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Qualitative modelling of kinematic robots for fault diagnosis

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This study presents an approach, the unit circle (UC), to qualitative representation of robots. A robot is described as a collection of constraints holding among time-varying, interval-valued parameters. The UC representation is presented, and the continuous motion of the end-effector is evaluated by the change of directions of qualitative angle and qualitative length. Analytical formulas of qualitative velocity and qualitative acceleration are derived. The characteristic mapping is introduced for fault detection and diagnosis in terms of the UC. In the end, simulation results demonstrate the feasibility of the UC approach in the domain of robotic fault diagnosis, where a fault is defined as a deviation from expected behavior. The UC representation of robots concerns a global assessment of the systems behaviour, and it might be used for the purpose of monitoring, diagnosis, and explanation of physical systems. This is the first step to fault diagnosis and remediation for Beagle 2 using qualitative methods.

Keywords: Qualitative modeling; Fault diagnosis; Robotics

1. Introduction

The research on robotic diagnosis and fault tolerance is a considerable challenge for both the artificial intelligence community and the robotics community (Patton and Frank 1989). Many contributions have been made to this topic in the past two decades (De Kleer and Williams 1987, Blanke *et al.* 2003, Polycarpou and Trunov 2003). Visinsky *et al.* (1995) provided a layered fault tolerance framework consisting of servo, interface and supervisor layers. The layers form a hierarchy of fault tolerance which provides different levels of detection and tolerance capabilities for structurally diverse robots. Schroder (2003) proposed a qualitative approach to fault diagnosis of dynamical systems, mainly process control systems. Qualitative and quantitative methods are two ways of looking at the world and solving problems. They have their advantages and disadvantages when used as solutions to particular problems. When the tradeoff between computational complexity and accuracy is a major problem, qualitative reasoning methods are usually considered as the preferred solutions. Qualitative kinematics is a branch of qualitative mechanics concerned with motion in qualitative space without reference to force or mass. There is

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little research literature in qualitative kinematics, let alone in qualitative robotic kinematics. However, there does exist a huge growing interest in qualitative spatial reasoning, qualitative physics and even cognitive science. They have made contributions to qualitative representation of geometry, which is the basis for qualitative kinematics of physical systems.

In the area of qualitative analysis of physical systems, a number of approaches have been developed. Artificial intelligence methods for qualitative reasoning about mechanisms were first developed by Rieger and Grinberg (1977), whose system produces realistic qualitative simulations of the behaviour of mechanisms based on their knowledge representation consisting of events, tendencies, states, and state changes, related by several different types of causal links. McDermott (1982) created an extended representation on a better logical foundation that is capable of addressing a larger set of issues based on the knowledge representation of Rieger and Grinberg. Nielsen (1988) described a theory of qualitative mechanics, the symbolic analysis of the motions and the geometric interaction of physical objects, for analysis of rigid body mechanisms. The most significant work on qualitative mechanism analysis is that of Faltings (1992). He built upon his and Forbust *et al.*'s (1987) work on qualitative kinematics, and developed a first-principles algorithm for analysing planar mechanisms. He introduced a 'theory of place vocabulary' which formed the basis for an envisionment of the qualitative behaviour of a device under external influences. However, this work suffered from the limitation that certain problems could not be solved without including quantitative information. Therefore, in contrast to the configuration space approach of Faltings, Olivier *et al.* (1995) have proposed a qualitative kinematics reasoning method based upon the use of occupancy arrays. This approach does not require inference rules. It works simply on the constraint that no two objects occupy the same occupancy array position, and can be extended to including semi-quantitative information.

Kramer (1992) proposed 'The Linkage Assistant' kinematic simulator which demonstrated that mechanism kinematic analysis did not solely have to rely on exact geometric mechanism information, i.e. a qualitative approach could be adopted. Liu (1996) presented a qualitative representation and reasoning approach based upon the formalism of qualitative trigonometry, qualitative arithmetic and qualitative spatial inferencing. The formalism has been applied successfully to both closed-chain constrained, and open-chain underconstrained, 2D multiple linkage problems. Stahovich *et al.* (2000) presented a theory of qualitative rigid-body mechanics to demonstrate a program, SKETCHIT that uses this theory to compute qualitative rigid-body dynamic simulation. SKETCHIT can handle devices that are composed of an arbitrary number of fixed-axis components and springs, with driving inputs coming from both applied motions and forces. Engineering design, like robotic navigation, ultimately normally requires a fully metric description. However, at the early stages of the design process, a reasonable qualitative description would suffice. The field of qualitative kinematics is largely concerned with supporting this type of activity (Faltings 1992).

The problem domain related to monitoring, diagnosis and explanation can be easily resolved by taking advantage of qualitative reasoning methods rather than quantitative methods. This paper proposes a novel qualitative modelling scheme for the representation of planar robots, the approach is expected as a general qualitative modelling of kinematic robots, though this approach is applied to the domain of robot fault diagnosis in this paper. This is the first attempt to define clearly the qualitative representation of robots, whose end-effector's position can be described by a qualitative length and a qualitative orientation angle within a unit circle. Qualitative analysis of a robot is constructed in terms of subsets of a unit circle with link sequence constraints. The UC approach also derives qualitative velocity and qualitative acceleration based on qualitative length and orientation angle (Liu and Coghill 2004a). The characteristic mapping presents the mapping relation between inputs and outputs of a physical system using characteristic values, which are characteristic quantities extracted from quantitative intervals of a domain to describe their corresponding qualitative information (Liu and Coghill 2004b). The selection of characteristic values is application-dependent, it is determined by the mean, minimum and maximum of quantitative intervals in this paper. The characteristic mapping basically provides approximate solutions to that may be used to guide the application of quantitative methods.

The paper is organized as follows. The related work is given in section 2. The UC qualitative representation of planar robots is presented in section 3. The characteristic mapping is addressed in section 4. A case study is given in section 5 to prove the proposed approach in the domain of robots fault diagnosis, discussions and conclusion are drawn at the end.

2. Qualitative position representation of planar robots

In this section, the UC approach is proposed for qualitative analysis of planar robots. An n-link serial robot, combined by links and joints, can be decomposed into n link-based segments, each of which consists of one link and its corresponding joint. Each segment can be described by a qualitative length and a qualitative orientation angle, furthermore the qualitative representation of the end-effector of the robot is provided by the qualitative information of each link segment. With respect to the n-link robot, the direct kinematic of the robot can be described with the following equations:

$$\begin{aligned} x &= p(\theta) \\ \dot{x} &= \mathbf{J}(\theta)\dot{\theta} \\ \ddot{x} &= \mathbf{J}(\theta)\ddot{\theta} + \dot{\mathbf{J}}(\theta, \dot{\theta})\dot{\theta} \end{aligned} \tag{1}$$

where

$$p(\theta) = \begin{bmatrix} p_y(\theta) \\ p_x(\theta) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} l_i \cos\theta_i \\ \sum_{i=1}^{n} l_i \sin\theta_i \end{bmatrix}$$

where p is an n-dimensional vector function representing direct kinematics. Compared with quantitative methods, the description of the ith link segment in qualitative reasoning requires qualitative position parameters, qp^i, the qualitative length of the ith link, qp_l^i, and the qualitative angle, qp_θ^i. Then the qualitative description is given in terms of constraint information:

$$\begin{cases} qp_l^i | qp_l^i \in [0, l_i] \\ qp_\theta^i | qp_\theta^i \in [0, 2\pi]. \end{cases} \tag{2}$$

Further, the intervals of the length and the orientation angle of the ith link segment are described by length parameter r_i and orientation parameter s_i. The setting of the

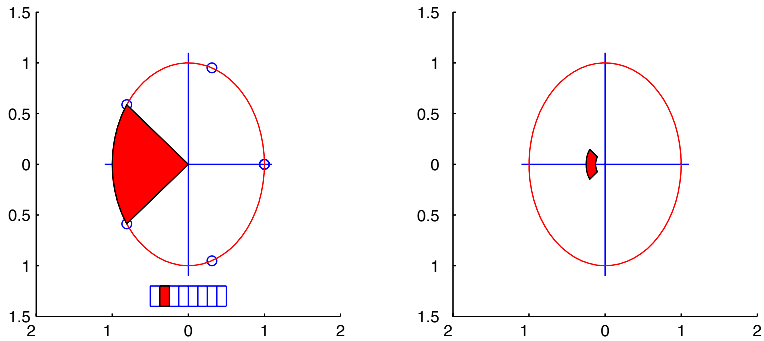

Figure 1. Separated and integrated UC representation.

two parameters is subject to systems requirement such as joint offset and so forth. The value of s_i and r_i are application-dependent.

$$\begin{cases} qp_l^i | qp_l^i \in [0, l_{i1}, l_{i2}, \cdots, l_{i(r_i-1),} l_{ir_i}] \\ qp_\theta^i | qp_\theta^i \in [0, q\theta_{i1}, q\theta_{i2}, \cdots, q\theta_{i(s_i-2)}, q\theta_{i(s_i-1)}, 2\pi] \end{cases} \quad (3)$$

where

$$0 \leq l_{i1} < l_{i2} < \cdots < l_{i(r_i-1)} < l_{ir_i} \leq l_i$$
$$0 \leq q\theta_{i1} < q\theta_{i2} < \cdots < q\theta_{i(s_i-1)} \leq 2\pi.$$

The length intervals and the orientation intervals of the UC approach is used to meet the system requirement such as qualitative position and qualitative orientation. The comparison between the desired data and the actual data in the UC representation can provide sufficient evidence for fault detection and diagnosis. For example, the link before deformation can be shorter, equal, longer in comparison with the link after deformation, meanwhile some of the domain can be defined as bending-safe area, the others as bending-damaged area for diagnostic purposes of link bending analysis. The separated description of qualitative information of the position and the orientation can be integrated into a unit circle, the active area is highlighted shown in figure 1.

2.1 *Qualitative position of unit circle representation*

For a global assessment of a system behaviour, the functional rule of qualitative constraints, $(Y = M^+(X))$, is applied so that the interval-valued parameters of the ith link segment are replaced by the proportion of the interval-valued parameters of the ith link segment to the addition of the lengths of all link segments. It is noted that the link segments are connected by the link sequence constraints, $l_1, l_2, \ldots, l_i, \ldots, l_n$, based on which, the unit circle qualitative representation of the position of the end-effector of the n-link robot can be derived:

$$\begin{cases} qp_i = \oplus qp_l^i | qp_l^i \in UC_{ql} \times \sum_{i=1}^{n} l_i \\ qp_\theta = \oplus qp_\theta^i | qp_\theta^i \in UC_{q\theta_i} \times 2\pi \end{cases} \quad (4)$$

where

$$\begin{cases} \mathrm{UC}_{qpl} = \left[0, \dfrac{l_{11}}{\Sigma_{i=1}^{n} l_i}, \dfrac{l_{12}}{\Sigma_{i=1}^{n} l_i}, \ldots, \dfrac{l_{i1}}{\Sigma_{i=1}^{n} l_i}, \ldots, \dfrac{l_{n(r_n-1)}}{\Sigma_{i=1}^{n} l_i}\right] \\ \mathrm{UC}_{qp_\theta^i} = \left[0, \dfrac{q\theta_{i1}}{2\pi}, \dfrac{q\theta_{i2}}{2\pi}, \ldots, \dfrac{q\theta_{i(s_i-1)}}{2\pi}, \dfrac{2\pi}{2\pi}\right] \end{cases}$$

where UC_{qpl} is the qualitative length of the ith link segment of a unit circle and $\mathrm{UC}_{qp_\theta^i}$ is the qualitative orientation angle of the unit circle.

Definition 1: The qualitative position of a point in domain space can be described by a pair of qualitative parameters such as a qualitative length and a qualitative orientation relative to a fixed reference coordinate system.

Definition 2: The qualitative position of the end-effector of a robot can be described by a pair of qualitative position and qualitative orientation in a unit circle, which are provided by combination of qualitative parameters of all segment links.

Each interval-valued area of the unit circle, UC_{ql}, $\mathrm{UC}_{q\theta_i}$ corresponds to the qualitative meaning in domain knowledge representation. The representation conversion of a particular position 'Q' of the end-effector of a n-link serial robot from quanitative to qualitative description is given in figure 2. The robot is described in terms of Cartesian coordinates, its qualitative representation of the UC is in terms of the qualitative angle and the qualitative length of the end-effector. It is defined by a qualitative vector $\vec{Q}'$ from the origin to the position Q. For example, the qualitative orientation can be divided into front, back, left and right. The qualitative length can be identified by less, equal and larger three regions in qualitative description.

In position mapping from the qualitative representation to continuous spatial quantities, one of the standard assumptions in traditional qualitative reasoning is that change is continuous. That is, in addition to qualitative magnitude such as qualitative angles and qualitative lengths in the quantify space, we need to know the direction of change of each variable. Thus for each variable, we describe its qualitative state in terms of its magnitude in the quantity space and its direction of change: increasing, decreasing or steady:

$$[\Delta qp^k] = \mathrm{sign}(qp^{k+1} - qp^k) = \begin{cases} + & \Delta qp^k > 0 \\ - & \Delta qp^k < 0 \\ 0 & \Delta qp^k = 0 \end{cases}$$

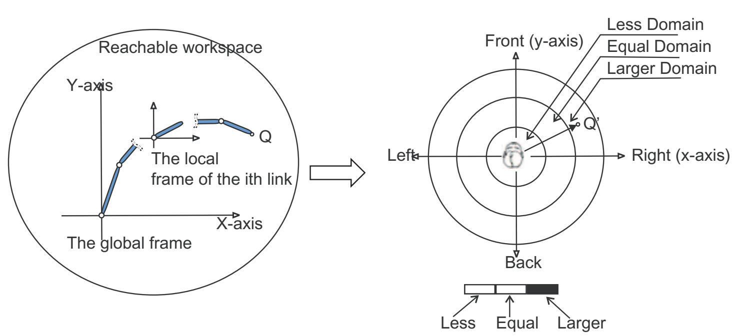

Figure 2. Representation conversion from a quantitative viewpoint to a qualitative viewpoint.

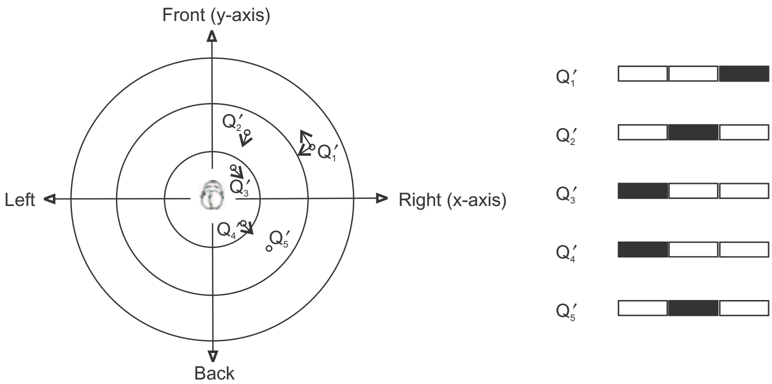

Figure 3. Mapping definition from a qualitative representation to continuous spatial quantities.

Table 1. Qualitative description of a continuous motion.

	$[q\theta]$	$[ql]$	$[\Delta q\theta]$	$[\Delta ql]$
Q'_1	Frontright	Large	Increasing (+)	Decreasing (−)
Q'_2	Frontright	Equal	Steady (0)	Decreasing (−)
Q'_3	Frontright	Small	Decreasing (−)	Steady (0)
Q'_4	Backright	Small	Steady (0)	Increasing (+)
Q'_5	Backright	Equal	Steady (0)	Steady (0)

where qp_i^k is the qualitative position parameters within the kth interval. Then the qualitative positions of the end-effector of a robot can be described as follows:

$$[\Delta qp] = \oplus[\Delta qp^k] = \oplus[\Delta qp_l^k] \oplus [\Delta qp_\theta^k].$$

For robotic qualitative kinematics, the continuous motion of robots can be described by the combination of magnitudes of qualitative parameters with their direction of change. The direction of change of a qualitative orientation angle is defined by a qualitative orientation vector, whose direction is perpendicular to the corresponding qualitative vector; the direction of change of a qualitative length is defined by a qualitative length vector, whose direction is vertical to the qualitative vector. The anticlockwise direction of qualitative orientation angles is denoted as positive, and the direction of facing the origin of qualitative lengths as position (figure 3).

The continuous motion, from the position Q'_1 to Q'_5, of an end-effector is given in figure 3, the qualitative description is in table 1, the direction of change can be decided by the qualitative magnitudes of continuous motion states, or predefined by robotic planners.

2.2 *Qualitative velocity representation*

Qualitative velocities can be used for describing the rate of change of qualitative positions.

Definition 3: The qualitative velocity of a point in domain space is the derivative of the qualitative positions of the point relative to any given reference system.

$$qv = \frac{\mathrm{d}qp}{\mathrm{d}t} \approx \frac{\Delta qp}{\Delta t} \tag{5}$$

Definition 4: The qualitative velocity of the end-effector of a robot, qv, consisting of qualitative linear velocity, qv_l, and qualitative angular velocity, qv_θ is the derivative of the qualitative position of a state such as qualitative length, qp_l, and qualitative orientation, qp_θ. For the robotic velocity of a robot, we have the following:

$$\begin{aligned} qv_l &= \frac{\mathrm{d}qp_l}{\mathrm{d}t} \approx \frac{\Delta qp_l}{\Delta t} \\ qv_\theta &= \frac{\mathrm{d}qp_\theta}{\mathrm{d}t} \approx \frac{\Delta qp_\theta}{\Delta t}. \end{aligned} \tag{6}$$

As the relationship between the certainty values of particular values is characterised by the partial derivative:

$$\begin{aligned} \Delta qv_l &= \frac{\partial qv_l}{\partial qp_l} \Delta qp_l \\ \Delta qv_\theta &= \frac{\partial qv_\theta}{\partial qp_\theta} \Delta qp_\theta. \end{aligned}$$

Assuming an initial state of the end-effector of the robot is qp_{l0}, $qp_{\theta 0}$, qv_{l0}, $qv_{\theta 0}$, and then we have the following in terms of the mean value theorem of differentiation:

$$\begin{aligned} \Delta qv_l &= qv_l - qv_{l0} = \frac{\partial qv_l}{\partial qp_l} \Delta qp_l = \frac{\partial qv_l}{\partial qp_l}(qp_l - qp_{l0}) \\ qv_l &= \frac{\partial qv_l}{\partial qp_l}(qp_l - qp_{l0}) + qv_{l0}. \end{aligned}$$

The interval-based value of qp_l can be substituted from equation (6). The direction of the velocity can be calculated from the following:

$$\begin{aligned} [\Delta qv_l] &= \left[\frac{\partial qv_l}{\partial qp_l}\right][\Delta qp_l] \\ [\Delta qv_\theta] &= \left[\frac{\partial qv_\theta}{\partial qp_\theta}\right][\Delta qp_\theta]. \end{aligned}$$

Further, the qualitative description of general velocity is derived:

$$[\Delta qv] = \oplus[\Delta qv_l] \oplus [\Delta qv_\theta] = \oplus \frac{\partial qv_l}{\partial qp_l}[\Delta qp_l] \oplus \frac{\partial qv_\theta}{\partial qp_\theta}[\Delta qp_\theta]. \tag{7}$$

2.3 *Qualitative acceleration representation*

Definition 5: The qualitative acceleration of a point in domain space is the double derivative of the positions of the state,

$$qa = \frac{\mathrm{d}qv}{\mathrm{d}t} = \frac{\mathrm{d}^2 ql}{\mathrm{d}^2 t} \approx \frac{\Delta qv}{\Delta t} \approx \frac{\Delta ql}{\Delta^2 t}$$

Definition 6: The qualitative acceleration of the end-effector of a robot is the derivative of the velocities of the state, or the double derivative of the state. Firstly we have:

$$qa_l = \frac{dqv_l}{dt} = \frac{d^2qp_l}{d^2t} \approx \frac{\Delta qv_l}{\Delta t} \approx \frac{\Delta qp_l}{\Delta^2 t}$$
$$qa_\theta = \frac{dqv_\theta}{dt} = \frac{d^2qp_\theta}{d^2t} \approx \frac{\Delta qv_\theta}{\Delta t} \approx \frac{\Delta qp_\theta}{\Delta^2 t}.$$

Then,

$$[\Delta qa_1] = \frac{\partial qa_l}{\partial qv_l}[\Delta qv_l] = \frac{\partial qa_l}{\partial qp_l}[\Delta qp_l]$$
$$[\Delta qa_\theta] = \frac{\partial qa_\theta}{\partial qv_\theta}[\Delta qv_\theta] = \frac{\partial qa_\theta}{\partial qp_\theta}[\Delta qp_\theta]$$

where $[\Delta qa] = \text{sign}(qa_{k+1} - qa_k)$. Finally, the qualitative description of general acceleration is derived:

$$\begin{aligned}[\Delta qa] &= \oplus [\Delta qa]_l \oplus [\Delta qa_\theta] = \oplus \frac{\partial qa_l}{\partial qv_l}[\Delta qv_l] \oplus \frac{\partial qa_\theta}{\partial qv_\theta}[\Delta qv_\theta] \\ &= \oplus \frac{\partial qa_l}{\partial qp_l}[\Delta qp_i] \oplus \frac{\partial qa_\theta}{\partial qp_\theta}[\Delta qp_\theta].\end{aligned} \tag{8}$$

3. Characteristic mapping

The goal of qualitative reasoning is to provide approximate solutions that may be used to guide the application of quantitative methods. The characteristic mapping basically extracts characteristic quantities from quantitative interval to describe the corresponding qualitative information. Stability of the characteristic mapping is obvious for linear systems; for nonlinear systems, Kawamura and Shima (1996) proved the robust stability with the condition that real and imaginary part of their characteristic polynomial $F(s)$ are monotonic parameters in the frequency domain. The methods used to select characteristic values are application dependent such as landmark methods.

For robotic fault diagnosis, let $f(\theta_t^d)$ be the desired trajectory of the position of the end-effector, $f(\theta_l^a)$ that from the sensors and the corresponding error $\Delta f(t)$. The following formula differentiates faulty and non-faulty intervals:

$$|\Delta f(t)| \leq \varepsilon \tag{9}$$

where ε is the fault index determined by system requirement such as joint offsets.

The characteristic quantities of each fault, interval, $\hat{\theta}s$, are determined by the time instants, $t_{l\min}$, $t_{l\max}$, where the local maximum and minimum of $\Delta f(t)$ are achieved. Note that the selection of the intervals is application-dependent in order to make sure that suitable local characteristic values are chosen.

So far the characteristic values of each faulty interval are extracted, which can describe the input qualitative states of the robot in each faulty interval. These include fault and noise signals as well if the inputs are from measurement. The output qualitative states, the positions of robot end-effector, can be calculated based on robots. The position of the end-effector of a robot can be calculated in terms of equation (1) as;

$$\begin{aligned}\hat{\alpha}_p &= \tan^{-1}\left(\frac{p_y(\hat{\theta}_i)}{p_x(\hat{\theta}_i)}\right) = \tan^{-1}\left(\frac{\Sigma_{i=1}^n l_i \sin\hat{\theta}_i}{\Sigma_{i=1}^n l_i \cos\hat{\theta}_i}\right)\\ \hat{r}_p &= \sqrt{\left(\Sigma_{i=1}^n l_i \sin\hat{\theta}_i\right)^2 + \left(\Sigma_{i=1}^n l_i \cos\hat{\theta}_i\right)^2}\end{aligned} \tag{10}$$

where $i = t_{l\max}\ t_{l\min}$. Thus, the qualitative information of each interval are $qp_\theta = [\hat{\alpha}_p]$ and $qp_l = [\hat{r}_p]$. Hence, the kth state position can be defined by two characteristic values

$$\begin{aligned}\mathrm{SP}_{k,1} &= \left(qp_\theta\left(\left[\alpha_p^{2k-1}\right]\right), qp_l\left(\left[r_p^{2k-1}\right]\right)\right)\\ \mathrm{SP}_{k,2} &= \left(qp_\theta\left(\left[\alpha_p^{2k}\right]\right), qp_l\left(\left[r_p^{2k}\right]\right)\right).\end{aligned}$$

The state change of continuous motion in the ith state position,

$$\Delta\mathrm{SO}_k = \mathrm{sign}(\mathrm{SP}_{k,2} - \mathrm{SP}_{k,1}) = \oplus\left[\Delta qp_\theta^k\right] \oplus \left[\Delta qp_l^k\right]$$

where

$$\begin{aligned}\left[\Delta qp_\theta^k\right] &= \mathrm{sign}\left(qp_\theta\left(\left[\alpha_p^{2k}\right]\right) - qp_\theta\left(\left[\alpha_p^{2k-1}\right]\right)\right)\\ \left[\Delta qp_l^k\right] &= \mathrm{sign}\left(qp_l\left(\left[r_p^{2k}\right]\right) - qp_l\left(\left[r_p^{2k-1}\right]\right)\right).\end{aligned}$$

4. Case study

A case study of the UC representation of a three-link robot in the domain of robot fault diagnosis is addressed in this section. This study demonstrates the proposed qualitative model, how it works in the fault diagnosis of the kinematic robot and its analysis of the accuracy of the approach. The kinematic model of the robot is as follows:

$$P(\Theta) = \begin{bmatrix} p_x(\theta_1, \theta_2, \theta_3)\\ p_y(\theta_1, \theta_2, \theta_3)\end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{3}\left(l_k \cos\left(\sum_{i=1}^{k}\theta_i\right)\right)\\ \sum_{k=1}^{3}\left(l_k \sin\left(\sum_{i=1}^{k}\theta_i\right)\right)\end{bmatrix}$$

where l_1, l_2, l_3, are link lengths and, θ_1, θ_2, θ_3, are orientation angles, respectively. The desired motion description of the robot is shown in figure 4, and its desired joint trajectories is given in figure 5. It clearly shows that the end-effector of the robot moves along an ellipse trajectory whose starting position and ending at $P(1.3, 0.0)$.

The accuracy of the robotic model depends on the setting of length and orientation parameters of a UC representation, which is application-dependent. That is to say, an operator can define faults in an application by controlling a deviation from expected behaviour to a certain degree. It allows the operator to clear faulty alarms by defining different setting of length and orientation parameters of a unit circle. Based on the proposed qualitative modelling of kinematic robots, a MATLAB

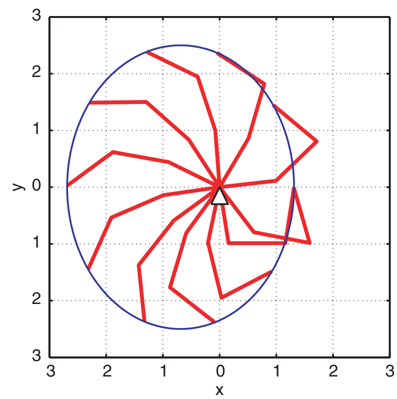

Figure 4. Desired motion of a three link robot.

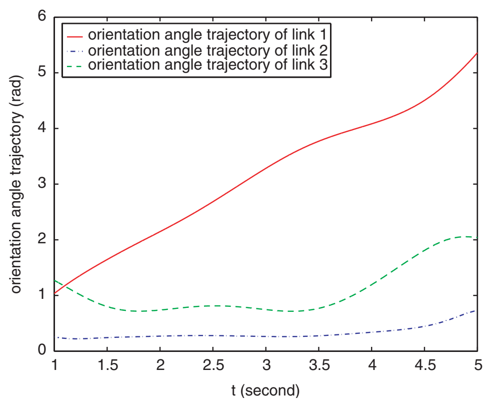

Figure 5. Desired orientation trajectories of the robot, θ_1, θ_2, θ_3.

toolbox named unit circle (UC) has been developed. Though the toolbox is designed for the purpose of general dynamic systems including process control systems and physical systems, it has been applied to the domain of robot fault diagnosis in this paper. Two versions of qualitative model of the robot in figure 4 have been shown in figures 6 and 7. The translation and orientation parameters of a unit circle are set as 6 and 5 in figure 6, those are set as 19 and 20 in figure 7. The former is provided for the purpose of comparison because such setting cannot diagnose any faults at all, the latter is determined by two facts, first, the parameter r is set as 19 due to that the maximum offset of the end-effector position is given by 0.08 m, second the parameter of s as 20 because two characteristic values are chosen in the interval of 0.25 seconds. The corresponding UC representations are illustrated in red area in unit circles, the large elliptic trajectories are those of the end-effector.

4.1 *Fault detection*

The first principle for the fault detection of robots is that no fault occurs if and only if the actual qualitative states remain within the coverage of the UC representation

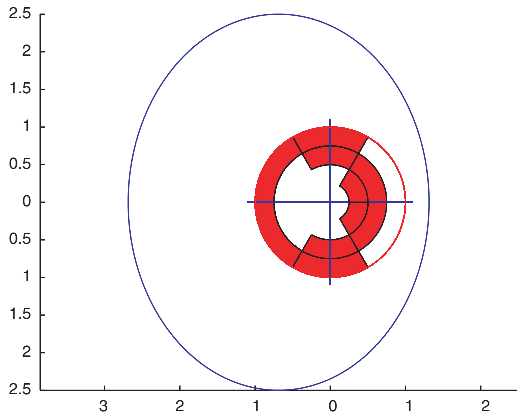

Figure 6. UC representation ($r = 6$, $s = 5$).

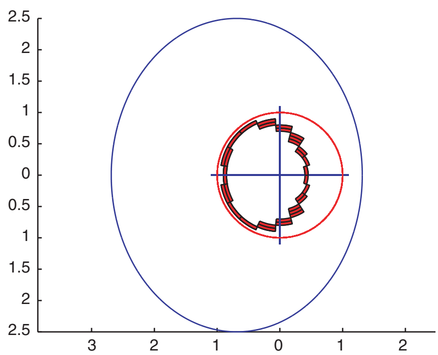

Figure 7. UC representation ($r = 19$, $s = 20$).

of the desired trajectories. Compared with figure 5, figure 8 provides faulty joint trajectories with θ_1 carrying faulty signal. The comparison of the two UC representation versions in the parameter setting, $r = 19$, $s = 20$, is demonstrated in figure 9, The faults of the end-effector are clearly identified by the four deep dark segments, which is generated by characteristic mappings from fault joint trajectories. The fault area in UC representation describes the dash-line fish-shaped faulty trajectory of the end-effector. Hence, the fault global assessment is sufficiently reconstructed using the UC fault version. It should be noted that no fault is detected when a unit circle uses such setting as $r = 6$, $s = 4$, because its deviations are about 3 times in the orientation of a unit circle and five times in the translation as the deviations in the setting ($r = 19$, $s = 20$).

How to locate faults position is another issue of fault diagnosis. Fault isolation has been made a lot easier by isolating functions of the UC. The inference is carried out based on characteristic value of fault segments in the UC. The faults can be classified into three types, single faults, multiple fault not happening in the same interval of the UC, multiple faults whose characteristic values are not in the same time instant. For single fault herein, the analysis is shown in table 2, in which, [DT], [AT] are the desired actual qualitative values of joint trajectories as shown in figures 5 and 7. From table 2, the faulty link segment 1 is detected by the fact that the actual

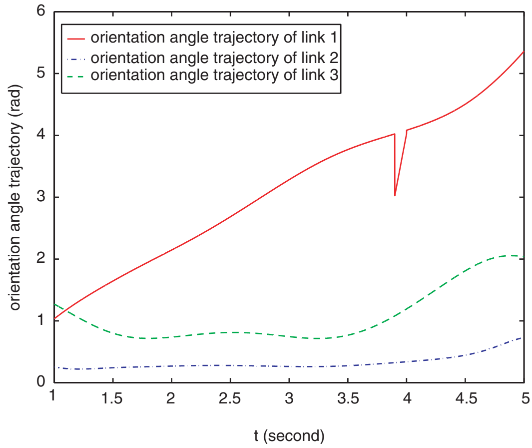

Figure 8. Faulty orientation trajectories, θ_1, θ_2, θ_3, of which, θ_1 carries faulty signal.

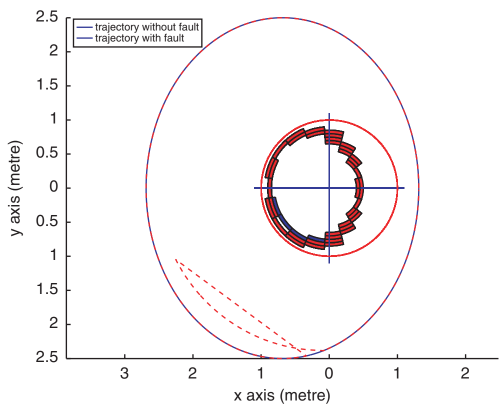

Figure 9. Comparison of UC representations based on signals from figures 5 and 8.

Table 2. Fault isolation analysis based on characteristic mapping.

Joint 1	Joint 2	Joint 3	Fault location
$=$[AT], $\neq$[DT]	[DT]	[DT]	Link segment 1
[DT]	[DT]	$\neq$[AT], $\neq$[DT]	No fault
[DT]	$\neq$[AT], $\neq$[DT]	[DT]	No fault

results of joint 1 are equal to the corresponding actual results of the faulty characteristic mapping in terms of the desired inputs of joint 2 and 3 rather than the desired results of joint 1. The same analysis is effective for the other two types of faults. Further research will examine multiple faults whose characteristic values exist on the same-time point, even though the possibility of those faults happening is very small. The research focuses on fault diagnosis at the link segment level, further fault isolation can be identified based on ontology representation, which is beyond the scope of this paper. The case study proves that the UC representation can not only generate qualitative models for robotic systems, but also detect and isolate faults.

5. Conclusion

A novel qualitative modelling has been proposed for the general purpose of kinematic robots, though the model has been implemented in the domain of robots fault diagnosis. Position and orientation properties of the end-effectors of robots and their link segments have been derived, and analytical formulas of qualitative velocity and qualitative acceleration have been derived based on qualitative position information. Characteristic mapping has been introduced to transfer qualitative information between system inputs and outputs in terms of characteristic values of the UC intervals. Finally a case study of a three-link robot has demonstrated how the qualitative robotic model works in robotic fault diagnosis and its advantages over quantitative approaches.

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