

# Weighted OFDM for Wireless Multipath Channels\*

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**SUMMARY** In this paper the novel method of “weighted OFDM” is addressed. Different types of weighting factors (including Rectangular, Bartlett, Gaussian, Raised cosine, Half-sin and Shanon) are considered. The impact of weighting of OFDM on the peak-to-average power ratio (PAPR) is investigated by means of simulation and is compared for the above mentioned weighting factors. Results show that by weighting of the OFDM signal the PAPR reduces. Bit error performance of weighted multicarrier transmission over a multipath channel is also investigated. Results indicate that there is a trade off between PAPR reduction and bit error performance degradation by weighting.

**key words:** *Orthogonal Frequency Division Multiplexing (OFDM), peak-to-average power ratio (PAPR), multipath channels, bit error performance*

## 1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) also called Multicarrier (MC) technique is a modulation method that can be used for the high speed data communications. In this modulation scheme transmission is carried out in parallel on the different frequencies. This technique is desirable for the transmission of the digital data through the multipath fading channels. Since by the parallel transmission, the deleterious effect of fading is spread over many bits, therefore, instead of a few adjacent bits completely destroyed by the fading, it is more likely that several bits only be slightly affected by the channel. The other advantage of this technique is its spectral efficiency. In the MC method the spectra of subchannels overlap each other while satisfying orthogonality, giving rise to the spectral efficiency. Because of the parallel transmission in the OFDM technique the symbol duration is increased. This has the added advantage of this technique to work in the channels having impulsive noise characteristics. Other advantage of the MC method is its implementation with the fast Fourier

transform (FFT) algorithm which provides full digital implementation of the modulator and demodulator.

The idea of the multi-carrier transmission appeared about three decades ago [1], [2]. However, with the advent of new technologies in Digital Signal Processing, there has been a great interest in using this technique in the digital communication systems [3]. The cellular mobile system based on the OFDM technique has been analyzed and simulated in [4]. The advantage of using MC transmission over a multipath fading mobile channel is reported in [5]. The performance of OFDM/FM modulation for a digital mobile Rayleigh fading channel is evaluated in [6]. In [7] the bit error performance of the MC system with differential detection scheme is analyzed and simulated, and in [8] the bit error probability of the OFDM technique in a Rician multipath fading environment is analyzed. The transmission characteristics of MC signal in digital broadcasting systems are investigated in [9] and in [10] the bit error performance of the OFDM system in a mobile fading channel is presented. In [11] and [12] performance of OFDM transmission over realistic indoor radio propagation channels is evaluated. MC transmission with nonuniform carriers is discussed in [13] and its performance is evaluated over a multipath channel. The optimal waveform for the OFDM transmission over a wireless multipath channel is designed in [14].

There are disadvantages regarding the OFDM technique. Besides from sensitivity of multicarrier signal to phase noise and frequency offset [15], the MC technique suffers from the non-constant envelope characteristics of the OFDM signal. Usually efficient power amplifiers in the transmitter are working in the nonlinear region. This nonlinearity added with the non-constant envelope OFDM signal cause intermodulation products which in turn increases the intercarrier interference and degrades the performance of the system. Different techniques have recently been reported to reduce the peak to average power ratio (PAPR) of the OFDM signal. In [16] clipping and filtering of OFDM signal is suggested to reduce the PAPR, and its influence on the performance of the system is investigated. Block coding technique is proposed to reduce the PAPR [17]–[20] and with the maximum-length sequences in [21], [22]. A selective scrambling technique is reported in [23] to lessen the PAPR problem of OFDM signal.

In this paper the novel method of weighted OFDM

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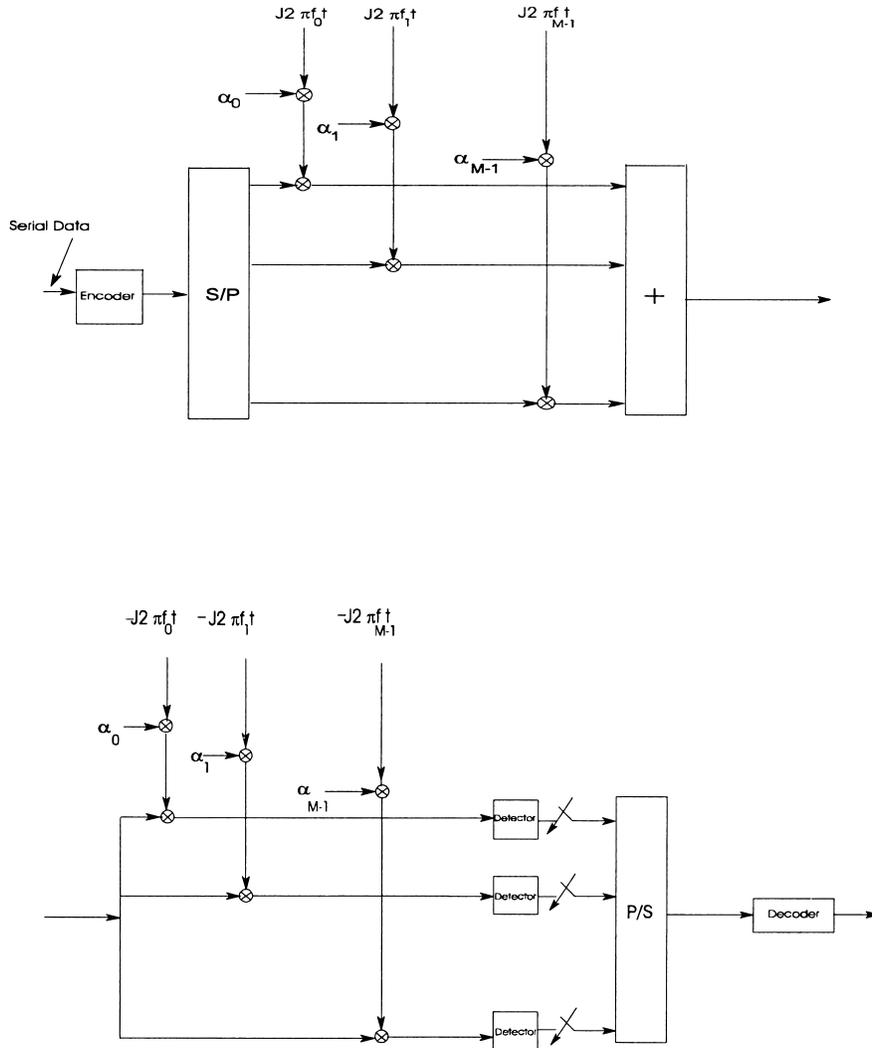


Fig. 1 Block diagram of the weighted multicarrier system.

is proposed and the PAPR reduction associated with this technique is reported. The paper is organized as follows: In Sect. 2 the weighted multicarrier modulation technique is explained. In Sect. 3 different weighting functions (Rectangular, Bartlett, Gaussian, Raised cosine, Half-sine and Shanon), are described and are used for weighting of the OFDM signal. The PAPR of the MC signal with these weightings is investigated in Sect. 4 and simulation results are provided. In Sect. 5 the impact of the above mentioned weighting functions on the bit error performance of the MC transmission over a multipath channel is investigated and compared. Concluding remarks appear in Sect. 6.

## 2. Weighted Multicarrier Modulation

In serial data transmission, sequences of data are transmitted as a train of serial pulses. However, in parallel transmission each bit of a sequence of  $M$  bits modulates a carrier. In the multicarrier technique transmission is

parallel. The block diagram of the weighted multicarrier technique is similar to the conventional MC method but with a difference that each carrier is weighted by a real factor  $\alpha_m$ ,  $m = 0, 1, 2, \dots, M - 1$ , as shown in Fig. 1. In the modulator the input data with the rate  $R$  is divided into the  $M$  parallel information sequences with the rate  $R/M$ . Each sequence modulates a weighted subcarrier. In the OFDM method the frequency of  $m$ th carrier is

$$f_m = f_0 + \frac{m}{T} \quad m = 0, 1, 2, \dots, M - 1 \quad (1)$$

where  $f_0$  is the lowest frequency (and without loss of generality can be considered zero),  $M$  is the number of carriers and  $T$  is the OFDM symbol duration.

The weighted MC transmitted signal is

$$x(t) = \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} b_m(i) \alpha_m e^{j2\pi \frac{m}{T}(t-iT)} p(t-iT) \quad (2)$$

where, as mentioned earlier,  $\alpha_m$  is the real weighting

factor of the  $m$ th carrier,  $b_m(i)$  is the symbol of the  $m$ th subchannel at time interval  $iT$ , which is  $\pm 1$  for BPSK modulation and  $(\pm 1 \pm j)/\sqrt{2}$  for QPSK,  $p(t)$  is a rectangular function with amplitude one and duration  $T$ .

### 3. Different Weighting Factors for OFDM

In this section several weighting factors for weighting of the OFDM signal is described.

**Rectangular:** This weighting function has a rectangular shape and is expressed by

$$\alpha_{1,m} = \begin{cases} A & 0 \leq m \leq M-1 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

**Bartlett:** This weighting function has simply a triangular shape for  $0 \leq m \leq M-1$ , i.e.,

$$\alpha_{2,m} = \begin{cases} A \left( 1 - \frac{|m - \frac{M}{2}|}{\frac{M}{2}} \right) & \text{if } 0 \leq m \leq M-1 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

**Gaussian:** These factors are generated based on the Gaussian function, i.e.,

$$\alpha_{3,m} = \begin{cases} A \exp \left[ -\frac{\left( m - \frac{M}{2} \right)^2}{2s^2} \right] & \text{if } 0 \leq m \leq M-1 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

where  $s$  is the spread or standard deviation (std.) of the weighting factors around  $M/2$ .

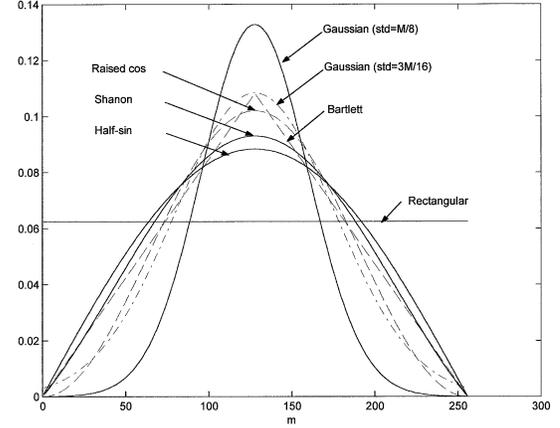
**Raised cosine:** The shape of this function in the interval  $[0, M-1]$  is described by  $1 - \cos(2\pi m/M)$  or equivalently

$$\alpha_{4,m} = \begin{cases} A \sin^2 \left( \pi \frac{m}{M} \right) & \text{if } 0 \leq m \leq M-1 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

**Half-sin:** This weighting function is explained by

$$\alpha_{5,m} = \begin{cases} A \sin \left( \pi \frac{m}{M} \right) & \text{if } 0 \leq m \leq M-1 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

**Shanon:** The shape of this weighting factors is the sinc function i.e.,  $\text{sinc}(x) = (\sin(\pi x))/(\pi x)$ , and is written as



**Fig. 2** Different shapes for the weighting factors  $\alpha_m$  of the OFDM signal ( $M = 256$ ).

$$\alpha_{6,m} = \begin{cases} \text{Asinc} \left( \frac{2m - M}{M} \right) & \text{if } 0 \leq m \leq M-1 \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

These waveforms are sketched in Fig. 2. For the sake of simplicity in this figure the weighting functions are plotted continuous, however, it should be noted that the weighting factors are discrete and have value at the integer  $m$ . Meanwhile, in order to compare the performance of the OFDM system with the above mentioned weighting factors, the amplitude  $A$  in (3)–(8) is selected in such a way that the power of all weighting factors be constant, i.e.,

$$\sum_{m=0}^{M-1} \alpha_{i,m}^2 = 1 \quad i = 1, 2, \dots, 6 \quad (9)$$

### 4. PAPR of Weighted OFDM

In this section the impact of weighting of OFDM signal on the PAPR is investigated. The OFDM signal of (2) in the time interval of  $0 \leq t \leq T$  can be written as

$$x(t) = \sum_{m=0}^{M-1} b_m \alpha_m e^{j2\pi \frac{m}{T} t} \quad (10)$$

For the calculation of PAPR first by using (10) we obtain the instantaneous power of OFDM signal as

$$P(t) = |x(t)|^2 = \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} b_m \alpha_m (b_n \alpha_n)^* e^{j \frac{2\pi(m-n)}{T} t} \quad (11)$$

Which can be written as

$$P(t) = \sum_{m=0}^{M-1} |b_m \alpha_m|^2$$

$$+ \sum_{m=0}^{M-1} \sum_{n=0, n \neq m}^{M-1} b_m \alpha_m (b_n \alpha_n)^* e^{j \frac{2\pi(m-n)}{T} t} \quad (12)$$

Averaging the power  $P(t)$  yields

$$E[P(t)] = \sum_{m=0}^{M-1} |b_m|^2 |\alpha_m|^2 + \sum_{m=0}^{M-1} \sum_{n=0, n \neq m}^{M-1} E[b_m b_n^*] \alpha_m \alpha_n^* e^{j \frac{2\pi(m-n)}{T} t} \quad (13)$$

The symbols on different carriers are assumed to be independent i.e.,  $E[b_m b_n^*] = \delta(m - n)$ , therefore, the second term in (13) is zero and accordingly, by using (9) the average power becomes

$$E[P(t)] = \sum_{m=0}^{M-1} |\alpha_m|^2 = 1 \quad (14)$$

The variation of the instantaneous power of OFDM signal from the average is

$$\Delta P(t) = P(t) - E[P(t)] = \sum_{m=0}^{M-1} \sum_{n=0, n \neq m}^{M-1} b_m \alpha_m (b_n \alpha_n)^* e^{j \frac{2\pi(m-n)}{T} t} \quad (15)$$

Averaging of  $(\Delta P(t))^2$  over a symbol period of  $T$  yields [23]

$$\rho = \frac{1}{T} \int_0^T (\Delta P(t))^2 dt = \sum_{i=1}^{M-1} |R_{cc}(i)|^2 \quad (16)$$

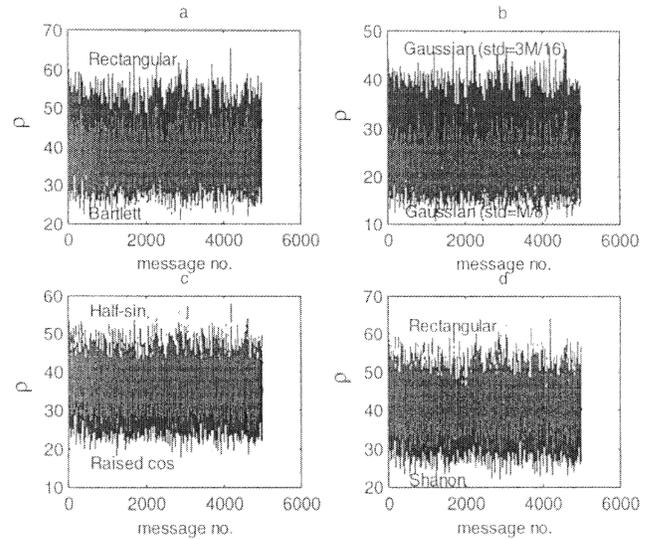
Where  $R_{cc}(i)$  is the autocorrelation function of the complex sequence  $c_m = b_m \cdot \alpha_m$

$$R_{cc}(i) = \sum_{m=0}^{M-1-i} c_m c_{m+i}^* \quad (17)$$

The parameter  $\rho$  is the power variance of the OFDM signal and as is described below is a good measure of the PAPR. Using (14) the PAPR of the OFDM signal is written as

$$PAPR = \frac{\text{Max}\{P(t)\}}{\text{Mean}\{P(t)\}} = \text{Max}\{P(t)\} = P_{\max} \quad (18)$$

Referring to (12) and by considering a large number of terms in the summations, and by using the Central Limit Theorem,  $P(t)$  can be approximated as a Gaussian random process with mean 1 and variance  $\rho$ . (See (14)–(16)). PAPR can be related to the power variance  $\rho$ . Let  $\beta$  denote the probability that  $P(t)$  be less than or equal to  $P_{\max}$ , i.e.,



**Fig. 3** Power variance  $\rho$  of OFDM signal with different weighting factors. BPSK modulation,  $M = 256$ . a) Black curve: Rectangular, Gray curve: Bartlett, b) Black curve: Gaussian ( $s = 3M/16$ ), Gray curve: Gaussian ( $s = M/8$ ), c) Black curve: Raised cos, Gray curve: Half-sin, d) Black curve: Shanon, Gray curve: Rectangular.

$$\beta = \text{Prob}[P \leq P_{\max}] = \int_0^{P_{\max}} \frac{1}{\sqrt{2\pi\rho}} \exp\left\{-\frac{(P-1)^2}{2\rho}\right\} dP \quad (19)$$

Using (18) and (19) it can be easily shown that PAPR has the following relationship with the power variance  $\rho$ :

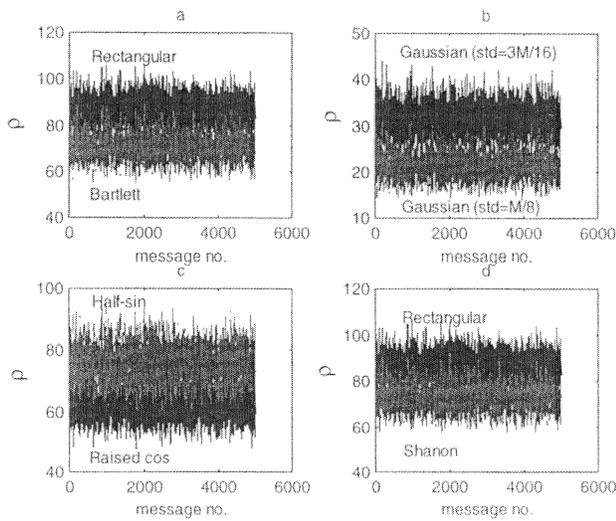
$$Q\left(\frac{PAPR-1}{\sqrt{\rho}}\right) + Q\left(\frac{1}{\sqrt{\rho}}\right) = \beta \quad (20)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2} du \quad (21)$$

Using (20) and knowing  $\rho$  and  $\beta$  the exact value of PAPR can be calculated. From (20) it is seen that for a fixed  $\beta$  the MC signal with high PAPR has a high value of  $\rho$ . In this paper we concentrate on the power variance  $\rho$  and assess its value when different weighting functions—described in Sect. 3—are used for weighting of the MC signal. The BPSK and QPSK modulations are considered and simulations are carried out for 5000 symbols weighted by proper weighting functions. In Figs. 3 and 4 the parameter  $\rho$  versus number of the messages for BPSK and QPSK modulations and for different weightings of the OFDM signal are sketched, respectively. From these figures it is clear that the power variance for BPSK is lower than QPSK. For both modulations the weighting of OFDM signal reduces the power variance  $\rho$ . The level of the reduction of  $\rho$  depends on the shape of the weighting factors. In Table 1 the aver-

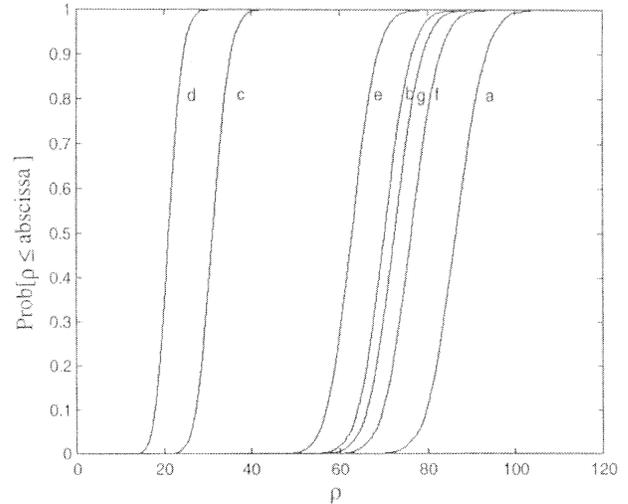
age value of  $\rho$  over a period of 5000 symbols for different weighting factors and different modulations is depicted. Figure 5 illustrates the CDF of the power variance for different weighting functions. It is quite obvious that weighting of the MC signal shifts the CDF of the  $\rho$  to the left. For both modulations the Gaussian weighting provides the least level of power variance. Referring to Table 1, for the OFDM signal with 256 carriers and BPSK modulation, by applying the Gaussian weighting (with the standard deviation of  $M/8$ ), the  $\rho$  is reduced by a factor of 3.2 dB and for the QPSK modulation by a factor of 6.1 dB. In Fig. 6 the influence of changing the spread of Gaussian weighting functions on the power variance has been shown. By decreasing the spread of the Gaussian weights the average  $\rho$  decreases remarkably. In the limiting case when spread of the Gaussian weights tends to zero the weighted OFDM system tends to the single carrier system which will have the power variance of 0 (or no PAPR problem).



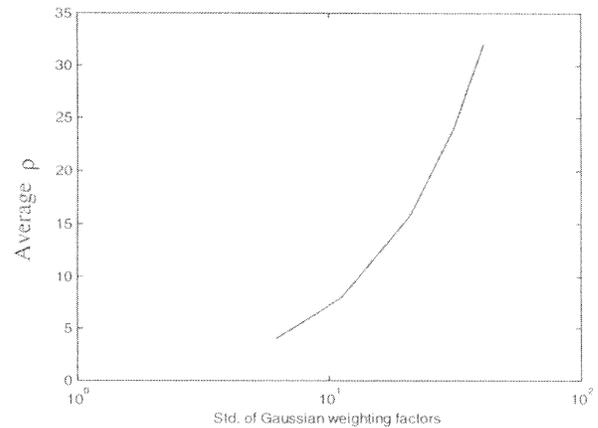
**Fig. 4** Power variance  $\rho$  of OFDM signal with different weighting factors. QPSK modulation,  $M = 256$ . a) Black curve: Rectangular, Gray curve: Bartlett, b) Black curve: Gaussian ( $s = 3M/16$ ), Gray curve: Gaussian ( $s = M/8$ ), c) Black curve: Raised cos, Gray curve: Half-sin, d) Black curve: Shanon, Gray curve: Rectangular.

## 5. Performance Results

In the previous section we saw how weighting of the MC



**Fig. 5** CDF of the power variance  $\rho$  of OFDM signal with different weighting factors. QPSK modulation,  $M = 256$ . a) Rectangular, b) Bartlett, c) Gaussian ( $std = 3M/16$ ), d) Gaussian ( $std = M/8$ ), e) Raised cos, f) Half-sin, g) Shanon.



**Fig. 6** Average power variance  $\rho$  versus standard deviation of the Gaussian weighting factors, QPSK modulation,  $M = 256$ .

**Table 1** Average power variance  $\rho$  (in dB), for different weighting factors of MC signal,  $M = 256$ .

Weighting Factors\ Average $\rho$ (dB)	BPSK	QPSK
Rectangular	16.4	19.4
Bartlett	15.5	18.5
Gaussian (std=3M/16)	14.9	15.0
Gaussian (std=M/8)	13.2	13.3
Raised cos	15.0	18.0
Half-sin	15.6	18.8
Shanon	15.6	18.6

signal reduces the power variance or peak-to-average power ratio of the OFDM. In this section we investigate the influence of the weighting of the OFDM signal on the performance of the system.

The OFDM signal of (2) is passed through the multipath channel. In our study, for the sake of simplicity, multipath channel is modeled as two impulse responses with equal amplitudes which are separated by some time  $\Delta$ . The rms delay spread of the channel (i.e., the square root of the second central moment of power delay profile) is calculated as  $\Delta/2$ . By transmitting the OFDM signal of (2) through this channel, the received signal  $y(t)$ , (i.e., the output of the channel with the impulse response  $h(t)$ ), is

$$y(t) = x(t)^*h(t) + \nu(t) \quad (22)$$

where  $\nu(t)$  is additive white Gaussian noise (AWGN) with a double side spectral density height of  $N_0/2$ . The first term of (22) is written as

$$x(t)^*h(t) = \sum_{n=0}^l \sum_{m=0}^{M-1} \sum_{i=-\infty}^{\infty} b_m(i)\alpha_m e^{j2\pi f_m(t-\tau_n)} \cdot p(t-iT-\tau_n) \quad (23)$$

where  $\tau_n$  is the time of arrival of the  $n$ th path component of the impulse response of the channel  $h(t)$ . In the receiver the recovery of data associated with the carrier  $f_k$  is performed by taking the decision variable  $z_k$  as

$$z_k = \int_0^T y(t)\alpha_k e^{-j2\pi f_k t} p(t) dt \quad (24)$$

which can be written as

$$z_k = \sum_{m=0}^{M-1} \sum_{n=0}^1 \left\{ b_m(-1) \int_0^{\tau_n} \alpha_m \alpha_k e^{j[2\pi(f_m-f_k)t-\phi_{m,n}]} dt + b_m(0) \int_{\tau_n}^T \alpha_m \alpha_k e^{j[2\pi(f_m-f_k)t-\phi_{m,n}]} dt \right\} + \int_0^T \nu(t)\alpha_k e^{-j2\pi f_k t} dt \quad (25)$$

where  $\phi_{m,n} = 2\pi f_m \tau_n$ . Equation (25) can be written as

$$z_k = \sum_{n=0}^1 \left\{ b_k(-1) \int_0^{\tau_n} \alpha_k^2 e^{-j\phi_{k,n}} dt + b_k(0) \int_{\tau_n}^T \alpha_k^2 e^{-j\phi_{k,n}} dt \right\} + \sum_{m=0, m \neq k}^{M-1} \sum_{n=0}^1 \left\{ b_m(-1) \cdot \int_0^{\tau_n} \alpha_m \alpha_k e^{j[2\pi(f_m-f_k)t-\phi_{m,n}]} dt + b_m(0) \int_{\tau_n}^T \alpha_m \alpha_k e^{j[2\pi(f_m-f_k)t-\phi_{m,n}]} dt \right\}$$

$$+ \int_0^T \nu(t)\alpha_k e^{-j2\pi f_k t} dt \quad (26)$$

Referring to the impulse response of the channel  $\tau_0 = 0$ , and  $\tau_1 = \Delta$ . Hence,

$$z_k = b_k(0) \left[ \int_0^T \alpha_k^2 dt + \int_{\Delta}^T \alpha_k^2 e^{-j\phi_{k,1} t} dt \right] + b_k(-1) \int_0^{\Delta} \alpha_k^2 e^{-j\phi_{k,1} t} dt + \sum_{m=0, m \neq k}^{M-1} \sum_{n=0}^1 \left\{ b_m(-1) \cdot \int_0^{\tau_n} \alpha_m \alpha_k e^{j[2\pi(f_m-f_k)t-\phi_{m,n}]} dt + b_m(0) \int_{\tau_n}^T \alpha_m \alpha_k e^{j[2\pi(f_m-f_k)t-\phi_{m,n}]} dt \right\} + \int_0^T \nu(t)\alpha_k e^{-j2\pi f_k t} dt \quad (27)$$

Equation (27) can be expressed as the following

$$z_k = b_k(0) \left[ \int_0^T \alpha_k^2 dt + \hat{X}_{k,k}^1 \right] + b_k(-1) X_{k,k}^1 + \sum_{n=0}^1 \sum_{m=0, m \neq k}^{M-1} \{ b_m(-1) X_{m,k}^n + b_m(0) \hat{X}_{m,k}^n \} + w_k \quad (28)$$

where  $b_m(-1)$  and  $b_m(0)$  indicate the previous and current symbols respectively, which are transmitted at the carrier  $f_m$  with weighting  $\alpha_m$ , and

$$X_{m,k}^n = e^{-j\phi_{m,n}} R_{m,k}(\tau_n) \quad \hat{X}_{m,k}^n = e^{-j\phi_{m,n}} \hat{R}_{m,k}(\tau_n) \quad (29)$$

and the partial cross correlations are given by

$$R_{m,k}(\tau_n) = \int_0^{\tau_n} \alpha_m \alpha_k e^{-j2\pi(f_m-f_k)t} dt$$

$$\hat{R}_{m,k}(\tau_n) = \int_{\tau_n}^T \alpha_m \alpha_k e^{-j2\pi(f_m-f_k)t} dt \quad (30)$$

The last term in (28) is due to noise and is expressed by

$$w_k = \int_0^T \nu(t)\alpha_k e^{-j2\pi f_k t} dt \quad (31)$$

From (28) it is seen that the first term is the desired signal, the second term is due to intersymbol interference (ISI) caused by the multipath channel. The third term relates to the loss of orthogonality between subcarriers also due to the multipath channel, which is the intercarrier interference ICI. Accordingly, (28) can be

rewritten as

$$z_k = \text{desired signal} + \text{ISI} + \text{ICI} + w_k \quad (32)$$

where

$$\text{desired signal} = b_k(0) \left[ \int_0^T \alpha_k^2 dt + \hat{X}_{k,k}^1 \right] \quad (33)$$

and

$$\text{ISI} = b_k(-1) X_{k,k}^1 \quad (34)$$

and

$$\text{ICI} = \sum_{n=0}^1 \sum_{m=0, m \neq k}^{M-1} \{b_m(-1) X_{m,k}^n + b_m(0) \hat{X}_{m,k}^n\} \quad (35)$$

For the BPSK modulation the bit error rate performance of the OFDM system with utilization of the above mentioned weighting factors is evaluated as the sampled signal of (28) or (32) be less than zero assuming a 1 has been transmitted, i.e.,

$$P(E) = \text{Prob}(z_k < 0 | b_k(0) = 1) \quad (36)$$

As mentioned above the ISI, ICI and noise contribute as interference on the desired signal. Ignoring noise and assuming independent interference components, the total power of the interference is obtained as

$$\text{Var}(z_k) = \sigma_{\text{ISI},k}^2 + \sigma_{\text{ICI},k}^2 \quad (37)$$

referring to the (35) and by considering a large number of components, the decision variable can be approximated as a Gaussian random variable. With this assumption the error probability in detection of data bit on the  $k$ th subcarrier is calculated as

$$P(E|k) = Q \left\{ \sqrt{\frac{P_{\text{desired},k}}{\sigma_{\text{ISI},k}^2 + \sigma_{\text{ICI},k}^2}} \right\} \quad (38)$$

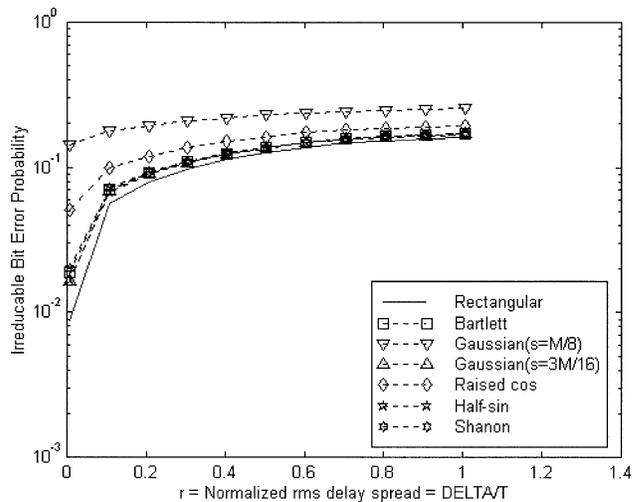
where  $Q(\cdot)$  is expressed by (21). The  $P_{\text{desired},k}$  is the power of the  $k$ th desired signal and by using (33) is computed as  $\alpha_k^4 T^2 [1 + (1-r)^2 + 2(1-r) \cos 2\pi kr]$  where  $r = \Delta/T$ . As shown in the Appendix A, the variance of ISI is

$$\sigma_{\text{ISI},k}^2 = |X_{k,k}^1|^2 \quad (39)$$

Referring to the Appendix B the variance of ICI is calculated as

$$\begin{aligned} \sigma_{\text{ICI},k}^2 = & \sum_{m=0, m \neq k}^{M-1} \{ |\hat{X}_{m,k}^0|^2 + |\hat{X}_{m,k}^1|^2 + |X_{m,k}^1|^2 \\ & + \hat{X}_{m,k}^1 (\hat{X}_{m,k}^0)^* + \hat{X}_{m,k}^0 (\hat{X}_{m,k}^1)^* \} \end{aligned} \quad (40)$$

Now according to the total probability theory, the probability of bit error is calculated as



**Fig. 7** Irreducible bit error probability for different weighting factors vs. rms delay spread of the the channel ( $M = 256$ ).

$$P(E) = \frac{1}{M} \sum_{k=0}^{M-1} P(E|k) \quad (41)$$

In Fig. 7 the irreducible bit error probability of the OFDM signal with different weighting functions versus rms delay spread of the channel is illustrated. From this figure it is clear that by weighting of the MC signal the bit error probability deteriorates. Among the considered weighting functions the Gaussian weightings with  $s = M/8$  has the worst and the rectangular function the best bit error performance. Referring to Fig. 5, the former provides the best PAPR and the later the worst PAPR result. Accordingly, a compromise is found in the PAPR reduction and performance degradation. That means we can reduce the PAPR of the OFDM signal by weighting of the subcarriers but this will degrade the bit error performance of the system. From Fig. 7 it is also observed that the bit error probability of OFDM system with Bartlett, Gaussian (with  $s = 3M/16$ ), Half-sin and Shanon weighting factors are close to each other. Meanwhile, from Fig. 7 it is also seen that for small values of  $r$ , the bit error probability increases by increasing the rms delay spread of the channel.

## 6. Conclusion

Peak-to-average power reduction of OFDM transmission by weighting was addressed. Different weighting factors including Rectangular, Bartlett, Gaussian— with different spreads—, Raised cos, Half-sin and Shanon were considered. For OFDM signal with  $M = 256$  carriers and QPSK modulation the Bartlett weighting reduces the power variance by 0.9 dB, Gaussian weights (with spread of  $M/8$ ) by a factor of 6.1 dB, raised cosine by a factor of 1.4 dB, Half-sin weighting by a factor of 0.6 dB and Shanon weights by 0.8 dB. By

reducing the spread of Gaussian weights the power variance further reduces. Bit error performance of weighted OFDM transmission over multipath fading was investigated. It was shown that weighting of OFDM signal degrades the bit error probability of the system. In fact this is the price paid for reducing the PAPR.

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## Appendix A

Derivation of (39). Since  $E[b_k(-1)] = 0$ , the mean value of the ISI is simply zero. Using (34) the variance of ISI is obtained as

$$\sigma_{ISI,k}^2 = E\{b_k(-1)X_{k,k}^1 \cdot [b_k(-1)X_{k,k}^1]^*\} \quad (\text{A.1})$$

Which can be easily written as

$$\sigma_{ISI,k}^2 = \overline{|b_k(-1)|^2} |X_{k,k}^1|^2 = |X_{k,k}^1|^2 \quad (\text{A.2})$$

which is (39).

## Appendix B

Derivation of (40).

Using (35) and with the same argument as given in Appendix A, the mean of ICI is calculated zero. The variance of ICI is calculated as

$$\begin{aligned} \sigma_{ICI,k}^2 = E & \sum_{n_1=0}^1 \sum_{m_1=0, m_1 \neq k}^{M-1} \sum_{n_2=0}^1 \sum_{m_2=0, m_2 \neq k}^{M-1} \\ & \cdot \{b_{m_1}(-1)X_{m_1,k}^{n_1} + b_{m_1}(0)\hat{X}_{m_1,k}^{n_1}\} \\ & \cdot \{b_{m_2}^*(-1)(X_{m_2,k}^{n_2})^* + b_{m_2}^*(0)(\hat{X}_{m_2,k}^{n_2})^*\} \end{aligned} \quad (\text{A.3})$$

which is

$$\begin{aligned} \sigma_{ICI,k}^2 = E & \sum_{n_1=0}^1 \sum_{m_1=0, m_1 \neq k}^{M-1} \sum_{n_2=0}^1 \sum_{m_2=0, m_2 \neq k}^{M-1} \\ & \cdot \{ b_{m_1}(-1) b_{m_2}^*(-1) X_{m_1,k}^{n_1} (X_{m_2,k}^{n_2})^* \\ & + b_{m_1}(-1) b_{m_2}^*(0) X_{m_1,k}^{n_1} (\hat{X}_{m_2,k}^{n_2})^* \\ & + b_{m_1}(0) b_{m_2}^*(-1) \hat{X}_{m_1,k}^{n_1} (X_{m_2,k}^{n_2})^* \\ & + b_{m_1}(0) b_{m_2}^*(0) \hat{X}_{m_1,k}^{n_1} (\hat{X}_{m_2,k}^{n_2})^* \} \quad (\text{A} \cdot 4) \end{aligned}$$

Since data symbols at different time intervals and on different carriers are uncorrelated, i.e.,  $E[b_i(j)b_k(l)] = 1$  if  $i = k$  and  $j = l$ , and zero otherwise, (A·4) reduces to

$$\begin{aligned} \sigma_{ICI,k}^2 = \sum_{m=0, m \neq k}^{M-1} \sum_{n_1=0}^1 \sum_{n_2=0}^1 \\ \cdot \{ X_{m,k}^{n_1} (X_{m,k}^{n_2})^* + \hat{X}_{m,k}^{n_1} (\hat{X}_{m,k}^{n_2})^* \} \quad (\text{A} \cdot 5) \end{aligned}$$

(A·5) can be written as

$$\begin{aligned} \sigma_{ICI,k}^2 = \sum_{m=0, m \neq k}^{M-1} \{ |\hat{X}_{m,k}^0|^2 + |\hat{X}_{m,k}^1|^2 + |X_{m,k}^1|^2 \\ + \hat{X}_{m,k}^1 (\hat{X}_{m,k}^0)^* + \hat{X}_{m,k}^0 (\hat{X}_{m,k}^1)^* \} \quad (\text{A} \cdot 6) \end{aligned}$$

which is (40).



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