Model-based feedback control of distributed air-conditioning loads for fast demand-side ancillary services

Julio H. Braslavsky, Cristian Perfumo and John K. Ward

Abstract—Load control (LC) of distributed populations of air conditioners (ACs) can provide effective demand-side ancillary services while reducing emissions and network operating costs. Pilot trials with ACs typically deploy model-free, open-loop strategies, which cannot deliver the full potential of LC as a network resource. Seeking more advanced strategies, much research in recent years has targeted the development of accurate models and LC approaches for this type of loads. Most existing approaches, however, are restricted to scenarios involving large numbers of ACs, which may not work in small populations, or require two-way communications with the controlled devices, which may come at high costs in widely distributed populations. This paper exploits a previously developed dynamic model for the aggregate demand of populations of ACs to design a simple controller readily implementable in such LC scenarios. The proposed feedback scheme broadcasts thermostat set-point offset changes to the ACs, and requires no direct communications from the devices to the central controller, using instead readings of total aggregate demand from a common power distribution connection point, which may include demand of uncontrolled loads. The scheme is validated on a numerical case study constructed by simulating a distributed population of ACs using real power and temperature data from a 70-house residential precinct, and is shown to deliver robust fast load following performance. The simulation results highlight the practical potential of the proposed model and feedback control scheme for analysing and shaping demand response of ACs using standard control techniques.

I. INTRODUCTION

A load management strategy of growing importance in recent years is the LC of distributed populations of thermostatically controlled loads (TCLs), such as ACs, fridges, and water heaters [1], [10], [14], [17]–[19]. ACs are especially interesting for LC because they are increasingly predominant and can provide fast responses with minimal impact to users, if carefully managed [4], [22]. In countries like Australia, summer peak electricity demand is driven by ACs, which typically account for one-third of the total peak demand, and as much as half of the total demand in states such as South Australia, where 90% of the households have ACs [9].

Demand-response pilot trials run by utilities [3], [7], [10], [11], [13], [16] have demonstrated in practice that the control of AC loads can be very effective in reducing peak demand. However, the model-free strategies typically deployed in these trials cannot deliver the full potential of these loads. Several authors have made compelling cases for TCLs as a resource for demand-side ancillary services such as load following, required to balance intermittency in renewable energy generation [4], [5], [13], [15], [17], [25]. ACs could indeed play a key role in the integration of renewable energy, but arguably not without exploiting knowledge of population dynamics to deliver the fast shaping of aggregate demand response to balance intermittency within short time scales.

A number of model-based LC strategies for TCLs have been proposed in recent years, with promising results [1], [4], [14], [17], [18], [22], [26]. For example, Callaway shows the potential of feedback LC for load following of variable wind generation using a minimum-variance controller based on a linearised model of aggregate demand response [4]. Bashash and Fathy propose in [1] a nonlinear feedback controller to regulate aggregate demand based on a finite-difference model. Sinitsyn and coauthors introduce in [26] a model-based control scheme to generate aggregate demand responses with arbitrary profiles without requiring feedback.

Two common assumptions that underly many existing LC strategies pose obstacles to their practical implementation. Firstly, the demand of (all or a representative subset of) the controlled TCLs is assumed measurable for feedback [4], [5], [17]. Besides privacy issues that could impact customer engagement [24], [26], accessing individual AC demand requires two-way communications infrastructure, which comes at high costs in LC scenarios involving large, widely distributed populations. Secondly, the population of controllable devices is commonly assumed to be in the order of thousands. While this assumption is convenient to work with probabilistic TCL models [4], [17], [18], [22], it does not apply in small-scale programs, such as LC for minigrids or precincts, which may involve just a few tens of devices.

A relaxation of the requirement of two-way communications for LC is discussed in [21], where the authors suggest using load forecasts and a Kalman filter to estimate aggregate TCL demand from feeder readings. Such forecasts, however, may not be available, or they may not have the levels of prediction accuracy required. In contrast, the model-based open-loop LC proposed in [26] circumvents the requirement of feedback altogether by introducing some local control actions in the devices. While such open-loop control scheme is promising for peak demand reduction, it is unclear how it could be implemented to shape demand to follow rapidly-changing output of renewable generation.

The present paper discusses a model-based feedback LC scheme that achieves fast demand response for peak reduction or load following, in small or large populations of ACs, and which can be implemented without recourse to load forecasts. Our LC scheme exploits a dynamic model developed.
in [22], [23] for the aggregate demand of populations of ACs in the design of a simple controller directly parameterised by the distributed parameters of the target population. The robust performance of this controller is validated in a numerical LC case study constructed by aggregating real power demand and temperature data for a 70-house residential precinct in Australia with the simulated demand of a corresponding heterogeneous population of ACs.

The ACs modelled in the case study are connected to a common power distribution point (e.g., a feeder) and are assumed to be responsive to small thermostat set-point offsets broadcast by a central aggregator-controller. Readings of the total aggregate residential demand (comprising ACs and other, uncontrolled loads) are transmitted to the central controller, which computes the new temperature set-point offsets. The controller-ACs communications are thus effectively one-way, as the power of individual devices is not measured.

The case study shows that the proposed strategy can effectively operate with feedback from aggregate demand and achieve fast demand response, with quantifiable impact to end-use operation. Demand is accurately dynamically shaped to achieve fast peak reduction, or to follow fast variations in wind generation even for the small population studied.

II. A SECOND-ORDER LTI MODEL FOR THE DYNAMIC DEMAND RESPONSE OF AC LOADS

The cornerstone of the LC scheme discussed in this paper is a linear time-invariant (LTI) model developed in [22], [23] for the aggregate demand response of an heterogeneous population of ACs. This model represents the aggregate power demand response $D_{ac}(t)$ of $n$ ACs (kW) to a common simultaneous step change in their temperature set-points as

$$D_{ac}(t) = D_{ss}(\theta^r) - D_{ss}(t),$$

where $D_{ss}(\theta^r)$ is the asymptotic mean steady-state aggregate demand of the population with constant temperature set-point $\theta^r$ (assumed uniformly distributed in the population), and $D_{ss}(t)$ is the unit (temperature offset) step response of a second order transfer function model of the form

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + 2\xi \omega_n s + \omega_n^2}. \tag{2}$$

The model parameters $b_2, b_1, b_0, \omega_n$ and $\xi$ in (2) are given as explicit functions of the means and variances of the physical parameters (such as thermal power, resistance and capacitance) distributed in the modelled population of ACs. For illustration (see [22], [23] for the full set of formulas),

$$\omega_n = \frac{\pi \mu_v}{\sqrt{6 - a} \sqrt{1 - \xi^2}}, \quad \xi = \frac{\log(r)}{\sqrt{\pi^2 + \log^2(r)}}, \tag{3}$$

where $\mu_v = \text{mean}[(\theta_a - \theta^r)/(CR)], a = \exp[\log(\sqrt{2}) - \xi^2 \log(\xi^2)]$, and

$$r = \frac{\text{erf} \left(0.9 + \sqrt{8\sigma_{\theta_a}}\right) - \frac{1}{2}}{\text{erf}[0.9] - \frac{1}{2}},$$

where erf[·] is the Gauss error function.

These expressions are obtained in [22], [23] by analytically fitting the step response of the second order transfer function (2) to a simplified parametric stochastic model for the aggregate demand of a distributed population of $n$ ACs. Such aggregate demand can be calculated at each instant $t$ from the (normalised) expression

$$D_{ac}(t) = \frac{\sum_{i=1}^{n} m_i(t) P_i}{\sum_{i=1}^{n} P_i \text{COP}_i}, \tag{4}$$

where $P_i$ and COP$_i$ represent the thermal power and coefficient of performance (cooling) of the $i$-th AC. Each AC is assumed to autonomously regulate temperature via a thermostat and hysteretic relay switching to the state $m_i(t) = 1$ when the compressor is on, and $m_i(t) = 0$ when it is off.

The exact aggregate dynamic behaviour of such a population of loads can be rather complex and has motivated a number of modelling approaches [4], [17], [18], [20], [27]. These models, however, have been in general difficult to translate into implementable control designs. In contrast, Equations (1)-(2) capture essential population dynamics in a simple LTI model that enables the analysis and design of robust controllers using well-understood control techniques.

A central feature in the proposed model is that its parameters are explicit functions of the distribution parameters of the population, which enables one to analyse fundamental behaviour (e.g., stability and performance) for general populations on a very simple model structure. This model also facilitates rapid ballpark control design without necessarily having to run identification experiments (compare [4], [21]), for example by using sample means and variances of the thermal capacities, resistances and power ratings of the ACs in the population by exploiting prior knowledge, such as statistics on building and AC ratings for the target population.

The LTI model given by Equations (1)-(2) produces responses that can be (given its simplicity) surprisingly close to those obtained from (4) and the simulation of an array of $n$ ACs with distributed thermal characteristics and independently operating relays. Figure 1 shows demand responses to a simultaneous step change of half a degree Celsius in temperature set-point: the response of the LTI model given by (1) and (2), and the responses of populations comprising $n = 60$ and $n = 10000$ ACs obtained from (4) by simulating $n$ independent devices, each with dynamics [6]

$$\frac{d\theta_i(t)}{dt} = -\frac{1}{C_i R_i}[\theta_i(t) - \theta_a(t) + m_i(t) R_i P_i], \tag{5}$$

$$m_i(t^+) = \begin{cases} 0, & \text{if } \theta_i(t) \leq \theta_i^+ + u(t), \\ 1, & \text{if } \theta(t) \geq \theta_i^+ + u(t), \\ m_i(t), & \text{otherwise}. \end{cases}$$

The LTI model and the distributed populations were simulated using the parameters given by Table I, for which the formulas in [22, 23, §5.3] yield in (1), (2) $b_2 = D_{ss}(\theta^r) = 0.5, b_1 = 0.11, b_0 = 1.5 \times 10^{-4}, \xi = 0.02$ and $\omega_n = 1.7 \times 10^{-3}$. The simulations were implemented on the event-driven simulation environment PowerDEVs, which is
especially formulated for numerically efficient computations in hybrid (continuous and discrete-state) dynamic systems.

### TABLE I

**SIMULATION PARAMETERS.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>$\mu_R = 2$</td>
<td>Thermal resistance (°C/kW), log-normally distributed with mean $\mu_R$ and variance $\sigma^2_R = \mu^2_R e^{\mu_R^2/2}$,</td>
</tr>
<tr>
<td>C</td>
<td>$\mu_C = 3.6$</td>
<td>Thermal capacitance (kWh°C), log-normally distributed with mean $\mu_C$ and variance $\sigma^2_C = \mu^2_C e^{\mu_C^2/2}$,</td>
</tr>
<tr>
<td>P</td>
<td>$\mu_P = 6$</td>
<td>Thermal power (kW), log-normally distributed with mean $\mu_P$ and variance $\sigma^2_P = \mu^2_P e^{\mu_P^2/2}$,</td>
</tr>
<tr>
<td>$\theta^-$</td>
<td>$\in [19, 20]$</td>
<td>Lower end of hysteresis band, uniformly distributed in $[19, 20]$ (°C).</td>
</tr>
<tr>
<td>$\theta^+$</td>
<td>$\in [20, 21]$</td>
<td>Higher end of hysteresis band, uniformly distributed in $[20, 21]$ (°C).</td>
</tr>
<tr>
<td>$\theta^t$</td>
<td>$\in [19.5, 20.5]$</td>
<td>Temperature set-point for the ACs (°C), uniformly distributed.</td>
</tr>
<tr>
<td>$\theta_a$</td>
<td>Variable</td>
<td>Ambient temperature (°C).</td>
</tr>
<tr>
<td>$H$</td>
<td>1</td>
<td>Hysteresis width ($\theta^+ - \theta^-$) (°C).</td>
</tr>
<tr>
<td>$\sigma_{rel}$</td>
<td>0.2</td>
<td>Standard deviation of log-normal distributions as fraction of the mean value for R, C and P.</td>
</tr>
<tr>
<td>COP</td>
<td>2.5</td>
<td>Coefficient of performance (thermal power on electrical power)</td>
</tr>
<tr>
<td>$n$</td>
<td>60</td>
<td>Number of ACs in the population.</td>
</tr>
</tbody>
</table>

### III. MODEL-BASED LOAD CONTROL DESIGN

The LTI second order model (2) is exploited in [22] to design a robust internal model controller (IMC) that regulates aggregate power of a population of ACs assuming availability of direct measurements of the aggregate demand of the population. In this section we propose the very simple integral controller

$$C(s) = \frac{K_I}{s}$$

in the feedback control scheme of Figure 2. The controller uses the error between a reference for the desired aggregate demand $D^r(t)$ for the feeder during the LC event, and the measured feeder demand $D(t)$, which aggregates the demand of the ACs, $D^{ac}(t)$, and that of non-controllable loads, $W(t)$. The computed temperature set point offset $u(t)$ is broadcast as a common control signal to the population of ACs.

The closed-loop transfer function from $D^r(t)$ to $D(t)$ obtained with the integral controller (6) (replacing the dashed block representing the ACs in Figure 2 by the LTI model (1), (2)) is the complementary sensitivity function (CSF)

$$T(s) = \frac{K_I (b_2 s^2 + b_1 s + b_0)}{s^3 + (2 \xi \omega_n + K_I b_2) s^2 + (\omega_n^2 + K_I b_1) s + K_I b_0}.$$ (7)

Standard control techniques may be used to study the system closed-loop response from (7) as $K_I$ varies, and design a suitable value. It is not hard to verify that the closed-loop system is asymptotically stable for any positive value of $K_I$ and, by integral action, that the output will asymptotically converge to any constant reference value, provided there is enough controllable AC demand to achieve it.

Figure 3 shows simulated closed loop step responses [aggregate demand (closed-loop output) and broadcast temperature set-point offsets (control input)] obtained for a population of 10000 ACs with the distribution parameters given in Table I. Aggregate load is controlled with the integral controller (6). The plot compares the population response against the step response of the target CSF (7) for the values of $K_I = 0.5$ and $K_I = 2$. These simulations assume constant ambient temperature $\theta_a$, disregarding uncontrolled demand $W$, so that $D(t) = D^{ac}(t)$ in the schematics of Figure 2. The closeness of the responses in both cases confirms that the LTI model (2) indeed captures the essential dynamics required for tight shaping of aggregate demand.

Figure 3 illustrates the closed-loop response overshoot and settling time for the two values of $K_I$. For example, Figure 3(a) shows that for $K_I = 0.5$, the response exhibits overshoot and, more importantly, takes hundreds of minutes to reach the desired value. On the other hand, it can be seen in Figure 3(b) that for $K_I = 2$ the overshoot is substantially reduced and the settling time is only a couple of minutes.

Improving tracking performance with larger values of $K_I$ comes at the cost of requiring faster control action. The right-hand side sub-plots of Figure 3 show that for the same interval of time (200 minutes), the larger value of $K_I = 2$ produces a more fine-grained, rapidly-varying control signal $u(t)$ than $K_I = 0.5$. In a real-world implementation, the
best achievable closed-loop performance would be limited by communication delays and the bandwidth of the communications channels used to broadcast the control signal to the population and to transmit the aggregate demand measurements to the controller. Communications thus naturally set an upper limit to the implementable value of $K_I$.

IV. CASE STUDY: DEMAND SHAPING PERFORMANCE ON A POWER NETWORK

We now construct a case study that combines real power distribution data with AC population responses simulated on PowerDEVS [2] to validate the performance of the proposed integral controller on a realistic LC scenario with a small population of $n = 60$ ACs, operating under variable ambient temperature and in the presence of non-neglectable demand $W$ from uncontrollable loads in feeder measurements.

The integral controller (6) is implemented in the configuration shown in Figure 2, so that only total aggregate demand readings $D(t) = D^{ac}(t) + W(t)$ are available for feedback. The demand of uncontrolled loads $W(t)$ corresponds to real demand data logged from a distribution feeder supplying 70 customers in a residential area on the east coast of Australia. The ACs are simulated on PowerDEVS using real ambient temperature data $\theta_{ac}(t)$ logged at location in the same period. These power and temperature readings are shown in Figure 4.

We assume that load profiles of mild-temperature days (e.g., Friday 11/11 and Tuesday 15/11 in Figure 4) do not include any AC load (temperatures then peaked at 21.9 and 23.5 C). We then construct the load profile for the hot Monday 14/11 by aggregating the total (assumed temperature uncorrelated) load of a mild-temperature day with AC load simulated from (4), (5) with the parameters of Table I and using the temperature profile for Monday 14/11.

Figure 5 shows the real and simulated load for the hot Monday 14/11. The temperature-uncorrelated load component (non-AC) is assumed to be the total load of the mild Tuesday 15/11. We assume that 60 out of the 70 customers in the residential area have an AC operating during the simulated day. This assumption aligns with studies that indicate that 90% of the houses in the Sydney area operate their ACs on the warmest days [12]. The ACs are randomly turned on from 12:30 to 16:00 to simulate the arrival of people at their houses, and turned off between 21:00 and 00:30, as people go to sleep (following statistics on residential AC usage in New South Wales, Australia [8]).

Figure 5 illustrates the extent of peak demand shaping achievable by LC using the integral controller (6) ($K_I = 2$).
by manipulating all 60 ACs connected to the feeder: at 18:00 the total demand $D(t)$ is lowered by more than 40% and maintained at that level for 1.5 hours, after which it is progressively ramped up to a predefined level, where it is maintained for another hour to allow for end-use comfort recovery. It can be observed that at all times during the control period, the load is tightly maintained.

The temperature set point change $u(t)$ used as a control signal is depicted in the bottom plot of Figure 6. Note that $\theta_i^+ + u(t)$ (where $\theta_i^+$ is the upper bound of the hysteresis width of the $i$th device) is an upper bound for the average temperature of the $i$th AC in the population. If $u(t)$ is maintained constant for long enough, then $\theta_i^+ + u(t) \leq \theta_i(t) \leq \theta_i^+ + u(t)$. Thus, an advantage of using temperature set point offsets as a control signal is that the comfort impact of a LC scenario can be easily estimated [22].

In a real-world scenario, the impact of raising the temperature set point of the ACs by up to 4°C, as done in Figure 6, could be unacceptable for the customers. This extreme case is chosen to illustrate the demand reduction achievable by LC of this small population with the proposed model-based controller. The peak reduction performance holds even though the assumptions under which the model was developed do not hold, including the presence of (unmodelled) loads perturbing the output signal used for feedback. In practice, the achievable demand shaping by LC will be limited by the number of responsive ACs operating, and will be higher, and with less end-use impact, in larger populations.

![Figure 6](image)

**B. Load following**

Another scenario of interest for model-based LC of ACs is that of load following, addressed in [4]. In this scenario the control target is to shape the ACs demand to match the variable power of a renewable energy generation source. Figure 7 shows the simulation results for the same population of ACs considered above, now controlled to follow the variable output of wind generation. The reference signal is the sum of 200 kW of assumed constant generation plus twice the aggregate output of three 20-kW wind turbines (1-minute data sampling) at the CSIRO Energy Technology site in Newcastle, Australia. The output of the wind turbines is doubled to simulate a higher level of wind penetration, which represents a higher-variability scenario than what would be observed for six independent equivalent 20-kW turbines.

Figure 7 shows how the integral controller can make the total demand tightly follow the variable output of renewable generation with a maximum change in temperature set-point of 1.5°C. The proposed model-based control design approach is also a practical enabler of load following of TCLs to effectively “smooth out” variability in renewable generation, seemingly with comparatively smaller comfort impact to customers than in peak reduction scenarios.

![Figure 7](image)

Controlling 100% of the ACs in the population is admittedly an ideal case. In practice, the number of ACs available would be a function of ambient temperature, with a higher percentage of the population available at higher temperatures, which is when LC could offer greater benefits.

**V. CONCLUSIONS**

We have presented a simple design methodology for LC of heterogeneous populations of ACs based on a novel parametric LTI model developed by the authors in [22]. We have illustrated the robustness and performance of such controller in a LC case study constructed using real distribution data for a medium voltage residential network of 70 houses. The controller is directly calculated using the distributed parameters characterising the population, namely, the means and variances of the thermal capacitances, resistances and power ratings the ACs, which are assumed stochastically distributed in the population. While the exact distribution of these parameters might be difficult to obtain, in practice sample means and variances from prior statistical knowledge of the population (for example, building and AC ratings) could be used as reasonable estimates.

Using aggregate demand readings from a substation for feedback, the controller is shown to achieve tight performance in peak demand shaping and load following scenarios.
By introducing offsets to the users temperature set-points, this LC strategy clearly has a direct impact to the thermal behavior of the system. However, such impact is directly quantified by the amplitude of the control signal broadcast — precisely the sequence of temperature offsets introduced during a LC event — and could be moderated by limiting the controller gain. See [22] for more discussion on tradeoffs between controllability and comfort using the predicted percent dissatisfied metric.

Existing feedback control strategies (with the exception of that in [21]) typically require measurements of the power output of all the controlled ACs for feedback. In reality, such requirement currently translates into expensive two-way communications infrastructure for each AC for implementation. While such two-way communications infrastructure may become more cost-effective in the future with large smart meter roll-outs, the achievable sample rates might not be as high as required for tight feedback LC.

In contrast, readings of aggregate power usage at substations and transformers are readily available. The obvious limitation of using these readings is that, even under the assumption that all the ACs connected to the feeder have a demand response enabling device, such power readings still include other (uncontrolled) loads. We have shown that the proposed feedback controller can overcome such limitation and may be implemented using one-way (broadcast) communications, which shows great potential for financially viable deployments even at small network scales.

The results in this paper demonstrate the practical potential of the proposed model and model-based control design approach, bridging a gap between open loop LC trials currently carried out by electricity utilities [3], [7], [11], [13], [16] and advanced modelling and feedback control algorithms proposed in the literature [1], [4], [14], [17], [18], [22].

REFERENCES


