Identification of Differentially Driven Wheeled Mobile Robot using Neural Networks

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ABSTRACT: In the field of control theory system identification is an important way of investigating and understanding the system. Several distinct approaches of system identification produce the different form of knowledge about the system. In system identification, neural network plays an important role. A novel technique has been considered to identify the nonlinear multi input multi output (MIMO) crossed coupled system using neural network. A very interesting mechatronics platform, namely, wheeled mobile robot, is a highly nonlinear cross coupled dynamic system and pose challenges for many classical control techniques, has been considered as a candidate system to illustrate the proposed neural networks based system identification. Static and dynamic back propagation methods are used for system identification. This paper verifies the ability of the neural networks and its applicability for the system identification of practical nonlinear systems.

Keywords: Identification, neural network, back propagation, adaptive identification, nonlinear systems, wheel mobile robot.

I. INTRODUCTION

Mathematical modeling of a practical system is an attractive field of research and a grown up subject with a variety of applications. The model can be very useful for system analysis, i.e., getting the better knowledge of systems, predict and simulate the system behavior. In control theory, models are essential for designing controllers for new processes and for analysis of existing processes. It is a very difficult task to develop mathematical modeling of complicated practical systems considering all its possible high level non linearity and cross couple dynamics. Although a mathematical system theory of nonlinear systems is sound quite interesting but there is no general solution for analysis and synthesis of the nonlinear dynamical systems. In the past decades, the best developed aspect to solve the real systems is defined by linear operators using a well known technique like linear algebra, complex theory and the linear differential equations. The stability of the system is closely related to the mathematical modeling of this linear time-invariant system, and some well-known technique has been established for such a system.

The modeling of the system is obtained from the first principles. The parameters within the model are normally unknown and need to estimate from the experimental data [1]. The main purpose of the system identification is to determine a suitable parametric model in a system, which is effective for predicting the behavior within the system under different operating conditions. In control system theory, many classical approaches are available for the system identification; most of them are off-line identification, but in some practical circumstance where the adaptive control law is applicable throughout the system, it is essential to obtain the system model by applying on-line system identification [2]. In this process, the estimation of the system model with input output data when the system is under operation. On-line identification in the system is more effective if it is accessible by using the fast and economical digital computers.

In non parametric system identification, artificial neural network newly developed technique and widely used in nonlinear system identification due to its adaptability, robustness and inherent capability to handle the nonlinear complex systems [3]. Neural network generates mapping between input and output of any systems and determine a static function empirically with knowing any fundamental physic of the systems. Still dynamic model identification is still a challenging task in neural network based system identification. System identification of nonlinear MIMO system using neural network involves dynamic differential equations for structural dynamic model identification [4]. Few nonlinear system identification techniques already been reported in the literature [5]. The neural network is used effectively for the identification and control of nonlinear dynamical systems [6-8].

In adaptive system identification technique, the estimated output of the model is compared with the true output of the plant, and error fed back to the learning element, which modifies the parameters within the model to reduce the error. In neural network based adaptive identification; parameter of the estimated model is adjusted by changing the weight of the neural network [9]. A fully adaptive normalized non-linear gradient descent (FANNGD) algorithm for neural adaptive filters employed for nonlinear system identification is proposed [10].
A nonlinear MIMO system with cross coupled dynamics is quite common to many applications like industrial robot manipulator, walking robots, wheeled mobile robots, skid-steered vehicles, aerospace, submarine, ship, space vehicles, rockets, etc. Wheeled mobile robot (WMR) is a very popular cross couple non linear MIMO systems. It is becoming more popular for performing the tasks that are dangerous or tedious for humans and with the advancement of the technology it is more cost-effective to replace human effort by the wheeled mobile robot. Laboratory scale wheeled mobile robot has become a benchmark laboratory test setup in control system engineering for its highly nonlinear cross coupled dynamics, attracted a special attention to the control system engineers to the development of robust controllers for achieving robust stability and control performance [11].

Wheeled mobile robot basically a mechanical mobile robot has minimum three wheels. Two drive wheels have a common horizontal axis and one or more wheels named castor wheels assure the robot equilibrium. Three wheels WMR introduced isometric equilibrium; more than three wheels ensure a better stability and control for the robot body. The entire control of the mobile robot depends upon the control of the angular velocities of the two drive wheels and the control laws for mobile robots with nonholonomic constraints that arise from constraining the wheels of the robot to roll without slipping [12]. WMR is a very useful platform to validate the feasibility of the control laws developed for the nonlinear system.

This paper contributes an interesting methodology of system identification and adaptive system identification of nonlinear dynamic system using neural networks is to involve the dynamic differential equation into each of the neural networks. Knowledge of the system dynamics is essential for the structural dynamic model identification, and the vital information from the system is taken from the knowledge of the system physics.

The rest of the paper has been organized as follows. The neural network has been discussed in section 2. The identification of a nonlinear dynamic system has been presented in section 3 and adaptive system identification has been presented in section 4. Section 5 and 6 described the mathematical modeling and discrete modeling of WMR. Finally, Simulation results and conclusions are drawn on the section sections 7 and section 8 respectively.

II. NEURAL NETWORKS

Neural networks are very power full tool for the identification of systems typically encountered in the structural dynamic fields. Neural networks are generating complex mapping between the input and output space. It mainly consists a number of neurons and performs the weighted sum of the input signal and the connecting weight. The neural network is a machine that is designed to model the way in which the brain performs a particular task or function of interest; the network is simulated and it is usually implemented by using electronic components or software on a digital computer. It employed a massive interconnection of simple computing cells referred to as “neurons” or “processing units.” According to the structure, neural network may be a single layer, or it may be a multi layer and recurrent network. The multi layer neural network consists of more than one hidden layer, and each hidden layer consists of one or more neurons. Each neuron of one layer is connected to every single neuron of the next layer.

A. Multi layer Neural Network

A two – layer NN, shown in the figure 1 has \( n \) inputs and two layers of neurons, with first layer having \( m \) neurons that fed into the second layer having \( q \) neurons. The first layer is known as the hidden layer, with \( me \) the number of hidden-layer neurons; the second layer is known as the output layer, with \( q \) the number of outputs-layer neurons. It is common for different layers have different numbers of neurons. Neural networks with multiple layers are called multilayer perceptrons; their computing power is significantly enhanced over the one-layer NN.

\[
z_j = \sigma \left( \sum_{i=1}^{n} w_{ji} x_i + w_{j0} \right); j = 1, 2, \ldots, m \quad \ldots (1)\]

Where

\[
w_j \equiv [w_{j1}, w_{j2}, \ldots, w_{jk}]
\]

In vector-matrix notation, the hidden layer has \( m \times l \) output vector

\[
z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix}
\]

\( m \times n \) weight matrix

\[
W = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\
 w_{21} & w_{22} & \cdots & w_{2n} \\
 \vdots & \vdots & \ddots & \vdots \\
 w_{m1} & w_{m2} & \cdots & w_{mn} \end{bmatrix}
\]

Fig. 1. Multi-layer neural network.
and \( m \times 1 \) bias vector
\[
\mathbf{w}_0 = \begin{bmatrix}
  \mathbf{w}_{10} \\
  \mathbf{w}_{20} \\
  \vdots \\
  \mathbf{w}_{m0}
\end{bmatrix}
\]

The output
\[
z = \mathbf{\Gamma}(\mathbf{Wx} + \mathbf{w}_0) \quad \ldots(2)
\]
where
\[
\mathbf{\Gamma}(.) = \begin{bmatrix}
  \sigma(.) & 0 & \cdots & 0 \\
  0 & \sigma(.) & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \cdots & \sigma(.)
\end{bmatrix}
\]

Defining the second – layer weight matrix as
\[
\mathbf{V} = \begin{bmatrix}
  \mathbf{v}_{11} & \mathbf{v}_{12} & \cdots & \mathbf{v}_{1m} \\
  \mathbf{v}_{21} & \mathbf{v}_{22} & \cdots & \mathbf{v}_{2m} \\
  \vdots & \vdots & \ddots & \vdots \\
  \mathbf{v}_{q1} & \mathbf{v}_{q2} & \cdots & \mathbf{v}_{qm}
\end{bmatrix}
\]
and bias vector as
\[
\mathbf{v}_0 = \begin{bmatrix}
  \mathbf{v}_{10} \\
  \mathbf{v}_{20} \\
  \vdots \\
  \mathbf{v}_{q0}
\end{bmatrix}
\]

One may write the NN output as
\[
\hat{y}_j = \left( \sum_{j=1}^{m} \mathbf{v}_{ij} \mathbf{z}_i + \mathbf{v}_{j0} \right); \quad j = 1,2,\ldots,q \quad \ldots(3)
\]
Where
\[
\mathbf{v}_j = \begin{bmatrix}
  \mathbf{v}_{j1} \\
  \mathbf{v}_{j2} \\
  \vdots \\
  \mathbf{v}_{jm}
\end{bmatrix}
\]

The output vector
\[
\hat{y} = \begin{bmatrix}
  \hat{y}_1 \\
  \hat{y}_2 \\
  \vdots \\
  \hat{y}_q
\end{bmatrix}
\]
is given by the expression
\[
\hat{y} = \mathbf{V}\mathbf{z} + \mathbf{v}_0 = \mathbf{V}(\mathbf{\Gamma}(\mathbf{Wx} + \mathbf{w}_0) + \mathbf{v}_0) \quad \ldots(4)
\]

Figure 2 shows the input-output map.

**Fig. 2.** Input– output map of a two–layer network.

**III. IDENTIFICATION**

System identification is the experimental approach towards the modeling of the systems or an identification of the plant with an unknown parameter. The mathematical modeling of a system is characterized in terms of suitable descriptions such as transfer function, impulse response or power series expansions. The characterization may be parametric or non parametric. The parametric characterizations are a functional form to the input output relation whereas the nonparametric characterization is function mapping can determine empirically without knowing any fundamental physics of the system by using the neural network technique. In control system theory the major problems are system characterization and the identification of the mathematical representation of practical systems.

In static system identification the pattern recognition is most important problem. Compact sets \( U_i \subset \mathbb{R}^n \) are mapped into elements \( y_i \in \mathbb{R}^m \) in the output space by a decision function \( P \). The elements of \( U_i \) denote the pattern vectors corresponding to class \( y_i \). In dynamical systems, the operator \( P \) defining a given plant is implicitly defined by the input-output pairs of time functions \( u(t), y(t), t \in [0,T] \). In both cases the objective is to determine \( \hat{P} \) so that
\[
\| \hat{y} - y \| = \| P(u) - P(u) \| \leq \varepsilon, u \in \mathcal{U} \quad \ldots(5)
\]
for some desired \( \varepsilon > 0 \) and a suitably defined norm (denoted by \( \| \| \) ) on the output space. \( \hat{P}(u) = \hat{y} \) denotes the output of the identification model and hence \( y - y \ \| \ \varepsilon \) is the error between the output generated by \( \hat{P} \) and the observed output \( y \) [6].

The time invariant, causal discrete–time dynamical plant consists the uniformly time bounded input function \( u(.) \) and output \( y(.) \). It is assumed that the candidate system is stable with all its known parameterization but with unknown values of the parameters. The main objective is to develop an identification model which produces the output \( \hat{y}_p(k) \) is approximately equal to the plant output \( y_p(k) \) identical signal \( u(k) \) apply to both plant and the identification model.

**Fig. 3.** Identification Model.
Based on the error between the plant and the identification model outputs a optimize performance function is introduced for adjusting the parameters of the model for setting up a suitable parameterized identification model. The representations of the plant of any nonlinear systems are assumed to belong to a known class in the domain of interest. The identification model structure is identical to the plant model, is achieved by assuming or by using weight matrices of the neural networks in the identical model. The plant and the identification model have the same output for any specified input. In the identification process the parameters of the model are achieved by adjusting the weights of the neural network, based on the error between the plant and model outputs but precautions should be taken to ensure that the results in the convergence of the identification model parameters to their values.

**A. Parallel Identification Model**

The parallel identification model is described by the equation and it is shown in the Fig. 4.

\[
\hat{y}_p(k+1) = \hat{\alpha}_0 \hat{y}_p(k) + \hat{\alpha}_1 \hat{y}_p(k-1) + N[u(k)] \quad \ldots(6)
\]

The entire signals in the plant are uniformly bounded. The stability of the identification model, described by the neural network cannot be assured and the parameters may be converging. There is no guarantee that the error tends to zero. The limitation of the parallel model is overcome by using the series-parallel model.

**B. Series-Parallel Identification Model:** In the series-parallel model the plant output is fed back to the identification model described by the equation no. 7 and shown in the Fig. 5.

\[
\hat{y}_p(k+1) = \hat{\alpha}_0 y_p(k) + \hat{\alpha}_1 y_p(k-1) + N[u(k)] \quad \ldots(7)
\]

Series-parallel identification model has several advantages over the parallel model. All signals used in the identification are bounded and static back propagation is use to adjust the parameters to reduce the error between the plant and the identification model.

**IV. WHEELED MOBILE ROBOT**

The most popular constructions of the mobile robots are two wheeled mobile robot with differential drive as shown in the Fig. 7. Two independent DC motors are left and right drive wheels of the robot in a horizontal axis and one free caster wheel is used to keep the platform of the robot stable. The robot configuration is represented by the position of the center of the axis between the two wheels in the Cartesian space (x and y) and by its orientation \( \theta \) (angle between the vector of the robot orientation and the abscissas axis). Kinematic and the dynamic modeling of the mobile robot are taken into the consideration [13]. The kinematic modeling described the relations between the derivatives of the robot position and orientation and the robot linear and angular speeds, \( \mathbf{v} \) and \( \mathbf{w} \).
The dynamic model is derived from the physics laws and all the physical parameters are taken into the consideration in the dynamic modeling of the WMR.

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix}
= \begin{bmatrix}
\cos \theta & 0 & v_x \\
\sin \theta & 0 & v_y \\
0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
v_x \\
v_y \\
\omega
\end{bmatrix}
\]

This equation models the non-holonomic restrictions of the robot, due to the fact of its wheels not allowing lateral movements.

### B. Dynamic Modeling

The parameterized dynamic model of a differential-derived wheeled mobile robot is derived from the physical laws that govern the robot subsystems, including the actuator dynamics like electrical and the mechanical characteristics of the actuators, friction and robot dynamics (movement equations) [14]. The second order dynamic modeling of WMR expressed by

\[
M \ddot{\mathbf{v}} + B \dot{\mathbf{v}} = K \mathbf{u}
\]  

Where \( V = [v \ w]^T \) represents the robot linear and angular speeds, \( u = [e_1 \ e_2]^T \) represents the input signals to the right and the left motor of the driving wheels, \( M \) is the generalized inertia matrix, \( B \) is the generalized damping matrix, consists of the viscous friction and electrical resistance and \( K \) is the matrix which transforms electrical signals \( u \) into the forces to be generated by the rotor wheel.

### C. Complete modeling of wheel mobile rotor

The complete modeling of the wheel is obtained by combining the kinematic and the dynamic modeling represented by the equations 8 and 9 respectively.

\[
\begin{bmatrix}
\dot{v} \\
\dot{\omega} \\
\dot{l} \\
\dot{\theta}
\end{bmatrix}
= \begin{bmatrix}
-M^{-1}B & 0 & 0 \\
0 & \omega & 0 \\
0 & 0 & 0 \\
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
v \\
\omega \\
l \\
\theta
\end{bmatrix}
\]

where the system output \( y = [x \ y \ \theta]^T \) corresponding to the robot configuration.

To obtain linear representation of the wheel mobile robot the position \( x \) and \( y \) and the orientation \( \theta \) is described in terms of the robot linear displacement \( l \) and the robot orientation \( \theta \).

\[
\begin{bmatrix}
\dot{v} \\
\dot{\omega} \\
\dot{l} \\
\dot{\theta}
\end{bmatrix}
= \begin{bmatrix}
-M^{-1}B & 0 & 0 & e_v \\
0 & \omega & 0 & e_w \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
ev \\
ew \\\nell \\
\theta
\end{bmatrix}
\]

where \( z = [l \ \theta]^T \)

Without any simplification assumption the linear model of the WMR is formulated from the nonlinear model. Thus the linear model is an exact equivalent representation of the original nonlinear representation.

### V. DISCRETE MODELING OF WHEELED MOBILE ROBOT

The system identification is concerned with the discrete time systems which can be represented by discrete state space model. The transformation of the state space model to a continuous transfer matrix is the first step towards the formulation of the discrete modeling of the WMR [14]. The equivalent transfer matrix representation of the WMR is

\[
\begin{bmatrix}
L(s) \\
\theta(s)
\end{bmatrix} = \begin{bmatrix}
G_{r1}(s) & G_{r2}(s) \\
G_{z1}(s) & G_{z2}(s)
\end{bmatrix}
\begin{bmatrix}
E_r(s) \\
E_z(s)
\end{bmatrix}
\]  

\[
\begin{bmatrix}
E_r(s) \\
E_z(s)
\end{bmatrix} = \begin{bmatrix}
G_{r1}(s) & G_{r2}(s) \\
G_{z1}(s) & G_{z2}(s)
\end{bmatrix}
\begin{bmatrix}
L(s) \\
\theta(s)
\end{bmatrix}
\]  

\[
L(s) = \begin{bmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\theta(s) = \begin{bmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
where each term $G_{ij}(s)$ has the following structure:

$$G_{ij}(s) = \frac{N_{ij}(s)}{D(s)} = \frac{\alpha_{ij}s + \beta_{ij}}{s^2 + ks + k_2} + \frac{\gamma_{ij}}{s}$$

Now the transfer matrix is represented into a discrete transfer matrix

$$\begin{bmatrix} L(z) \\ \theta(z) \end{bmatrix} = \begin{bmatrix} G_{11}(z) & G_{12}(z) \\ G_{21}(z) & G_{22}(z) \end{bmatrix} \begin{bmatrix} E_1(z) \\ E_2(z) \end{bmatrix}$$

and each term $G_{ij}(z)$ has the following expression

$$G_{ij}(z) = \frac{N_{ij}(z)}{D(z)} = \frac{\gamma_{ij}z^2 + \delta_{ij} + \epsilon_{ij}}{(z-1)(z^2 + \alpha_i z + \alpha_j)}$$

<table>
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<tr>
<th>$ij$</th>
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<th>$\delta_{ij}$</th>
<th>$\epsilon_{ij}$</th>
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<td>$\alpha_2$</td>
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</tbody>
</table>

Discrete state model of the WMR is derived from the discrete transfer matrix for identification of the differential driven WMR.

### VI. SIMULATION RESULT

In this section the simulation results for nonlinear cross couple dynamic MIMO plant identification model by using neural network. The series –parallel model is used to identify the given plant and static back propagation technique is used to adjust the parameter of the neural networks. The plant of the WMR is described by the equations

$$\begin{bmatrix} x_n(k+1) \\ x_p(k+1) \end{bmatrix} = \begin{bmatrix} f[x_n, x_n(k-1), x_p(k-2)] \\ f[x_p, x_n(k-1), x_p(k-2)] \end{bmatrix} + 0.25 \begin{bmatrix} u_n(k) \\ u_p(k) \end{bmatrix}$$

where

$$f[x_n, x_n(k-1), x_p(k-2)] = 1.212x_n + 0.3237x_n(k-1) - 0.5362x_n(k-2)$$

The series parallel model consists of two neural networks $N^1$ and $N^2$ is described by the equations

$$\begin{bmatrix} \hat{x}_n(k+1) \\ \hat{x}_p(k+1) \end{bmatrix} = \begin{bmatrix} N^1[x_n, x_n(k-1), x_p(k-2)] \\ N^2[x_p, x_n(k-1), x_p(k-2)] \end{bmatrix} + 0.25 \begin{bmatrix} \hat{u}_n(k) \\ \hat{u}_p(k) \end{bmatrix}$$

The neural network belongs to the class $\mathcal{F}^3_{1,2,10,1}$ and the gradient method is employed with a step size of $\eta = 0.1$. The control of the robot depends on controlling the angular velocities of the two drive wheels. If the angular velocities of the two wheels are identical in values, the robot makes a linear motion. The identical sinusoid input signals $u_n(k) = \sin(2\pi k / 25)$ and $u_p(k) = \sin(2\pi k / 25)$ are applied to the two drive wheels of the robot for linear motion.
After completion of the training of multi-layer neural network there is considerable difference between the plant and the identification model of the WMR. The RMS error between the plant and the identification model are 1.7420 and 1.7396 for the outputs of the WMR. The output error between the plant and the identification model is shown in the Fig. 10. and Fig. 11. The error between the estimated outputs and the desired output can be reduced by the adaptive identification methods. In the adaptive identification method the parameters of the identification model are adjusted by using the algorithm, which actually modify the weight of the identification model depending on the error between the plant and the identification model.

![Fig. 12. Output of the plant and Identification Model.](image1)

![Fig. 13. Output of the plant and Identification Model.](image2)

![Fig. 14. Error between Plant and Identification Model.](image3)

![Fig. 15. Error between Plant and Identification Model.](image4)

VII. CONCLUSION

This paper presented a systematic approach toward the identification of the highly nonlinear MIMO system with cross coupled dynamics and further applying the adaptive identification technique to generate more accurate identification model relating to the plant model of the WMR. In non parametric system identification a newly developed neural network base technique is widely used due its strong mapping capability and adaptability. Multilayer neural network is used for adjusting the parameter of the nonlinear system identification and back propagation technique is introduced in this context to generate partial derivatives of a performance index with respect to adjustable parameters.

The simulation studies of the nonlinear dynamical system reveal that the identification generates a considerable error between the estimated identification model and the desire plant model. The RMS error between the plant and the identification model are 1.7420 and 1.7396 respectively. In adaptive identification more precise estimated model is generated by adjusting the parameter of the identification model by neural network, it actually modifies the weight of the estimated model depending on the error between the model and the plant. The simulation results of the adaptive identification of WMR established the effectiveness of the algorithm shown in the Fig.12 and Fig. 13. The RMS error between the estimated and actual are model 0.1337 and 0.1380 respectively. The adaptive identification method is more effective than the normal identification method.
REFERENCE


