Software Reliability Analysis using Queuing-based Model with Testing Effort

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Abstract—In this paper, we investigate software fault detection and fault correction processes based on infinite server queuing model which incorporate testing effort functions. Some researches proposed in the literature to study fault detection and fault correction processes. However, most of them do not consider the amount of resources consumed during fault detection and fault correction processes. The consumption amount of resources is usually depicted by testing effort functions which can largely influence fault detection speed and the time to correct a fault detected. Therefore, we will show that new models incorporate testing effort functions into the fault detection and fault correction processes. In addition, we study how to use queuing models to explain the fault detection and fault correction processes during software development. Parameters are estimated and experiments on actual fault data sets are illustrated. The results show that the proposed models in this paper can estimated the number of initial faults better than the model without testing effort functions.

Index Terms—software reliability, fault detection process, fault correction process, testing effort function, queuing model

I. INTRODUCTION

Software reliability represents a user-oriented view of software quality, which is defined as the probability of fault-free software operation for a specified period of time in a specified environment [1]. To evaluate and predict software reliability, many mathematical models called software reliability growth models (SRGMs) have been proposed to help software engineers to manage software debugging process quantitatively [2-5]. Non-homogeneous Poisson process (NHPP) models, as a class of SRGMs, are extensively used. Moreover, NHPP SRGMs have been quite successful tools in practical software reliability engineering [2].

These SRGMs were developed based on the common assumptions that faults detected are corrected immediately and no new faults are introduced during software debugging process [2-5]. In practice, it is not the case. Each fault is reported, diagnosed, corrected, and then verified [6]. There is a considerable time delay between fault detection and fault correction processes. The time to correct a fault depends on the complexity of the fault, the skill of the debugging team, the available manpower, and the software development environment and so on. Therefore, the time delay between the fault detection and fault correction is not negligible. In the past, some research activities showed how to use queuing theory to explain software debugging process [7-11]. For example, Wallace and Coleman [8] modeled the time delay between fault detection and correction by the concept of a fault correction queuing service with exponentially distributed delay - a highly statistically significant empirical result based on Shuttle data. Dohi et al. [9] presented an approach to treat both finite and infinite software reliability models in a unified modeling framework. By introducing an Infinite Server Queuing (ISQ) model to describe the software debugging behavior, they showed that it can involve representative NHPP models with a finite, and an infinite number of faults. Gokhale et al. [10] proposed a single-server queue to evaluate the fault correction activity and showed the benefits of applying multi-priority queuing models to the software defect resolution process. In addition, Huang et al [11] proposed an extended infinite server queuing model with multiple change-points to help managers and developers measure software reliability.

These queuing-based models have achieved a great improvement in the accuracy of assessment of software reliability, however, the amount of resources consumed were ignored during software fault detection and fault correction processes. The consumption amount of resources is the key elements for developers and managers during software debugging phase, which is described by testing effort function (TEF) curve. The testing effort is measured by the man-power, the number of CPU hours, and the number of executed test cases and so on [12]. In recent years, researchers have proposed SRGMs to describe the relationship among the testing time, the amount of testing effort expended, and the
number of software faults detected [12–16]. However, the influence of testing effort on fault correction process did not consider in these models discussed above [17]. Musa et al. [2] pointed out the resources that governed the pace of software testing were fault identification personnel, fault correction personnel and computer time, and the fault correction personnel resource had the greatest effect on calendar time prediction. Thus, it is reasonable to incorporate the amount of resources consumed into queuing-based models when describing fault detection and fault correction processes.

In this paper, new SRGMs are derived with consideration of TEF during the fault detection and fault correction processes. ISQ models are used to describe software debugging process, where faults detected and fault correction resources are represented as arrival customers and infinite servers, respectively. Moreover, we assume that fault correction commenced with fault detection, i.e., the time delay between fault detection and fault correction is equal to fault correction time. The proposed models are initially formulated for the generally models, and then special models are given to simplify the forms of the general models. The new models relax the unrealistic assumptions of conventional SRGMs, and thus are capable of improving the quality of software reliability prediction and assessment. The remainder of this paper is organized as follows. Section 2 gives models of testing effort functions. Section 3 presents the formulation of ISQ with TEFs. Section 4 provides an overview of two types of testing effort functions (TEFs): Exponentiated Weibull TEF and Logistic TEF. The Exponentiated Weibull TEF has a great flexibility in accommodating the forms of the consumption rate function and can be used with a wide variety of possible expenditure patterns in actual software projects. Although the Weibull type curve can fit the data well under the general software development environment, it will have an apparent peak phenomenon when the shape parameter \( m > 3 \). Therefore, Huang [15] used the testing effort consumption using a Logistic curve.

A. Exponentiated Weibull TEF

In recent years, Bokhari and Ahmad used an Exponentiated Weibull curve to describe the amount of effort spent on testing [14]. The Exponentiated Weibull curve includes the Exponential, Rayleigh, Weibull, generalized Exponential and generalized Rayleigh (Burr Type X) curves. The current testing effort consumption at time \( t \) is

\[
w(t) = \alpha \beta m t^{m-1} \exp(-\beta t^m) \left[1 - \exp(-\beta t^m)\right]^{m-1}
\]

where \( \alpha \) is the total amount of testing effort expenditures; \( \beta \) is the scale parameter, \( m \) and \( \theta \) are shape parameters. The cumulative testing effort consumed in \((0, t]\) is

\[
W(t) = \int_0^t w(x) dx = \alpha \beta t^m \left[1 - \exp(-\beta t^m)\right]^m
\]

We have the following special cases:

For \( \theta = 1 \) \& \( m = 1 \), there is an Exponential TEF, and the current testing effort consumption at time \( t \) is

\[
w_{ex}(t) = \alpha \beta \exp(-\beta t)
\]

The cumulative testing effort consumed in \((0, t]\) is

\[
W_{ex}(t) = \alpha t \left[1 - \exp(-\beta t)\right]
\]

For \( \theta = 1 \) \& \( m = 2 \), there is a Rayleigh TEF, and the current testing effort consumption at time \( t \) is

\[
w_r(t) = 2\alpha \beta \exp(-\beta t^2)
\]

The cumulative testing effort consumed in \((0, t]\) is

\[
W_r(t) = \alpha t \left[1 - \exp(-\beta t^2)\right]
\]

For \( \theta = 1 \), there is a Weibull TEF, and the current testing effort consumption at time \( t \) is

\[
w_w(t) = \alpha \beta m t^{m-1} \exp(-\beta t^m)
\]

The cumulative testing effort consumed in \((0, t]\) is

\[
W_w(t) = \alpha \beta m t^m \left[1 - \exp(-\beta t^m)\right]^{m-1}
\]

For \( m = 1 \), there is a generalized Exponential TEF, and the current testing effort consumption at time \( t \) is

\[
w_{ge}(t) = \alpha \beta \theta \exp(-\beta t) \left[1 - \exp(-\beta t)\right]^{\theta-1}
\]

The cumulative testing effort consumed in \((0, t]\) is

\[
W_{ge}(t) = \alpha \beta \theta t \left[1 - \exp(-\beta t)\right]^\theta
\]

For \( m = 2 \), there is a generalized Rayleigh TEF, and the current testing effort consumption at time \( t \) is

\[
w_{gr}(t) = 2\alpha \beta \theta \exp(-\beta t^2) \left[1 - \exp(-\beta t^2)\right]^{\theta-1}
\]

The cumulative testing effort consumed in \((0, t]\) is

\[
W_{gr}(t) = 2\alpha \beta \theta t \exp(-\beta t^2) \left[1 - \exp(-\beta t^2)\right]^\theta
\]

B. Logistic TEF

In this paper, new SRGMs are derived with consideration of TEF during the fault detection and fault correction processes. ISQ models are used to describe software debugging process, where faults detected and fault correction resources are represented as arrival customers and infinite servers, respectively. Moreover, we assume that fault correction commenced with fault detection, i.e., the time delay between fault detection and fault correction is equal to fault correction time. The proposed models are initially formulated for the generally models, and then special models are given to simplify the forms of the general models. The new models relax the unrealistic assumptions of conventional SRGMs, and thus are capable of improving the quality of software reliability prediction and assessment. The remainder of this paper is organized as follows. Section 2 gives models of testing effort functions. Section 3 presents the formulation of ISQ with TEFs. Section 4 provides an overview of two types of testing effort functions (TEFs): Exponentiated Weibull TEF and Logistic TEF. The Exponentiated Weibull TEF has a great flexibility in accommodating the forms of the consumption rate function and can be used with a wide variety of possible expenditure patterns in actual software projects. Although the Weibull type curve can fit the data well under the general software development environment, it will have an apparent peak phenomenon when the shape parameter \( m > 3 \). Therefore, Huang [15] used the testing effort consumption using a Logistic curve.

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w_{ex}(t) = \alpha \beta \exp(-\beta t)
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The cumulative testing effort consumed in \((0, t]\) is

\[
W_{ex}(t) = \alpha t \left[1 - \exp(-\beta t)\right]
\]

For \( \theta = 1 \) \& \( m = 2 \), there is a Rayleigh TEF, and the current testing effort consumption at time \( t \) is

\[
w_r(t) = 2\alpha \beta \exp(-\beta t^2)
\]

The cumulative testing effort consumed in \((0, t]\) is

\[
W_r(t) = \alpha t \left[1 - \exp(-\beta t^2)\right]
\]

For \( \theta = 1 \), there is a Weibull TEF, and the current testing effort consumption at time \( t \) is

\[
w_w(t) = \alpha \beta m t^{m-1} \exp(-\beta t^m)
\]

The cumulative testing effort consumed in \((0, t]\) is

\[
W_w(t) = \alpha \beta m t^m \left[1 - \exp(-\beta t^m)\right]^{m-1}
\]

For \( m = 1 \), there is a generalized Exponential TEF, and the current testing effort consumption at time \( t \) is

\[
w_{ge}(t) = \alpha \beta \theta \exp(-\beta t) \left[1 - \exp(-\beta t)\right]^{\theta-1}
\]

The cumulative testing effort consumed in \((0, t]\) is

\[
W_{ge}(t) = \alpha \beta \theta t \left[1 - \exp(-\beta t)\right]^\theta
\]

For \( m = 2 \), there is a generalized Rayleigh TEF, and the current testing effort consumption at time \( t \) is

\[
w_{gr}(t) = 2\alpha \beta \theta \exp(-\beta t^2) \left[1 - \exp(-\beta t^2)\right]^{\theta-1}
\]

The cumulative testing effort consumed in \((0, t]\) is

\[
W_{gr}(t) = 2\alpha \beta \theta t \exp(-\beta t^2) \left[1 - \exp(-\beta t^2)\right]^\theta
\]
\[ W_{gr}(t) = \alpha \left[ 1 - \exp \left( -\beta t^2 \right) \right]^\theta \]  

(15)

**B. Logistic TEF**

Huang et al. [15] found that although a Weibull-type curve can well fit the data often used in the field of software reliability modeling, it display a “peak” phenomenon when the shape parameter \( m > 3 \). Hence, they modeled the testing effort consumption using a Logistic curve [15]. Logistic TEF was originally proposed by Parr [16] and it exhibits similar behavior to the Rayleigh curve except for the initial stage of the project. The Logistic current testing effort consumed at time \( t \) is given by

\[ W_l(t) = \frac{\alpha \beta \exp(-\beta t)}{1 + \alpha \exp(-\beta t)} \]  

(16)

where \( \alpha \) is the total amount of testing effort expenditures, \( \beta \) is the consumption rate of testing effort expenditures, and \( A \) is a constant. The cumulative testing effort consumption in \([0, t]\) is

\[ W_c(t) = \frac{\alpha}{1 + A \exp(-\beta t)} \]  

(17)

**C. Comparisons among Different TEF Models**

To compare the performance of different TEFs, the actual software fault data set was used to these TEFs. The data set, reported by Musa et al. [2], is from the System T1 of the Rome Air Development Center (RADC) Project, and shown in Table I. The comparison criteria for evaluation are described as following [15, 18]:

1. **Prediction Error (PE)**: \( PE_i = (Actual_i - Predicted_i) \)
2. **Bias**: \( \frac{1}{n} \sum_{i=1}^{n} PE_i \)
3. **Variation**: \( \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (PE_i - Bias)^2} \)
4. **Root Mean Square Prediction Error (RMSPE)**: \( \sqrt{Bias^2 + Variation^2} \)

The comparisons among Logistic TEF, Exponential Weibull TEF and Generalized Exponential TEF are illustrated in Table II and Figs.1-2. Figs.1-2 illustrate the comparisons between the observed actual software fault data and the data estimated by the Exponential Weibull TEF and Logistic TEF. The computed the PE, Bias, Variation, and RMSPE based on the actual data set are shown in Table II. Figs 1-2 and Table II show that (1) the Logistic TEF and Exponential Weibull TEF yield a batter fit for the data set chosen; (2) the Logistic TEF and Exponential Weibull TEF are adopted for further analysis.

**TABLE I.**

<table>
<thead>
<tr>
<th>Weeks</th>
<th>CPU hour</th>
<th>Identified Faults</th>
<th>Corrected faults</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00917</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.010</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0.003</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.023</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0.041</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
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<td>2</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.025</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0.302</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>0.973</td>
<td>13</td>
<td>6</td>
</tr>
<tr>
<td>10</td>
<td>0.020</td>
<td>2</td>
<td>4</td>
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<tr>
<td>11</td>
<td>0.450</td>
<td>11</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>0.250</td>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>13</td>
<td>0.94</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>14</td>
<td>1.34</td>
<td>14</td>
<td>19</td>
</tr>
<tr>
<td>15</td>
<td>3.32</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>16</td>
<td>3.56</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>17</td>
<td>2.66</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>18</td>
<td>3.77</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>19</td>
<td>3.40</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>20</td>
<td>2.40</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>21</td>
<td>1.80</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

**TABLE II.**

<table>
<thead>
<tr>
<th>TEF</th>
<th>( PE_{21} )</th>
<th>Bias</th>
<th>Variation</th>
<th>RMSPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logistic</td>
<td>0.2035</td>
<td>-0.0746</td>
<td>0.4354</td>
<td>0.4417</td>
</tr>
<tr>
<td>Exponential Weibull</td>
<td>0.4666</td>
<td>-0.0431</td>
<td>0.4553</td>
<td>0.4573</td>
</tr>
<tr>
<td>Generalized Exponential</td>
<td>0.6431</td>
<td>0.0800</td>
<td>0.5229</td>
<td>0.5290</td>
</tr>
</tbody>
</table>

![Figure 1. Observed/estimated current testing effort by Exponentiated Weibull TEF vs. time](image1)

![Figure 2. Observed/estimated current testing effort by Logistic TEF vs. time](image2)
III. PROPOSED MODELS FRAMEWORK

This section studies how to integrate testing effort into the fault detection and fault correction processes. The formulation of the proposed model is based on the following assumptions [2-5, 7, 11, 15]:

(1) The fault detection process follows a NHPP.
(2) The software is subject to faults at random times, caused by the manifestation of remaining faults in the system.
(3) Testing effort expenditures are described by an EW curve or a Logistic curve.
(4) The mean number of faults detected in the time interval \((t, t+\Delta t)\) by the current testing effort is proportional to the mean number of remaining faults in the system.
(5) The fault correction time is non-negligible so that the number of corrected faults lags behind total number of detected faults.
(6) The proposed queuing model for describing fault detection and correction processes is an ISQ with NHPP arrival and general service time distribution.
(7) No new faults are introduced, when faults are completely corrected in the fault detection and fault correction processes. The formulation of the proposed model is based on the fault data to illustrate that the exponential distribution is a reasonable model for the distributions of fault correction times. Moreover, Musa et al. [2] pointed out that a real software testing effort expenditures to interpret. So, the service rate can be obtained

Based on assumptions (1)-(3), we have

\[
\frac{dm_d(t)}{dt} \times \frac{1}{w(t)} = b(t) \times [a - m_d(t)]
\]

where \(m_d(t)\) is the expected number of faults detected by time \(t\), \(a\) is the expected total number of faults, \(b(t)\) is the fault detection rate per unit detection effort, and \(w(t)\) is the current effort expenditure at time \(t\). Furthermore we have

\[
d\left[m_d(t)\exp\left(\int_0^t b(s)w(s)ds\right)\right] = \frac{d}{dt}\left[a\exp\left(\int_0^t b(s)w(s)ds\right)\right]
\]

Solving Eq. (19) under boundary condition \(m_d(0) = 0\), we get

\[
m_d(t) = a\left[1 - \exp\left(-\int_0^t b(s)w(s)ds\right)\right] = \frac{1}{1 + \lambda_d(t)}
\]

- \(b(s) = b\), \(w_{ew}(t) = \alpha\beta\eta_{ew}^{-1}\exp[-\beta\eta_{ew} t]\)
- \(m_d(t) = a(1-\exp[-bW_{ew}(t)+bW_{ew}(0)]) = a(1-\exp[-bW_{ew}(t)])\)

Where \(N_d(t), N_c(t)\) be the cumulative number of faults detected and faults corrected at \(t\), respectively. We assume that there are \(n\) faults detected and \(k\) faults completely corrected in \([0,t]\). We have

\[
\Pr[N_d(t) = k, N_c(t) = n] = \frac{n!}{k!(n-k)!} \cdot \frac{1}{(1+\lambda_d(t))} \cdot \left(\int_0^t b(s)w(s)ds\right)^n \cdot \exp[-a(1-\exp[-bW_{ew}(t)])]
\]

So, we obtain

\[
\Pr[N_c(t) = k] = \frac{1}{(1+\lambda_d(t))} \cdot \left(\int_0^t b(s)w(s)ds\right)^n \cdot \exp[-a(1-\exp[-bW_{ew}(t)])]
\]

The mean value function of \(N_c(t)\) is given by

\[
E[N_c(t)] = \frac{m_c(t)}{\lambda_d(t)} \cdot \frac{1}{\lambda_d(t)} \cdot \left(\int_0^t b(s)w(s)ds\right)^n \cdot \exp[-a(1-\exp[-bW_{ew}(t)])]
\]

The probability that a fault is detected at time \(t\) is

\[
p(t) = \int_0^t p\{T_s \leq t - \tau \cap T_i = \tau\} d\tau
\]

Substituting (26) and (27) into (25), we can get

\[
m_c(t) = \int_0^t G(t - \tau)\lambda_d(\tau) d\tau
\]

where \(g(\bullet)\) is the probability density function of the service time.

It is well known that human queue service time can be approximated with an exponential distribution [7]. Moreover, Musa et al. [2] pointed out that a real software fault data to illustrate that the exponential distribution is a reasonable model for the distributions of fault correction times. Hence, we assume that the service time is exponential distribution [11]. That is

\[
G(t) = 1 - \exp(-\eta t)
\]

\[
g(t) = \eta \exp(-\eta t)
\]

where \(\eta\) is the service rate. That is to say, \(\eta\) is fault correction rate, and is a constant. In reality, fault correction rate strongly depends on the skill of debuggers or programmers, the amount of faults detected, and the number of correction resource consumed and so on. Particularly, the influence of resource on fault correction should be considered in correction process. Thus, we treat the fault correction rate as the time-dependent behavior of testing effort expenditures to interpret. So, the service rate can be obtained.
\[ \eta(t) = \eta^m(t) \]  

where \( \eta(t) \) is time-dependent fault correction rate function per unit effort at time \( t \). Substituting (28b) and (21), and (22), we can obtain respectively

\[ m_e(t) = \int_0^t m_d(x) \eta w_c(x) \exp(-\eta W_e(x) + \eta W_c(x)) dx \] (31)

\[ m_c(t) = \int_0^t m_d(x) \eta W_c(x) \exp(-\eta W_e(x) + \eta W_c(x)) dx \] (32)

IV. NUMERICAL EXAMPLE

A. Data Description and Comparison Criteria

The real data set is from the System T1 of the RADC Project in [2] and shown in Table 1. System T1 was used for a real time command and control application. The size of software was about 21700 object instructions. It took 21 weeks’ testing, 25.3 CPU hours, 9 programmers, and 136 software faults corrected.

To give quantitative comparisons, some criteria were used to judge the performance of the proposed model. The comparison criteria are the MSE and R-squared which are defined as follows [2].

1. Mean Squared Errors (MSE)

\[ \text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (m_i - \hat{m}(t_i))^2 \] (33)

2. R-squared

\[ R - \text{squared} = 1 - \frac{\sum_{i=1}^{n} (\hat{m}(t_i) - \bar{m})^2}{\sum_{i=1}^{n} (m_i - \bar{m})^2}, \quad \bar{m} = \frac{1}{n} \sum_{i=1}^{n} m_i \] (34)

where \( m_i \) is the total number of fault detected by \( t_i \), and \( \hat{m}(t_i) \) is the estimated cumulative number of faults by time \( t_i \) obtained from the fitted mean value function. The lower MSE value means the better goodness-of-fit of the model. The R-squared can take on any value between 0 and 1, with a value closer to 1 indicating a better fit.

B. Performance Analysis

This section evaluates the performance of proposed models and several existing models. First of all, parameters of selected models are estimated and the related mean value functions are obtained. Secondly, all the selected models are compared with each other based on objective criteria. In the past, fault detection and correction resources are not considered as separate resources, and the data sets of effort are not treated as separate. So, we assumed the detection effort and the correction effort were called testing effort. According to the comparisons among various TEFs are illustrated in section 2, the Exponentiated Weibull TEF and Logistic TEF are chosen to fit the actual software fault data set curve. The parameters \( \alpha, \beta, m, \theta \) and \( A \) in TEFs can be estimated by least squares estimation. The estimators for \( \hat{\alpha}, \hat{\beta}, \hat{m}, \hat{\theta} \) and \( \hat{A} \) are investigated for testing effort \( W_i \) spent at testing time \( t_i (i = 1, 2, \ldots, 21) \) and are listed in Table III.

<table>
<thead>
<tr>
<th>TEF</th>
<th>Estimated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponentiated Weibull TEF</td>
<td>( \hat{\alpha} = 26.83, \hat{\beta} = 6.165e-9, \hat{\theta} = 0.95, \hat{m} = 6.5508 )</td>
</tr>
<tr>
<td>Logistic TEF</td>
<td>( \hat{\alpha} = 29.1065, \hat{\beta} = 0.493515, \hat{m} = 4624.89 )</td>
</tr>
</tbody>
</table>

Using the estimated parameters \( \hat{\alpha}, \hat{\beta}, \hat{m}, \hat{\theta} \) and \( \hat{A} \), the other parameters \( a, b \) and \( \eta \) can be estimated by the method of least squares estimation. Table IV compares the performance of various SRGMs for the actual fault data set investigated in this paper. Due to the limitations of paper size, only 2 models [13, 19] are used for detailed discussions. In [13], Huang et al. proposed a SRGM incorporated the Logistic TEF. However, the time to correct a fault is negligible. In [19], Lo developed a general framework of the fault detection and fault correction processes, but the amount of resources consumed was ignored during these two processes. These have better performance as shown in Table IV. As shown in Table IV, the proposed models in this paper provide the lower value of MSE and the highest value of R-squared than other models. Figs.3-4 plotted the actual fault data and the estimated mean value function up to time \( t \). Fig.3 show that the Exponentiated Weibull TEF incorporates into ISQ SRGM. Fig.4 show that the Logistic TEF incorporates into ISQ SRGM. From Figs. 3-4, we can see that these models fit the observed data better, and predict the future behavior well for this actual fault data set. Thus, from these simulation/comparison results, the proposed model performs appreciably better than the others.

V. CONCLUSION

Many resources are consumed by a software development project. Most papers assumed that the consumption rate of testing resource expenditures during the testing and debugging phase is a constant or even do not consider such testing effort. In reality, software reliability models should be developed by incorporating different testing-effort functions. In this paper, we have shown how to apply infinite server queuing models with TEFs to model the fault detection and correction processes. New SRGMs with an Exponentiated Weibull TEF and a Logistic TEF are proposed that can consider the influence of resources on software debugging phase, and enhance the prediction and assessment of software reliability. Experiments are performed based on real software fault data set. Comparing with some selected SRGMs, experimental results show that the proposed models give a better fit to the observed data.

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<table>
<thead>
<tr>
<th>Models</th>
<th>Estimated Parameters</th>
<th>MSE</th>
<th>R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1(t) ) in Equation (21)</td>
<td>( \hat{a} = 139.6 )  ( \hat{b} = 0.1297 )  ( \hat{\eta} = 0.8794 )</td>
<td>MSE = 42.94</td>
<td>R-square = 0.9826</td>
</tr>
<tr>
<td>( m_1(t) ) in Equation (31)</td>
<td></td>
<td>MSE = 60.4762</td>
<td>R-square = 0.9745</td>
</tr>
<tr>
<td>( m_2(t) ) in Equation (22)</td>
<td>( \hat{a} = 141.3 )  ( \hat{b} = 0.1293 )  ( \hat{\eta} = 0.8628 )</td>
<td>MSE = 32.17</td>
<td>R-square = 0.9869</td>
</tr>
<tr>
<td>( m_2(t) ) in Equation (32)</td>
<td></td>
<td>MSE = 58.6667</td>
<td>R-square = 0.9753</td>
</tr>
<tr>
<td>Huang et al. [13]</td>
<td>( \hat{a} = 138.026 )  ( \hat{b} = 0.1451 )  ( \hat{\eta} = 0.7825 )</td>
<td>MSE = 62.41</td>
<td>R-square = 0.9758</td>
</tr>
<tr>
<td>Lo [19]</td>
<td>( \hat{a} = 133.119 )  ( \hat{b} = 0.1597 )  ( \hat{c} = 0.7825 )</td>
<td>MSE = 58.70</td>
<td>R-square = 0.9812</td>
</tr>
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Figure 3. Observed/estimated cumulative number of faults using the Exponentiated Weibull TEF

Figure 4. Observed/estimated cumulative number of faults using the Logistic TEF

**REFERENCES**


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