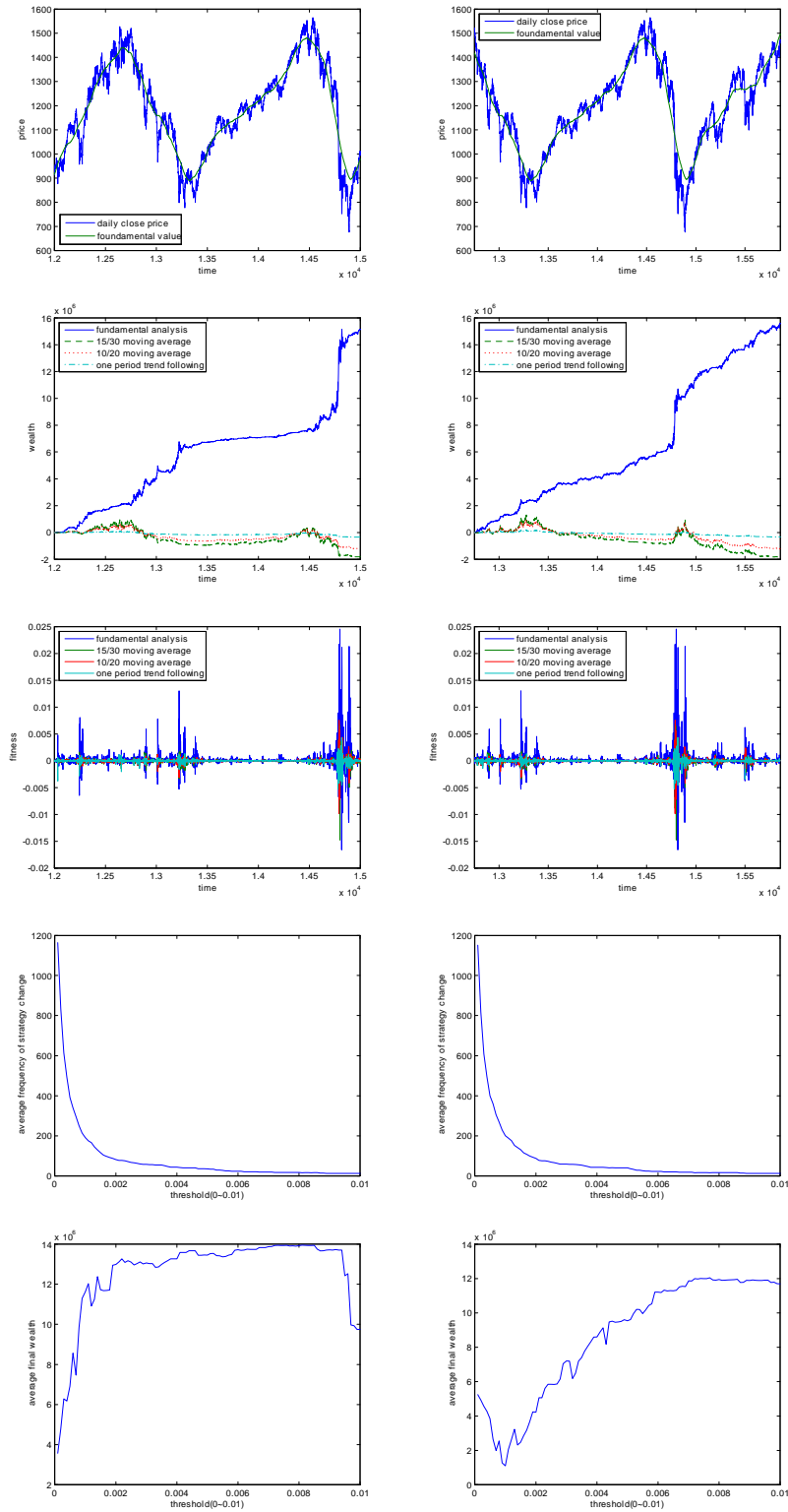


Figure 8: Analysis of markets with ups and downs. Left panels: from 9/10/1997 to 8/13/2009. Right panels: from 8/30/2000 to 1/14/2013.

$$\begin{aligned}
W_{i,t+1} &= S_{i,t+1} + P_{t+1}Z_{i,t+1} \\
S_{i,t+1} &= (S_{i,t} - P_t D_{i,t})R \\
Z_{i,t+1} &= Z_{i,t} + D_{i,t}
\end{aligned} \tag{12}$$

$$\begin{aligned}
\Delta W_{i,t+1} &= W_{i,t+1} - W_{i,t} \\
&= S_{i,t}r + (P_{t+1} - P_t)Z_{i,t} + (P_{t+1} - P_t R)D_{i,t}
\end{aligned} \tag{13}$$

After iterating Eq. 12 twice, we get agent  $i$ 's wealth at period  $t+2$   $W_{i,t+2}$  and the change of wealth ( $W_{i,t+2} - W_{i,t}$ ).

$$W_{i,t+2} = S_{i,t}R^2 + Z_{i,t}P_{t+2} + D_{i,t} \cdot (P_{t+2} - R^2P_t) + D_{i,t+1} \cdot (P_{t+2} - RP_{t+1}) \tag{14}$$

$$W_{i,t+2} - W_{i,t} = S_{i,t}(R^2 - 1) + Z_{i,t}(P_{t+2} - P_t) + D_{i,t} \cdot (P_{t+2} - R^2P_t) + D_{i,t+1} \cdot (P_{t+2} - RP_{t+1}) \tag{15}$$

After iteration of Eq. 12  $n$  times, we can get the value of investor  $i$ 's portfolio after  $n$  period as  $W_{i,t+n}$ .

$$\begin{aligned}
W_{i,t+n} &= S_{i,t}R^n + Z_{i,t}P_{t+n} + D_{i,t} \cdot (P_{t+n} - R^n P_t) \\
&\quad + D_{i,t+1} \cdot (P_{t+n} - R^{n-1}P_{t+1}) + \dots + D_{i,t+n-1} \cdot (P_{t+n} - RP_{t+n-1})
\end{aligned} \tag{16}$$

The first two parts on the right hand side of Eq. 16 are determined by the investor's portfolio at time  $t$  ( $S_{i,t}, Z_{i,t}$ ), the risk-free return  $R$  and the latest price of risky asset  $P_{t+n}$ . All rest parts are determined by investor's historical transactions ( $D_{i,t}, D_{i,t+1}, \dots, D_{i,t+n-1}$ ). They show the impact of investor's past transactions on his future wealth. However, such impact is out of the investor's control, because the weight of every past transaction contains future information ( $P_{t+n}$ ) which is unavailable to him when he made his decision ( $D_{i,t+\tau}, 0 \leq \tau \leq n-1$ ).

Reform Eq. 16 as follows.

$$W_{i,t+n} = F(S_{i,t}, Z_{i,t}, R, P_t, P_{t+1}, \dots, P_{t+n}, D_{i,t}, D_{i,t+1}, \dots, D_{i,t+n-1})$$

The value of investor  $i$ 's portfolio at time  $t+n$  is a function of his initial portfolio ( $S_{i,t}, Z_{i,t}$ ), the risk-free return  $R$ , the price series of risk asset ( $P_t, P_{t+1}, \dots, P_{t+n}$ ) and his past transactions ( $D_{i,t}, D_{i,t+1}, \dots, D_{i,t+n-1}$ ). A close look at the right hand side of Eq. 16 gives us more details. If investor  $i$  does not execute any transactions, all the increase of his wealth comes from the interest of his risk-free asset. If investor  $i$

places a market order at any time  $t+x$ , the transaction  $D_{i,t+x}$  changes the value of his portfolio at time  $t+y$  by  $D_{i,t+x} \cdot (P_{t+y} - R^{y-x}P_{t+x})$ . The impact of a past transaction on the investor's current wealth is weighted by  $(P_{t+y} - R^{y-x}P_{t+x})$ , i.e. current benefit minus opportunity cost. Since the opportunity cost of every transaction is different, the weight of every transaction in the calculation of current wealth is different. That is why we argue that investors' wealth is path dependent on his past transactions. For example, two investors with identical initial portfolio  $(S_{i,t}, Z_{i,t}) = (S_{j,t}, Z_{j,t})$  must act exactly the same in every period to get identical value of portfolio at any time. Another example: two investors with identical initial portfolio  $(S_{i,t}, Z_{i,t}) = (S_{j,t}, Z_{j,t})$  may get identical value of portfolio at a time  $W_{i,t+n} = W_{j,t+n}$ ; but if they do not share the same transaction history, the values of their portfolio will diverge later.

Previously, the investor adjust his portfolio first and then get the interest from his holdings of risk-free asset. If we change the order, i.e. get risk-free interest first and then buy or sell risky asset, our argument on path dependence still stands.

$$\begin{aligned} W_{i,t+1} &= S_{i,t+1} + P_{t+1}Z_{i,t+1} \\ S_{i,t+1} &= S_{i,t}R - P_tD_{i,t} \\ Z_{i,t+1} &= Z_{i,t} + D_{i,t} \end{aligned} \tag{17}$$

$$\begin{aligned} W_{i,t+n} &= S_{i,t}R^n + Z_{i,t}P_{t+n} + D_{i,t} \cdot (P_{t+n} - R^{n-1}P_t) \\ &\quad + D_{i,t+1} \cdot (P_{t+n} - R^{n-2}P_{t+1}) + \dots + D_{i,t+n-1} \cdot (P_{t+n} - R^0P_{t+n-1}) \end{aligned} \tag{18}$$

## 5 Conclusion

Whether a fast strategy change helps to accumulate more wealth? Out of the curiosity, we put artificial investors into a real stock market to see how their strategy change behavior affects their wealth accumulation. Fundamental analysis, 15/30 moving average, 10/20 moving average and one period trend following form a strategy pool from which agents pick their strategies. Agents either trade the risky S&P 500 index or hold risk-free cash. Agents change their current strategies only when other strategies surpass their old ones by an amount over their strategy change thresholds. Agents with higher thresholds change strategy less often. As for the relation of their final wealth and their threshold, no simple clear relation is revealed. In general, there is a threshold range within which agents with higher threshold gain higher final wealth, that is, agents change strategy less frequently accumulate more wealth. Such a range starts from the minimum threshold in bear markets and markets with multiple trends. In bull markets, a piece of negative relation of threshold and wealth happens before the range where agents' threshold and final wealth show positive relation. We could not find a clear explanation for this, and further study is required. Under all markets,

when agents' thresholds are so high that they barely change their strategies, agents tend to stick to their initial strategies. After averaging the final wealths of agents with better or worse initial strategies, the average final wealth drops when agents' threshold further increases. Besides, we decompose the process of wealth accumulation and find that wealth accumulation is path dependent on investors' past transactions.

In this chapter, artificial investors are assumed trivial that they have no influence on the stock price. Their trading does not affect the formation of future price. Some may wonder when investors are nontrivial, that is, when investors' aggregate demand drives the future price, whether the negative relation between their strategy change frequency and their final wealth still exists. In the next chapter, we build a heterogeneous agent model to show that the answer is positive.

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