Radio Resource Management for Optimizing Energy Efficiency of D2D Communications in Cellular Networks

Tuong Duc Hoang, Long Bao Le, and Tho Le-Ngoc

Abstract—This paper deals with the energy-efficient resource allocation for device-to-device (D2D) communication underlay cellular networks. Specifically, our design objective is to maximize the weighted energy-efficiency (EE) of D2D links while guaranteeing the minimum data rate for each cellular link. To solve this problem, we first characterize the optimal power allocation solution for the cellular links based on which the original resource allocation problem can be transformed into the joint subchannel and power allocation problem for D2D links. We then propose a relaxation-based algorithm to solve the transformed problem. We prove that the proposed algorithm converges to the optimal solution of the original problem if no D2D link utilizes the maximum transmitted power. Extensive numerical results demonstrate that the proposed algorithm achieves the optimal performance and it outperforms existing algorithms.

Index Terms—D2D communication, cellular networks, energy efficiency, resource allocation, subchannel and power allocation

I. INTRODUCTION

Enabling device-to-device communication has been proposed as an important technology for future wireless cellular networks [1] where efficient resource allocation algorithms must be developed to limit the negative impacts on existing communications between users and base-stations (BSs). On the other hand, green communication has also attracted a lot of attention in recent years where maximization of energy efficiency has become an important design objective [2]. In general, downlink energy efficiency would be less critical as compared to that in the uplink since the energy consumption of BSs in the downlink can be supported by various efficient energy sources including renewable energy. Moreover, uplink energy-efficient resource allocation is very important since mobile devices are supported by energy-limited batteries.

There have been some initial efforts in developing energy-efficient resource allocation techniques for D2D underlaying cellular networks [4]–[6]. In [4], the authors propose a resource allocation solution based on non-cooperative game theory in which each D2D link selfishly performs power and subchannel allocation to maximize its own energy-efficiency considering the fixed resource allocation of other links. The authors in [5] develop the energy-efficient algorithm by using the two-stage Stackelberg game in which the cellular BS is the leader and the D2D links are the followers. On the other hand, our previous work in [6] propose an optimal energy-efficient algorithm for D2D links where each D2D link is allowed to reuse resource of one cellular link.

In this paper, we study the joint subchannel and power allocation problem that maximizes the weighted energy-efficiency of all D2D links and guarantees the minimum data rate of each cellular link. In particular, we consider this resource allocation problem for a general setting with multiple cellular and D2D links where each D2D link can reuse the spectrum of multiple cellular links.

Toward solving the problem, we first characterize the optimal power allocation solution for a cellular link, which is a function of optimal powers of D2D links. Based on this result, we show how the original resource allocation problem can be transformed into the resource allocation problem for D2D links. We then propose a relax-based algorithm to solve the problem. We prove that, if the transmitted powers of all D2D link are all smaller than the maximum values, the obtained solution from our algorithm is the optimal one. Extensive numerical results are presented to evaluate the performance of the developed algorithms. It is shown that the objective values achieved by our algorithm is very close to the optimal value. Moreover, the proposed algorithm outperforms the spectrum-efficient algorithm and the conventional algorithm.

The rest of paper is organized as follows. In Section II, we describe the system model, problem formulation, and problem transformation. In Sections III, we describe the relaxation-based algorithm. Section IV presents numerical results, followed by conclusions in Section V.

II. SYSTEM MODEL, PROBLEM FORMULATION, AND PROBLEM TRANSFORMATION

A. System Model

We consider the uplink resource allocation scenario where cellular links share the same spectrum with multiple D2D links in a single macro-cell. We assume that $K$ uplink cellular links in a set $\mathcal{K} = \{1, \cdots, K\}$ occupying $K$ orthogonal subchannels in the set $\mathcal{N} = \{1, \cdots, K\}$ in the considered cell. Moreover, we assume that the set $\mathcal{L} = \{1, \cdots, L\}$ of D2D links transmit data using the same set of subchannels.\(^1\) Here, $K = |\mathcal{K}|$, $L = |\mathcal{L}|$, and $N = |\mathcal{N}|$ denote the numbers of cellular links, D2D links, and subchannels, respectively where $|A|$ denotes the cardinality of set $A$.

Let $h_{kl}^{(t)}$ denote the channel gain from the transmitter of link $l$ to the receiver of link $k$ on subchannel $h$. We assume

\(^1\)The considered orthogonal subchannels can be sub-carriers or sub-channels in the OFDMA system or simply channels in the FDMA system.
that the subchannel allocation for cellular links has been pre-determined and we are interested in allocating these subchannels to D2D links efficiently. Without loss of generality, we assume that cellular link \( k \) has been allocated subchannel \( k \).

We introduce vector \( \rho_l = [\rho^1_l, \cdots, \rho^K_l] \) to describe subchannel allocation decisions for D2D link \( l \) where \( \rho^k_l = 1 \) if subchannel \( k \) is allocated for D2D link \( l \) and \( \rho^k_l = 0 \), otherwise. Let \( \rho = [\rho^1, \cdots, \rho_L] \) denote the subchannel allocation variables for all D2D links.

We denote the allocated power vectors as \( p = [p_C^1, \cdots, p_C^K] \) for all the links, where \( p_C = [p_{C1}, \cdots, p_{C^K}] \) for \( K \) cellular links, \( p_D = [p_{DL1}, \cdots, p_{DL}] \) for \( L \) D2D links, and the variables \( p_C^k \) and \( p_D^k \) denote the allocated transmit powers on subchannel \( k \) of cellular link \( k \) and D2D links respectively. Then, the signal to interference plus noise ratio (SINR) achieved by cellular link \( k \) on its allocated subchannel \( k \) can be expressed as

\[
\Gamma_{Ck}^k(p, \rho) = \frac{p_C^k h_k}{\sigma^k + \sum_{l \in \mathcal{L}} \rho_l p_D^k h_{kl}},
\]

where \( \sum_{l \in \mathcal{L}} \rho_l p_D^k h_{kl} \) represents the interference due to the D2D link using subchannel \( k \) and \( \sigma^k \) denotes the noise power on subchannel \( k \). Similarly, the SINR of D2D link \( l \) on subchannel \( k \) can be written as

\[
\Gamma_{Dk}^l(p, \rho) = \frac{p_D^l h_{kl}}{\sigma^k + p_C^k h_{kl}}.
\]

The data rates in b/s/Hz (i.e., normalized by the subchannel bandwidth) of cellular link \( k \) on its subchannel \( k \), D2D link \( l \) on subchannel \( k \), and D2D link \( l \) on all the subchannels can be calculated as

\[
R_{Ck}^k(p, \rho) = \log_2 \left( 1 + \Gamma_{Ck}^k(p, \rho) \right),
\]

\[
R_{Dk}^l(p, \rho) = \log_2 \left( 1 + \Gamma_{Dk}^l(p, \rho) \right),
\]

\[
R_{Dl}^l(p, \rho) = \sum_{k \in \mathcal{K}} \rho^k_l R_{Dk}^l(p, \rho).
\]

We assume that the total consumed power of D2D link \( l \) can be expressed as [7]

\[
P_{Dl}^\text{total} = P_0^l + \alpha_l \sum_{k \in \mathcal{K}} \rho^k_l P_{Dl}^k,
\]

where \( P_0^l \) represents the fixed circuit power of D2D link \( l \) transmitter, and \( \alpha_l > 1 \) is a factor accounting for the transmit amplifier efficiency and feeder losses.

### B. Problem Formulation

In this paper, we consider the resource allocation design with the following constraints. First, it is required to maintain the minimum rate of each cellular link \( k \) (on its allocated subchannel \( k \)), i.e.,

\[
R_{Ck}^k(p, \rho) \geq R_{Ck}^\text{min}, \quad \forall k \in \mathcal{K}.
\]

Second, the power constraints of individual links are given as

\[
p_C^k \leq P_C^\text{max}, \quad \forall k \in \mathcal{K},
\]

\[
\sum_{k \in \mathcal{K}} \rho^k_l P_{Dl}^k \leq P_D^\text{max}, \quad \forall l \in \mathcal{L}.
\]

Third, we require that each subchannel can be reused by at most one D2D link to guarantee the performance of the cellular links, i.e.,

\[
\rho^k_l \leq 1, \quad \forall k \in \mathcal{K}, l \in \mathcal{L}.
\]

Finally, subchannel allocation variables are binary, i.e.,

\[
\rho^k_l \in \{0, 1\}, \quad \forall k \in \mathcal{K}, l \in \mathcal{L}.
\]

Our design objective is to maximize the weighted EE of all D2D links. Therefore, we consider the following energy-efficient resource allocation problem

\[
\max_{p, \rho} \frac{\sum_{l \in \mathcal{L}} w_l R_{Dl}(p, \rho)}{\sum_{l \in \mathcal{L}} P_{Dl}^\text{total}}
\]

subject to (7), (8), (9), (10), (11),

where \( \frac{\sum_{l \in \mathcal{L}} w_l R_{Dl}(p, \rho)}{\sum_{l \in \mathcal{L}} P_{Dl}^\text{total}} \) is the ratio between the weighted sum rate of D2D links and total consumed power of D2D transmitters. The weight parameters \( w_l \) can be employed to control the relative priorities among different D2D links (e.g., different classes of D2D links). We assume that the cellular BS can collect all the necessary information about channel gains and QoS constraints, then it performs the resource allocation algorithm and broadcasts the decision to all the users.

### C. Problem Transformation

To solve problem (12), we first characterize the optimal power allocation for D2D link \( l \) on subchannel \( k \) in the following proposition.

**Proposition 1.** If D2D link \( l \) is allowed to reuse subchannel \( k \) of cellular link \( k \), then its optimal power on subchannel \( k \) is \( p_{Dl}^k = \frac{1}{h_{kl}} \left( \frac{p_C^k h_k}{2^\rho^k_l - 1} - \sigma^k \right) \in [0, P_{Dl}^\text{max}] \), where \( p_C^k \) is the power of cellular link \( k \), and \( P_{Dl}^\text{max} = \min \{P_{Dl}^\text{max}, \frac{1}{h_{kl}} \left( \frac{p_C^k h_k}{2^\rho^k_l - 1} - \sigma^k \right) \} \).

**Proof.** The proof is given in Appendix A.

From Proposition 1, the data rate of D2D link \( l \) on subchannel \( k \) given in (4) can be re-written as

\[
\hat{R}_{Dl}^k(p_D, \rho) = \log_2 \left( 1 + \frac{p_D^k h^k_{kl}}{\sigma^k + (2^\rho^k_l - 1) h^k_{kl}} \right).
\]

For convenience, let us define

\[
a_{kl} = \frac{\sigma^k}{h^k_{kl}} + \frac{(2^\rho^k_l - 1) h^k_{kl}}{h^k_{kl} h^k_{kl}},
\]

\[
b_{kl} = \frac{2^\rho^k_l - 1}{h^k_{kl} h^k_{kl}}.
\]
Then, the data rate of D2D link \( l \) on subchannel \( k \in \mathcal{N} \),
\( R_{DL}(p_d, \rho) \) in (13), and the total rate over all subchannels,
\( R_{DL}(p_d, \rho) \) in (5), can be rewritten, respectively, as

\[
R_{DL}(p_d, \rho) = \rho^k \log_2 \left( 1 + \frac{p_d^k}{a_k \rho^k + b_k s_{DL}^k} \right),
\]
(16)

\[
R_{DL}(p_d, \rho) = \sum_{k \in \mathcal{N}} \hat{R}_{DL}(p_d, \rho),
\]
(17)

where the transmitted power must satisfy
\[
p^k_{DL} \leq P^\text{max}_{DL}, k \in \mathcal{N}, l \in \mathcal{L}.
\]
(18)

Therefore, problem (12) is equivalent to the following

\[
\begin{align*}
\max_{(p_d, \rho)} & \quad \sum_{l \in \mathcal{L}} w_l \hat{R}_{DL}(p_d, \rho) \\
\text{s.t.} & \quad (9), (10), (11), (18).
\end{align*}
\]
(19)

III. RELAXATION-BASED ALGORITHM

In general, problem (19) is the Mixed Integer Nonlinear Programming (MINLP) problem, which is difficult to solve. In the following, we employ a relaxation-based approach to solve problem (19). The relaxed version of problem (19) is obtained by assuming that subchannel allocation variables previously defined in (11) are continuous as follows:

\[
\rho^k_l \in [0, 1], \forall k \in \mathcal{K}, \forall l \in \mathcal{L}.
\]
(20)

Note that (20) allows the subchannel variables to take real values which correspond to the time sharing solution of each subchannel among different involved links. Consequently, the relaxed version of problem (19) can be defined as

\[
\begin{align*}
\max_{p_d, \rho} & \quad \sum_{l \in \mathcal{L}} w_l \hat{R}_{DL}(p_d, \rho) \\
\text{s.t.} & \quad (9), (10), (18), (20).
\end{align*}
\]
(21)

In order to solve this problem, we introduce a new vector \( s_D \), which corresponds to the power vector of D2D links \( p_d \). The data rate and consumed power of D2D link \( l \) and all D2D links can be described as functions of \( s_D \) in the following

\[
\begin{align*}
\hat{R}_{DL}(s_{DL}, \rho) & = \sum_{k \in \mathcal{N}} \rho^k_l \log_2 \left( 1 + \frac{s^k_{DL}}{a_k \rho^k_l + b_k s^k_{DL}} \right), \\
\hat{R}_{D}(s_{DL}, \rho) & = \sum_{l \in \mathcal{L}} \hat{R}_{DL}(s_{DL}, \rho), \\
\hat{P}^\text{total}_{DL} & = P_0^l + \alpha_l \sum_{k \in \mathcal{K}} s^k_{DL}, \\
\hat{P}^\text{total}_{D}(s_{DL}, \rho) & = \sum_{l \in \mathcal{L}} \hat{P}^\text{total}_{DL}(s_{DL}, \rho).
\end{align*}
\]

Therefore, we arrive to the following resource allocation problem for D2D links

\[
\begin{align*}
\max_{s_D, \rho} & \quad \sum_{l \in \mathcal{L}} w_l \hat{R}_{DL}(s_{DL}, \rho) \\
\text{s.t.} & \quad \sum_{l \in \mathcal{L}} s^k_{DL} \leq P^\text{max}_{DL}, \forall l \in \mathcal{L}, \\
& \quad \sum_{l \in \mathcal{L}} \rho^k_l \leq 1, \forall k \in \mathcal{K}, \\
& \quad s^k_{DL} \leq \rho^k_l P^\text{max}_{DL}, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}.
\end{align*}
\]
(26)

Now we state the relationship between (21) and (26) in the following proposition, which proof is given in Appendix B.

**Proposition 2.** Problem (21) and (26) are equivalent.

Because problem (21) and (26) are equivalent, we can solve problem (26) instead of tackling problem (21). On the other hand, from the proof of Proposition 2 in Appendix B, if \( (s^*_D, \rho^*) \) is the optimal solution of problem (26), \( (p^*_D, \rho^*) \) will be the optimal solution of problem (21) in which the components of \( p^*_D \) is obtained by the following rule

\[
p^k_{DL} = \begin{cases} 0, & \text{if } \rho^*_l = 0 \\ \frac{s^*_D}{P^*_D}, & \text{otherwise}. \end{cases}
\]
(27)

In the following, we propose an algorithm to solve problem (26). At first, we consider the following optimization problem

\[
\begin{align*}
\max_{s_D, \rho, \zeta} & \quad \eta(s_D, \rho, \zeta) = \hat{R}_{D}(s_D, \rho) - \zeta \hat{P}^\text{total}_{D}(s_D, \rho) \\
\text{s.t.} & \quad (26b), (26c), (26d).
\end{align*}
\]
(28)

Suppose that \( \eta^*(\zeta) = \eta(s^*_D, \rho^*, \zeta) \) where \( (s^*_D, \rho^*) \) is the optimal solution of problem (28), and \( D \) denotes the set of feasible solutions of problem (28). Then, we can characterize the optimal solution of problem (26) in the following theorem, which is adopted from [8].

**Theorem 1.** \( \eta^*(\zeta) \) is a decreasing function of \( \zeta \). In addition, if we have

\[
\begin{align*}
\max_{(s_D, \rho) \in D} & \quad \hat{R}_{D}(s_D, \rho) - \zeta \hat{P}^\text{total}_{D}(s_D, \rho) \\
= & \hat{R}_{D}(s^*_D, \rho^*) - \zeta \hat{P}^\text{total}_{D}(s^*_D, \rho^*) = 0,
\end{align*}
\]
(29)

then \( \zeta^* = \frac{\hat{R}_{D}(s^*_D, \rho^*)}{\hat{P}^\text{total}_{D}(s^*_D, \rho^*)} \) is the optimal objective value of (26).

This theorem allows us to transfer a general fractional problem (26) to a non-fractional optimization problem with the parameter \( \zeta \). In addition, the optimal solution of problem (26), \( \zeta^* \), can be found if \( \eta^*(\zeta^*) = 0 \). Since \( \eta^*(\zeta) \) is a decreasing function of \( \zeta \), it can be seen that \( \zeta^* \) can be indeed determined by the gradient or bisection method. Therefore, the remaining task to solve problem (26) is (i) solving problem (28) for given \( \zeta \), and (ii) update \( \zeta \) to obtain the optimal solution of problem (21). On the other hand, it can be verified that problem (28) is convex; therefore, it can be solved optimally by the standard interior point method [10].

Algorithm 1 describes the procedure to find a feasible solution of problem (19). In this algorithm, steps 1-6 are
employed to determine the optimal solution of problem (26). Based on this solution, step 7 allows us to obtain the optimal solution of problem (21). Finally, in step 8, the fractional sub-channel allocation variables are rounded to achieve a feasible solution of problem (19). We now confirm the optimality of the obtained solution in the following proposition.

**Proposition 3.** The obtained solution \((p^*_D, \rho^*)\) in the step 7 of Algorithm 1 is the optimal solution of problem (21).

**Proof.** The proof is given in Appendix C.

It can be seen that the optimal solution of problem (21) is an upper bound of the solution obtained from problem (19). Nevertheless, the optimal sub-channel allocation, \(\rho^*\) acquired in Algorithm 1 can be fractional, which is not a feasible sub-channel allocation solution for problem (21). Therefore, the rounding procedure is necessary to obtain a feasible solution of problem (19). We have the following proposition which states the optimality of the solution obtained from Algorithm 1 whose proof is given in Appendix D.

**Proposition 4.** If no D2D link uses its maximal transmitted power at optimality, Algorithm 1 converges to the optimal solution of problem (19).

It is intuitive that each D2D link does not use maximal transmitted power in the energy-efficient resource allocation design. Therefore, from Proposition 3, the optimal solution of problem (19) is almost surely obtained.

**IV. Numerical Results**

We consider the system configuration shown in Fig. 1, with the base-station at the center. \(K_C = 20\) cellular users randomly placed in the 500m x 500m cell and \(N = 20\) subchannels for uplink communications. In addition, the sub-channel power gain is modeled as \(h_{kl}^0 = K(d_0)(d_{kl})^{-3} \delta\), where \(K(d_0) = 1\) is the path loss at the reference distance, \(d_{kl}\) is the distance between the receiver of link \(k\) and the transmitter of link \(l\), \(\delta\) represents the Rayleigh fading, which follows exponential distribution with the mean value of 1. We set the noise power equal to \(10^{-13} W\) for any link. The circuit power of each cellular and D2D link \(P_0\) is 0.5W, the efficiency factor \(\alpha_l\) is 1.5 for each D2D link, the maximum transmitted power of each D2D link \(P_{D,k}^\text{max}\) is 0.5W. In addition, the weight parameters of D2D links are set as \(w_l = 1, \forall l \in L\), the maximum distance of D2D links \(d_{\text{max}}\) is 50m, and the minimum rate of each cellular link is \(R_{C}^\text{min}\) is 2b/s/Hz.

We compare the performance of our algorithm with the SEE algorithm in our previous work in [6], in which each D2D link is allowed to use the resource of one cellular link. This is referred to as the conventional algorithm in this section. We also evaluate the performance of the proposed algorithms and the spectrum-efficient maximization, which is obtained by solving problem (28) for \(\zeta = 0\). Furthermore, to verify the efficiency of our algorithms, we also compare the objective values achieved by our algorithm with the corresponding upper-bound obtained from the optimal solution of problem (21). All numerical results are obtained by averaging over 1000 random realizations of D2D and cellular locations and subchannel gains. The EE of D2D links corresponding to the relaxation-based algorithm, upper-bound, the spectrum-efficiency (SE) maximization solution, and conventional algorithm are indicated by “Relax-based Alg.”, “Upper bound”, “SE solution”, and “Conventional Alg.”, respectively.

Fig. 2 indicates the fast convergence of Algorithm 1, which is within 10 iterations for a particular system realization where \(d_{\text{max}}\) is equal to 10m, 50m, 100m, and 150m, respectively.

Fig. 3 shows the EE of all D2D links versus the maximum distance of each D2D link. It can be seen that the curve corresponding to the proposed algorithm is identical to the curve showing its upper bound. This indicates the optimality of the proposed algorithm. Moreover, the EE of all D2D links obtained from our algorithm is much higher than that of the SE solution, which means that considering energy-efficient design leads to significantly higher energy-efficiency of the system. On the other hand, the objective value obtained from our algorithm is much higher than that of the conventional algorithm. This is because in the conventional algorithm, each D2D link is restricted to use the resource of one cellular link. This restriction lead to significantly lower EE for all D2D links.

Fig. 4 describes the EE of all D2D links versus the minimum data rate of cellular links, \(R_{C}^\text{min}\). As \(R_{C}^\text{min}\) increases, the EE...
of all D2D links decreases slightly. This can be explained as follow. As $R_{Ck}^{\text{min}}$ increases, each cellular link has to increase its transmitted power to satisfy the QoS constraints. However, since the proposed algorithm enables the D2D links to intelligently utilize the resource of cellular links, it can choose the cellular link which has small mutual interference. Consequently, when a cellular link increases its transmitted power, it does not cause much degradation to the performance of the D2D link which reuses its resource.

Fig. 5 illustrates the EE of all D2D links versus the circuit power of the D2D transmitter. It can be seen that our algorithm offers an excellent performance, which is equal to the upper-bound. For small circuit power, the minimum EE of our algorithms is about 400% higher than those of the SE solution and conventional algorithm. However, when circuit power increases, the performance gap between the proposed algorithm and the SE-maximization solution is reduced since the total consumed power is dominated by the circuit power.

Finally, Fig. 6 illustrates the variation in the EE of all D2D links with the number of D2D links. In our algorithm, the EE of all D2D links decreases as the number of D2D links increases. On the other hand, the performance gap between the proposed algorithm and the SE-maximization solution decreases as the number of D2D links increases. This is because when the system supports more D2D links, the available resource for D2D links becomes smaller, which results in the decrease in the EE of all D2D links. Fig. 6 demonstrates that as number of D2D links changes, the EE of all D2D links in the conventional algorithm is nearly constant. This is because each D2D link is allocated resource of one cellular link. As the number of D2D links varies, D2D links always efficiently utilize the spectrum resource, which explains the similar EE of all D2D link for varying number of D2D links.

V. CONCLUSIONS

In this paper, we have developed an efficient resource allocation algorithm for D2D underlaying cellular systems, which maximizes the weighted energy-efficiency of D2D links while guaranteeing the QoS of cellular links. In particular, we have proposed a relaxation-based algorithm to solve the relaxed version of the problem, and then perform rounding operation to obtain a feasible solution to the original resource allocation problem. We have studied the theoretical performance of the presented algorithm. Numerical results have confirmed that the proposed algorithm achieves excellent performance, which is identical to that achieved by the upper bound.

**APPENDIX A**

**PROOF OF PROPOSITION 1**

We first show that the min-rate constraints of cellular link $k$ must be met at equality

$$\log_2 \left(1 + \frac{p_{Ck}^k \sigma_k^k}{\sigma_k^k + p_{Dl}^{kl} h_{kl}^k}\right) = R_{Ck}^{\text{min}}.$$  \hspace{1cm} (30)

Note that if $\sigma_k^k (2R_{Ck}^{\text{min}} - 1) > P_{Ck}^{\text{max}}$, the required minimum rate of cellular link $k$ cannot be supported; hence, problem (12) is infeasible. On the other hand, if $\sigma_k^k (2R_{Ck}^{\text{min}} - 1) \leq P_{Ck}^{\text{max}}$, the cellular link $k$ can allow D2D link $l$ to reuse its resource. Therefore, the power of D2D link $l$ on subchannel $k$ can be presented as

$$p_{Ck}^k = \frac{\sigma_k^k}{h_{kl}^k} + \frac{p_{Dl}^{kl} h_{kl}^k}{\sigma_k^k} (2R_{Ck}^{\text{min}} - 1).$$ \hspace{1cm} (31)

Moreover, the power of cellular link $k$ must satisfy the maximum power constraint, which can be expressed as

$$p_{Ck}^k \leq \frac{1}{h_{kl}^k} \left(\frac{P_{Ck}^{\text{max}} \sigma_k^k}{2R_{Ck}^{\text{min}} - 1} - \sigma_k^k\right).$$ \hspace{1cm} (32)

Let us now define

$$P_{Dl}^{\text{max}} = \min \left\{P_{Dl}^{\text{max}} \cdot \frac{1}{h_{kl}^k} \left(\frac{P_{Ck}^{\text{max}} h_{kl}^k}{2R_{Ck}^{\text{min}} - 1} - \sigma_k^k\right)\right\}.$$ \hspace{1cm} (34)

Then, it can be verified that if D2D link $l \in \mathcal{L}$ reuses the resource of cellular link $k$, we have $p_{Dl}^{kl} \in [0, P_{Dl}^{\text{max}}]$, and $p_{Dl}^{kl} = \frac{1}{h_{kl}^k} \left(\frac{P_{Ck}^{\text{max}} h_{kl}^k}{2R_{Ck}^{\text{min}} - 1} - \sigma_k^k\right).$
APPENDIX B
PROOF OF PROPOSITION 2

If \((\mathbf{p}_D^{*}, \rho^*)\) is the optimal solution of problem (21) with the optimal objective value \(z^{*}\), we can express \(s_D^{*}\) as \(s_D^{*} = \rho_k^{*} p_D^{k*}, \forall l \in \mathcal{L}, \forall k \in \mathcal{N}\). Therefore, \((s_D^{*}, \rho^*)\) is a feasible solution of problem (26) and the achieved objective value is \(z^{*}\). On the other hand, if \((s_D^{*}, \rho^*)\) is the optimal solution of problem (26) with the objective value \(zeta^{*}\), \(p_D^*\) is described as

\[
p_{D}^{k*} = \begin{cases} 0, & \text{if } \rho_k^{*} = 0 \\ \frac{s_k^{*}}{\rho_k^{*}}, & \text{otherwise.} \end{cases}
\]

Consequently, \((\mathbf{p}_D^{*}, \rho^*)\) is a feasible solution of problem (21) and its objective value is \(zeta^{*}\). Hence, \(zeta^{*}\) is the optimal objective value of problem (21) iff it is the optimal objective value of problem (26), which means that problem (21) and (26) are equivalent.

APPENDIX C
PROOF OF PROPOSITION 3

Assume that \((s_D^{(t-1)}, \rho^{(t-1)})\) and \((s_D^{(t)}, \rho^{(t)})\) are the solution of problem (26) in iterations \(t - 1\) and \(t\) respectively. In addition, let us define \(\zeta_t = \frac{R_D(s_D^{(t-1)}, \rho^{(t-1)})}{P_D^{\text{total}}(s_D^{(t-1)}, \rho^{(t-1)})}\) and \(\zeta_{t-1} = \frac{R_D(s_D^{(t-1)}, \rho^{(t-1)})}{P_D^{\text{total}}(s_D^{(t-1)}, \rho^{(t-1)})}\). Moreover, we have

\[
\max_{(s_D, \rho) \in D} \tilde{R}_D(s_D, \rho) - \zeta_{t-1} P_D^{\text{total}}(s_D^{(t-1)}, \rho^{(t-1)}) \geq \max_{(s_D, \rho) \in D} \tilde{R}_D(s_D, \rho) - \zeta_{t} P_D^{\text{total}}(s_D^{(t)}, \rho^{(t)}) \geq \max_{(s_D, \rho) \in D} \tilde{R}_D(s_D, \rho) - \zeta_{t-1} P_D^{\text{total}}(s_D^{(t-1)}, \rho^{(t-1)}) = 0.
\]

Therefore, \(\tilde{R}_D(s_D^{(t)}, \rho^{(t)}) - \zeta_{t} P_D^{\text{total}}(s_D^{(t)}, \rho^{(t)}) \geq 0\), which means that \(\zeta_{t} = \frac{R_D(s_D^{(t)}, \rho^{(t)})}{P_D^{\text{total}}(s_D^{(t)}, \rho^{(t)})} \geq \zeta_{t-1}\). This implies that Algorithm 1 creates a sequence of feasible solutions of problem (21) whose objective values monotonically increase over iterations; therefore, the algorithm converges.

Assume that at convergence \(\zeta_{t-1} = \zeta_{t} = zeta^{*}\), therefore the following must hold

\[
\max_{(s_D, \rho) \in D} \tilde{R}_D(s_D, \rho) - \zeta_{t-1} P_D^{\text{total}}(s_D^{(t)}, \rho^{(t)}) = \tilde{R}_D(s_D^{(t)}, \rho^{(t)}) - \zeta_{t} P_D^{\text{total}}(s_D^{(t)}, \rho^{(t)}) = \tilde{R}_D(s_D^{(t)}, \rho^{(t)}) - \zeta_{t} P_D^{\text{total}}(s_D^{(t)}, \rho^{(t)}) = 0.
\]

Since \((s_D^{*}, \rho^*, zeta^{*})\) satisfies the sufficient condition of Theorem 1, \((s_D^{*}, \rho^*)\) is the optimal solution of problem (26). On the other hand, since problems (21) and (26) are equivalent, \((\mathbf{p}_D^{*}, \rho^*)\) is the optimal solution of problem (21).

APPENDIX D
PROOF OF PROPOSITION 4

As \((s_D^{*}, \rho^*)\) is the optimal solution of problem (28) at \(\zeta = zeta^{*}\), \(\rho^*\) is the optimal solution of the following problem

\[
\max_{\rho \geq 0} \tilde{R}_D(s_D^{*}, \rho) - \zeta P_D^{\text{total}}(s_D^{*}, \rho) \tag{38a}
\]

s.t.

\[
\sum_{k \in \mathcal{N}} \rho_k^{k*} p_D^{k*} \leq P_D^{\text{max}}, \forall k \in \mathcal{N}, \tag{38b}
\]

which is originated from problem (28) for given \(s_D = s_D^{*}\). Moreover, problem (38) is a linear program. According to the Rank Lemma theorem [12], the number of non-zero variables is always equal to the number of linearly independent equality constraints in the problem. Since for the optimal solution of problem (38), there is no D2D link using maximum transmitted power, the optimal solution of problem (38) has at most \(N\) constraints holding with equality in (38c). Therefore, there are at most \(N\) non-zero elements of vector \(\rho^*\), which means that \(\rho^*\) is an integer vector. As a consequent, \((\mathbf{p}_D^{*}, \rho^*)\) is the optimal solution of problem (19), and Algorithm 1 converges to the optimal solution.

REFERENCES