Modeling the Substrate Skin Effects in Mutual RL Characteristics of General Asymmetric Coupled Interconnects

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Abstract. The goal of this work was to model the influence of the substrate skin effects on the distributed mutual impedance per unit length parameters of multiple coupled on-chip interconnects. The proposed analytic model is based on the frequency-dependent distribution of the current in the silicon substrate and the closed form integration approach. It is shown that the calculated frequency-dependent distributed mutual inductance and the associated mutual resistance are in good agreement with the results obtained from CAD-oriented circuit modeling technique.

Keywords
On-chip interconnects, lossy silicon, RL line characteristics, analytic model, skin effects.

1. Introduction

High-frequency effects such as delay, distortion and crosstalk, are found to be a limiting factor in the development of VLSI silicon integrated circuits. Due to the continuing decrease of the distances between components and the simultaneous increase of operating frequency, the crosstalk between components and/or blocks through the lossy substrate becomes stronger. As the operating speed of VLSI increases beyond few tens of GHz, the simple modeling approach for distributed RL parameters of on-chip interconnects including frequency-dependent behavior is crucial for the design and validation phase of high-speed VLSI circuits [1]. The ultimate performance of high-speed VLSI circuits is strongly influenced by the materials used for the substrate [3]. The advanced technology of multilayer multichip modules (MCM), especially MCM-D on silicon substrate, has many advantages such as smaller size, lighter systems and improved performance, making it suitable for system-on-chip implementations. Silicon substrates have lower resistivity due to the requirements for the formation of active devices. In this case losses in the semiconductor substrate become significant and should be taken into account. In order to accomplish this, it is necessary to analyze and model the broadband characteristics [2, 4 - 9] of the on-chip interconnects on a silicon oxide-silicon substrate. With respect to the substrate conductivity and signal frequency, in general three wave propagation modes can be distinguished which lead to different formulas for the line parameters (quasi-TEM, slow-wave and skin-effect mode, respectively) [9]. In recent works [2, 4, 5] the authors succeeded to describe the frequency-dependent transmission line parameters of on-chip interconnects on lossy silicon substrate in a unified way. Therefore, simple and fast methods and formulas will be necessary to generate electrical models for the on-chip interconnects that accurately account for such effects as delay, crosstalk and resistive voltage drops.

In this letter, a new model that takes into account the influence of the substrate skin effects on distributed RL characteristics of coupled interconnects on a lossy silicon substrate is presented. To illustrate the accuracy of the model, the frequency-dependent mutual inductance and resistance of multiple coupled interconnects on a silicon substrate were computed using proposed analytic formulas, and compared with the solution by CAD-oriented circuit modeling approach.

2. New Frequency-Dependent Model

To develop an expression for series mutual impedance of general asymmetric coupled conductors on a lossy silicon substrate (see Fig. 1), the quasi-TEM wave is considered, and the magnetic vector potential A determined as

\[ \mathbf{H} = \frac{1}{\mu} \text{rot} \mathbf{A} \] (1)

has only a z-component and satisfies the magnetic vector potential equation

\[ \nabla^2 A_z - j \omega \mu \sigma A_z = -\mu J_z(x, y) \] (2)
where \( J(x,y) \) is unknown current density on the interconnect conductors. Results obtained from the full-wave analysis [2, 4] have shown that the influence of the finite substrate thickness \( t_s \) can be neglected for practical dimensions (i.e. \( t_s \gg w_1, w_2, t_{eq} \gg t_{ox}, t_1 \gg T_1, T_2 \), and \( t_s \gg s \)). The silicon substrate is therefore assumed to be infinitely thick in the following derivation. The \( w_1, w_2 \) are the widths and \( T_1, T_2 \) are the thickness of the metal strip conductors, \( s \) is the distance between the interconnect conductors and \( t_{ox} \) is the thickness of the silicon oxide.

The silicon substrate impedance given here makes the expression more tractable for use in micrometer and submicrometer designs.

Applying the method of separation of variables and expressing the magnetic field components of the line current as Fourier integrals, the solution of (2) that can lead to eqn. (6) for perturbation part of mutual impedance per unit length is given in the Appendix.

If a lossy silicon substrate has conductivity \( \sigma \) and relative permeability \( \mu_r \), then resulting integral expression (part of mutual impedance due to the “shadow currents” induced in the conductive silicon substrate beneath the interconnect lines) can be written as [4]

\[
Z_{\text{sim}}(\omega) = j \omega \mu_r \mu_0 \int_0^\infty \frac{e^{2\omega \mu \eta \cos(\theta \xi)}}{\sqrt{\lambda^2 + j \mu \lambda + \eta^2}} d\lambda
\]

where \( \lambda \) denotes the spectral domain variable (Fourier Transform domain), \( t_{ox} \) is the thickness of the silicon oxide, \( \mu \) is the magnetic permeability of the substrate and \( s \) the distance between the coupled interconnect conductors.

It is convenient to reduce the number of the explicit parameters in the integral in (6) by setting \( \xi = \frac{m r}{\mu_0}, \) and \( \eta = 2 m r_{eq} \) to obtain

\[
Z_{\text{sim}} = \frac{\omega m r_{eq}}{\mu_0} I_\mu(\xi, \eta),
\]

\[
I_\mu(\xi, \eta) = j \int_0^\infty e^{-\eta \cos(\xi \tau)} \sqrt{\tau^2 + j + \mu \tau} d\tau.
\]

Here \( K(z) \) is shorthand for \( H_2(z) - Y_1(z) \), where \( Y_1 \) is the Bessel function of second kind and order 1, and \( H_2 \) is the Struve function of order 1 (a particular integral of Bessel’s equation). The function \( H_1 \) may be evaluated using the power series [15]

\[
H_1(z) = \frac{2}{\pi} \left( \frac{z^2}{3} - \frac{z^4}{3^2 5} + \frac{z^6}{3^3 5^2 7} - \cdots \right).
\]

3. Numerical Results

In order to validate the derived new formulas, the frequency-dependent mutual per-unit-length inductance and
resistance \[ Z_{\text{sm}}(\omega) = R_m(\omega) + j\omega L_m(\omega) \] for a coupled interconnect structure on a heavily doped CMOS substrate (resistivity \( \rho_{\text{si}} = 0.01 \ \Omega\cdot\text{cm} \)) with a 3 \( \mu \text{m} \) oxide layer are considered. The cross sections of the conductors are 2 \( \mu \text{m} \) by 1 \( \mu \text{m} \) and 1 \( \mu \text{m} \) by 1 \( \mu \text{m} \), and the thickness of the silicon substrate is 300 \( \mu \text{m} \). The spacing between the two conductors is 2 \( \mu \text{m} \). Fig. 2 shows the variation in the distributed mutual resistance, \( R_m(\omega) \), as a function of frequency. Similarly, Fig. 3 shows the change in the distributed mutual inductance, \( L_m(\omega) \), as a function of frequency. It is observed that the values of the mutual inductance and resistance per unit length, calculated from the new formulas, are found to be in good agreement with those of [8] (quasi-magnetostatic analysis and circuit modeling technique). The agreement is excellent over the entire frequency range of 0.4 - 10 GHz. The maximum deviations observed between the new analytic formulas and the EM simulation results of Figs. 2 and 3 correspond to relative errors less than 2 \%. The relative errors between the curves in Figs. 2 and 3 for \( R_m \) and \( L_m \) are shown in Fig. 4. As expected, a lossy silicon substrate has a significant impact on the frequency-dependent characteristics of multiple coupled on-chip interconnects and must be attributed to the skin effects in the substrate.

4. Conclusion

A simple and accurate frequency-dependent analytic model is proposed to calculate the distributed mutual per unit length impedance components of general asymmetric coupled interconnects on a lossy silicon substrate over the entire frequency range. The model includes the frequency-dependent distribution of the induced currents in the conductive silicon substrate beneath the interconnect lines as the return current path. It is clear that the model developed in this letter can be used for the accurate estimation of the distributed mutual per unit length impedance parameters of the CMOS on-chip interconnects.

5. Appendix

Integral expression for the perturbation part of the mutual impedance per unit length (6) is derived as follows [11, 12].

For this reason a straight filament (unit current source) parallel to a semiconducting half-space as shown in Fig. 5 will be first regarded. The conductivity, permittivity and permeability of the silicon substrate are designated as \( \sigma \), \( \varepsilon_r \), \( \mu \), respectively.

A general solution of eqn. (2) may be looked for in the form
\[ A_i(x, y) = \frac{1}{2\pi} \int_0^\lambda \left[ C_1(\lambda)e^{i\omega(x-x')} + C_2(\lambda)e^{-i\omega(x-x')} \right] \cos[\lambda(y-y' - q)]d\lambda, \quad \text{for} \quad y \geq 0 \] (A1.a)

\[ A_i(x, y) = \frac{1}{2\pi} \int_0^\lambda \left[ C_1(\lambda)e^{i\omega(x-x')} \right] \cos[\lambda(y-y')]d\lambda, \quad \text{for} \quad y \leq 0 \] (A1.b)

where \( q = (\lambda^2 + j\omega\mu\epsilon)^{1/2} \) and having considered that \( A_2 \) must vanish at large distance from the current filament, when \( y \) tends to \(-\infty\). In order to determine the unknown coefficients \( C_1, \lambda_1, \) and \( C_2, \) we have to take into account, that when the silicon semiconducting half-space is absent, the magnetic flux density \( B_0 \) of a single current filament can be expressed in the form

\[ B_{0x} = \pm \frac{\mu_0 I}{2\pi} \int_0^\lambda e^{i\omega(x-x')} \cos[\lambda(y-y')]d\lambda \] (A2.a)

\[ B_{0y} = \frac{\mu_0 I}{2\pi} \int_0^\lambda e^{i\omega(x-x')} \sin[\lambda(y-y')]d\lambda \] (A2.b)

Using these expressions and imposing at the boundary surface \( y = 0 \), the continuity of the tangential components of the magnetic flux density and of the normal component of the magnetic flux density, the following expressions were obtained:

\[ A_1(x, y) = \frac{\mu_0 I}{2\pi} \ln \left[ \frac{(x-x')^2 + (y-y')^2}{(x-x')^2 + (y+y')^2} \right] + \mu_0 J e^{i\omega(x-x')} \cos[\lambda(y-y')]d\lambda \] (A3.3)

\[ A_2(x, y) = \frac{\mu_0 I}{2\pi} \ln \left[ \frac{(x-x')^2 + (y-y')^2}{(x-x')^2 + (y+y')^2} \right] + \mu_0 J + \pi \left[ \frac{e^{i\omega(x-x')} \cos[\lambda(y-y')]d\lambda}{\mu_0 \lambda + q} \right] \] (A4.4)

As we can see above the magnetic vector potential is introduced in Maxwell’s equations for the horizontal magnetic field intensity above the lossy silicon semiconducting substrate. This leads to the quasi-static voltage drop \( \partial V/\partial z \) in the z-direction that also appears in the classical line equations,

\[ \frac{\partial V}{\partial z} = -[Z_j]I \] (A5.5)

Hence, the total series impedance per unit length \( Z_j \) (self or mutual) of the interconnect lines can be derived. This method provides the best insight into the relation between the purely mathematical equations and the physical reality of the problem.

The magnetic vector potential is used in order to find the quasi-static potential drop \( \partial V/\partial z \), at any point \((x,y)\) in space along the line parallel to the z direction. This allows the total series self and mutual impedances per unit length, eq. (A. 5), to be evaluated.

The axial electric field intensity along the lossy silicon substrate is

\[ E_{\alpha}(x, y = 0) = -j\omega A_i(x, y = 0) - \frac{\partial V(x, y = 0)}{\partial z} \] (A6.6)

and at any point \((x,y)\) above the lossy silicon substrate

\[ E_{\alpha}(x, y) = -j\omega A_i(x, y) - \frac{\partial V(x, y)}{\partial z}. \] (A7.7)

Subtracting eqn. (A. 6) from eq. (A. 7), we get

\[ E_{\alpha}(x, y) = E_{\alpha}(x, y = 0) - j\omega [A_i(x, y) - A_i(x, y = 0)] \]

\[ - \frac{\partial}{\partial z} [V(x, y) - V(x, y = 0)]. \] (A8.8)

The last term in eqn. (A.8) represents the total scalar voltage drop, in the axial z-direction, of the distributed parameter circuit consisting of interconnects conductor and lossy silicon substrate, as in equation (A. 5).

From the definition of the magnetic vector potential

\[ A_i(x, y) - A_i(x, y = 0) = \mu_0 \int_0^\lambda H_{ik}(x, y) dy \] (A9.9)

where the \( x \)-component of the magnetic field intensity above the lossy silicon substrate is given by

\[ H_{ik}(x, y) = \frac{1}{\mu_0} \frac{\partial A_i(x, y)}{\partial y}. \] (A10.10)

The final simplified equation for electric field component in z-direction above the lossy silicon substrate is

\[ E_{\alpha}(x, y) = - \frac{\partial V(x, y)}{\partial z} - \frac{\partial V(x, y = 0)}{\partial z} - j\omega \frac{\mu_0 I}{2\pi} \ln \left[ \frac{(x-x')^2 + (y+y')^2}{(x-x')^2 + (y-y')^2} \right] \]

\[ - j\omega \frac{\mu_0 I}{2\pi} \left[ \frac{e^{i\omega(x-x')} \cos[\lambda(y-y')]d\lambda}{\mu_0 \lambda + q} \right]. \] (A11.11)

In order to develop expression for the mutual impedance per unit length of coupled interconnect conductors above the lossy silicon substrate a current \( I \) flowing in the interconnect conductor with width \( w_1 \) and thickness \( T_1 \), as depicted in Fig. 1, is now regarded. It can be assumed that the current density is constant over the conductor’s cross section because the skin-effect in the strip or rounded conductor can be neglected due to its small dimensions (up to a few tens GHz). For the interconnect conductor with current \( I \) the axial electric field intensity is equal to the potential drop per unit length on the surface of the interconnect conductors.

The mutual impedance per unit length \( Z_{ik} \) between two circuits (conductors) is given by

\[ Z_{ik} = \frac{E_{\alpha}^{\text{ind}}}{I_k}. \] (A12.12)

where \( E_{\alpha}^{\text{ind}} \) is the z component (axial) of the inductive part of the electric field produced by the \( k \)-th circuit (conductor \( k \)) carrying current \( I_k \) at the axis location of the \( i \)-th circuit (conductor \( i \)).

Substituting the coordinates for the interconnect conductors in equation (A.11), rearranging the terms, we get
\[
\frac{\partial V(x, y)}{\partial z} - \frac{\partial V(x, y = 0)}{\partial z} = \frac{\partial V}{\partial z} = -Z_m I
\]

\[
= -I \left[ j \omega \frac{\mu_0}{2\pi} \ln \left( \frac{(s_q + r_{eq} + r_{2eq})^2 + (H_{eq} + r_{eq} + H_{2eq} + r_{2eq})^2}{(s_q + r_{eq} + r_{2eq})^2 + (H_{eq} + r_{eq} - H_{2eq} - r_{2eq})^2} \right) + j \omega \frac{\mu_0}{\pi} \int_0^\infty \frac{e^{-2\mu_\lambda}}{\mu_\lambda + \sqrt{\alpha^2 + j\omega \mu_\sigma}} \cos(s_\lambda) d\lambda \right]
\]

(A.13)

Dividing both sides of eqn. (A.13) by current \( I \), it can be observed that the total series mutual impedance per unit length consists of two parts. The first term is the per unit length external mutual inductance of coupled on-chip interconnects on a lossy silicon substrate with zero resistivity \( (L_m \text{ given by eqn. (5)}) \), whereas the remaining integral constitutes the perturbation part of the mutual series per unit length impedance that takes into account skin-effect phenomena in a lossy silicon substrate \( (Z_{sim} \text{ given by (6)}) \).

References


