MIMO Channel Modeling and Capacity Analysis for 5G Millimeter-Wave Wireless Systems

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Abstract—This paper presents a 3-D multiple-input multiple-output (MIMO) statistical channel model of the impulse response based on 28 GHz millimeter-wave channel measurements, which generates the channel coefficients between each transmitter and receiver antenna element pair over a local area. Individual multipath voltage amplitudes are found to be Rician distributed, and the spatial autocorrelation of multipath powers can be conveniently modeled by an exponential function. The MIMO channel model is used to evaluate the system capacity in a realistic wideband millimeter-wave urban outdoor environment for fifth generation (5G) air interface design, and results are compared using the traditional independently and identically distributed Rayleigh coefficients. Results indicate that depending on the SNR, Rician channels may exhibit equal or even greater capacity compared to Rayleigh channels.

Index Terms—28 GHz; millimeter-wave; small-scale fading; spatial autocorrelation; SSCM; MIMO; wideband capacity; channel impulse response; multipath.

I. INTRODUCTION

A rich scattering multipath environment can be effectively utilized through multiple-input multiple-output (MIMO) wireless systems to dramatically increase system capacity throughputs [1], by simultaneously exploiting many parallel subchannels distributed in space, as compared to a single-input single-output (SISO) communication system. Antenna diversity can be fully exploited assuming the subchannels to be uncorrelated, while a fully correlated channel only provides one subchannel, thereby significantly decreasing system throughput [2]. The MIMO channel capacity is limited by the antenna element spacing and the spatial autocorrelation of multipath amplitudes between each transmitter (TX) and receiver (RX) antenna element pair, requiring accurate models to estimate MIMO channel coefficients and total channel capacity between multi-antenna base station and mobile terminal. The small wavelengths at millimeter-wave (mmWave) frequencies allow hundreds of electrically-steerable antennas to be placed at the base station, enabling a massive MIMO wireless communication system [3].

Recent work has investigated the MIMO channel capacity for a wideband (i.e., frequency-selective) channel to enable the design of broadband systems for very high data rates. The widespread 3GPP and WINNER MIMO spatial channel models (SCM) assume channel coefficients to be Rician and Rayleigh distributed in LOS and NLOS channels, respectively [4], [5]. The effects of spatial and temporal correlations of multipath amplitudes at different antenna elements affect MIMO capacity results, and so must be appropriately modeled to enable realistic multi-element antenna simulations that are expected to drive future mmWave technologies.

Work in [6] demonstrates the usefulness in taking fading correlations in a MIMO communication system into account, showing a large increase in capacity over SISO systems when the fades connecting pairs of TX and RX antennas are independent and identically distributed. Key findings show that the angle spread is a critical parameter in determining capacity, where the MIMO capacity decreases as the global angle spread of multipaths decreases [6]. In [2], [7], the capacity of a MIMO system is extracted from narrowband and wideband measurements and reconstructed from channel models and spatial correlation matrices obtained from arbitrary antenna patterns, and shown to agree relatively well. In [8], a joint space-time cross-correlation function is derived describing spatial and temporal correlations in a MIMO system, for a scenario that considers motion of the receiver and nonisotropic scattering at the TX and RX. It was shown that outage capacity increases linearly with the number of antennas, even when taking spatial and temporal correlations into account. Various schemes may be implemented on a MIMO radio channel to further enhance system capacity, for instance, antenna selection [9].

In this paper, the mmWave SISO modeling approach presented in [10] is extended to MIMO for arbitrary antenna pattern using measurement-based spatial autocorrelation functions and small-scale spatial fading distributions of multipath amplitudes [11], to generate power delay profiles (PDPs) over a local area. The system capacity of a MIMO system at mmWave frequencies is then investigated using Monte Carlo simulations in a realistic mmWave MIMO mobile radio channel based on the channel models in [10], [11], to enable next generation mmWave air interface design.

II. 3-D MIMO CHANNEL IMPULSE RESPONSE

The omnidirectional radio propagation channel can be described using the double-directional time-invariant baseband channel impulse response [12], also known as a parametric channel model [13] and commonly expressed as in [14],

\[ h_{\text{mmR}}(t, \Theta, \Phi) = \sum_{k=1}^{K} a_k e^{j\varphi_k} \delta(t - \tau_k) \cdot \delta(\Theta - \Theta_k) \cdot \delta(\Phi - \Phi_k) \]

where \(a_k, \varphi_k,\) and \(\tau_k\) are the path voltage amplitude, phase, and absolute propagation delay of the \(k\)th multipath component; \(\Theta_k\) and \(\Phi_k\) are the vectors of azimuth/elevation angle of departure (AOD) and angle of arrival (AOA), respectively; \(K\) is the total.
number of multipath components. Statistical distributions for \(|a_k|^2, \varphi_k, \tau_k, \Theta_k, \text{ and } \theta_k\) shown in (1) are provided in [10], using a 12-step procedure for multi-frequency and arbitrary antenna patterns, using the concepts of time clusters and spatial lobes, following a 3GPP-like approach.

Nonparametric channel models [13] are commonly used to describe the stochastic evolution of the MIMO channel matrix \(H_i\), where \(H_i\) denotes the \(N_r \times N_t\) MIMO channel matrix of the \(i^{th}\) multipath component in an omnidirectional channel impulse response, expressed as in (2) [14]:

\[
H_i = R_r^{1/2} H_{i} R_t^{1/2}
\]

where \(R_r\) and \(R_t\) denote the receive and transmit spatial correlation matrices, respectively, for user-defined antenna pattern, and \(H_{i}\) is a matrix whose entries correspond to small-scale path (voltage) amplitudes and phases. Note that \(R_r\) and \(R_t\) collapse to the identity matrices when disregarding the spatial correlation of multipath across the antenna elements. The entries of \(H_{i}\) are commonly assumed independently and identically Rician and Rayleigh distributed in LOS and NLOS environments, respectively. The entries of \(H_{i}\) retain the spatial autocorrelation of multipath amplitudes specified through \(R_r\) and \(R_t\), while exhibiting the small-scale distribution specified in \(H_{i}\).

The entries of \(R_r\) and \(R_t\) are related to the AODs, AOAs, and global angular spreads (ASs) at the receiver and transmitter. The \((i, k)^{th}\) entry of the receive correlation matrix \(R_r\) is given in Eq. (3) [14]:

\[
[R_r]_{i,k} = \int_{-\pi/2}^{\pi/2} \int_{-\pi}^{\pi} P(\theta, \phi)v_i(\theta, \phi)v_k(\theta, \phi)d\theta d\phi
\]

where \(P(\theta, \phi)\) is the received power angular spectrum (PAS) over the \(4\pi\) steradian sphere, \(\theta\) is the azimuth AOA ranging from \(-\pi\) to \(\pi\), \(\phi\) is the elevation AOA ranging from \(-\pi/2\) to \(\pi/2\), and \(v_i(\theta, \phi)\) is the array response for the \(i^{th}\) antenna element in the array. The transmit correlation matrix \(R_t\) can be expressed in a similar way. The array response vector of a uniform linear array (ULA) is expressed as in (4):

\[
v(\theta) = [1, e^{-j2\pi \frac{\lambda}{d} \sin(\theta)}, \ldots, e^{-j2\pi \frac{\lambda}{d} (N-1) \sin(\theta)}]^T
\]

where \(\lambda\) is the wavelength long track, emulating a virtual array with antenna element spacing [13], \(N\) is the total number of antenna elements in the ULA. For a uniform rectangular array (URA), the array response vector can be written as:

\[
v(\theta, \phi) = [1, e^{-j2\pi \frac{\lambda}{d} \sin(\phi) \cos(\theta)}, \ldots, e^{-j2\pi \frac{\lambda}{d} (\sin(\phi) \cos(\theta) + \phi(\sin(\phi) \sin(\theta)))},
\]

\[
e^{-j2\pi \frac{\lambda}{d} ((W-1) \sin(\phi) \cos(\theta) + (N-1) \sin(\phi) \sin(\theta)))]^T
\]

where \(\lambda\) is the azimuth AOA of a propagation path, \(\phi\) is the elevation AOA of the propagation path, \(W\) is the number of antenna elements in the azimuth dimension, \(N\) is the total number of antenna elements in the URA. Therefore, if a ULA is used at the receiver, the receive correlation matrix \(R_r\) in (3) becomes:

\[
[R_r]_{i,k} = \int_{-\pi}^{\pi} P(\theta)e^{-j2\pi \frac{\lambda}{d} (i-k) \sin(\theta)} d\theta
\]

where the term \(2\pi \frac{\lambda}{d} (i-k) \sin(\theta)\) accounts for the phase difference between the \(i^{th}\) and \(k^{th}\) array element due to spacing [13]. and the PAS \(P(\theta)\) is normalized such that \(\int_{-\pi}^{\pi} P(\theta) d\theta = 1\).

The transmit and receive spatial correlation matrices can also be obtained from empirically-derived spatial autocorrelation functions [15].

For a URA at the receiver, \([R_r]_{i,k}\) is given by (7), where \(i, k = 0, \ldots, N-1\) with \(N\) denoting the total number of antenna elements in the URA, \(W\) is the number of antenna elements in the azimuth dimension in the URA. \(\mod(a, b)\) yields the integer remainder after division of \(a\) by \(b\), and \(\fix(a/b)\) results in the integer part of the quotient after division of \(a\) by \(b\).

The steps for calculating the MIMO channel capacity are as follows:

1) Generate an omnidirectional PDP from a SISO model using (1).
2) For each path, generate \(N_r \times N_t\) local “copies” using (2) such that the voltage amplitudes obey the spatial correlation specified by \(R_r\) and \(R_t\) and the small-scale distribution specified by \(H_{i}\).
3) Compute the frequency response \(H_f\) of the MIMO channel impulse response \(H_i\) using a discrete Fourier transform operation.
4) Compute the total wideband capacity from (8) [7]:

\[
C = \frac{1}{BW} \int_{f_{\text{min}}}^{f_{\text{max}}} \log_2 \det \left( I + \rho \frac{H_f H_f^H}{N_t} \right) df
\]

where \(BW\) denotes bandwidth, \(\rho\) represents the average SNR, \(f_{\text{min}}\) and \(f_{\text{max}}\) denote the minimum and maximum narrowband sub-carrier frequencies, respectively.

### III. SMALL-SCALE MEASUREMENT DESCRIPTIONS

The 28 GHz ultrawideband propagation measurements were performed using a 400 megahertz-per-second (Mpcps) broadband sliding correlator channel sounder, and a pair of high gain 15 dBi directional horn antennas (28.8° and 30° half-power beamwidths in azimuth and elevation, respectively) in a base-to-mobile scenario [11]. The transmitter-receiver (T-R) separations ranged from 8 m to 12.9 m. The maximum measurable path loss was 157 dB, with a measurement time resolution of 2.5 ns (800 MHz RF null-to-null). The RX antenna was moved over a local area at one TX and four RX locations to investigate the statistics of small-scale spatial fading and spatial autocorrelation of individual multipath voltage amplitudes. At each RX location, the RX antenna was moved over a 33-wavelength long track, emulating a virtual array with antenna spacing of \(\lambda/2 = 5.35\) mm, where each antenna position was situated on a cross (i.e., 66 antenna positions on each axis of the cross). The TX and RX antennas were kept fixed in azimuth and elevation, and PDPs were acquired for each step increment with fixed RX antenna during the captures. The RX and TX antennas were located 1.4 m and 4 m above ground level, respectively, well below surrounding rooftops. Directional antennas were employed to emulate a typical realistic mmWave base-to-mobile scenario, where both the TX and RX beamform towards the strongest angular directions to maximize signal-to-noise (SNR) ratio.
\[
\mathbf{R}_{i,k} = \int_{-\pi/2}^{\pi/2} \int_{-\pi}^{\pi} P(\theta, \phi)e^{-j2\pi\left((\text{mod}(i,W)-\text{mod}(k,W))\sin(\phi)\cos(\theta)+(\text{fix}(i/W)-\text{fix}(k/W))\sin(\phi)\sin(\theta)\right)} \, d\theta d\phi
\]  

(7)

IV. MEASUREMENT-BASED STATISTICAL MODELS

A. MILLIMETER-WAVE SMALL-SCALE SPATIAL FADING

Small-scale fading denotes the fluctuations in received signal levels over short, sub-wavelength receiver distances, and physically corresponds to the coherent phasor sum of many random multipath components arriving within the measurement system resolution [17]. Fig. 1 shows the cumulative distribution functions (CDFs) for \( K \) factors as a function of environment and polarization configurations.

![Fig. 1: CDF of 28 GHz measurement-based small-scale spatial fading distributions in LOS and NLOS scenarios.](image)

TABLE I: Table summarizing the ranges of \( K \)-factors for the Rician distributions, describing the path (voltage) gains \( a_i \) in (1), obtained from 28 GHz directional small-scale fading measurements over a local area in different environments, for V-V and V-H polarization configurations.

<table>
<thead>
<tr>
<th>Environment</th>
<th>( K_{VV} ) [dB]</th>
<th>( K_{VH} ) [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS</td>
<td>9 - 15</td>
<td>3 - 7</td>
</tr>
<tr>
<td>NLOS</td>
<td>5 - 8</td>
<td>3 - 7</td>
</tr>
<tr>
<td>LOS-to-NLOS</td>
<td>4 - 7</td>
<td>6 - 10</td>
</tr>
</tbody>
</table>

B. AVERAGE SPATIAL AUTOCORRELATION OF MULTIPATH AMPLITUDES

The spatial autocorrelation of individual multipath component voltage amplitude indicates the level of similarity in signal levels between antennas \( i \) and \( j \) with sub-wavelength spacing \( \Delta r \). The spatial correlation values were computed from (10) using all co-polarized and cross-polarized measurements in LOS, NLOS and LOS-to-NLOS environments, where \( E[] \) is the expectation operator, \( \Delta X \) is the physical separation between two adjacent track positions, and is equal to integer multiples of \( \lambda/2 = 5.35 \text{ mm} \), \( A_K(T_k, X_t) \) is the multipath voltage amplitude at the track position \( l \) and bin delay \( K \) [16]. Fig. 2 and Fig. 3 show the average (over excess delay) spatial autocorrelation function obtained from the V-V measurements in LOS and NLOS scenarios, and the corresponding best fit exponential model of the form (13),

\[
F(\Delta X) = Ae^{-B\Delta X} - C
\]  

(9)

where \( A, B, \) and \( C \) are constants that were determined using the minimum mean square error (MMSE) method, by minimizing the error between the empirical curve and theoretical exponential model shown in (9). In Fig. 2 and Fig. 3, the constants were determined to be \( A = 0.99, B = 1.95, C = 0 \), and \( A = 0.9, B = 1, C = -0.1 \), respectively. Table II summarizes the model coefficients as a function of polarization and environment type.

![Fig. 2: Empirical spatial autocorrelation of resolvable multipath amplitudes, and mean exponential model obtained at 28 GHz vertical-to-vertical LOS scenario.](image)

TABLE II: Table summarizing the model parameters (A, B, C) in (9) obtained using the MMSE method, to estimate the empirical spatial autocorrelation functions.

<table>
<thead>
<tr>
<th>Environment</th>
<th>( A, B, C )</th>
<th>V-V</th>
<th>V-H</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS</td>
<td>(0.99, 1.95, 0)</td>
<td>(1.0, 0.9, 0.05)</td>
<td></td>
</tr>
<tr>
<td>NLOS</td>
<td>(0.9, 1, -0.1)</td>
<td>(1, 2.6, 0)</td>
<td></td>
</tr>
<tr>
<td>LOS-to-NLOS</td>
<td>(0.9, 1.75, -0.3)</td>
<td>(0.9, 1.05, 0)</td>
<td></td>
</tr>
</tbody>
</table>
\[
\rho(i\Delta X) = \frac{E\left[\left( A_K(T_K, X_i) - A_K(T_K, X_i) \right) \left( A_K(T_K, X_i + i\Delta X) - A_K(T_K, X_i + i\Delta X) \right) \right]}{\sqrt{E\left[\left( A_K(T_K, X_i) - A_K(T_K, X_i) \right)^2 \right] E\left[\left( A_K(T_K, X_i + i\Delta X) - A_K(T_K, X_i + i\Delta X) \right)^2 \right]}}, \quad i = 0, 1, 2, \ldots \tag{10}
\]

\[28 \text{ GHz Average Spatial Correlation } \sim \sim \sim \sim \text{ V--V, NLOS}\]

**Fig. 3:** Empirical spatial autocorrelation of resolvable multipath amplitudes, and mean exponential model obtained at 28 GHz vertical-to-vertical NLOS scenario.

**V. MIMO SIMULATION**

**A. Simulation Settings**

Monte Carlo simulation was performed to simulate the wideband capacity using a realistic mmWave empirically-derived single-input multiple-output (SIMO) and MIMO channel model. The carrier frequency is centered at 28 GHz with a bandwidth of 800 MHz, which is uniformly divided into 100 narrowband sub-carriers. For correlated small-scale distribution in a ULA, the spatial correlation matrices are calculated using (6) and (11):

\[
[R_{i,k}] = e^{-j\Theta}(Ae^{-B|i-k|d} - C) \tag{11}
\]

where \(\Theta\) is a random phase assigned to each coefficient in \([R_{i,k}],\) and \(\Theta = 0\) when \(i = k\). A key difference between (6) and (11) is that (6) jointly yields the amplitude and phase information contained in the spatial correlation of signal voltages, while (11) considers the amplitude and phase separately. For the Rician distribution using (11), the parameter values corresponding to the NLOS V-V scenario \((A = 0.9, B = 1, C = -0.1)\) are adopted.

First, it is assumed that a ULA with 20 antenna elements and a URA with 20\times3 elements with a spacing of half-wavelength are employed at the receiver, respectively, while a ULA with one antenna element is used at the transmitter, which essentially compose SIMO channels. Then, the number of transmit antenna elements \(N_t\) is increased to two to form a MIMO channel together with the 20 receive antenna elements in the ULA. Comparisons are made between different small-scale distributions, including Rician distribution as shown by the measurements described in this paper, and Rayleigh distribution that has been widely used in previous literature.

**B. Simulation Results**

**Fig. 4** compares the SIMO channel capacity using Rayleigh and Rician distributed small-scale fading coefficients with Rician \(K\)-factors of 5 dB and 15 dB. As shown by Fig. 4, the capacities for the Rician distribution using the measured correlation expressed in (11) exceed the capacity with the correlation calculated using the conventional method given by (6) for both Rician and Rayleigh distributions, where the improvement ranges from 1 to 2 b/s/Hz on average, indicating that the spatial correlation expression (11) derived in this paper results in less significant spatial correlation compared to (6). Furthermore, the Rician distribution yields slight improvement (about 0.3 b/s/Hz) in capacity compared to the Rayleigh distribution, indicating that the Rician distribution may increase channel capacity, as confirmed by the analysis in [19], which shows that Rician channels may result in greater or smaller capacity compared to Rayleigh channels, depending on the number of antennas.

**Fig. 5** illustrates the SIMO channel capacity as a function of SNR when a URA with 20\times3 elements are used at the receiver. Compared to the corresponding capacities using a ULA with traditional correlation matrices in Fig. 4, it is observed that the capacity increases and the capacity gap (varying from 1 to 2 b/s/Hz) becomes larger as the SNR increases.

**Fig. 6** compares the MIMO channel capacity between Rayleigh and Rician small-scale distributions with Rician \(K\)-
The small-scale channel coefficients are Rician-distributed, with exponential spatial autocorrelations of multipath amplitudes. Further, the SIMO channel matrix has been extended to the MIMO case where both the transmitter and receiver are equipped with ULAs. Monte Carlo simulations were performed to evaluate the capacity of SIMO and MIMO channels, where the wideband capacities were computed from 1,000 simulated PDPs over a local area. Results indicated that the Rician distribution may even result in higher channel capacity, as compared to Rayleigh distribution, depending on the SNR. In addition, the exponential model of the spatial correlation tends to underestimate the spatial correlation between antenna array elements, which leads to higher estimated capacity.

**VI. CONCLUSION**

This paper presented a 3-D statistical channel model for mmWave MIMO as an extension of [10] for a local area. The small-scale channel coefficients are Rician-distributed, with exponential spatial autocorrelations of multipath amplitudes. Further, the SIMO channel matrix has been extended to the MIMO case where both the transmitter and receiver are equipped with ULAs. Monte Carlo simulations were performed to evaluate the capacity of SIMO and MIMO channels, where the wideband capacities were computed from 1,000 simulated PDPs over a local area. Results indicated that the Rician distribution may even result in higher channel capacity, as compared to Rayleigh distribution, depending on the SNR. In addition, the exponential model of the spatial correlation tends to underestimate the spatial correlation between antenna array elements, which leads to higher estimated capacity.

**REFERENCES**