A New Adaptive Variational Model for Liver Segmentation with Region Appearance Propagation

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Abstract—Liver segmentation is a crucial step for aiding in liver surgery. Due to intensity overlapping, blurred edges, large variability in shape and appearance, and complex context with clutter features, it is still a challenging task. In this paper, we address this problem with an integrated variational model based on the idea of adaptive region growing and region appearance propagation, with which we can focus on the target liver region regardless of the complex but uninterested backgrounds. Our model consists of an edge based term and two novel region based terms, with which both region intensity and appearance information are integrated and weak liver boundaries can be stably delineated. An adaptive weight is introduced to spatially balance them and control their respective advantages and disadvantages. Moreover, the proposed model is robust to parameters, initialization and noises, and can greatly alleviate the requirement of the scanning protocol and data quality. Last but not least, our model is a nearly automatic one which needs only an arbitrary initialization inside the liver. While segmenting slice by slice neglects the consecutiveness between slices, we directly segment the liver from the 3D volume data. Experimental results show that the liver can be accurately and effectively distinguished, and vessels can also be simultaneously isolated with accuracy. Our system is promising for stable practical use and can be also used to segment other abdominal organs.

Index Terms—Liver segmentation, Bi-direction force, Region appearance propagation, Region based active contour, Vessels segmentation.

I. INTRODUCTION

Liver disease is one of the most common internal malignancies and also one of the leading death causes, and liver intervention has become one of the most demanding fields in surgery. The treatment of malignant liver diseases targets at the complete destruction or removal of all tumors together with a sufficient safety free margin, at the same time life-critical anatomical structures must be saved. Liver transplantation, the replacement of a diseased liver with a healthy liver allograft, has emerged in recent decades as a critical surgical option for patients with end stage liver disease and acute liver failure. It is also one of the most expensive treatments in modern medicine. Numerous anastomoses and sutures, and many disconnections and reconnections of abdominal and hepatic tissue, must be made for the transplant to succeed. Computerized medical imaging analysis aims at detecting and delineating anatomical structures for surgery planning and diagnosis, which would substantially increase the safety and success rates of surgery. Recently it has gained more attention and has become more useful for doctors to make preoperative decisions for liver cancer diagnose and liver transplantation.

Liver and vessels segmentation from Computed Tomography Angiography (CTA) images are vital for quantitative assessment of treatment options and virtual resection planning. It should be convenient to provide the radiologist with an automatic segmentation system which takes few time, but segments the liver and vessels accurately enough. However, fully automated segmentation is known to be an ill-posed problem due to the fact that there is no clear definition of a correct segmentation without high level prior information. In fact, it has been a growing field with open research problems. Firstly, low contrast and blurred edges often characterize the CTA abdominal images. They are caused by partial volume effects and also heavily influenced by the administration of contrast media and different machine setup conditions. Secondly, complex context is often the case. There is ambiguity of boundaries between the liver and the complex backgrounds and adjacent organs (such as kidney, heart, spleen, stomach, etc) share similar intensities as part of the liver. The complex context also lies in that the backgrounds consist of many abdominal organs and soft tissues with diverse intensities and shapes. Further difficulties arise from the large variability in appearance, size and shape of liver.

A. Previous Work

There are many approaches for liver segmentation, which can be grouped into two categories:

Low level information based methods such as intensity based region-growing [18], [26], [27], histogram processing [29], [30], voxel-classification algorithms [15], thresholding [10], [11], graph cut [4], etc with some pre- and post-processing steps. All these methods requires an accurate estimate of the liver intensity range. A liver binary volume is created by simple or iterative thresholding, after that, some attached neighbor organs and irrelevant tissues are deleted by operations such as morphological operators, holefilling and connected component analysis. Although intensity ranges can be roughly obtained by histogram analysis, but precise threshold values are hard to determine. Actually in many cases there are no optimal thresholds. As a result, it is difficult for these methods to isolate the liver effectively without including neighboring tissues. Moreover, they are sensitive to threshold values and can only get coarse liver surface. To further get smooth liver surface, active contours have been used, e.g., Liu et al. [16] propose a method which combined a GVF snake [33] with edge detectors to refine the coarse liver surface.
obtained by intensity peak analysis and thresholding. Besides, they segment the liver slice by slice and the user is requested to select a proper starting slice.

Besides, several authors have also designed new active contour models to segment the liver, e.g., Pan et al in [24] proposed to explicitly drive an active contour with a dynamic speed function which changes according to the past history of the front. Their approach is to progressively slow the front down as it passes over the boundary points. If the front passes over one boundary point it slows down some. If it passes over two boundary points in sequence, it slows down more, etc. In order to reduce leakage at the liver-rib interface, the contour is constrained by a-priori anatomic information regarding the distance between liver and skin/ribs, which are segmented beforehand through thresholding, morphology, and region labeling. However, the contour speed of their intensity range based method is always positive and a critical stop parameter should be introduced. Thus it is hard to prevent leakage on weak boundaries and the computation complexity is also the weakness. Moreover, previous accurate segmentation of skin/ribs is also not easy to take.

Prior information aided methods such as shape model based methods [8], [12], probabilistic atlas based methods [25] and so on. Prior information can further alleviate the ill-posedness of the segmentation. While prior geometric model has achieved great success in some segmentation tasks, several authors have extended it to the liver segmentation task and have shown some interesting results. Most of these methods treat segmentation firstly as a statistical estimation problem, but the quality and the support of the training set’s exemplars are often ignored. Building the training set is really a hard work, which needs a lot of training data. Moreover, due to the large variability in appearance, size and shape of liver, model based liver segmentation methods often fall short of accurate segmentation and so is still an interesting but challenging task.

In summary, most of previous methods segment the liver intuitively in several individual steps with heavy pre-and post-processing and employ different cues in scattered way, which makes the segmentation model less robust and often leads to over-segmentation problem. Besides, with only few pixel level cues, e.g., intensities or gradients, it is impossible to accurately delineate weak liver boundaries and distinguish the liver from other anatomical structures. An interesting alternative is to use a unified variational model, with which multi-cues can be simultaneously integrated and it is easy to control the smoothness of the liver surface. However, few variational models have been designed to segment objects simultaneously with blurred edges, large intensity overlapping and complex backgrounds. In the present paper, we will investigate these difficulties and try to solve this complicate liver segmentation problem with a new variational method.

B. The Variational Framework and Motivations

A great class of segmentation tasks can be stated in the variational framework through the minimization of a functional, where the solution is given by the evolution equation of an active contour. And usually they can be classified as edge based and region based models. In this direction, two models, namely geodesic active contour model [6] and Chan-Vese(CV) model [7], stand out respectively as the paradigms for edge-based and region-based segmentation methods. Also there is a class of hybrid models [19], [34] that integrate both region information and edge information. A grave drawback is the difficulty to properly balance different terms.

The geodesic active contour (GAC) model is defined as the variational problem

\[
\min_C \{ E(C) = \int_0^L g(|\nabla I(C(s))|)ds \},
\]

where \( I \) is the image which can be 2D or 3D, \( L \) is the evolving contour or surface \( C(s) \), \( s \) the arc length parameter and \( g \in (0, 1] \) is the edge detection function, which is positive in homogeneous regions, and near zero at the edges

\[
g = \frac{1}{1 + \beta |\nabla I|^2},
\]

where \( \beta \) is an arbitrary positive constant. The gradient descent method gives the flow that minimizes \( E(C) \):

\[
\frac{\partial C}{\partial t} = (\kappa g - \langle \nabla g, \vec{N} \rangle) \vec{N},
\]

where \( \kappa \) is the curvature of \( C \) and \( \vec{N} \) is the inward normal of the contour. The equation (1) is a curve shorten flow and thus we should initialize a contour to enclose the object. Then the contour evolves towards sharp edges, while at the same time acting as a smoothing term. The advantage of this model is that it only requires the partial homogeneity of the image, i.e., the homogeneity of regions enclosed by the initial contour and the desired object boundaries. However, this model is only edge based and sensitive to the initialization position and various image artifacts, e.g., spurious edges and noises. Moreover, its convergence speed is very slow and it often hard to capture blurred edges and very concave boundaries. A positive constant balloon force term [5] has been introduced to enlarge its capture range and resist noises,

\[
\frac{\partial C}{\partial t} = (g(\kappa + a) - \langle \nabla g, \vec{N} \rangle) \vec{N}
\]

where \( a \) is a constant. A sufficiently negative constant will react on the shrinking behavior and force the contour to expand. Unfortunately, the GAC model with balloon force is sensitive to the choice of the constant \( a \) and will lead to more serious leakage over weak boundaries.

Comparatively speaking, region based models are more robust to initialization and noises, and no edge information is required. In this framework, the image is divided into a number of disjoint regions such that the pixels have high similarity inside each region and high contrast between regions. The CV model is based on the piecewise constant assumption and segments the image into two regions. There are many improvements [9], [13], [35] based on local or global statistics in the this paradigm, i.e., the Bayesian generalization of the CV model. All these method model the foreground and background with two different intensity distributions no matter they are parametric or non-parametric. The same idea can be
generalized to multi-phase segmentation problems. However, the number of region classes should be predefined and they model every region class with an intensity distribution.

Unfortunately, these region based methods are not feasible for liver segmentation. Because the complex backgrounds do not meet homogeneity assumption. Although it is intuitive to address the liver segmentation with complex backgrounds by multi-phase models, a great number of region classes are needed, which will cause heavy computational cost especially for 3D huge volume data. On the other hand, what we want is only the liver region which has similar appearances or intensities and it is too elaborate to simultaneously segment all organs. Moreover, there is large intensity overlapping between the liver and other tissues and thereby methods relying on intensity homogeneity can not well discriminate them. In fact, they segment the whole image without any spatial order. On the contrary, the radiologist recognizes the liver mostly relying on edge information. Analogously, we can expect that segmenting with a spatial order as the GAC model can help to avoid uninterested background tissues. However, the requirement of the initialization close to the object boundaries is too strict.

In this paper, we propose to conduct the segmentation based on the result of an Information Propagation Process with the GAC model and continues form region growing being prototypes of this new framework. This process iteratively classifies new pixels and is adaptively driven by both region and edge information. After a contour is initialized inside the object, the most discriminative features are learned from the initial region. Then the learned information is propagated out until encountering reliable boundaries, which are delineated by these multi feature cues. It is important to note that this new framework is only relying on the homogeneity of the foreground.

Our model is partially inspired by other three models. The first one is region growing [1] which examines neighboring pixels of initial seed points and determines whether the pixel neighbors should be added to the region according to some hard threshold values. While the computation is consuming, no matter the time or power, it is sensitive to threshold values and hard to get regular surface. The second one is the hybrid segmentation model proposed by Zhang [34], where they proposed a new region term to locally segment objects with higher intensity values than some manually define global threshold. Another underlying assumption is that the surround context must have obviously lower intensity values than the foreground object. So it is not feasible for the complicated liver segmentation from complex context. Besides, their model is sensitive to the threshold. Although he combined this region with an edge term, there is a constant balance between them. As a result, only when a proper threshold is chosen, the two term can balance between each other nearby boundaries. The third one is the model proposed by Ni [20], where they encourage partitioning the image domain so that the local histograms of intensity within each region are approximately homogeneous. They have applied their model to texture image segmentation problems. But piecewise homogeneity is assumed by this model.

C. Overview of Our Method

In this paper, we deal with the problem of liver segmentation with complex context, blurred boundaries and large intensity overlapping between the liver and adjacent organs. To tackle these problems and the aforementioned difficulties, we propose a new hybrid variational model based on the idea of adaptive region growing and region appearance propagation in our new Information Propagation framework. Our model only relies on partial homogeneity of the liver image, i.e., the homogeneity of the liver region.

The general idea behind is to firstly initialize a contour/surface inside the liver region and then a combined force adaptively drives the contour out to the right boundary. In contrast to previous active contour models, reliable intensity and region appearance information learned from initial region and integrated to help regularize the ill-posedness of the segmentation and delineate weak liver boundaries. On the other hand, when encountering non-liver organs or tissues no matter what the organs are, the contour will move back and thus further prevent great deal over-segmentation. In this bidirectional propagation manner, we can take full advantage of the partial homogeneity and also the most discriminative cues in different regions. Thereby, we can focus on the liver region as local as possible and need not to directly build complex model for the whole image.

Another contribution of our model is the spatial adaptive balance between the region term and boundary term which generally has not received enough attention. Most hybrid models take constant balance parametric for simple segmentation task and have poor performance in complicate cases, where image properties such as intensity, gradient information, etc, are not uniformly strong. Actually, the low contrast, blurred edges and complex intensity context are often the case for CTA liver images and at the same time, most part of liver boundaries have strong gradients. So adaptive balancing weight is more favorable and the region based part of the model and edge based part should dominate the contour evolving in proper regions.

Last but not least, our model can be fast computed and is robust to model parameters, initialization and image artifacts, which are favorable properties for stable practical use.

D. Organization of the Work

The paper is organized as follow. In Section II, we discuss our model in detail and analysis the properties of our two new region terms. In Section III, we briefly discuss the computation of our model and show the procedures to segment both the liver and vessels. Experimental results on some 2D slices and 3D CTA volume images and their analysis are given in Section IV followed by conclusion in Section V.

II. OUR PROPOSED MODEL

A. Key Observations for the Liver Segmentation

In spite of complex backgrounds, four key observations about the liver region can facilitate the correct and robust segmentation and also the foundation of our model.
1) Liver boundaries/edges are the most reliable cues to delineate the liver region. However, edges, i.e., the gradients, are not uniformly strong. Thus, boundaries weak in intensity gradients but locally prominent should also be captured by our model.

2) Intensities in the liver region are homogeneous and roughly lie in a range $[\mu, \eta]$. It is worth noting that most variational algorithms are in the the philosophy of the CV model and assume the homogeneity of both foreground and backgrounds and little intensity distribution overlapping between them. But neither of the assumptions is the case for the liver segmentation.

3) The liver region has similar feature patterns which are not pixel level features and adjacent organs can be further discriminated by region appearances.

4) Image features learned from initial region can be regarded as reliable information to constrain the ill-posedness of the liver segmentation. On the contrary, due to the large appearance variances in different livers, image features learned from training sets are not quite reliable.

While intuitively taking into account multi-cues can result in more robust segmentation, the properly balance of these cues are much more important for correct and robust segmentation. Because image features are non-uniformly distributed, every discriminative cue should dominate the segmentation in regions, where it is most prominent.

**B. Model I: Integrating Edge Information and Region Intensity Information**

Here we propose to deal with the liver segmentation with a new hybrid variational model. Suppose for now we have obtained a rough estimate about the liver intensity range $[\mu, \eta]$. A novel region based term is introduced to adaptively guide the segmentation process according to local image information and softly constrain the intensity of the foreground to $[\mu, \eta]$. The GAC term is also employed to capture strong liver boundaries. Our new variational model named Model I reads

$$\min \{ E_{R}(C) = \int_{0}^{L} g ds + \int_{\Omega_{in}} w(x) \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^{2}} d\Omega \},$$

(3)

where $g$ is the edge detector defined as Section I. B, $\Omega_{in}$ is the foreground region enclosed by the contour $C$, $w(x)$ is the balancing weight which is spatially varying according to local image properties, and $\mu$ and $\eta$ are the estimated lower and upper threshold respectively.

The optimal segmentation minimizing $E_{R}(C)$ is computed through evolving an active contour. The corresponding gradient flow equation is

$$\frac{\partial C}{\partial t} = \{ g\kappa - \langle \nabla g, \overrightarrow{N} \rangle + w(x) \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^{2}} \} \overrightarrow{N},$$

(4)

where $\overrightarrow{N}$ is the inward normal and $\kappa$ is the curvature of the contour $C$. Let

$$v = g\kappa - \langle \nabla g, \overrightarrow{N} \rangle + w(x) \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^{2}},$$

which is the contour speed. The first two terms in $v$ correspond to the first term in energy functional $E_{R}(C)$ and the third term corresponds to our new region based term.

To avoid directly tackling complex backgrounds, a contour in 2D (a surface in 3D) is firstly initialized inside the liver region. Then our new region based force would react on the shrinking behavior of the contour and force it to expand until encountering reliable boundaries or nonliver tissues. As the liver region is homogenous in intensity, the liver intensity range can be estimated from the region inside the initial contour and this individual based prior knowledge will further constrain the segmentation. Suppose the mean and variance in the initial region are $m$ and $\sigma$ respectively. A bit wider range $[m - 3\sigma, m + 3.5\sigma]$ can be chosen as the estimated liver intensity range and experiments in Section IV show that this estimation has stable performance for all our data sets. It worth indicating that a rough intensity range estimation is sufficient for correct segmentation, which will be discussed later in detail.

**Analogous to Adaptive Region Growing.** It is obvious that only minimizing the second region based term encourages the contour $C$ to enclose regions where gray-levels lie $g$ in $[\mu, \eta]$. In fact, when setting $w(x) = Constant$, the second term amounts to continuous form of region growing with predefined accepting range $[\mu, \eta]$. The pixels enclosed by the initial contour can be seen as the seed points. As the evolving of the contour, pixels on it is accepted as the foreground according to their intensities. However, with a spatial weight $w(x)$ and the balancing of the GAC term, pixels with intensities lying in $[\mu, \eta]$, may be also classified as backgrounds when $w(x)$ is small. As a results, our new model can be regarded as a continues form of Adaptive Region Growing term. Similar to the GAC and region growing, this propagation process locally segment part of the image $I$. However, in this variational framework, this adaptive region growing like process can lead to more elegant and robust results than traditional region growing and the GAC model. Moreover, in contrary to region growing, it can reduce computation cost and result in smoother liver surface.

**The Choice of $w(x)$**. There are many choices for the adaptive weight $w(x)$. In the liver segmentation, because the strong gradient information is the most reliable cue to recognize the liver, it is intuitively reasonable that reliable boundary is the sign to stop the thresholding process. Thus we take the weight to spatially vary according to boundary information,

$$w(x) = \alpha g(x) = \frac{\alpha}{1 + \beta |\nabla I|^2},$$

where $\alpha$ and $\beta$ are arbitrary positive constants. The weight $w(x)$ goes to zero in regions with great gradients and to $\alpha$ in homogeneous regions. Other choice such as local intensity variance information, can also be made.

**C. Property Analysis of Model I**

As mentioned before, we first initialize a contour or surface inside the liver region and the new region based force will drive it evolving out. As $\overrightarrow{N}$ is the inward normal, the active contour will shrink when $v > 0$ and expand when $v < 0$. As
When either the intensity is near the threshold bounds \( f \rightarrow 0 \) or the gradients are sufficiently great \( w(x) \rightarrow 0 \), the contour/surface will move slowly and even stop. As a result, we can grasp not only sharp boundaries but also blurred edges on the intensity bounds. We analysis this below in two cases:

**Case 1.** In boundary regions with strong gradients, \( g \) and \( w(x) \) will decay quickly to zero (i.e., \( g \rightarrow 0 \) and \( w \rightarrow 0 \)). The second term in \( v \) will be strong in homogenous regions and, force the contour to evolve out where the boundary fall in to the liver intensity range, great local contrast can also stop the contour. Besides, when intensities outside the boundaries are out of the liver intensity range, the bidirectional region force also act as a boundary attracting force.

**Case 2.** In blurred boundary regions where gradients are weak, the region based term dominates the evolving contour and will attract it to the boundary, e.g., the bottom row of Fig.1. As we can definitely recognize a foreground object from backgrounds when either they are divided by obviously boundaries, which corresponds to Case 1 or there are mild gradients and blurred edges but outside intensity values are obviously different from the inside intensity values which is the circumstance in this case. As a result, when the contour is outside the boundary, the region term force will be positive and force the contour to contract. When inside the boundary, the region term force is negative and drive the contour to expand. So the bidirectional region term will dominate and capture blurred edges, which is reasonable to be indicated by the intensity range. On the other hand, the second convection term will still weakly attract the contour to the boundary.

So summing above up, our model alternatively act as edge based model and region based model in different regions. Thus we can properly capture non-uniformly distributed image features. The bidirectional force is also the prominent property of our model. Fig.1 shows some difficult cases where our model can segment with subpixel accuracy. Besides, because of the GAC term and the spatially adaptive speed of the contour, our model is robust to initialization and image artifacts.

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*Fig. 1. 2D segmentation results of our Model I, i.e., equation (3) on two convolved slices. Top right: Segmentation results with tissues that are glued to the liver. Bottom right: segmentation results with blurred edges between the liver and cholecyst.*
combination of region growing and the GAC model, which is analogues to Zhang et al’s model [34] and is a generalization to the liver segmentation problem

$$\min_{C} \{E_R(C) = \int_0^L gds + \alpha \int_{\Omega_{in}} \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^2} d\Omega \}.$$  (5)

The contour will tend to leak out because of poor balance between the region term and edge term. As it is shown in Figure 2, while the adaptive Model I (3) segments this slice perfectly, the contour with equation (5) leaks out both on the right to the inferior vena cava (IVC) and on the left to some soft tissues which share the same intensity range as the liver. Moreover, the model is very sensitive to thresholds, see Fig.6.

In fact, region based active contour models implemented with level set tend to leak out when there are objects that not only share similar intensities but also are very close to the liver. Because unlike snakes implemented by traditional methods, it embeds the contour as the zero level set of a one dimensional higher function. Although arbitrary variation in region topology can be handled automatically, the side effect is that the states of all the image pixels are updated in every step. On the contrary, traditional contours only update points on the front. This drawback will be more serious for models with intensity based region terms, for the intensity include no spatial information. Although with either frequently re-initializing the level set function as sign distance function or narrow band methods [2] we can eliminate this high dimensional effect for pixels far away from the front, for pixels near by the zero level front we need very small time steps to alleviate the leaking out problem, which will lead to the overall evolving process very slow. On the contrary, its spatially adaptive weight \(w(x)\) amounts to a spatially varying time steps of the contour.

In a narrow band of strong boundaries, the region force is very weak and the edge based force dominate the contour, which accounts for our segmentation results. In homogenous region, the contour will still evolve with full speed. Further stability nearby weak boundaries can obtained with patch level information.

D. Model II: Enhancing Model I by Region Appearance Propagation (PAP)

Our proposed Model I is not only simple enough but also effective for a large class of images. However for some cases, its ability and robustness of correct segmentation are limited. The complex context and large intensity distribution overlapping between different organs often confuse people without anatomical knowledge and thus only pixel level information such as intensities and gradients of intensity can not well discriminate the liver from other tissues. For these cases, we have to tune the parameters carefully and even in some complicate cases we can not get good segmentation results, see Fig.3. However, we note that most organs have different appearances such as intensity statistics, texture distribution and so on, which would help to distinguish them further. On blurred edges, though intensity gradients are weak, most often the local appearances on both sides are very different. Even though adjacent organs share similar intensities, the region appearances around the weak boundaries are still quite different.

To further address the oversegmentation problem, we propose to employ additional information, e.g., patch level features, to delineate weak boundaries and control the homogeneity of the liver region. As our previous region term can be seen as a process of Intensity Information Propagation (IIP). Keeping the underlying idea of Model I, we propagate out reliable patch level features learned from initial region.
We call this process Region Appearance Propagation (PAP), where we define region appearance as the distribution of some discriminative features over a region.

In this Region Appearance Propagation framework, we set the region appearances learned initially as the backbone for correctly segmentation. Then there is great freedom to choose feature descriptors (i.e., features that describes local structures of the image), which can be either point-wise or patch-wise. Suppose the image is \( I(x) : \Omega \rightarrow R \). In order to describe a unified way, we define a feature mapping

\[
\mathcal{F}_I : x \in \Omega \mapsto \mathcal{F}_I(x) \in R^b
\]

which maps an image \( I(x) \) to its feature space and \( b \) is the dimension of the feature space. The simplest choice of \( \mathcal{F}_I \) is the identity mapping \( \mathcal{I} \) and \( b = 1 \):

\[
\mathcal{F}_I(x) := \mathcal{I}(I(x)) = I(x).
\]

A good choice is the Local Binary Pattern (LBP) [22] which detects microstructures (e.g., edges, lines, spots, flat areas) and models a specific local structure with a unique integral number. So it is a patch-wise feature but with \( b = 1 \). Other choice can be \( \mathcal{F}_I(x) := f(x) \), where \( f \) is defined as Section II. B and \( b = 1 \); \( \mathcal{F}_I(x) := \nabla I(x) \), in which case \( b = 2 \), to name a few.

The distribution of the feature “signatures” \( \mathcal{F}_I(x) \) over an image region provides a discriminative tool to both segment the liver from adjacent organs and control the region appearance consistency of the foreground region. To achieve this, firstly we estimate the rough global liver appearance, i.e., the feature distribution \( P_0 \) of the liver, from the initial region. Secondly, we define the local region appearance around a pixel \( x \in \Omega \) as the distribution of a chosen feature over a neighborhood of it. The region appearance of arbitrary region in the image can be similarly described. Then we can build a potential field \( \mathcal{P}(x) \) for \( I(x) \), which signifies the similarity \( \mathcal{P}(x) \) between \( P_x \), i.e., local region appearance of very pixel \( x \) and the estimated global region appearance \( P_0 \).

Probability density function (PDF) can be estimated by parametric or nonparametric method. It is worth noting that estimation of PDF in high-dimensional spaces is notoriously challenging because the available data populates such spaces very sparsely regarded as the curse of dimensionality. Instead, the feature image can be seen as a multichannel image and thereby we model a distribution of one dimension for every channel. Obviously, low dimensional features would result in low computational cost. To illustrate the concept here, we set \( \mathcal{F}_I(x) = I(x) \) in this paper. However, experimental results beyond this paper have show that LBP is much more favorable but the computation complexity is higher. In addition, we choose histogram as the rough approximate of PDF. Thus each pixel is initially assigned a local estimated histogram (i.e., a normalized histogram of the pixel intensities in a neighborhood of that pixel). We have noted that the local region histogram can be computed fast by integral images [23], which is specially useful for high dimensions. Suppose that image \( I : \Omega \rightarrow [0, N] \). Let \( \mathcal{N}_{x,r} \) be the ball of radius \( r \) centered at \( x \) and \( P_x \) be the local region appearance around \( x \). The corresponding local cumulative distribution function is defined by

\[
F_x(y) := \frac{|z \in \mathcal{N}_{x,r} \cap \Omega : I(z) \leq y|}{|\mathcal{N}_{x,r} \cap \Omega|},
\]

for \( 0 \leq y \leq N \). The cumulative distribution \( F_0 \) in the initial region is computed similarly.

To measure the similarity of \( P_0 \) and \( P_x \), we select the distance measure introduced by Chan et al [20], i.e., the particular Wasserstein distance as the measure of two PDFs \( P \) and \( Q \).

\[
W(P, Q) = W(F, G) = \int_0^N |F(y) - G(y)|dy,
\]

where \( N \) is the number of the gray levels and \( F(y) \) and \( G(y) \) is the cumulative distribution functions of \( P \) and \( Q \) respectively. This distance, which corresponds to a special closed form solution of the Monge-Kantorovich problem, defines a metric and is insensitive to oscillations [31]. So we can build the potential field \( \mathcal{P} \) through computing the distance between the predefined liver histogram and the local histogram for every pixel \( x \),

\[
\mathcal{P}(x) = W(F_0, F_x), \forall x \in \Omega,
\]

where \( F_0 \) is the cumulative histogram of the liver region and estimated from the initial region and \( F_x \) is the local cumulative histogram around point \( x \). In general, \( \mathcal{P}(x) \) tends to be zero inside the liver and has higher values on pixels with different local region appearance. The more different from the liver region the region appearance is, the higher the value of \( \mathcal{P}(x) \) gets. So it can be used to further “regularize” the ill-defined liver segmentation problem in these regions and help to control the homogeneity of the liver region. Our enhanced Model II reads

\[
\min_C \left\{ \mathcal{E}_{ER}(C) = \int_0^L g(|\nabla I(C(s))|)ds + \int_{\Omega,n} w(x) \left[ \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^2} + \gamma \mathcal{P}(x) \right] d\Omega \right\}.
\]

The corresponding gradient flow equation is

\[
\frac{\partial C}{\partial t} = \{g \kappa - \langle \nabla g, \vec{N} \rangle + w(x) \left[ \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^2} + \gamma \mathcal{P}(x) \right] \} \vec{N}.
\]

As \( \mathcal{P}(x) \) is positive and \( \vec{N} \) is the inward normal, \( \mathcal{P}(x) \) acts as a driven back force.

In the liver segmentation problem, blurred edges are often the case and gradient or intensity based active contours tend to leak out. However, we have observed that although the intensity gradients are weak, the local appearance of boundary points are quite different from that of the liver region. Then \( \mathcal{P}(x) \) will have higher values and act as a driven back force to stop the contour and here \( w(x) \) forces the region appearance based term to play a part only when the gradient is not sufficiently high. As a result, weak edges are delineate by multi-cues, which is intuitively much more robust, see Fig.3 and Fig.4 for comparisons results.
Moreover, people can also recognize the liver from adjacent tissues when they reveal different region appearances. Analogous resonances explain the performance of our Model II. When confronting tissues with different region appearance, the potential field value \( \mathcal{F} \) will increase and drive the contour back. Fig.5 and Fig.8 show some results using the enhanced model. With this region appearance based term we can get more reliable results, especially more reliable vessels segmentation, for example in Fig.5. Moreover, our model becomes much more robust to thresholds, see comparison results in Fig.6. More detail see Section IV.

III. Model Computation and Segmentation Procedures

A. Model Computation

Our models can be reformulated into a level set formulation [21], which handles automatically arbitrary variation in region topology and yield elegant and stable representation of the region membership and boundary. Then Equation (8) can be written as:

\[
\frac{\partial \phi}{\partial t} = |\nabla \phi| \left[ \nabla \left( g \frac{\nabla \phi}{|\nabla \phi|} \right) + w(x) \frac{(I - \mu)(I - \eta)}{(\eta - \mu)^2} + \gamma \mathcal{F}(x) \right]
\]

where \( \phi \) is the level set function embedding the active contour \( C \). While implicit active contour models avoid several of the difficulties encountered with classical deformable models, the main drawback is its high computational cost. However, our model can be solved with an Additive Operator Splitting (AOS) [32] scheme, which is unconditional stable and allows the decomposition of the multidimensional problem into several one-dimensional ones. So our model can be solved quickly even in 3D case.

B. Segmentation of the Liver and Vessels

As we have noted that our method can simultaneously segment out the liver without vessels. Let \( R_r(x) \) is the binary function:

\[
R_r(x) = \begin{cases} 
1 & \mathcal{L}(x) = r \\
0 & \text{otherwise} 
\end{cases}
\]

where \( \mathcal{L}(x) \) is the label of pixel \( x \) and \( r \in \{ \text{liver}, \text{vessel}, \text{pure} \} \). Both the liver \( R_{\text{liver}} \) and vessel \( R_{\text{vessel}} \) can be extract with ease and accuracy. The procedures of simultaneously segmentation of liver and vessels are as follow:

1) Segment the pure liver region \( R_{\text{pure}} \) without vessels by our proposed hybrid model;
2) Compute the intensity lower bound \( \mu_1 \) of the liver region \( R_{\text{pure}} \);
3) Fill the holes of the liver region \( R_{\text{pure}} \) slice by slice and get the region \( R_{\text{fill}} \). Threshold it with \( \mu_1 \) and then we get the liver region with vessels \( R_{\text{liver}} \);
4) Compute the \( R_{\text{vessel}} \) with

\[
R_{\text{vessel}} = R_{\text{liver}} - R_{\text{pure}}.
\]

IV. Results and Discussion

Our methods are validated on 12 volume data-sets. Due to the page limit, we show only the segmentation results of several key slices from 4 subjects. The data sets used for the evaluation are obtained from the First Affiliated Hospital, Zhejiang University College of Medicine. Every volume consist of about 250 to 450 slices with \( 512 \times 512 \) resolution for every slice. The algorithm is developed using matlab 2009 and C++ language. To illustrate concepts, we firstly test our model on some difficult and representative 2D slices and show some comparisons results of our Model I (3) and Model II (6), the GAC model with balloon force (2) and the simplified version of our model (5). Then we compare our 3D results with the “ground truth” and show some other application.

A. Parameters

Suppose that in the initial region, the estimated mean is \( m \) and variance is \( \sigma \). As we estimate the liver intensity range from part of the liver, a bit wider range \([m - 3\sigma, m + 3.5\sigma]\) is chosen. It is important to note that this choice stays fixed for most cases in our practical use. The parameter \( \beta \) in the edge detector \( g \) is set as 0.1. The window size for computing local histogram is \( 4 \times 4 \) square neighborhood in 2D and \( 4 \times 4 \times 2 \) in 3D. The stopping criterion is that the total difference of the level set function \( \phi \) between the current step and last step is less than \( \varepsilon \), i.e., \( \| \phi^{n+1} - \phi^n \| \leq \varepsilon \), where \( \varepsilon = 0.000005 \) in 2D, and \( \varepsilon = 0.00001 \) in 3D.
B. Experiments

For the tested 2D slices, our models, especially Model II, can obtain results as accurate as ground truth. It is worth indicating that, as our model is region based, the initialization can be arbitrary inside the liver. To further reduce the negative effects of initialization to the estimated intensity bounds, we iteratively update a new estimation in the first ten steps.

- **The performance of our models for complex context segmentation and blurred edges.** In Fig.2 where some soft tissues are close to the liver and share similar intensities, we compare the our Model I with Model (5) in which a constant balance is chosen. The results show that our Model I can effectively prevent leaking out problems because of its adaptive weight. Surprisingly, our model can even prevent the leakage to the inferior vena cava (IVC). Fig.3 shows the effect of the region appearance information in tackling blurred boundaries and complex context. Although some soft tissues are glued to the liver and intensities are also similar, the local feature distribution (i.e., local region appearance) is different. Thus with region appearance information, we can segment it perfectly. In Fig.4, where there are blurred edges near the heart and very weak boundary near the spleen, we compared the performance of the GAC model with balloon force (2) with our models. The GAC model with balloon force require careful parameter tuning and tends to leak our near weak boundaries. Model I can capture a part of weak boundaries, where intensities outside are out of the intensity bounds. In contrast, our Model II is superior in capture weak boundaries and its segmentation is as accurate as the ground truth.

- **Vessels segmentation.** One of the most important aspects is that our method can simultaneously segment out liver vessels with accuracy. Depending on the quality of the original image data, the requirements for vessel analysis methods can be very high, especially in the case of small or closely located vessels, or if the intensity in the vessel lumen or the contrast between the lumen and surrounding structures strongly varies. While a great class of models penalize high curvature for smoothness, part of the vessels have strikingly high curvature. The thin structure also prohibit many algorithms. But with region appearance information, we can segment exclude out vessels which will greatly ease the difficulty of vessel segmentation. In Fig.5, we compare our Model I and II with the GAC model with balloon force (2), and show that Model II with patch level region appearance information can accurately distinguish vessels from low contrast images.

![Fig. 5. Comparison of 2D vessels segmentation results by model(2), Model Iand Model II. Top left: original image. Top right: Segmentations with model (2)with finely tuned parameters. Bottom left: Segmentations with our Model I. Bottom right: Segmentations with our enhanced Model II, which can be regarded as ground truth.](image1)

![Fig. 6. The effect of the intensity range. 2D segmentation results of spleen with different intensity range. The estimated σ is 15. First row left: the original image. First row right: The ground truth. Second row: Results by model (5) with intensity range \([m - 3.0\sigma, m + 3.5\sigma]\) (left) and \([m - 4.5\sigma, m + 3.5\sigma]\) (right). Third column: Results by our Model I with intensity range \([m - 3.0\sigma, m + 3.5\sigma]\) (left) and \([m - 4.5\sigma, m + 3.5\sigma]\) (right). Forth row: Results by our Model II with intensity range \([m - 3.5\sigma, m + 3.5\sigma]\) (left) and \([m - 7.5\sigma, m + 3.5\sigma]\) (right).](image2)

- **The robustness of our model to model parameters, image noise and image quality.** In order to test the sensitiveness of our model to the estimated thresholds values (i.e., \(\eta\) and \(\mu\)), we consider the experiment in Fig.6, where there are soft tissues that are glued to the spleen. These soft tissues have slightly lower intensities. When varying the intensity
low bound, Model II can always stably segment the spleen with accuracy, even with $\mu = m - 7.5\sigma$. On the contrary, the simplified version of Model I will quickly leak out and in fact, it is hard to select a proper intensity bounds for this model. Besides, with Model I, we have certain degree of freedom to choose intensity bounds.

- **Validation of the accuracy of the segmentation results.** Validation of the accuracy of the segmentation results is difficult because ground truth is not available. For comparison, we have to refer to the manual correction by the radiologist as ground truth. Fig.7 shows several 2D slices from 3D segmentation results. It is worth noting that we have filled the holes caused by vessels as in Section.III. B. The results by our model results are plotted with green contours and manual corrected by red contours. Our results are comparable with those produced by expert raters in these difficult cases. Nearby the heart, our model can successfully exclude the heart in most cases, e.g., the forth column of Fig.7. The cholecyst and the spleen can also be excluded, such as the second and third column of Fig.7. However, our model often oversegments the inferior vena cava (IVC), as part of the boundaries between it and the liver are often totally missed, such as the fist three columns in Fig.7.

- **Segmentation results of other abdominal organs.** As above noted that although our model is proposed for liver segmentation, it can be used to segment other organs under similar assumptions. Fig.8 shows the segmentation results of two slices of the spleen. We have noted that the spleen also has neighbor tissues with similar intensity level but our Model II can segment the spleen accurately. Fig.9 shows the segmentation results of the cholecyst.

V. CONCLUSION

In this paper, we have formulated the segmentation of liver from complex context as an Information Propagation Process. The GAC model and region growing are shown to be the prototypes of our model. One of the contributions of our work is the formulation of the liver segmentation by an unified variational model which integrates multi-cues, e.g., intensities range, gradients and region appearance. To overcome the sensitivity of the estimated intensity bounds and leakage near weak boundaries, we proposed a spatially adaptive weight to balance the intensity based region term and the edge term, and at the same time weak boundaries are enhanced by region appearance information. Another advantage of our model is that it is much more robust to initialization and image quality and can be implemented with fast speed. Furthermore, vessels are distinguish out simultaneously. Our model is validated on 12 volumes and experimental results show promising results for stable practical usage.

The main limitation of our model is that, when part of the liver boundaries are totally missed, our low level information based model can not avoid leakage problem. As for future work, we plan to incorporate geometric information to constrain the segmentation.
REFERENCES