Statistical Analysis and Data Analysis of Stock Market by Interacting Particle Models

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Abstract—The statistical analysis of Chinese stock market fluctuations modeled by the interacting particle systems has been done in this paper. The contact model and voter model of the interacting particle systems are presented in this paper, where they are the continuous time Markov processes. One interpretation of contact model is as a model for the spread of an infection. One interpretation of voter model is, an individual reassesses his view by choosing a neighbor at random with certain probabilities and then adopting his position. In the first part of this paper, based on the contact process, a new stochastic stock price model of stock markets is modeled. From it, the statistical properties of Shenzhen Composite Index are studied. The data of Shenzhen Stock Exchange (SZSE) Composite Index is analyzed, and the corresponding simulation is made by the computer computation, and we further investigate the statistical properties, fat tails phenomena and the power-law distributions of returns. The methods of Skewness-Kurtosis test, Kolmogorov-Smirnov test are applied to study the fluctuation behavior of the returns for the stock price and Index. In the second part of this paper, based on the voter model, we study the statistical properties of prices changes for the different dimensions, intensity of the model and initial density $\theta$.

Index Terms—data analysis, contact model, voter model, statistical properties, computer simulation, market fluctuation, prices changes

I. INTRODUCTION

In recent years, the empirical research in financial market fluctuations has been made with a great progress. Some statistical properties for market fluctuations uncovered by the high frequency financial time series, such as fat tails distribution of price changes, the power-law of logarithmic returns and volume, volatility clustering which is described as on-off intermittency in literature of nonlinear dynamics, and multifractality of volatility, etc, for example, see [1-2,4-13,15-22]. More specifically, the empirical research has shown the power-law tails in price fluctuations. Some research results show that the distribution of large returns follow a power law distribution with exponent 3, that is

$$P(r_i > x) \approx \frac{1}{x^\mu}$$

where $r_i$ is the returns of the stock prices in a given time interval $\Delta t$, $\mu \approx 3$, for example, see [4,6,7,10,11].

In this paper, we study the fluctuations of stock prices and the return process and prices changes in a stock market by applying the theory of contact model and voter model. Through computer simulations on these financial models, we discuss the tail behavior and the statistical behavior of fluctuations for the return processes and prices changes, especially we study the statistical properties of absolute value for the returns and prices changes. And also we analysis quotes data from Shenzhen Stock Exchange during the year 2000-2007, and study the fluctuations of SZSE Composite Index, since SZSE Composite Index is the most important security index in China. The original attempt of this work is to study the financial phenomena by statistical physics models.

Stauffer and Penna [18], Tanaka [19] has studied the market fluctuations by the percolation model [3,14]. In their study, they assume the information in the stock market leads to the stock price fluctuations and the investors in stock market follow the effect of sheep flock, that is, the investment decision-making by other investors dissemination of information, thus stock price fluctuations will eventually depend on the investor’s investment attitude of the securities markets. In this paper, we apply other statistical physics models (contact model and voter model) to study the fluctuations of the return processes and prices changes. For the details of the statistical physics models, see [3, 14].

II. THE CONTACT MODEL

The contact model is a crude model of the spread of a disease or a biological population. In one dimension space, one point is stood by one individual. The virus infects one proximate individual at a rate equal to $\lambda$, where $\lambda$ is an intensity which is called the “carcinogenic
advantage” in contact model. And the individual infected recovers at rate one. Speaking concretely, contact model is a continuous time Markov process \( \eta \) in the configuration \( \{0,1\}^d \). At some random time, one individual at the point \( x \) is deemed to be infected when \( \eta_x(t) = 1 \) and the infected individual recovers at rate one; if \( \eta_x(t) = 0 \), the individual at the point \( x \) is healthy and will be infected at a rate equal to \( \lambda \) times the number of the infected neighbors.

In the one-dimension contact model, for each \( x \) and \( y \) with \( |x-y|=1 \) let \( \{T_{n,x,y}\}, n \geq 1 \) be a Poisson process with \( \lambda \), and let \( \{U_{n,x}\}, n \geq 1 \) be a Poisson process with a rate one. At times \( T_{n,x,y} \), we draw an arrow from \( x \) to \( y \) and indicate that if \( x \) is infected then \( y \) will become infected (if it is not already). At times \( U_{n,x} \), we put a \( \delta \) at \( x \). The effect of a \( \delta \) is to recover the individual at \( x \) (if one is infected). For simplicity, we give the construction of graphical representation for one-dimension contact model, see Figure 1.

![Figure 1](image.png)

**Figure 1.** The graphical representation of the one-dimension contact model. The x-axis is the sites and the y-axis is the time. Right arrow and left arrow express the Poisson processes of infecting the right neighbor and of infecting the left neighbor. The \( \delta \) expresses the Poisson process of recovering himself.

III. MODELING THE FINANCIAL MODEL BASED ON THE CONTACT MODEL

**A. Modeling a Financial Price Model by the Contact Model**

We have introduced the contact model above, in the following, we make use of the contact model to construct the return process of stock price on d-dimensional integer lattice. In this paper, we also assume that the stock price fluctuates because of the information in the stock market. The stock price fluctuations will eventually depend on the investors’ investment attitude of the securities markets. We assume that the viruses of contact model represent the information in the stock market and construct the stock price process based on the contact model. The information is divided into buy information (or good news) and sell information (or bad news). At time \( t \), we compute the number of the investors with the buying positions \( \Sigma(t) \), and the number of investors with the selling positions \( \Sigma(t) \). Finally, the effective of the investment can be calculated as

\[
\Sigma(t) = \Sigma(t) - \Sigma(t).
\]

As we know, stock price \( S(t) \) is decided by the differential equation

\[
\frac{dS}{dt} = \alpha x(t)S(t)
\]

where we let \( x(t) = \frac{\Sigma(t)}{N} \), where \( N \) is a large integer and \( \alpha \) is the constant of proportionality. We can get the formula of stock price

\[
S(t+1) = \exp(S(t))
\]

But this definition produces one problem that the extent of price change going up is more than that of price change going down in the same absolute value of \( \Sigma(t) \). It will weaken the effect of the data of price change going down, leading to getting imprecise statistic result. So we improve the formula of stock price as

\[
S(t+1) = \begin{cases} 
\exp\left(\frac{\alpha \Sigma(t)}{N}\right) & \text{change up} \\
2 - \exp\left(\frac{\alpha \Sigma(t)}{N}\right) & \text{change down}
\end{cases}
\]

This formula ensures that the extent of stock price fluctuation is almost the same when the absolute values of \( \Sigma(t) \) (for positive value and negative value) are the same. So the results we get are credible. Then we gain the formula of stock logarithmic return

\[
\text{ln}\left(\frac{S(t+1)}{S(t)}\right) = r(t) = \ln(S(t+1)) - \ln(S(t)).
\]

In the paper, we analyze the logarithmic return for the daily price change.

**B. Simulation and Statistical Analysis of Data**

![Figure 2](image.png)

**Figure 2.** The semilog x plots of the cumulative distributions of normalized price returns with different values of \( \lambda \) when \( d=1 \), \( n = 400 \).
In order to investigate the distribution of price returns, four parameters of the contact model are discussed in this paper, the intensity $\lambda$, the lattice dimension $d$, the number of individual $n$. In the following, the double logarithmic plots for the absolute normalized price changes are used to show the computer simulations of the empirical data. For the different values of some parameter (for example, the intensity $\lambda$), we compare the fluctuations of the normalized price changes with Gaussian distribution, and study the statistical properties of the financial model. Next we consider the price changes for the different intensity values $\lambda$.

$\lambda$ is the intensity, and represents the rate of information spread in the model. In Figure 2 and Table I, for $d = 1$, $n = 400$, we discuss the statistical behavior of the price changes on the parameter $\lambda$. For $\lambda = 40$, the tail of probability distribution of the simulation data departs sharply from that of Gaussian distribution. For $\lambda = 30$ and $\lambda = 20$, the departure is less sharply. This shows, for the number of the individuals $\lambda$, the probability distribution of the price changes deviates from the Gaussian distribution. And we learn that the kurtosis and $\mu$ are increasing. The peak distribution of returns is obvious and the fat tail is also visible. This implies that the rate of infected speed can affect the behavior of fat tails.

We learn that when $d = 3$, $\lambda = 3$, $n = 4096$, the simulative data follows Gaussian distribution, this means that the large size of investment of stock market can weaken the fluctuations of the stock market.

Next we study the statistical properties of price changes for the different dimensions $d$.

$\lambda$ is the intensity, and represents the rate of infected speed. For $\lambda = 40$, the tail of probability distribution of the simulation data departs sharply from that of Gaussian distribution. For $\lambda = 30$ and $\lambda = 20$, the departure is less sharply. This shows, for the number of the individuals $\lambda$, the probability distribution of the price changes deviates from the Gaussian distribution. And we learn that the kurtosis and $\mu$ are increasing. The peak distribution of returns is obvious and the fat tail is also visible. This implies that the rate of infected speed can affect the behavior of fat tails.

In this section, we compare the data of returns of SZSE Composite Index and the financial model constructed by the contact model. In recent years, the probability distribution in financial market fluctuations has been studied, the empirical research results show that the distribution of large returns follow a power-law distribution with exponent 3, see Section I of this paper. According to the statistical methods and data analyzing methods (see [5,8,20]), we will study the cumulative probability distributions of daily returns and the power-law character of daily returns for Shenzhen stock market, the database which used in the present paper is from the webset of Shenzhen Stock Exchange (www.sse.org.cn). And we also simulate the corresponding cumulative probability distributions of returns by the financial model, which is modeled by the contact model.

### Table I. The Statistics of Change $\lambda$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>kurtosis</th>
<th>skewness</th>
<th>$\mu$</th>
<th>$\text{Var}(x(t))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>3.0333</td>
<td>-0.0423</td>
<td>5.462</td>
<td>0.0904</td>
</tr>
<tr>
<td>30</td>
<td>4.0499</td>
<td>0.0014</td>
<td>3.296</td>
<td>0.0649</td>
</tr>
<tr>
<td>40</td>
<td>6.6178</td>
<td>-0.0505</td>
<td>2.667</td>
<td>0.0459</td>
</tr>
</tbody>
</table>

### Table II. The Statistics of Change $n$

<table>
<thead>
<tr>
<th>$n$</th>
<th>kurtosis</th>
<th>skewness</th>
<th>$\mu$</th>
<th>$\text{Var}(x(t))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>729</td>
<td>6.0398</td>
<td>-0.1280</td>
<td>3.862</td>
<td>0.0390</td>
</tr>
<tr>
<td>2197</td>
<td>3.2494</td>
<td>-0.0641</td>
<td>5.550</td>
<td>0.0801</td>
</tr>
<tr>
<td>4096</td>
<td>3.0751</td>
<td>-0.0347</td>
<td>6.251</td>
<td>0.0574</td>
</tr>
</tbody>
</table>

### Table III. The Statistics of Change $d$

<table>
<thead>
<tr>
<th>$d$</th>
<th>kurtosis</th>
<th>skewness</th>
<th>$\mu$</th>
<th>$\text{Var}(x(t))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.0012</td>
<td>0.0231</td>
<td>8.348</td>
<td>0.0171</td>
</tr>
<tr>
<td>2</td>
<td>6.6178</td>
<td>-0.0505</td>
<td>5.422</td>
<td>0.0733</td>
</tr>
<tr>
<td>3</td>
<td>9.7987</td>
<td>0.1772</td>
<td>3.495</td>
<td>0.0869</td>
</tr>
</tbody>
</table>

$n$ is the number of individual investors, and also represents the size of investment in a stock market. In Figure 3 and Table II, we can see a more obvious fat-tail phenomenon in the plots, decreasing parameter $n$. We

![Figure 3](image-url)  
Figure 3. The semilog $x$ plots of the cumulative distributions of normalized price returns with different values of $n$ when $d = 3$, $\lambda = 3$.

![Figure 4](image-url)  
Figure 4. The semilog $x$ plots of the cumulative distributions of normalized price returns with different values of $d$ when $n = 729$, $\lambda = 3$.

$d$ is the lattice dimension. From the simulations in Figure 4 and Table III, we can see that, for fixed two parameters $\lambda = 3$, $n = 729$, the behavior of the price changes depends on the lattice dimension $d$. The more the dimension $d$ is, the more the interaction among individuals is. We learn that if the interaction is active enough, the peak distribution of returns is obvious and the fat tail is also visible.

### IV. The Statistical Comparison of SZSE Composite Index and the Financial Model

In this section, we compare the data of returns of SZSE Composite Index with the empirical data from the financial model constructed by the contact model. In recent years, the probability distribution in financial market fluctuations has been studied, the empirical research results show that the distribution of large returns follow a power-law distribution with exponent 3, see Section I of this paper. According to the statistical methods and data analyzing methods (see [5,8,20]), we will study the cumulative probability distributions of daily returns and the power-law character of daily returns for Shenzhen stock market, the database which used in the present paper is from the webset of Shenzhen Stock Exchange (www.sse.org.cn). And we also simulate the corresponding cumulative probability distributions of returns by the financial model, which is modeled by the contact model.
A. The Probability Distribution of Returns for SZSE and the Financial Model

We analyze quotes data from SZSE Composite Index during the year 2000-2007, and at the same time, we also analyze the simulative data from the financial model (modeled by the contact model) with the parameters \( d = 1, \lambda = 30, n = 400 \). According to the data, we plot the probability density of the daily returns figures as follows (Figure 5):

![Figure 5. The probability distribution of SZSE Composite Index (a) and simulative data (b).](image)

Comparing with the Gaussian distribution, the probability density of SZSE Composite Index and the simulative data obviously show the phenomena of the fail-tail and peak distribution in Figure 5.

The fluctuations of returns are believed to follow a Gaussian distribution for long time intervals but to deviate from it for short steps, so we try to study the probability distribution of returns for SZSE Composite Index and the simulative data. Comparing the daily returns of SZSE Composite Index and the simulative data with the normal distribution, we have the following Figure 6.

![Figure 6. (a) The comparison of SZSE Composite Index with a normal distribution. (b) The comparison of simulative data with a normal distribution.](image)

The plots above are the normal probability plots, which are the useful graph for assessing whether data comes from a normal distribution. From Figure 6, for each (a) and (b), we can see that the return processes of the data are normal in the middle part, but the parts (where the probability is above the 75th or below 25th percentiles of the samples) deviates from the dash line. This implies the fat-tail phenomenon of the returns for SZSE Composite Index and the corresponding simulative data modeled by the contact model.

B. Skewness-Kurtosis Test and Kolmogorov-Smirnov Test of Returns

In this section, we study the properties of skewness and kurtosis on the data of returns for SZSE Composite Index and simulative data. First, we give the definitions of skewness and kurtosis as following:

\[
Skewness = \frac{\sum (r_i - \mu)^3}{(n-1)\sigma^3}, \quad Kurtosis = \frac{\sum (r_i - \mu)^4}{(n-1)\sigma^4}
\]

where \( r_i \) denotes the return of \( i - \text{th} \) trading day, \( \mu \) is the mean of \( r \), \( n \) is the total number of data, and \( \sigma \) is the corresponding standard variance. The kurtosis shows the centrality of data. And the skewness shows the symmetry of the data. It is known that the skewness of standard normal distribution is 0, and kurtosis is 3. From the data of SZSE Composite Index and simulation, we have the following Table IV.

| Table IV. The Statistical Properties of Returns of SZSE Composite Index and Simulative Data |
|-----------------------------------------------|-----------------------------------------------|
| Sample size | SZSE Composite Index | Simulative data |
| 2035 | 5,9554e-004 | 2.9879e-004 |
| 5000 | 0.0165 | 0.0065 |
| 4,0381 | -0.1466 | -0.1218 |
| 9,3637 | 0.4442 |

In Table IV, the value of kurtosis of returns for the actual and simulative markets are both more than 3, and the values of skewness are close to -0.1. This implies that the statistical distribution of the data deviates from the Gaussian distribution, and it also implies that the fluctuation of returns for SZSE Composite Index and simulative data is greater than that of the corresponding Gaussian distribution.

For deeply analyzing the character of distribution of SZSE Composite Index and simulative data, we make another single-sample Kolmogorov-Smirnov test by the statistical soft, Matlab. We use the Matlab to calculate the value of the basic statistics of the returns of SZSE Composite Index and simulative data, see Table V.

| Table V. The Kolmogorov-Smirnov Test |
|-------------------------------------|-------------------------------------|
| Sample size | SZSE Composite Index | Simulative data |
| 2035 | H | 1 |
| 5000 | CV | 0.0300 | 0.0143 |
| | Two-tail test P | 0.000 | 0.000 |
| | Statistics of K-S | 0.4740 | 0.4442 |

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The value of two-tail test P is 0.000, thus the hypothesis is denied that the distribution of returns follow the Gaussian distribution.

Through the two tests above, the distribution of SZSE Composite Index and simulative data obviously have the fat-tail character, and have little distinct skewness. So if we properly choose the parameters of the financial model, we can fit the actual distribution of SZSE Composite Index well.

C. The Power-Law Behavior

Power law scaling is the universal property that characterizes collective phenomena that emerge from complex systems composed of many interacting units. Power law scaling has been observed not only in physical systems, but also in economic and financial systems. In this section, we study the cumulative probability distribution of returns for SZSE Composite Index and simulative data, and try to show the power law distribution of returns. In recent years, the empirical research has shown the power-law tails in the return fluctuations, that is, $P(r > x) \approx x^{-\mu}$, and some research results show that the distribution of large returns follow a power law distribution with exponent $\mu = 3$. We plot the cumulative probability distribution of returns for SZSE Composite Index and simulative data.

![Figure 7. The cumulative probability distributions of returns for SZSE Composite Index (a) and simulative data (b), and $\mu = 2.9485$ (Shenzhen), $\mu = 3.2963$ (Simulation)](image)

In Figure 7, the probability distribution of returns follows a power-law with the exponent $\mu = 2.9485$, which is determined by the ordinary least-squares regression in log-log coordinates. The distribution follows the power law behavior in the large price range, but gradually deviates from the power law as the absolute return becomes small.

V. THE VOTER MODEL

First, we give the brief definitions and properties of the voter model, for details see [14]. One interpretation for the voter model is, for a collection of individuals, each of which has one of two possible positions on a political issue, at independent exponential times, an individual reassesses his view by choosing a neighbor at random with certain probabilities and then adopting his position.

Specifically, the voter model is one of the statistical physics models, we think of the sites of the $d$-dimensional integer lattice as being occupied by persons who either in favor of or opposed to some issue. To write this as a set-valued process, we let $\{\xi(s), s \geq 0\}$ the set of voters in favor, we can also think of the sites in $\xi(s)$ as being occupied by cancer cells, and the other sites as being occupied by healthy cells. We can formulate the dynamics as follows: (i) An occupied site becomes vacant at a rate equal to the number of the vacant neighbors; (ii) An vacant site becomes occupied at a rate equal to $\lambda$ times the number of the occupied neighbors, where $\lambda$ is a intensity which is called the "carcinogenic advantage" in voter model. When $\lambda = 1$, the model is called the voter model, and when $\lambda > 1$, the model is called the biased voter model.

Let $\xi^d(s)$ ($s \in I$) denote the state at time $s$ with the initial state $\xi^d(0) = A$, then from [14], the voter model $\xi^d(s)$ approaches total consensus in $d = 1$ and $d = 2$. But in higher dimensions $d \geq 3$, the differences of opinion may persist. For more generally, we consider the initial distribution as $\nu_0$, the product measure with density $\theta$, that is, each site is independently occupied with probability $\theta$ and let $\xi^d(s)$ to denote the voter model with initial distribution $\nu_0$.

For the biased voter model ($\lambda > 1$), there is a "critical value" for the process, it can be shown that $\lambda_c = 1$ for the voter model, see [14]. This means that, on $d$-dimensional lattice, if $\lambda < \lambda_c$, then the process dies out (becomes vacant) exponentially fast, i.e.,

$$P(\xi^{[0]}(s) \neq \emptyset) \leq e^{-\rho s}$$

for some positive constant $\rho$. If $\lambda > \lambda_c$, then the process survives with positive probability, i.e., on $\{\xi^{[0]}(s) \neq \emptyset, \text{for all } s \geq 0\}$, we have

$$\frac{1}{s} \log P(\xi^{[0]}(s) \neq \emptyset) \rightarrow 2(\lambda - 1), \quad \text{a.s., } s \rightarrow \infty.$$

VI. THE SIMULATION OF THE FINANCIAL MODELS

A. Modelling a Financial Prices Changes by Voter Model

We have introduced the voter model in above section, in the following, we make use of the voter model to construct the price process for a stock market on $d$-dimensional integer lattice. Considering a model of auctions for a single stock in a stock market, we can derive the stock price process from the auctions of the investors. Assume that each trader can trade the stock several times at each day $i \in \{1, 2, \cdots, n\}$, but at most one unit number of the stock at each time. Let $I$ be the time length of trading time in each trading day, we denote the
stock price at time \( t \) in the \( t \)-th trading day by \( S(t,s) \) or \( S(t) \), where \( s \in [0, I] \). At the beginning of trading in each day, suppose that only the investor at the site 0 receives some news. We define a random variable \( X_t(0) \) for this investor, suppose that this investor taking buying position \( (X_t(0) = 1) \), selling position \( (X_t(0) = -1) \) or neutral position \( (X_t(0) = 0) \) with probability \( p_i, p_{-i}, \) or \( 1 - (p_i + p_{-i}) \) respectively. Then this investor sends bullish, bearish or neutral signal to his nearest neighbors. According to one-dimensional voter process system, investors can affect each other or the news can be spread during the daily trading time \([0, I]\), which is assumed as the main factor of price fluctuations for the investors. The stock price \( S \) changes from one trading day to the next trading day by an amount proportional to the investors' positions (which can be positive, negative or neutral), which defined by

\[
\Delta S = S(t) - S(t-1) \propto X_t \mid \xi(t) \mid, \quad s \in [0, I],
\]

where \( t \in \{1, 2, \ldots, n\} \).

In order to investigate the distribution of price changes \( \Delta S \) defined in above (1), three parameters of the voter model are discussed in this paper, the initial density \( \theta \), the intensity \( \lambda \), and the lattice dimension \( d \). In the following, the double-logarithmic plots for the absolute normalized price changes are used to show the computer simulations of the empirical data. For the different values of some parameter (for example, the intensity \( \lambda \)), we compare the fluctuations of the normalized price changes with the corresponding Gaussian distributions, and study the statistical properties of the financial model by the plots of the data, the statistical analysis and the comparison of the plots.

B. The Data Analysis of Price Changes for Different Intensity Values

In this subsection, we consider the price changes for the different intensity values \( \lambda \), we give three plots as following for three different values \( \lambda \). According to these plots, we explain the fat tails distribution of price changes.

From the simulations in Figure 8, we can see that, for fixed two parameters \( d = 2 \), \( \theta = 0.5 \), the behavior of the price changes \( \Delta S \) depends on the intensity value \( \lambda \). Figure 8(a) shows, at the subcritical case \( \lambda = 0.90 < \lambda_c = 1 \), the graph of voter model is near the dash line, which is the graph of the corresponding Gaussian distribution. It means that \( \Delta S \) strictly follows a normal distribution. In fact, the voter model dying out exponential fast at the subcritical case implies this property, see the properties of voter model in Section V. In the Figure 8(b), at the subcritical case \( \lambda = 0.99 < \lambda_c \), this figure is similar as that of Figure 8(a). In Figure 8(c), at the supercritical case \( \lambda = 1.01 > \lambda_c \), the distribution of price changes shows the fat-tail phenomenon, this phenomenon can also be implied by the property of the supercritical voter model, since the model survives with positive probability in this case. In summary, Figure 8 shows, for increasing intensity \( \lambda \), the probability distribution of price changes deviates from the Gaussian distribution in the tail parts.

Figure 8. The normalized price changes plot of \( |\Delta S| \) obtained when \( d = 2 \), \( \theta = 0.5 \), the time \( s = 2000 \), and the trading days \( t = 1, \ldots, 2000 \). The traders on 64 \( \times \) 64 lattices. In Figure 8(a), \( \lambda = 0.90 \) (circles). In Figure 8(b), \( \lambda = 0.99 \) (squares). Figure 8(c), \( \lambda = 1.01 \) (diamonds).

C. The Data Analysis of Price Changes for Different Initial Density Values

In this subsection, we consider the price changes for three initial distributions \( \nu_p \), we give three plots as following for three different values of density \( \theta \). The results also imply the fat tails and power law phenomena of price changes.

In Figure 9, for \( d = 2 \) and \( \lambda = 1.01 \) (around the critical point), we discuss the statistical behavior of the price changes \( |\Delta S| \) on the parameter \( \theta \). In Figure 9(a) and Figure 9(b), since the number of occupied sites is not dominant, the simulations show that price changes \( |\Delta S| \) follow the corresponding Gaussian distributions. In Figure 9(c), for initial density \( \theta = 0.5 \), since there are more occupied sites than vacant sites in the financial model, Figure 9(c) shows that the 'diamonds' line in the plot deviates from the dash line (especially the part in the bottom). This shows, around the critical point \( \lambda_c \), the probability distribution of the price changes deviates from...
the Gaussian distribution (in the tails) as we increase the value of density $\theta$.

In Figure 9, we can see (when increasing dimension $d$) a more obviously fat-tail phenomena in the plots. One reason is that the interaction among traders (neighbors) becomes more active in the financial voter model when we increase the number of dimension $d$, the other reason is $\lambda = 1.1 > \lambda_c$, since the voter model will not dies out at the supercritical condition.

CONCLUSION

In the present paper, in Section II—Section IV, we apply the statistical models—the contact model to construct a financial model (a price changes model). From this model, we try to understand the statistical properties (for example, the fat-tail phenomena, the power law behavior) of the price changes at the subcritical case, at the critical point and at the supercritical case. The statistical properties of SZSE Composite Index are also studied in this paper. And we compare the statistical properties of actual and simulative markets. This paper also shows that research results on the fluctuations of a stock market by applying the financial model based on the contact model. In Section V—Section VI, we apply another statistical model --- the voter model to construct a financial model, a price changes model. The simulation of this financial model shows some statistical properties (for example, the fat tails phenomena) of the price changes for the different dimensions $d$ and initial density $\theta$ at the subcritical case, the critical point, the supercritical case. This new approach to the fluctuations of stock market is useful for us to understand the statistical properties of a stock market.

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