A Simulator for Relative Descriptions[1]

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Abstract

This report deals with the qualitative simulation of physical systems based on descriptions relative to normal behavior. Relative descriptions are important because some kinds of system behavior cannot adequately be described by absolute descriptions. In particular, this is true for faulty behavior that is often viewed relative to the normal case. Most of the existing approaches use relative descriptions only to analyse static systems. In addition, a comparison of deviations in dynamic systems is not possible. In this report, a simulator, called RSIM, is presented, that predicts the effects of system deviations with a “less than normal” or “greater than normal” character. Aside from absolute descriptions, the deviations themselves and the resulting behavior are described with respect to the normal case, i.e. to a reference system and its reference behavior. The simulation of absolute behavior is carried out by a QSIM-like simulator, since the concepts of RSIM are oriented towards QSIM. RSIM can be viewed as an extension to QSIM.

In RSIM, deviations cannot only be described by “less than normal” or “greater than normal”, but additionally they can be compared with each other. In this way, a refined system description is achieved. Furthermore, some spurious behaviors are prevented on the relative description layer, and a more accurate prediction of behavior is possible.

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Contents

1. Introduction

The obvious attractiveness of qualitative simulations, their effectiveness, efficiency, and naturalness (compare [Struss 89]) has initiated a large amount of work in this field. The most influencing approaches are ENVISION [de Kleer, Brown 84], QPT [Forbus 84], and QSIM [Kuipers 86]. While relative descriptions have widely been used ([de Kleer 79], [Raiman 86], [Dague, Devès, Raiman 87], [Weld 87,88a,88b,90], [Downing 87], [Mavrovouniotis, Stephanopoulos 88], [Gallanti, Stefanini, Tomada 89], [Kockskämper, Neumann, et al. 93]), only few approaches deal with relative simulation of dynamical systems (e.g. [Weld 90])[1]. A relative simulation is valuable for system analysis and fault diagnosis. In this paper, concepts of relative simulation are described, and a relative simulator, called RSIM, is presented. RSIM's input and output, and some essential inference features are oriented towards QSIM. In fact, RSIM can be viewed as an extension to QSIM. Extending QSIM by RSIM leads to a refinement of input and output. In order to exploit existing QSIM techniques, RSIM is integrated into the system SLOD, which allows **s**imulation on different **l**ayers **o**f **d**escription. SLOD contains four description layers: sign and landmark descriptions, as QSIM, and two relative description layers belonging to RSIM.

The development of RSIM was initiated by the observation that there are kinds of faulty system behavior that cannot adequately be described by absolute descriptions. And even worse, a natural modeling of a faulty system and the corresponding correct system in some cases led to identical descriptions. Instead, terms like "less than normal" or "too high" characterized this sort of faulty behavior. Additionally, comparisons of too low or too high deviations turned out to be necessary. Therefore, we formalized these terms and developed an inference engine for them, the simulator RSIM. RSIM's modeling language offers a set of constraints that - due to relative descriptions - facilitates a more specialized system description than in usual simulators. Inferences about the relative duration of time intervals are carried out by RSIM's transition rules. Thus, the problems that were identified at the beginning of our work, are solved by the RSIM simulator.

The report is organized as follows: First we explain how qualitative simulation works and why relative descriptions are useful and necessary. Afterwards specific properties of simulation by relative values are discussed. In Chapter 3, the main part of the report, the relative simulator, RSIM, is presented. Finally some notes regarding future work are given. The appendix contains proofs of RSIM's transition rules.

1. [Neitzke 92a] gives an overview about relative descriptions in qualitative simulation.

1.1. How Does Qualitative Simulation Work?[1]

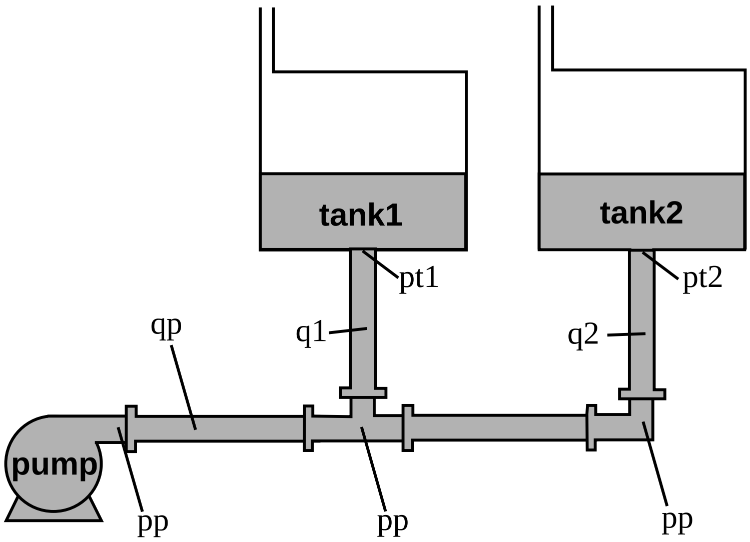
Qualitative simulation, as conventional numerical simulation, has the purpose to predict the behavior of a system. System behavior, however, is not described by exact numerical values but by qualitative descriptions. Qualitative descriptions have two characteristics: they are more general than quantitative descriptions and they are more imprecise. A qualitative system description does not concern a single system but a whole class of systems. Figure 1.1 shows a simple model for tank systems of two tanks and a pump. This and other tank systems will be used as examples throughout the report.

In a qualitative simulation, ambiguities arise in the prediction of behavior, so that in general not exactly one behavior is predicted, but a set of possible behaviors. One reason for ambiguities is that not all members of the simulated class of systems behave in the same way. In the 2-tank system above, for example, it cannot be decided which tank is full first. Another reason lies in the inexactness of qualitative descriptions. As a consequence of the latter not all of the predicted behaviors correspond to physically possible behavior. These impossible behaviors are called spurios behaviors. However, it can be guaranteed that every possible behavior is among the predicted behaviors.

While in a numerical simulation the state of a system is calculated at fixed equidistant time points, a qualitative simulation is guided by the occurrence of “interesting” events. In the kind of simulation we are concerned with, an interesting event occurs whenever a parameter reaches an interesting value (a so-called ***landmark value***). At each interesting event the system state, i.e. the qualitative values of all parameters, is recorded. Additionally, the system state is recorded for the time intervals lying between two events. So, the generation of behavior is a generation of a sequence, or - because of ambiguities - a tree of system states.

As input a simulator needs a system description and some information about its initial state, i.e. the values of some parameters. There are different ways to describe a system. We follow QSIM and describe a system by a set of constraints (compare Fig. 1.1a), i.e. a set of relations between the system parameters. The output of a simulation, the tree of system states, is generated in a cycle of two phases, called intrastate analysis and interstate analysis. Simulation starts with an intrastate analysis. In intrastate analysis, a set of completely described system states is derived from an incompletely specified system state. In QSIM-like simulators this is done by using the constraints of the system description (in a process called constraint propagation). For our example, an incomplete description of the initial state can be seen in Fig. 1.1b.

1. Excellent introductions to qualitative simulation are given by [Forbus 88] and [Struss 89]



(DERIV q1 vol1)	Flow *q1* is the derivative of volume of tank1 *vol1*.
(DERIV q2 vol2)	Flow *q2* is the derivative of volume of tank2 *vol2*.
(M+ pt1 vol1)	Pressure of tank1 *pt1* is an increasing function of *vol1*.
(M+ pt2 vol2)	Pressure of tank2 *pt2* is an increasing function of *vol2*.
(PRODUCT pf1 k1 q1)	"Friction pressure" of tank1 *pf1* is the product of friction coefficient *k1* and *q1*.
(PRODUCT pf2 k2 q2)	"Friction pressure" of tank2 *pf2* is the product of friction coefficient *k*2 and *q2*.
(SUM pp pt1 pf1)	The pressure of the pump *pp* is the sum of *pt*1 and *pf1*.
(SUM pp pt2 pf2)	The pressure of the pump *pp* is the sum of *pt2* and *pf2*.

Fig. 1.1a A simple model of a system of two tanks and a pump. For the sake of simplicity and reasons of demonstration, friction has only been modelled for vertical pipes and linearly dependent from flow.

vol1	**: 0**	Tank1 is empty.
vol2	**: 0**	Tank2 is empty.
pp	**: +**	Pressure of pump is positive.
k1	**: +**	Friction coefficient k1 is positive
k2	**: +**	Friction coefficient k2 is positive

Fig. 1.1b Initial information about the system's state.

In interstate analysis, an incomplete successor state is inferred from a complete system state. Interstate analysis works with continuity information about continuously differentiable functions. It is required that every system parameter is a continuously differentiable function of time. To determine the future course of a parameter, qualitative information about its derivative is needed. Therefore, a parameter is described by two numbers: its amount and its derivative. Figure 1.2 shows the tree of behaviors when the filling of the 2-tank system is simulated. The behavior tree is the result of a sign simulation, the simplest qualitative simula-

tion, working with just one landmark value, 0. Therefore, the only interesting event is reaching the landmark value 0.

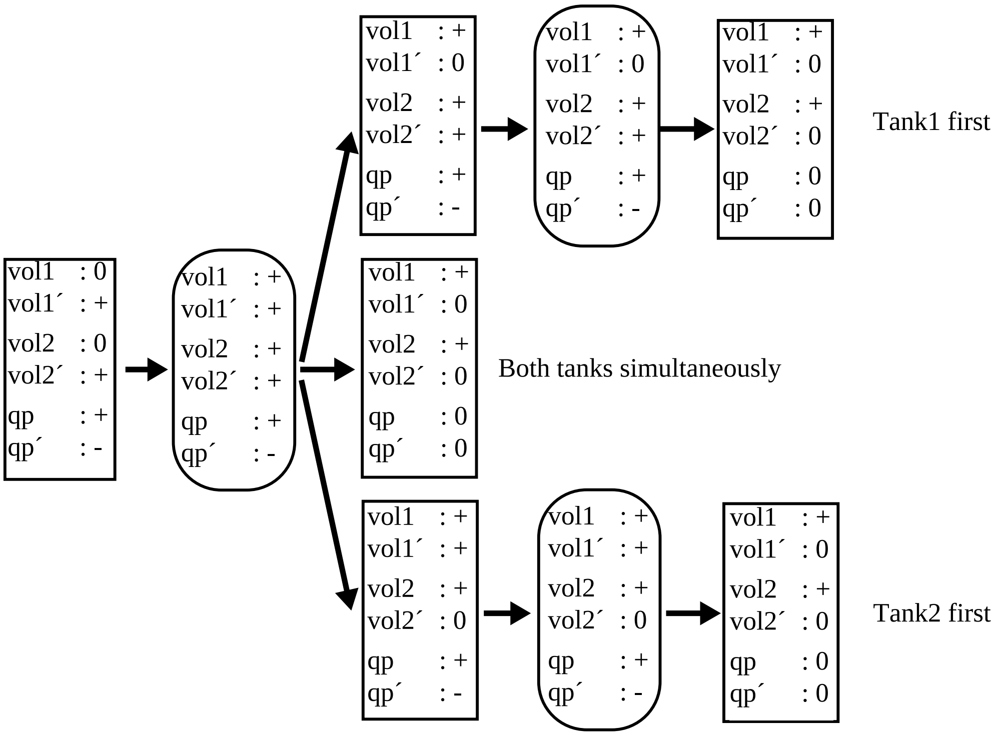


Fig. 1.2 Behavior tree for the 2-tanks system. For simplicity only three parameters have been listed. Time points are indicated by rectangles, time intervals by ovals.

1.2. Relative Descriptions and their Use[1]

1.2.1. Why Relative Descriptions?

There are kinds of system behavior that cannot adequately be described by absolute descriptions. Take for example a leaky water pipe. An adequate description of its behavior says that the output flow is less than the input flow. Or consider a pump that generates less pressure than in the normal case. The general meaning of “less” cannot be described by assigning absolute values, like signs or intervals between landmark values, to the parameters involved.

1.2.2. What are Relative Descriptions Used for?

Relative descriptions can be used for many different purposes in qualitative simulation. Our aim in using relative descriptions is to describe deviations with respect to a reference system and reference behavior. The deviations we are interested in are those that can be described as less than normal or greater than normal. Such descriptions can be used to characterize the

1. Section 1.2 overlaps with [Neitzke 92b] and [Neitzke 92c].

values of system parameters in a certain system state. Furthermore, they can be used to characterize the duration of a process taking place in a system. Before going into details, we have to take a closer look at the different types of deviations.

1.2.3. Types of Deviations

What are the characteristic features of deviations that can be described as less or greater than normal? Of course, not all deviations can be described in this way. Consider, for example, water in a pipe, that is flowing in the wrong direction, or an electrical charge that is positive, but normally negative. One property of less/greater-than-normal (LGTN) deviations is that they can be arbitrarily small. Weld calls arbitrarily small changes in the value of a parameter ***differential*** and more drastic changes ***non-differential***. The essential property of a differential change is that it could be arbitrarily smaller without falling into a qualitatively different area, i.e. no landmark value may lie between the deviating value and the normal value. Following Weld we call deviations that lie in the same qualitative area, i.e. have the same absolute description, as the normal value ***differential*** and deviations that lie outside this area ***non-differential***.

Unfortunately, the definition of differential does not capture all deviations that are called less or greater than normal. The voltage between the ports of a diode, for example, could be characterized as less than normal even if it is less than the diode's threshold voltage. Instead, expressions like less or greater describe deviations in the distance to a reference point, which we want to call ***basis landmark***[1]. If the deviating value lies on or beyond the basis landmark, as in the examples above, one cannot use the terms less or greater than normal. In most of all cases, the suitable basis landmark is given by the real number 0. But as the *quantity space* of {-, 0, +} in general corresponds to the regions $x < a$, $x = a$, and $x > a$, (where a is an arbitrary landmark value in the range of the parameter x (compare for example de Kleer)) the basis landmark may in principal be different from 0. We only require that basis landmark and landmark value, a, must be identical.

Definition: The deviation of a value v_1 from a value v_2 is called a ***continuous*** deviation with regard to a basis landmark l_a if v_1 is on the same side of l_a as v_2, that is $(v_1 < l_a \wedge v_2 < l_a) \vee (v_1 > l_a \wedge v_2 > l_a)$. Otherwise it is called a ***discontinuous*** deviation wrt l_a (Fig. 1.3).

1. The basis landmark may not be confused with the normal value, which we often call reference value.

Differential and continuous deviations are very similar. For quantity spaces with only one landmark like {-, 0, +} both classes are identical. In general, each differential deviation is a continuous deviation and each discontinuous deviation is a non-differential deviation.

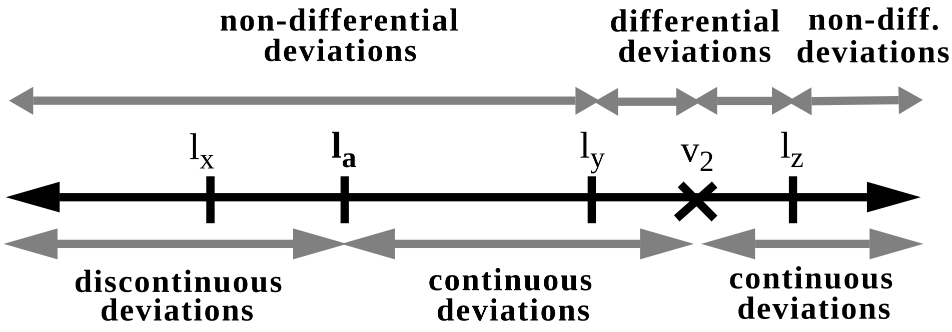


Fig. 1.3 Differential, non-differential, continuous and discontinuous deviations wrt l_a from a value v_2.

1.2.4. A Relative Description that allows Comparison of Deviations

The way and the extent a value v_1 differs from a normal value v_2 can be described by the quotient of the distances of v_1 and v_2 from the basis landmark. We call this quotient P-value.

Definition: $$P(f,t) = \frac{f(t) - l_a}{f_{ref}(t) - l_a}$$

f is a continuously differentiable function of time, f_{ref} is the corresponding reference function, l_a is f's basis landmark in the range of f. Deviations from a normal value that lie beyond l_a cannot be described as too-low or too-high.

We call the values of the function P ***P values***. Note, that P values are relative, but not qualitative. However, we are only interested in qualitative properties of P values. Therefore, we don't need exact values for the function P. The following qualitative areas of P values are relevant for us:

$P(f,t) > 1$ The distance of the deviating value to the basis landmark is greater than the normal value's distance. Usually, such deviations are called "too high".

$P(f,t) = 1$ There is no deviation.

$0 < P(f,t) < 1$ The distance of the deviating value to the basis landmark is less than the normal value's distance. Usually, such deviations are called "too low".

$P(f,t) = 0$ The deviating value lies on the basis landmark. This deviation is discontinuous. It can be expressed by absolute descriptions.

$P(f,t) < 0$	Deviating value and normal value lie on different sides of the basis landmark. This deviation is discontinuous. It can be expressed by absolute descriptions.

Now we want to define, what we exactly mean by “too low”, “normal” and “too high” by the function PQ. We call the values of PQ ***PQ values***.

$PQ(f,t) = \textit{too-low} \quad ::= \; 0 < P(f,t) < 1$

$PQ(f,t) = \textit{normal} \quad ::= \; P(f,t) = 1 \vee f(t) - l_a = f_{ref}(t) - l_a = 0$

$PQ(f,t) = \textit{too-high} \quad ::= \; P(f,t) > 1$

As an important extension of the above mentioned qualitative descriptions that are based on fixed intervals, we introduce comparisons of P values. For example, the P value of the output flow of a leaky pipe is always less than the P value of the input flow (provided that there is a flow). Besides the relations <, =, > the reciprocal relation can be useful too because if the product of a *too-low* value and a *too-high* value results in a *normal* value, the P value of the first factor is the reciprocal of the second. Comparison of P values gives us a handle to refine qualitative system descriptions and behaviors, to distinguish more specific classes of constraints between system parameters, and to diminish some unwanted spurious behaviors.

These two qualitative descriptions, PQ values and relations between P values, shall be the basis for our relative simulation. Before we come to the corresponding inference machine, the relative simulator RSIM, we want to examine the properties of a relative simulation in general.

2. Simulating with Relative Descriptions

As we have seen, a qualitative simulator produces a behavior description from a system description and information about the systems initial state. A relative simulator, as a special qualitative simulator, works the same way: As input it receives a system description that contains the deviations from a reference system, and as output it predicts the possible system behaviors, using descriptions that contain the deviations from the reference behavior. Of course, the input of our relative simulator must have this LGTN character we are interested in. But does it always follow from such input that an output has this character too?

2.1. Describing Deviations from a Reference System

The kind of qualitative simulators we deal with (the QSIM-type) gets as input a system description consisting of constraints between the system parameters (Fig. 1.1a) as well as information about the initial state of the system in form of some parameter values (Fig. 1.1b). We can distinguish three kinds of deviations from the reference system with a LGTN character:

1. Differential deviations of initial parameter values
2. Continuous deviations of initial parameter values
3. Deviations in the system description

2.1.1. Differential Deviations of Initial Parameter Values

Differential deviations cannot be expressed with absolute descriptions by definition, i.e. the absolute description of deviating parameters does not change. But, if there is no difference in the input concerning absolute descriptions, there cannot be any difference in the output. That is, the set of predicted behaviors is identical for the deviating system and for the reference system as long as one focusses on absolute descriptions. This does not mean that a differential deviation in the initial state cannot lead to changes of the absolute behavior of a system. But both, the reference behavior and the deviating behavior must be elements of the common set of absolute behaviors. Consider the 2-tank system of Fig. 1.1. If the friction coefficient, *k2*, of tank2 is higher than normal, then filling tank2 takes more time, and therefore, the order of interesting events may change. How are the three possible behaviors of the reference system (Fig. 1.2) affected, if the friction coefficient of tank2 is too high?

> <u>First behavior</u> (tank1 is full at first): The volume of tank2 will be too low when tank1 is full. However, the absolute description does not change: At first tank1 will be full and afterwards tank2.
>
> <u>Second behavior</u> (both tanks are full simultaneously): Tank1 will be the first. The deviating behavior is characterized by the first behavior. The absolute description of

behavior has changed. Weld calls this a change in the ***behavioral topology*** [Weld 87,88b,90].

Third behavior (tank2 is full at first): It cannot be decided whether the deviation of the friction coefficient is so heavy that now tank1 is the first to be full, or whether both tanks are full simultaneously, or whether tank2 still is the first. Thus each behavior of the tree may be a consequence of the deviation. That is, there exist solutions with and without a change in the behavioral topology.

2.1.2. Continuous Deviations of Initial Parameter Values

In contrast to differential deviations, continuous deviations allow changes of absolute descriptions. Therefore, the resulting absolute behavior can change too. Continuous deviations imply that the basis landmark of the deviating parameter is not reached. Since we require the identity of basis landmark and the landmark value defining the sign quantity space, continuous deviations cannot take place on signs. So a projection of absolute descriptions onto signs leads to identical behavior sets for the deviating and the reference system. That is, in principle one can transform continuous deviations into differential deviations by leaving out some landmark. However, there are situations where it is not adequate to do this. Suppose, we want to model the value of tank pressure, at the moment when the valve of a tank closes, as a special landmark value in the tank pressure quantity space. There are situations, where a pressure above, at, and below this landmark value must be considered as too high. Fig. 2.1 shows another example of a continuous deviation. The deviating behavior of Fig. 2.1a and the corresponding reference behavior of Fig. 2.1b describe the emptying of a tank. The quantity space of the tank level contains three landmark values, *0*, *half-full* and *full*. In the reference behavior, the initial value of the tank level is *half-full*, while in the deviating behavior it is *full*, i.e. the tank level is too high. The topology of the behaviors is different, but the deviations are continuous, because all amounts and derivatives keep their signs. (The problems concerning the different duration of the behaviors are discussed below.)

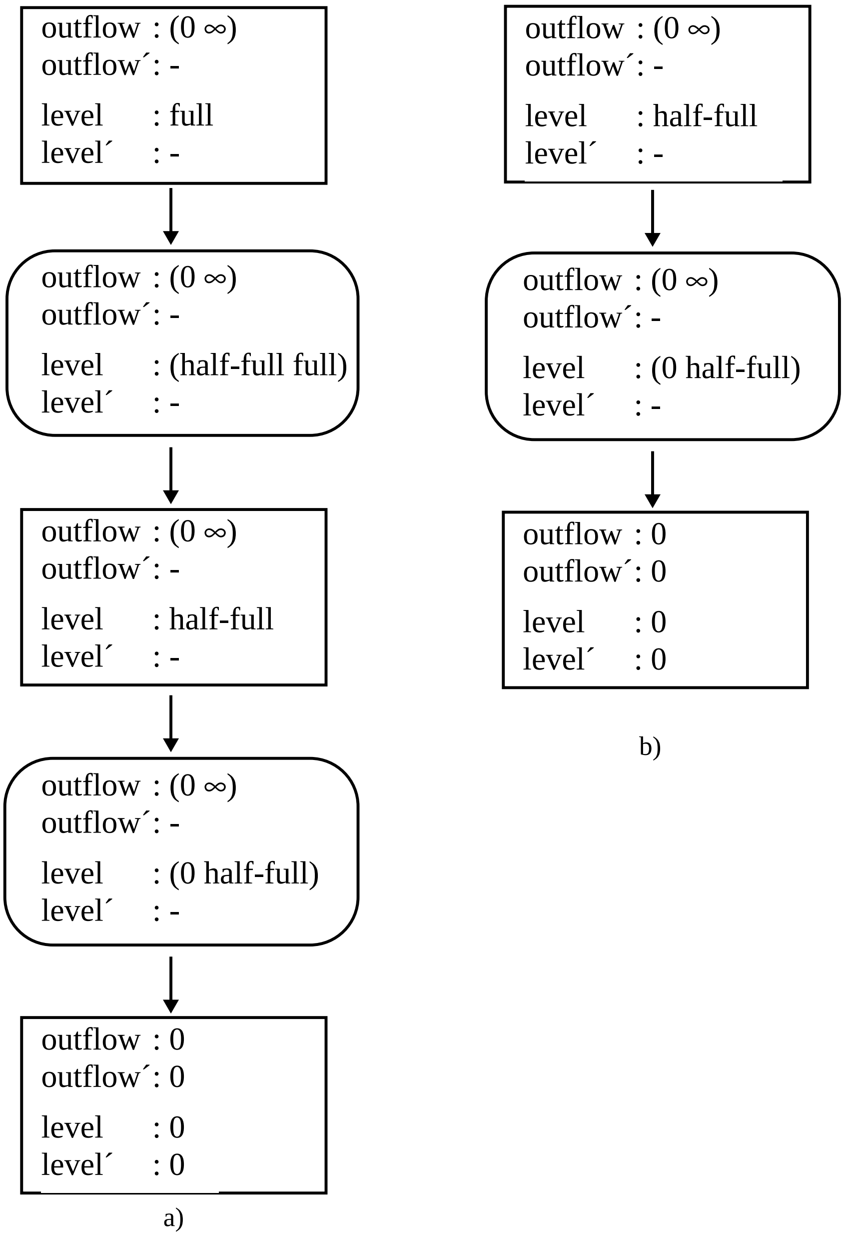


Fig. 2.1 Behaviors for emptying a tank. a) Deviating behavior
b) Reference behavior

2.1.3. Deviations in the System Descriptions

In general, changes in the system descriptions lead to plainly different behaviors, so that a comparison via less/greater descriptions is not possible. However, there exist some changes that have no significance on absolute descriptions but on relative descriptions. Fig. 2.2 shows the constraints used by the original QSIM. While ADD, MULT, MINUS and DERIV are "exact" constraints, M^+, M_0^+, M^-, and M_0^- have a qualitative character. They each refer to a

whole class of functional relationships. It would bring no advantages for absolute descriptions to distinguish between subclasses of M*, because the qualitative definitions of the corresponding constraints would be identical. The definition of P values, however, makes it possible to differentiate between some subclasses. On the one hand one can distinguish

ADD(f,g,h) : $\Leftrightarrow$ f(t) + g(t) = h(t)
MULT(f,g,h) : $\Leftrightarrow$ f(t) * g(t) = h(t)
MINUS(f,g) : $\Leftrightarrow$ f(t) = - g(t)
DERIV(f,g) : $\Leftrightarrow$ f´(t) = g(t)
M^+(f,g) : $\Leftrightarrow$ $f(t) = H(g(t)) \wedge H'(x) > 0$
M_0^+(f,g) : $\Leftrightarrow$ $M^+(f,g) \wedge H(0) = 0$
M^-(f,g) : $\Leftrightarrow$ $f(t) = H(g(t)) \wedge H'(x) < 0$
M_0^-(f,g) : $\Leftrightarrow$ $M^-(f,g) \wedge H(0) = 0$

Fig. 2.2 Constraints of QSIM. The description has been adopted from [Struss 89]

between linear, overlinear, and underlinear relationships. This feature will be discussed below. On the other hand one can express faulty relationships, for example a M_0^+ relationship with a greater-than-normal gradient. Fig. 2.3 shows the sign and PQ tupels of the standard M_0^+

M_0^+

Sign tupels: (- -)
(0 0)
(+ +)

PQ tupels:
If sign tupel is (- -) or (+ +):
(too-low too-low)
(normal normal)
(too-high too-high)

If sign tupel is (0 0):
(normal normal)

M_0^+TOO-STEEP

Sign tupels: (- -)
(0 0)
(+ +)

PQ tupels:
If sign tupel is (- -) or (+ +):
(too-low too-low)
(normal too-low)
(too-high too-low)
(too-high normal)
(too-high too-high)

If sign tupel is (0 0):
(normal normal)

Fig. 2.3 Sign and PQ tupels of the standard M_0^+ constraint and the M_0^+TOO-STEEP constraint. The sign tupels of both are identical.

constraint and a M_0^+ constraint with too-high gradient, called M_0^+TOO-STEEP. What are the consequences of such relative deviations in the system description? First, the set of absolute behaviors does not change, since the absolute system description does not change. And second, a change of the behavioral topology is possible. If, for example, two parameters *x* and *y* are related by a M_0^+TOO-STEEP relationship, (M_0^+TOO-STEEP *x y*), and normally reach

certain landmark values at the same time, then in the deviating system, *x* will reach its landmark value before *y*.

2.2. Comparison of Behaviors

In the last chapter, we have seen that under deviations with a LGTN character in the system description or the initial conditions, the set of possible absolute behaviors, or at least the set of possible sign behaviors, does not change. Additionally, we have seen that changes of the behavioral topology are possible. So, when a deviating behavior is compared to a reference behavior, two kinds of comparisons can be distinguished: comparisons with and without a change in the behavioral topology. Let us first consider the easier case:

2.2.1. Comparisons without a Change in the Behavioral Topology

No change in the behavioral topology means no change in the absolute behavior of the system. That is, for every state of the path of deviating behavior there exists a corresponding state in the reference behavior with identical absolute descriptions, and vice versa. All possible deviations are differential: parameter values of certain states may be too low or too high (or normal), and the duration of an interval state may be too low or too high (or normal) too[1]. The comparison of behaviors is a comparison of each state of the deviating behavior with the corresponding state of the reference behavior.

The comparison of point states is simple. For each parameter it must be stated, whether its amount and derivative are too low, normal or too high with respect to the reference state. The comparison of interval states is more complicated. Since the duration of an interval may change, either the deviating interval or the reference interval will end before the other. So a comparison with respect to time cannot concern both intervals in full length, unless the rest of the longer interval would be compared to states following the shorter interval. To cope with this difficulty, Weld has introduced the concept of ***perspective*** [Weld 87,88b,90]. A comparison of the values of a parameter x under the perspective of a parameter y means that those values of x are compared with each other that belong to the same values of y. Using appropriate parameters as perspectives often leads to simpler descriptions than using the standard perspective of time. Weld shows that relevant statements about the duration of an interval and the magnitude of values compared to reference values are possible through working with different perspectives. The corresponding method, Weld's ***DQ analysis***, determines analytically and not by simulation the consequences of differential deviations. The concept of perspective facilitates elegant and effective descriptions of differential deviations. In this paper, we show that a simulation assuming the standard case of time as perspective yields these results too, without having the problem of incompleteness of DQ analysis (see [Weld 90]). Additionally, due to higher exactness that is gained by relations between P values, some ambiguities can be avoided and more problems can be solved.

1. Deviations of the duration of an interval must be differential, because the duration is always positive and further absolute information is not given.

2.2.2. Comparisons with Changes in the Behavioral Topology

If the behavioral topology changes, the deviations become non-differential. However, LGTN descriptions still can be used, if the deviations are continuous. So one has to distinguish between continuous and discontinuous changes of the behavioral topology. An example of continuous, non-differential changes is given by Fig. 2.1a. Continuous changes of the behavioral topology can be handled in a similar way as differential changes of behavior (dealed with in the previous section) because the sign behavior does not change. Differences are related to the fact, that the mapping between the states of the deviating and the reference behavior in contrast to differential changes is not bijective (because of their different number). Further details will be explained in Section 3.5. In the following we will focus on discontinuous changes.

Discontinuous Changes of the Behavioral Topology

Discontinuous changes of the behavioral topology occur if the deviating and the reference behavior correspond to different paths of the behavior tree. The point where deviating and reference behavior split is a point of ambiguities. In principle at each branching of the behavior tree the deviating and the reference behavior can split. There are two types of branchings and correspondingly two types of topology changes. One reason for a branching is that it cannot be decided which event out of a number of possible ones will occur first. Every order of events must be taken into account. This type of branching is called ***occurrence branching*** (see e.g. [Fouché,Kuipers 91]). The other reason for a branching is that it cannot be decided in which direction the amount or derivative of a parameter moves. In this case, all possible developments, i.e. increase, decrease, or constancy, must be considered.

In the case of discontinuous topology changes, a sensible mapping between deviating and reference system states in general is not possible. But if the topology changes because of occurrence branching, parameter histories[1] still can be compared. In occurrence branching, the order of parameter-specific events changes, but the absolute descriptions of parameter histories themselves do not. The behavior tree of Fig. 1.2 is an example for occurrence branching.

But if the reason for branching is, that the further course of a parameter cannot be determined, the parameter history itself can change. For example, this type of branching happens, if in a system of coupled tanks it cannot be decided in which direction the water will flow. Now, the last chance of comparisons via LGTN descriptions is restricted to parameter histories that have not changed. But the essential characteristic of the deviation, the change of some parameter histories, cannot be described by this means.

1. A parameter history is the qualitative course of a parameter value.

2.3. Summary

If a deviating system shall be simulated, the deviations from the reference system can be expressed by changing the description of the reference system, i.e. the model, or by changing the initial parameter values of the reference system. Model changes with a LGTN character are restricted to the M* constraints. Deviations of the initial parameter values are subdivided into differential and continuous deviations. Differential deviations of the initial parameter values may cause changes of the behavioral topology, but the set of predicted behaviors does not change with respect to absolute descriptions. Continuous deviations, on the other hand, may cause changes of the set of absolute behaviors.

If a deviating behavior has to be compared to a reference behavior, the comparison may or may not have to deal with changes of the behavioral topology. If there is no change in the behavioral topology, a state-wise comparison is possible. The comparison has to determine for each state whether the parameter values are too low, normal, or too high, and it has to determine for each interval state whether the duration of the interval is too low, normal, or too high. Changes of the behavioral topology are subdivided into continuous and discontinuous changes. Continuous changes of the behavioral topology can be handled in a similar way as if no changes of the topology exist. Concerning discontinuous changes of topology, one has to distinguish between changes because of occurrence branching and changes because of ambiguities in the further course of a parameter. If the topology changes because of occurrence branching, LGTN description have to be restricted to parameter histories. For the other type of topology changes, no sensible comparison via LGTN descriptions is possible.

3. The Relative Simulator RSIM

In this chapter the relative simulator RSIM is presented. RSIM's input and output, and some essential inference features are oriented towards QSIM. In fact, RSIM can be viewed as an extension to QSIM. Extending QSIM by RSIM leads to a refinement of input and output. In order to exploit existing QSIM techniques, RSIM is integrated into a system, SLOD, which allows **s**imulation on different **l**ayers **o**f **d**escription. SLOD contains four description layers: sign and landmark descriptions, as QSIM, and two relative description layers belonging to RSIM.

In the following, the concepts of RSIM are explained. First, we will deal with system descriptions and state descriptions in RSIM. After that the two essential parts of a qualitative simulation, intrastate and interstate analysis, are explained. Finally we give some examples of output. But before we come to the concepts of RSIM, some words about SLOD have to be said.

3.1. Simulating on Different Layers of Description

In qualitative simulation, a parameter is usually described by two numbers: its amount and its derivative. The derivative carries dynamical information so that the future course of the amount can be computed. In SLOD, the different ways of describing a number qualitatively are represented in description layers. SLOD has four description layers: a coarse and a fine absolute layer, and a coarse and a fine relative layer.[1] SLOD's description layers are listed below.

<u>Absolute layers</u>

Sign layer: The simplest qualitative information about a real number is its sign. The sign layer is the basic description layer for the other layers.

Qval layer: On the qval layer the real number line or a part of it is divided into regions of interest by so-called landmark values. The landmark values themselves and the open intervals between them form a quantity space. The sign layer can be seen as a special qval layer with landmark value 0. In contrast to the other description layers, on the qval layer only the amount of a parameter is described, but not its derivative.[2]

1. In principal, more description layers can be added.

2. The name *qval* has been taken from QSIM since the qvals in SLOD have the same properties as in QSIM (totally ordered set of landmark values, dynamical generation of landmarks possible).

<u>Relative description layers</u> (belonging to RSIM):

PQ-layer: The PQ layer works with three qualitative values: *too-low, normal*, *too-high*. The semantics of PQ values has been explained in Section 1.2.4.

P-layer: On the P layer, relations between P values are collected. So far, RSIM deals with the relations *less*, *greater* and *equal*. In some few cases the reciprocal relation would be a helpful extension.

A description layer can be activated or deactivated. Therefore, depending on the status of description layers, simulation can produce different outputs. In the simplest case a pure sign simulation takes place. However, some dependencies between the description layers have to be taken into account (Fig. 3.1). The sign layer is required by all the other description layers, so it must not be deactivated. Besides that, the P layer needs the PQ layer.

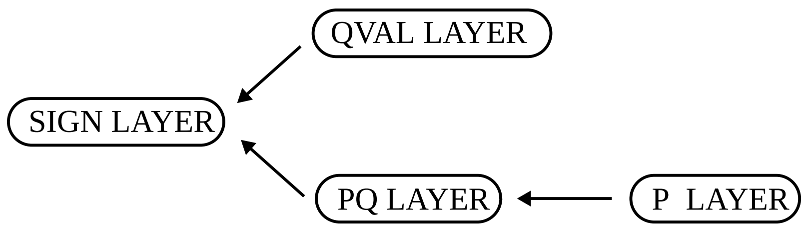


Fig. 3.1 Dependencies between description layers.

Each description layer performs its own intrastate and interstate analysis. The techniques of sign and qval layer are similar to existing simulators, in particular to QSIM, and therefore are not discussed further on.

3.2. System Description in RSIM

As in QSIM, a system is described in RSIM by a number of constraints. Aside from QSIM's constraint classes (Fig. 2.2) some specializations of the M* constraints are possible (Fig. 3.2). Two types of specializations can be distinguished. On one hand a faulty M* relationship can be expressed by the M*TOO-FLAT and M*TOO-STEEP constraints. They concern an M* relationship that is too flat or too steep on every point. For example, if the breather tube of a tank is clogged (the tanks of Fig. 1.1a and Fig. 3.3 have breather tubes), the air in the tank is compressed when filling the tank. From this follows, that the static pressure corresponding to a certain volume of water is greater than normal. The relationship between static pressure and volume level is M^{+}_{0}TOO-STEEP. A model of a tank with a clogged breather tube can be seen in Fig. 3.3.

On the other hand, an M* relationship can be stated more precisely by using the LINEAR*, UNDERLINEAR* or OVERLINEAR* constraints. The specializations of M* constraints have different definitions only on relative description layers. On absolute layers they have identical definitions. Under the assumption of straight tank sides, the relationship between static pressure and volume in a correct tank is $LINEAR^{+}_{0}$.

ADD(f,g,h)	$:\Leftrightarrow f(t) + g(t) = h(t)$
MULT(f,g,h)	$:\Leftrightarrow f(t) * g(t) = h(t)$
MINUS(f,g)	$:\Leftrightarrow f(t) = - g(t)$
DERIV(f,g)	$:\Leftrightarrow f'(t) = g(t)$
M^{+}(f,g)	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge PQ(H'(x)) = normal$
M^{+}TOO-FLAT(f,g)	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge PQ(H'(x)) = too\text{-}low$
M^{+}TOO-STEEP(f,g)	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge PQ(H'(x)) = too\text{-}high$
M_0^{+}(f,g)	$:\Leftrightarrow M^{+}(f,g) \wedge H(0) = 0$
M_0^{+}TOO-FLAT(f,g)	$:\Leftrightarrow M^{+}TOO\text{-}FLAT(f,g) \wedge H(0) = 0$
M_0^{+}TOO-STEEP(f,g)	$:\Leftrightarrow M^{+}TOO\text{-}STEEP(f,g) \wedge H(0) = 0$
M^{-}(f,g)	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge PQ(H'(x)) = normal$
M^{-}TOO-FLAT(f,g)	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge PQ(H'(x)) = too\text{-}low$
M^{-}TOO-STEEP(f,g)	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge PQ(H'(x)) = too\text{-}high$
M_0^{-}(f,g)	$:\Leftrightarrow M^{-}(f,g) \wedge H(0) = 0$
M_0^{-}TOO-FLAT(f,g)	$:\Leftrightarrow M^{-}TOO\text{-}FLAT(f,g) \wedge H(0) = 0$
M_0^{-}TOO-STEEP(f,g)	$:\Leftrightarrow M^{-}TOO\text{-}STEEP(f,g) \wedge H(0) = 0$
$LINEAR^{+}$(f,g)	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge H''(x) = 0$
$LINEAR_0^{+}$(f,g)	$:\Leftrightarrow LINEAR^{+}(f,g) \wedge H(0) = 0$
$LINEAR^{-}$(f,g)	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge H''(x) = 0$
$LINEAR_0^{-}$(f,g)	$:\Leftrightarrow LINEAR^{-}(f,g) \wedge H(0) = 0$
$UNDERLINEAR^{+}$(f,g)	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge H''(x) < 0$
$UNDERLINEAR_0^{+}$(f,g)	$:\Leftrightarrow UNDERLINEAR^{+}(f,g) \wedge H(0) = 0$
$UNDERLINEAR^{-}$(f,g)	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge H''(x) > 0$
$UNDERLINEAR_0^{-}$(f,g)	$:\Leftrightarrow UNDERLINEAR^{-}(f,g) \wedge H(0) = 0$
$OVERLINEAR^{+}$(f,g)	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge H''(x) > 0$
$OVERLINEAR_0^{+}$(f,g)	$:\Leftrightarrow OVERLINEAR^{+}(f,g) \wedge H(0) = 0$
$OVERLINEAR^{-}$(f,g)	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge H''(x) < 0$
$OVERLINEAR_0^{-}$(f,g)	$:\Leftrightarrow OVERLINEAR^{-}(f,g) \wedge H(0) = 0$

Fig. 3.2 Constraints of RSIM.

```
(DERIV inflow volume)
(SIGNED-SQUARE inflow2 inflow)
(PRODUCT p-pressure c-frict inflow2)
(M+0TOO-STEEP p-static volume)
(SUM p p-pressure p-static)
```

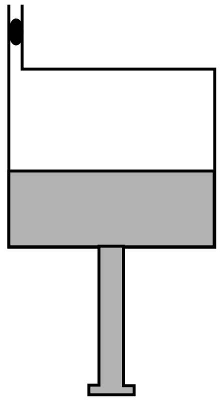


Fig. 3.3 A model for a tank with clogged breather tube. C-frict is a friction coefficient. The SIGNED-SQUARE constraint-class has been added to the basic constraints of Fig. 3.2. It is defined as follows:
SIGNED-SQUARE(f,g) : ⇔ f(t) = g(t) * |g(t)|

3.3. State Descriptions in RSIM

A system state is characterized by all parameter values of a certain time point or time interval. Compared with QSIM, state descriptions in RSIM additionally contain PQ values of the parameter amounts and derivatives, and relations between their P values. In addition the duration for reaching a certain point state is described by a PQ value. Fig. 3.4 gives an example for a state that describes an interval during the filling of the clogged tank. While the parameters on the sign, qval and PQ layers are described by qualitative values, qualitative information on the P layer is expressed by relations.

sign(volume) = +
sign(volume´) = +
qval(volume) = (0 half-full)
pq(volume) = normal
pq(volume´) = normal

sign(inflow) = +
sign(inflow´) = 0
qval(inflow) = (0 ∞)
pq(inflow) = normal
pq(inflow´) = normal

sign(inflow2) = +
sign(inflow2´) = 0
qval(inflow2) = (0 ∞)
pq(inflow2) = normal
pq(inflow2´) = normal

sign(c-frict) = +
sign(c-frict´) = 0
qval(c-frict) = c-frict*
pq(c-frict) = normal
pq(c-frict´) = normal

sign(p-pressure) = +
sign(p-pressure´) = +
qval(p-pressure) = (0 ∞)
pq(p-pressure) = normal
pq(p-pressure´) = normal

sign(p-static) = +
sign(p-static´) = +
qval(p-static) = (p-half-full p-full)
pq(p-static) = too-high
pq(p-static´) = too-high

sign(p) = +
sign(p) = +
qval(p) = (0 ∞)
pq(p) = too-high
pq(p) = too-high

p(p) < p(p-static)
p(p´) < p(p-static´)

Fig. 3.4 An interval state of the process of filling a tank with clogged breather tube. All parameters are described with sign, qval, and PQ values. Additionally some relations between P values can be stated.

3.4. Intrastate Analysis of RSIM

In intrastate analysis, constraint propagation takes place. As on the sign layer, the constraints on the pq layer are described by listing the possible tupels of values. Tables 1 and 2 show the definitions of the SUM and PRODUCT constraints. The left column contains the respective

PQ tupels and the right column assertions about relationships between the corresponding P values. The relationships between P values refine the information given by a PQ tupel. The tupels of the SUM-PQ-relation and the PRODUCT-PQ-relation are identical, but there are differences on the P layer. For example, if a too high value is added to or multiplied with a normal value, sum and product are both too high. But while the sum is less too high than the too high summand, the product is exactly as high as the too high factor.

If a certain PQ tupel is established during constraint propagation, the corresponding P assertions are recorded. On the P layer, the main task is to collect relationships between P values and to generate a consistent graph of P values. In this process, transitivity and symmetry have to be taken into account. Since RSIM uses the relations *less*, *equal* and *greater*, the graph represents a partial order of sets of P values. Each set consists of P values of the same magnitude. Constraining on the P layer means increasing the degree of order between the P values of all parameters. It is not the aim to reach a total order. Generating a total order would still entail ambiguities. The additional information on the P layer facilitates to avoid spurious behavior on the PQ layer. That is, if a system state is completely described on the PQ layer, it may happen that the corresponding P assertions are not consistent, for example because of cycles in a path of *less* relationships.

Table 1: Part of the definition of the SUM constraint

SUM-PQ-Relation $\subset$ PQ(f) $\times$ PQ(g) $\times$ PQ(h) (SUM(f,g,h) $\Leftrightarrow$ f(t)=g(t)+h(t))	Corresponding P assertions
(too-low, too-low, too-low)	
(too-low, too-low, normal)	P(g) < P(f)
(too-low, too-low, too-high)	P(g) < P(f)
(normal, too-low, too-high)	
(too-high, too-low, too-high)	P(f) < P(h)
(too-low, normal, too-low)	P(h) < P(f)
(normal, normal, normal)	
(too-high, normal, too-high)	P(f) < P(h)
(too-low, too-high, too-low)	P(h) < P(f)
(normal, too-high, too-low)	
(too-high, too-high, too-low)	P(f) < P(g)
(too-high, too-high, normal)	P(f) < P(g)
(too-high, too-high, too-high)	

Table 2: Part of the definition of the PRODUCT constraint

PRODUCT-PQ-Relation $\subset$ PQ(f) $\times$ PQ(g) $\times$ PQ(h) (PRODUCT(f,g,h) $\Leftrightarrow$ f(t)=g(t)*h(t))			Corresponding P assertions
(too-low,	too-low,	too-low)	P(f) < P(g) $\wedge$ P(f) < P(h)
(too-low,	too-low,	normal)	P(f) = P(g)
(too-low,	too-low,	too-high)	P(g) < P(f)
(normal,	too-low,	too-high)	
(too-high,	too-low,	too-high)	P(f) < P(h)
(too-low,	normal,	too-low)	P(f) = P(h)
(normal,	normal,	normal)	
(too-high,	normal,	too-high)	P(f) < P(h)
(too-low,	too-high,	too-low)	P(h) < P(f)
(normal,	too-high,	too-low)	
(too-high,	too-high,	too-low)	P(f) < P(g)
(too-high,	too-high,	normal)	P(f) = P(g)
(too-high,	too-high,	too-high)	P(g) < P(f) $\wedge$ P(h) < P(f)

3.5. Interstate Analysis in RSIM

In interstate analysis, transition rules are applied. A transition deduces information about the parameter values of successor states. Usually continuity information about continuously differentiable functions is used for this. Two types of transitions are distinguished: point transitions and interval transitions. A point transition is applied on point states and infers information about the following interval state. An interval transition works in the corresponding way. In contrast to absolute transitions, a relative transition can infer information about the duration of the interval.

3.5.1. Transitions on the PQ Layer

In the current version of RSIM, PQ transitions do not consider discontinuous changes of the behavioral topology (see Section 2.1.1). Therefore, they just use the values *too low*, *normal*, and *too high*.[1] PQ transitions, as sign transitions or qval transitions, like those defined in

[Kuiper 86], refer to the values of exactly one parameter. Although there are analogies between sign transitions and PQ transitions, the following differences have to be mentioned:

- In PQ transitions, information about the duration of a time interval can be deduced. This information is of relative nature and expressed by the values *too low*, *normal*, *too high*.
- In sign transitions (and qval transitions), qualitative information about the derivative is used to determine the further course of the amount. In PQ transitions, as they are formulated below, the PQ value of the derivative is used instead of qualitative information about the derivative of a P value. That is (for a basis landmark of 0):

$$PQ(f',t) = \frac{f'(t)}{f_{ref}'(t)} \quad \text{instead of} \quad P'(f,t) = \left(\frac{f(t)}{f_{ref}(t)}\right)'$$

- Because of the definition of P values, a PQ transition depends not only on the PQ values of amount and derivative of a parameter, but additionally on their signs. For example, if a parameter's amount is too low and its derivative is too high, the amount would come closer to normal if both, amount and derivative, are positive (or negative). But if the derivative is negative and the amount positive (or vice versa) the amount would become more and more too low.

Point Transitions

Table 3 shows the point transitions of the PQ layer. Information about the parameter values at a time point ti is used to determine the values in the following time interval. The first three transitions determine the derivative of the following interval, the next four transitions the amount. The remaining transitions handle the ambiguous case of normal amount and normal derivative at time point ti. For example the transition PANDN2 can be read in the following way:

> *If at some time point ti, the PQ value of a parameter's amount is normal and the PQ value of its derivative is normal too, and if the sign of its amount is positive and of its derivative negative or vice versa then in the following time interval, either the PQ value of the parameter's amount will be too high and the PQ value of its derivative too low, or the amount's PQ value will be too low and the derivative's PQ value too high, or both PQ values will be normal.*

1. An extension is suggested in Chapter 4.

Table 3: Point transitions on the PQ layer

Name of transition	$PQ(f,t_i)$	$PQ(f',t_i)$	$sign(f(t_i))$	$sign(f'(t_i))$	$PQ(f,(t_i,t_j))$	$PQ(f',(t_i,t_j))$	$sign(f',(t_i,t_j))$
PDL		L				L	
PDH		H				H	
PALHDN	L H	N N				unconstrained	
PAL	L				L		
PAH	H				H		
PANDLH1	N N N N N N	L L L L H H	- 0 0 + + -	- - + + - +	L		
PANDLH2	N N N N N N	L L H H H H	+ - - 0 0 +	- + - - + +	H		
PANDN1	N N N N N	N N N N N	- 0 0 0 +	- - 0 + +	L N H	L N H	
PANDN2	N N	N N	+ -	- +	H N L	L N H	
PANDN3	N	N	-	0	L L N H H	L H N L H	- + unconstrained + -
PANDN4	N	N	+	0	L L N H H	L H N L H	+ - unconstrained - +

Interval Transitions

Interval transitions use information about the parameter values of a time interval to determine the events that may finish the qualitative state described in the interval. The reason for a change of the qualitative state is, that a landmark is reached by a parameter's amount or derivative. On the PQ layer, the corresponding event is reaching *normal*, or more exactly the landmark 1 in the range of P values. Therefore, a qualitative behavior description can be refined by additional events on relative description layers. Or in other words: An interval on absolute descriptions layers can have a structure on relative description layers. In Fig. 3.5 deviating and reference behavior of a parameter *f* can be seen. On absolute layers the time intervals (t_a, t_p) or (t_a, t_s) describe the state of decreasing *f* . On relative layers, however, there is an event at time point t_i where the normal value is reached by *f*. So in a complete description

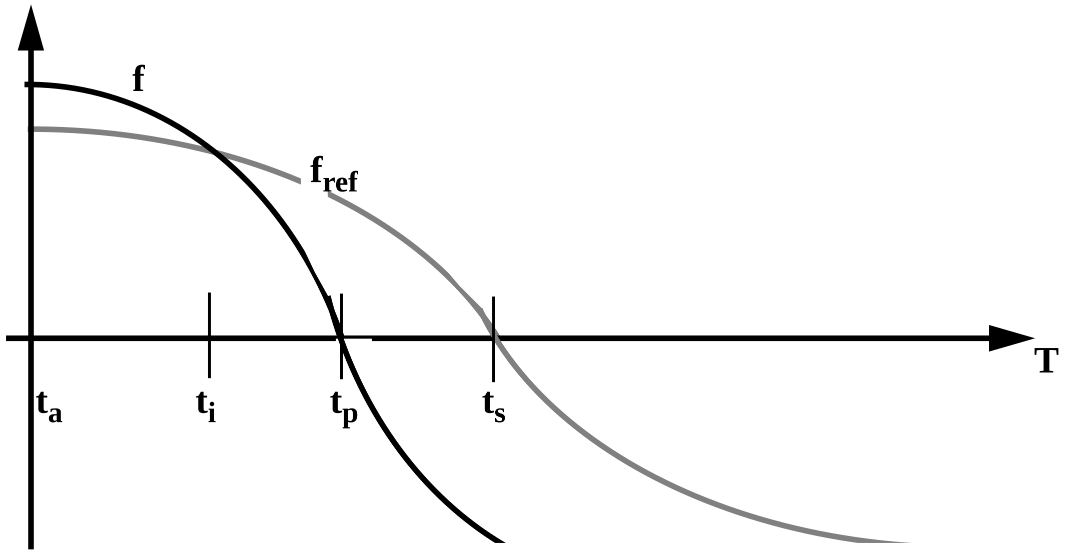


Fig. 3.5 A deviating parameter f and its reference course f_{ref}.

we have the following sequence of states:

Time point t_a:	*f* is positive and *too-high*.
Time interval (t_a, t_i):	*f* is positive and *too-high*.
Time point t_i:	*f* is positive and *normal*.
Time inteval (t_i, t_p):	*f* is positive and *too-low*.
Time point t_p:	*f* reaches 0...

The event at time point t_p entails a difficulty. Here, the deviation of *f* is discontinuous. And, in the time interval beginning at t_p, it remains discontinuous. In general it makes no sense, to compare *f* with f_{ref} during interval (t_p, t_s). Instead, the parts of *f* and f_{ref} where both are negative should be compared with each other. And *f* at time point t_p should be compared with f_{ref} at time point t_s. That means, a synchronization step is necessary. Fig. 3.6 shows the

synchronized behaviors. For f_{ref} no comparison takes place during (t_p, t_s). Now the list of qualitative states can be continued:

Time point t_{p*}: f is 0 and *normal*.

Time interval (t_{p*}, $t_?$) f is negative and *too high*[1].

In addition, it can be stated that the duration for reaching 0 is less than normal, i.e. *too-low*.

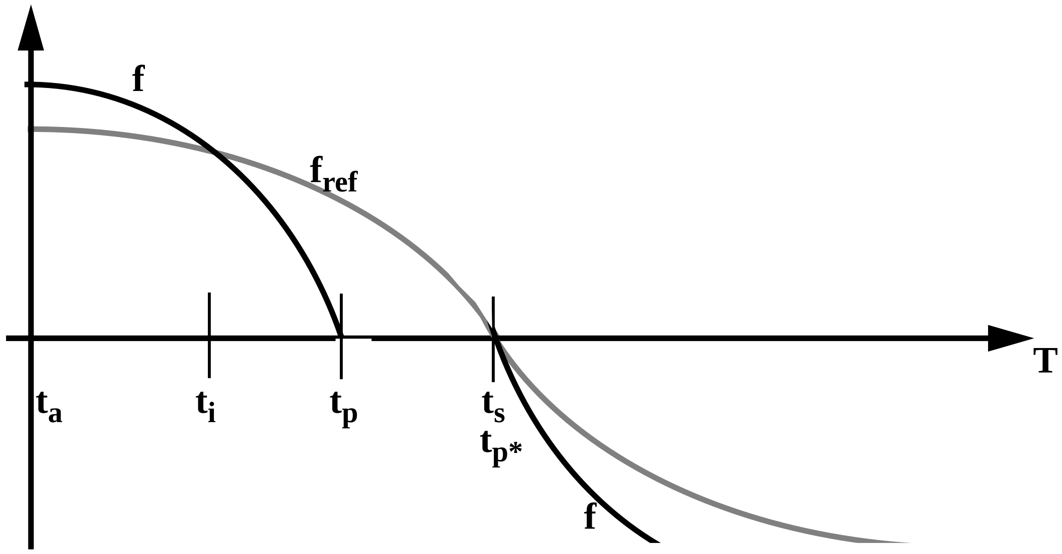


Fig. 3.6 Synchronized behaviors of Fig.3.5.

So, the event of reaching 0 requires a synchronization because otherwise the deviations would become discontinuous and sensible comparisons would not be possible. Reaching other landmarks than 0 does only requires a synchronization, if comparisons of behaviors shall be restricted to differential deviations. (If a landmark different from 0 is reached and no synchronization takes place, a deviation would become non-differential, but it would still remain continuous).

In Fig. 3.7. a parameter g first reaches a landmark l_k and afterwards a landmark l_m, both faster than normal. At the same time when g reaches l_m, parameter f reaches 0 (time point t_p). Therefore, a synchronization is necessary. Before that, at time point t_b, g reaches landmark l_k. And no other parameter reaches 0 at that time. The deviations of g at and after t_b are non-differential, but continuous. So, only if one wants to restrict oneself to differential deviations, a synchronization becomes necessary. Both qualitative behaviors, with and without synchronization, are listed below.

1. Note, a PQ value describes the magnitude of a value, so a negative value that is too low in a mathematical sense has the PQ value *too-high*.

Without synchronization at t_b (see Fig. 3.7):

Time point t_a:	*f* is positive and *too-high*.	g is 0 and normal
Time interval (t_a, t_b):	*f* is positive and *too-high*.	*g* is positive and *too-high*
Time point t_b:	is positive and *too-high*.	*g* is positive at l_k and ***too-high***
Time inteval (t_b, t_i):	*f* is positive and *too-high*.	*g* is positive in (l_k, l_m) and *too-high*
Time point t_i:	*f* is positive and *normal*.	*g* is positive in (l_k, l_m) and *too-high*
Time inteval (t_i, t_p):	*f* is positive and *too-low*.	*g* is positive in (l_k, l_m) and *too-high*
Time point t_p:	*f* is 0 and *normal*.	*g* is positive at l_k and *normal*[1]

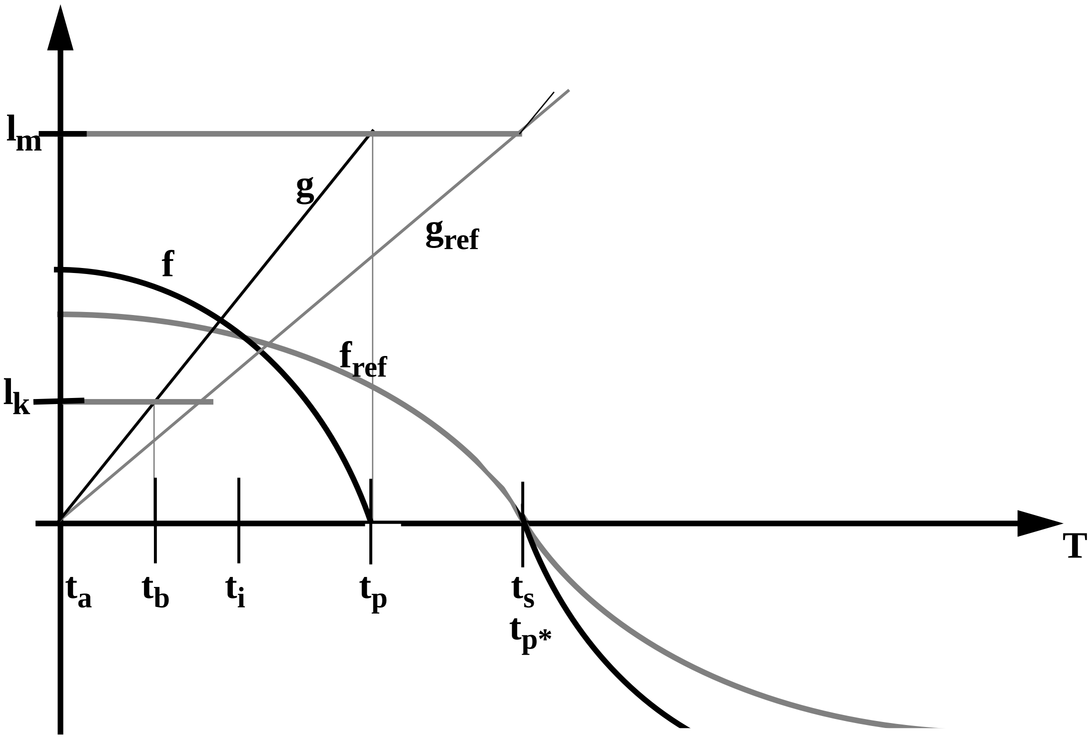


Fig. 3.7 Additional events at a parameter *g*.

With synchronization at t_b (see Fig.3.8):

Time point t_a:	*f* is positive and *too-high*.	g is 0 and normal
Time interval (t_a, t_b):	*f* is positive and *too-high*.	*g* is positive and *too-high*
Time point t_{b*}:	is positive and *too-high*.	*g* is positive at l_k and ***normal***
Time inteval (t_{b*}, t_{i*}):	*f* is positive and *too-high*.	*g* is positive in (l_k, l_m) and *too-high*

1. *G* is normal at t_p because g_{ref} and f_{ref} reach their landmarks at the same time.

Time point t_{i*}:	*f* is positive and *normal*.	*g* is positive in (l_k, l_m) and *too-high*
Time inteval (t_{i*}, t_{p*}):	*f* is positive and *too-low.*	*g* is positive in (l_k, l_m) and *too-high*
Time point t_{p**}:	*f* is 0 and *normal*.	*g* is positive at l_k and *normal*[1]

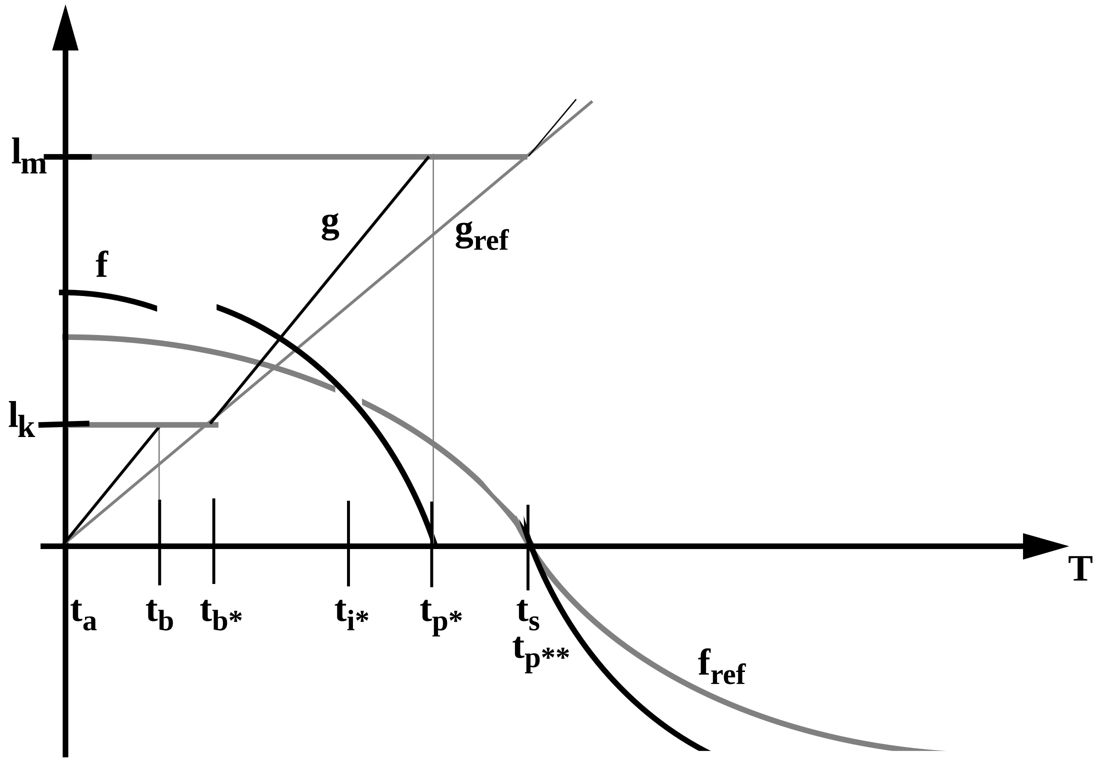


Fig. 3.8 Additional synchronization at t_b

In the example above the duration of the deviating behavior has been too low. A too high duration has similar consequences. Fig. 3.9 shows the difference between a too low and a too high duration of the deviating behavior. Here, synchronization takes place when a landmark value l_k is reached. In the deviating behavior b_l, l_k is reached earlier than normal, in deviating behavior b_h later than normal. The qualitative state describing the phase of increase refers to the full length of b_l, but for b_h it ends at t_n. So, for the interval (t_n, t_h) of b_h no description exists. However, the relevant information that the duration to reach l_k is *too-high*, is included.

1. *G* is normal at t_p because g_{ref} and f_{ref} at the same time reach their landmarks.

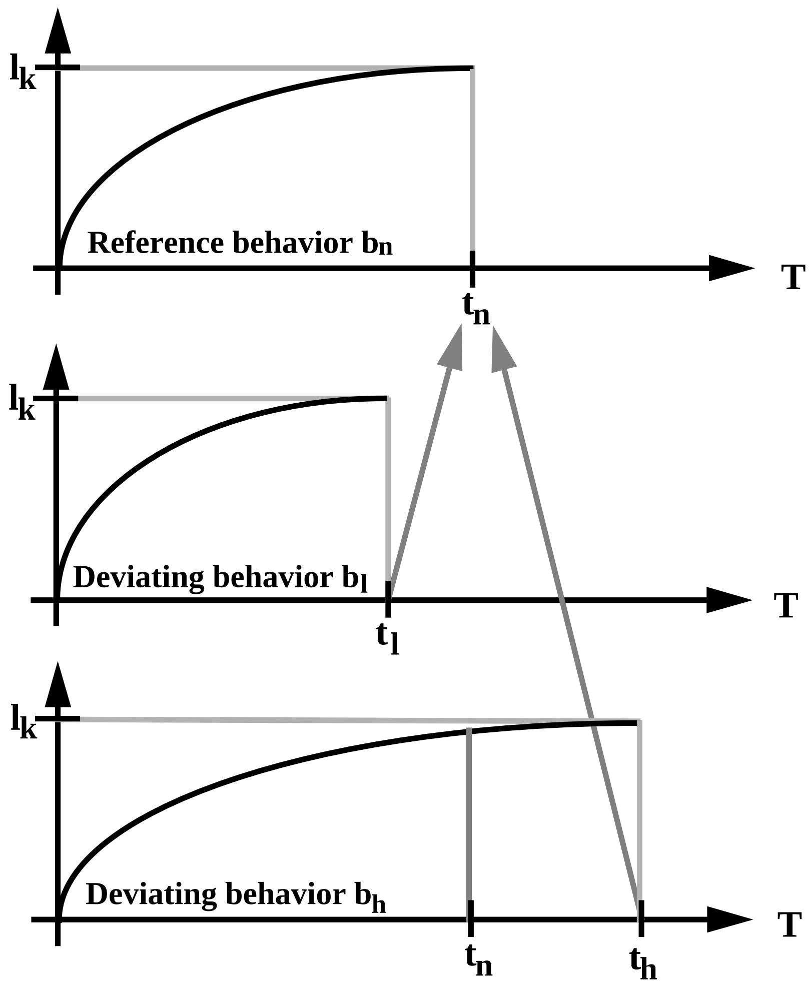


Fig. 3.9 Synchronization for a *too-low* and a *too-high* duration.

We have seen that a synchronization is necessary when landmark 0 or any landmark[1] respectively is reached, depending on whether continuous or merely differential deviations are allowed, . We want to call a landmark that requires a synchronization a ***synchronization landmark***. Whenever a synchronization landmark is reached by a parameter's amount or derivative, its PQ value becomes *normal* because of the synchronization. So, there are two reasons for reaching *normal*:

1. The amount or derivative of a parameter in the deviating system reaches a synchronization landmark.
2. The amount or derivative of a parameter in the deviating system reaches the normal value.

In qualitative simulation, in general, it cannot be determined which event occurs first, i.e. which parameter is involved in the next event. Therefore, transition rules (which are specific to a parameter) addionally have to supply the values of a parameter's amount and derivative

1. on an absolute description layer

for the case that the parameter is not involved in the next event, i.e. the next event concerns a different parameter. This “foreign” event may entail a synchronization. Therefore, an interval transition on the PQ layer has to cover five cases:

1. The parameter’s amount or derivative reaches a synchronization landmark.
2. The parameter’s amount or derivative reaches the normal value.
3. The next event occurs outside with normal duration.
4. The next event occurs outside with too low duration.
5. The next event occurs outside with too high duration.

Tables 4 and 5 show the PQ interval transitions[1]. The five situations listed above cannot be directly identified in the transition tables, because a situation can have different consequences as a result of ambiguities, or different situations can have the same consequences. In transition IAHDH, for example, the five situations have the following consequences:

Case 1: duration(t_i,t_j) = *normal*, PQ(f,(t_j) = *normal* or
duration(t_i,t_j) = *too-high*, PQ(f,(t_j) = *normal*

Case 2: duration(t_i,t_j) = *normal*, PQ(f,(t_j) = *normal*

Case 3: duration(t_i,t_j) = *normal*, PQ(f,(t_j) = *too-high*

Case 4: duration(t_i,t_j) = *too-low*, PQ(f,(t_j) = *too-high*

Case 5: duration(t_i,t_j) = *too-high*,
PQ(f,(t_j) = *too-low* or *normal* or *too-high* (= unconstrained)

Table 4: Interval transitions on the PQ layer (1)

Name of transition	PQ(f´,(t_i,t_j))	duration(t_i,t_j)	PQ(f´,(t_j)
IDL	L	L N N H	unconstrained L N unconstrained
IDN	N	L N H	unconstrained N unconstrained
IDH	H	L N N H	unconstrained N H unconstrained

1. PQ values are abbreviated by L, N, and H.

Table 5: Interval transitions on the PQ layer (2)

Name of transition	PQ(f,(t_i,t_j))	PQ(f´,(t_i,t_j))	sign(f,(t_i,t_j))	sign(f´,(t_i,t_j))	duration(t_i,t_j)	PQ(f,(t_j))
IAL	L			0		L
IAN	N			0		N
IAH	H			0		H
IANDN1	N N	N N	- +	- +	L N H	L N H
IANDN2	N N	N N	+ -	- +	L N H	H N L
IALDLN	L L L L	L L N N	- + - +	- + - +	L N H	L L unconstrained
IALDNH	L L L L	N N H H	+ - + -	- + - +	L N H	unconstrained L L
IAHDLN	H H H H	L L N N	+ - + -	- + - +	L N H	H H unconstrained
IAHDNH	H H H H	N N H H	- + - +	- + - +	L N H	unconstrained H H
IALDH	L L	H H	- +	- +	L N N H	L L N unconstrained
IAHDL	H H	L L	- +	- +	L N N H	unconstrained N H H
IALDL	L L	L L	+ -	- +	L N N H	unconstrained L N L
IAHDH	H H	H H	+ -	- +	L N N H	H N H unconstrained

3.5.2. Transitions on the P Layer

As explained above, an incomplete description is generated on the P layer, but one that supplies additional information and helps to avoid spurious behavior on the PQ layer. Accordingly it is not necessary for transitions on the P layer to cover all possible configurations of values. Instead, some special but very useful transitions are formulated. When required, more transitions can be added.

While sign transitions, qval transitions, or PQ transitions work with qualitative values of amounts and derivatives, transitions on the P layer refer to relations between P values. Relations between P values can be used in different ways: They can help to further constrain PQ transitions. They can deduce relationships between P values in the following time interval or time point. And because the relations used on the P layer are binary, they can concern different parameters. As a consequence, P transitions are not as homogeneous as the other types of transitions.

Point Transitions on the P Layer

One obvious point transition is given by the continuity rule, that if two P values are different at a time point t_i, there must exist an interval around t_i where they are different too. While the same consequences mostly can be derived from the P layer's intrastate analysis and from PQ transitions, the case of equality between P values is a more important one. In the transition P==[1] (Table 6), an interesting correlation between the P values of a parameter's amount and its first and second derivatives is formulated.[2] P== says:

> *If at a time point t_i on the one hand the P values of a parameter's amount and its second derivative are equal and on the other hand the P values of amount and first derivative are equal too, or the sign of the first derivative is 0, then in the following time interval (t_i, t_j) the P values of amount and derivatives must be equal, or the P values of amount and first derivative and amount and second derivative must be unequal.*

In the current version of RSIM, P== is the only point transition on the P layer. Further point transitions may follow.

Table 6: Point transition P==

Name of transition	relation (P(f,t_i), P(f´,t_i))	relation (P(f,t_i), P(f´´,t_i))	sign(f´,t_i)	relation (P(f,(t_i,t_j)), P(f´,(t_i,t_j)))	relation (P(f,(t_i,t_j)), P(f´´,(t_i,t_j)))
P= =	= undefined =	= = =	- 0 +	= ≠	= ≠

1. P== is proven for a basis landmark of 0 (see appendix B).
2. Since only the first derivative of a parameter is modeled, the rule can only be applied to parameters that are connected to another parameter via the DERIV constraint.

Interval Transitions on the P layer

The first group of interval transitions on the P layer treats the situation when the amount of a parameter moves to the landmark 0 and both amount and derivative of the parameter are too low or too high. The PQ transitions describing this case are IALDL and IAHDH. On the P layer, it can be expressed that one value is more too high or too low than another value. The more detailed information on the P layer facilitates a more constrained transition consequence. Table 7 shows the transitions IALDL and IAHDH augmented by P information. For each of them there are six transitions on the P layer.

Table 7: Interval transitions IALDL* and IAHDH*

Name of transition	PQ $(f,(t_i,t_j))$	PQ $(f',(t_i,t_j))$	relation $(P(f,(t_i,t_j)), P(f',(t_i,t_j)))$	sign $(f,(t_i,t_j))$	sign $(f',(t_i,t_j))$	duration (t_i,t_j)	PQ(f,$(t_j$)	sign (f,$(t_j$)
IALDL<-	L	L	<	-	+	L L N H	N unconstrained L L	0 - - -
IALDL<+	L	L	<	+	-	L L N H	N unconstrained L L	0 + + +
IALDL=-	L	L	=	-	+	L N N H	unconstrained L N L	- - 0 -
IALDL=+	L	L	=	+	-	L N N H	unconstrained L N L	+ + 0 +
IALDL>-	L	L	>	-	+	L N N H	unconstrained L N L	- - - -
IALDL>+	L	L	>	+	-	L N N H	unconstrained L N L	+ + + +
IAHDH<-	H	H	<	-	+	L N N H	H N H unconstrained	- - - -
IAHDH<+	H	H	<	+	-	L N N H	H N H unconstrained	+ + + +

Table 7: Interval transitions IALDL* and IAHDH*

Name of transition	PQ $(f,(t_i,t_j))$	PQ $(f',(t_i,t_j))$	relation $(P(f,(t_i,t_j)), P(f',(t_i,t_j)))$	sign $(f,(t_i,t_j))$	sign $(f',(t_i,t_j))$	duration (t_i,t_j)	PQ$(f,(t_j)$	sign $(f,(t_j)$
IAHDH=-	H	H	=	-	+	L N N H	H N H unconstrained	- 0 - -
IAHDH=+	H	H	=	+	-	L N N H	H N H unconstrained	+ 0 + +
IAHDH>-	H	H	>	-	+	L N H H	H H N unconstrained	- - 0 -
IAHDH>+	H	H	>	+	-	L N N H	H N H unconstrained	+ + 0 +

As an example, the P transition IAHDH=+ shall be explained.The transition IAHDH=+ (and the transition IAHDH=-) can be interpreted in the following way:

> *If at every time point of a time interval (t_i, t_j) the distance that has to be traveled is too high by the same factor as the velocity then it costs as much time as normally to travel the whole distance.*

Transition IAHDH=+ is illustrated in Figure 3.10. The reference behavior is represented by the black line and the deviating behavior by the dotted line. Let’s examine how the five possible endings (Section 3.5.1) of the time interval (t_i, t_j) look like.

1. A landmark is reached: One has to distinguish between a positive landmark l_k and the landmark 0. Every positive landmark l_k will first be reached by the reference function. Therefore, the duration to reach a positive landmark is too high. This case is covered by line 4 of the transition’s description in Table 7. On the other hand, the duration to reach the landmark 0 is the same as in the reference behavior (line 2 in transition table).
2. The normal value is reached: This happens at time point t_n. Transition IAHDH=+ handles the special case where landmark 0 is reached in normal duration. Reaching the normal value is covered by line 2.
3. The next event occurs outside with normal duration: In Figure 3.10 this case is represented by the vertical arrow. At every time point in the interval (t_i, t_j), the deviating value is *too-high* (line 3 in transition table).

4. The next event occurs outside with too low duration: In Figure 3.10 this case is represented by the arrow pointing to the right. At every time point in the interval (t_i, t_j), the deviating value is *too-high* (line 1 in transition table).
5. The next event occurs outside with too high duration: In Figure 3.10 this case is represented by three arrows pointing to the left. The deviating value may be *too-low*, *normal*, or *too-high*. (Line 4 in transition table.)

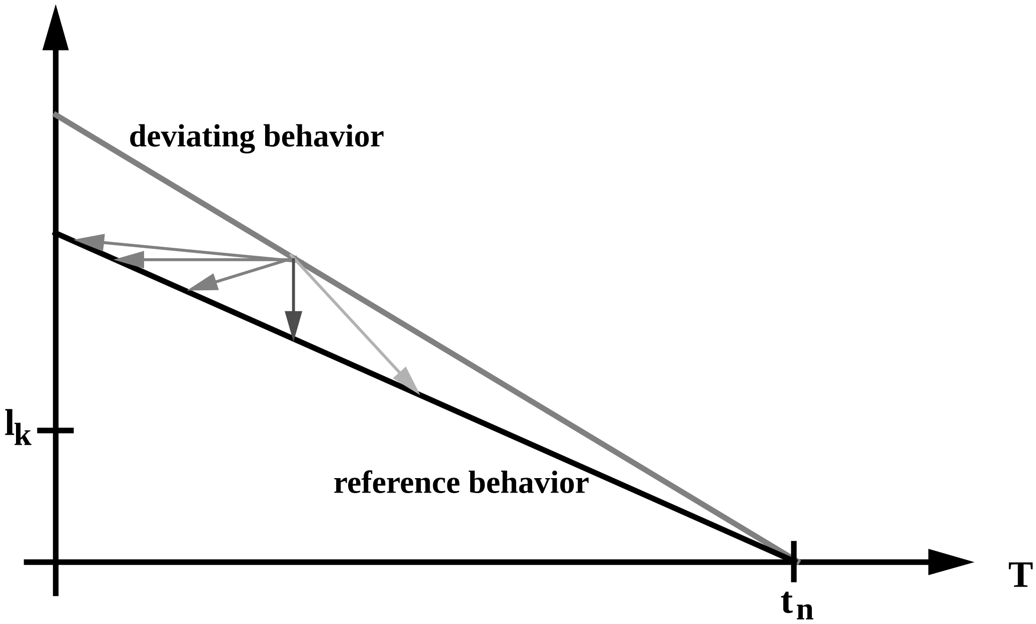


Fig. 3.10 An illustration of the interval transition IAHDH=+. Possible cases of comparisons are indicated by arrows.

Another type of transition makes predictions about the relations between P values at time

Table 8:

Name of transition	relation (P(f,(t_i,t_j)), P(f´,(t_i,t_j)))	duration (t_i,t_j)	relation (P(f,t_j), P(f´,t_j))	sign(f,t_j)	sign(f´,t_j)
I=	=	L	unconstrained	unconstrained	unconstrained
		N	=	{-, +}	{-, +}
		N	undefined	unconstrained	0
		N	undefined	0	unconstrained
		H	unconstrained	unconstrained	unconstrained

point t_j. Transition I= handles the case of equality between amount and derivative of a parameter. In words, I= expresses the following:

If during a time interval (t_i, t_j) the P values of amount and derivative of a parameter are equal then if the duration of (t_i, t_j) is normal and the signs of amount and derivative are unequal to 0, the P values of amount and derivative are equal at time point t_j.

3.6. The Output of RSIM

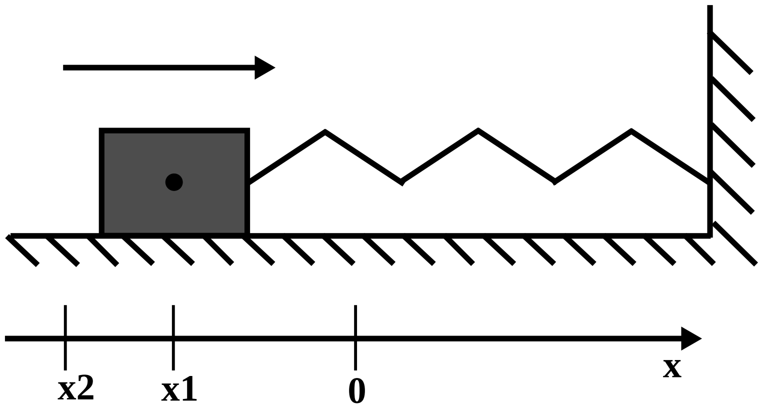
RSIM's output is a tree of behaviors. An RSIM behavior is more specific than a QSIM behavior, i.e. the description of behavior is more detailed. This means that

1. the description of a parameter and thus a state description is more detailed,
2. a behavior consists of more states, because additional events on relative description layers have to be integrated in a description of behavior,
3. the behavior tree contains more branchings, because the additional descriptions are not free of ambiguities either.

Especially the last point leads to time consuming computations and to extensive and sometimes confusing output. To cope with this problem, some methods have been added, that help to prevent irrelevant distinctions. It is possible, for example, to deactivate a description layer just for a certain amount or derivative. Another very effective method prevents distinctions for the PQ values of derivatives. It first generates fully described system states and then merges them afterwards.

We want to illustrate RSIM's output for the popular example system of a mass on a spring. Fig. 3.11 shows a model of this system and two different initializations. On the one hand, we want to examine a model with a too high mass. On the other hand we want to know what happens, if the amplitude is higher than normal. In Fig. 3.12 RSIM's output is listed for the position of the block, its velocity, and its acceleration, until the block reaches the point of no spring tension ($x = 0$).

Both behaviors of Fig. 3.12 consist of 5 states and have no ambiguities. In fact, there are ambiguities for the PQ values of some derivatives. But the simplifying strategies, mentioned above, merge these states. For a too high mass, it is derived that the duration of a period is too high, that the velocity is too low for $x = 0$, and that the acceleration at first is too low, then becomes normal, and afterwards is too high. For a too high amplitude it is unambiguously derived that the duration of a period is normal. This is true, because the deviations of x and v have the same strength, i.e. x and v have equal P values.




```
(create-model *mass-on-a-spring*
   :quantity-spaces ((k-qs (k* 0))              ;Definition of quantity spaces:
                     (m-qs (0 m*))              ;(name-of-quantity-space list-of-landmarks)
                     (energy-qs (0 te*))
                     (x-qs (x2* x1* 0)))
   :variables ((x x-qs) v vv a (f f-qs) (ke energy-qs) (pe energy-qs))
   :constants ((m m-qs) (k k-qs) (te energy-qs))
   :non-negatives (pe ke te m)
   :constraints ((deriv v x)        ;The velocity of the block is the derivative of its position.
                 (deriv a v)        ;The acceleration of the block is the derivative of its velocity.
                 (product f m a)    ;The force of the spring is the product of spring constant and position
                                     (Hooke's law).
                 (product f k x)    ;Force is the product of mass and acceleration (Newton's second law of
                                     motion).
                 (square vv v)      ;Vv is the square of v.
                 (product ke m vv)  ;Kinetic energy depends on the product of mass and the square of velocity.
                 (square pe f)      ;Weld calls it a "cheating definition of potential energy" [Weld 90, p.159].
                 (sum te pe ke)))   ;Total energy is the sum of kinetic and potential energy.
```

```
(initialize-model *mass-on-a-spring*
  (v  :sign-of-amount 0
      :pq-of-amount normal)
  (x  :qval-of-amount x1*
      :pq-of-amount normal)
  (k  :qval-of-amount k*
      :pq-of-amount normal)
  (m  :qval-of-amount m*
      :pq-of-amount too-high)
  (te :qval-of-amount te*))
```

```
(initialize-model *mass-on-a-spring*
  (v  :sign-of-amount 0
      :pq-of-amount normal)
  (x  :qval-of-amount x2*
      :pq-of-amount too-high)
  (k  :qval-of-amount k*
      :pq-of-amount normal)
  (m  :qval-of-amount m*
      :pq-of-amount normal)
  (te :qval-of-amount te*))
```

Fig. 3.11 A model for a mass on a spring and two different initializations in RSIM's language. In the left initialization the mass is two high, in the right initialization the amplitude is greater than normal.

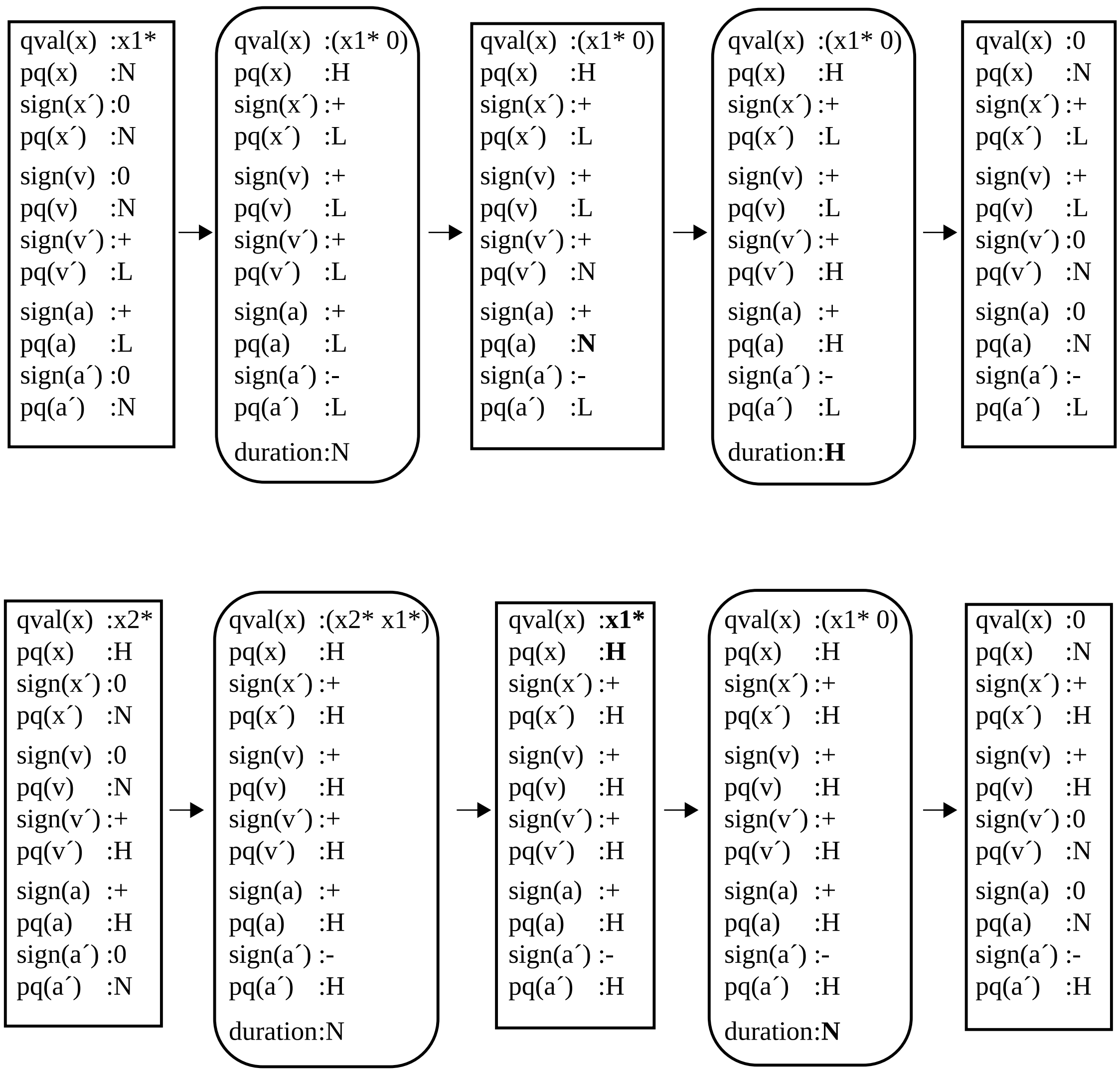


Fig. 3.12 The deviating behaviors for the mass-on-a-spring system, produced by RSIM. At the top the behavior for a too high mass, at the bottom the behavior for a too high amplitude.

4. Related Work

Relative descriptions are used in various approaches. [Raiman 86] reasons about orders of magnitude, in [Dague,Devès,Raiman 87] these concepts are used for a fault diagnosis. The *IQ analysis* of [de Kleer 79] is a qualitative sensitivity analysis of a system's steady state. Similar techniques that aim at a fault diagnosis can be found in [Downing 87], [Gallanti, Stefanini, Tomada 89] and [Kockskämper, Neumann, et al. 93]. [D'Ambrosio 89] describes a qualitative perturbation analysis that also is based on IQ analysis.

Dynamic systems, on the other hand, are analysed in Weld's approaches *DQ analysis* and *exaggeration* [Weld 87,88a,88b,90]. Both techniques predict the effects of differential changes as RSIM does. Therefore DQ analysis and exaggeration are the most relevant works for a comparison with RSIM. Since RSIM is oriented towards QSIM, we additionally will outline some differences to QSIM concerning the simulation techniques[1].

QSIM

RSIM's (or SLOD's) simulation techniques are similar to those of QSIM. Both simulators have an identical form of input and output: A system is described by constraints and a behavior is described by a sequence of states. However, RSIM's inferences have a more constructive character than QSIM's inferences. This becomes apparent in form and processing of transition rules. In QSIM, a single situation generally matches to several alternate transition rules. Constraint propagation in QSIM means filtering out the unsuitable transitions or the unsuitable combinations of transitions, which corresponds to filtering out impossible combinations of parameter values. That is, the set of possible values for a parameter is gradually reduced. In RSIM, on the other hand, transition rules are non-overlapping. A transition rule that can be applied will be applied. In general, the consequence of an RSIM transition rule contains a disjunction of value combinations. In order to fulfill the allowed combinations of an applicable transition rule, temporary *transition constraints* are generated. Constraint propagation in RSIM directly leads to single parameter values instead of a reduced set of values.

Weld's DQ Analysis

While the input/output behavior of RSIM and DQ analysis are similar, the underlying techniques are very different. DQ analysis uses a set of inference rules that analytically, and not by simulation, determine the effects of differential deviations. These inference rules refer to a model, expressed by QSIM constraints, and to a QSIM behavior. To facilitate a comparison of intervals of different length, DQ analysis introduces the concept of *perspective* (see Section 2.2.1). In RSIM, this problem is solved by interval transitions that perform a synchronization (see Section 3.5.1). DQ analysis sometimes produces no output, that is DQ analysis is incomplete. RSIM, on the other hand, always produces an output. But the current version of RSIM

1. Differences resulting from a refinement by means of relative descriptions have extensively been explained in the previous chapters.

does not capture changes in the behavioral topology.[1] That is, RSIM only is sound if topology changes can be excluded.

RSIM can answer some questions that DQ analysis cannot answer because DQ analysis has problems to predict the resulting effect of contrary deviations. If, for example, a longer distance has to be covered with a higher velocity, DQ analysis cannot decide, whether the duration increases, decreases or does not change. RSIM's relations between P values provide the means to answer such questions.

Exaggeration

In contrast to DQ analysis, exaggeration is based on simulation. Exaggeration's input and output, however, are different from RSIM. Exaggeration answers concrete questions about the effects of a certain differential deviation on a target parameter. For this purpose it first exaggerates the given deviation, e.g. a value higher than normal is considered as infinite. Then two simulations take place: a standard QSIM simulation and a simulation of the exaggerated system. The latter simulation works with hyperreal values, like *infinite* or *negligible*, and is carried out by a simulator called HR-QSIM. Finally, the exaggerated output is rescaled by a comparison of both simulation results. A negligible value, e.g., may be rescaled to "less than normal". In general, a QSIM simulation as well as a HR-QSIM simulation produces a set of behaviors because of ambiguities. As it is implemented, exaggeration compares the HR-QSIM behaviors to a single QSIM behavior and not to all behaviors that result from a QSIM simulation. The decision which behavior to choose is obtained from the input question. A general prediction of the effects of a deviation, as it is done by RSIM, would require a pairwise comparison of all exaggerated and all normal behaviors. Exaggeration is complete, but unfortunately only sound if all relationships between parameters are monotonous. The idea of an exaggeration conflicts with a comparison of deviations. Therefore, an important advantage of RSIM again lies in the comparison of deviations.

1. DQ analysis also has problems with topology changes. See, for example, Section 4 in [Weld 88b].

5. Future Work

At the moment, simulation by RSIM predicts all behaviors that entail no discontinuous changes of topology. Future work will investigate this topic. Occurrence branching shall be captured by making parameter-specific statements about duration instead of state-specific statements. The other type of branchings corresponds to discontinuous deviations in parameter histories. We want to integrate these changes of behavior by extending the set of PQ values, so that discontinuous deviations of parameter values can be described. In addition, the definitions of constraints have to be extended by the new PQ values.

RSIM takes all behaviors that result from the system description as reference behavior. In practice, often the correct behavior of a system is well known, such that this knowledge can be used to constrain the prediction of faulty behavior [Neitzke 91]. To realize this, RSIM needs additional input about the reference behavior. In the extreme, this can be the path of the behavior tree that corresponds to the reference behavior. (This is done in Weld's DQ analysis [Weld 87,88b,90].) During generation of behavior, RSIM then has to decide which behavior can result from the correct behavior.

6. Summary

Relative descriptions are necessary to characterize certain kinds of system deviations and the resulting behavior. These deviations are classified as differential or continuous. RSIM is a simulator that works with relative descriptions and is able to compare deviations with each other. RSIM cannot only deal with differential deviations but additionally accepts continuous deviations. In RSIM, the special properties of linear, overlinear and underlinear relationships between parameters are exploited to gain more accuracy in the prediction of system behavior. Additionally, faulty M^* relationships can be expressed. Due to these refined description facilities, some system behaviors can be distinguished that are identical under usual qualitative descriptions. The class of physical systems that can be described by RSIM corresponds to that of QSIM. In the current version of RSIM, a prediction of behaviors with discontinuous changes of topology is not possible. But RSIM's concepts allow an extension in this direction.

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A: Proofs of PQ Transition Rules

In Appendix A, it will be proven that the PQ transition rules are mathematically sound. The proofs are based on continuity conditions of continuously differentiable functions of time.

PQ values refer to the function P. (See the definition of P in Section 1.2.4.) Since $P(f,t) = (f(t) - l_a) / (f_{ref}(t) - l_a)$ is a continuously differentiable function of time for $(f_{ref}(t) - l_a) \neq 0$[1], PQ transitions have a similar character as sign transitions or qval transitions.

According to the definition of PQ values, we have to deal with the following situations:

1. $0 < P(f,t) < 1 \quad \Leftrightarrow \quad PQ(f,t) =$ *too-low*
2. $P(f,t) = 1 \quad \Rightarrow \quad PQ(f,t) =$ *normal*
3. $P(f,t) > 1 \quad \Leftrightarrow \quad PQ(f,t) =$ *too-high*
4. $f(t)-l_a = f_{ref}(t)-l_a = 0 \quad \Rightarrow \quad PQ(f,t) =$ *normal*

For the moment, the last situation will be left out. For the function P, interesting transitions are from the intervals $(0, 1)$ or $(1, \infty)$ to a value of 1 and vice versa. Discontinuous transitions are not considered at this place. We want to see, which transitions between time points and time intervals are possible. At a time point t_i, either

1. $P(f,t_i) \in (0, 1)$, or
2. $P(f,t_i) = 1$, or
3. $P(f,t_i) \in (1, \infty)$

If $P(f,t_i) = 1$, nothing can be said about time intervals (t_{i-1}, t_i) or (t_i, t_{i+1}) without additional information. In these intervals, $P(f,t)$ may be less, equal or greater than 1. But,

Lemma A: If $P(f,t_i) \in (0, 1)$ (or $P(f,t_i) \in (1,\infty)$), then there exists an environment of t_i with $P(f,t) \in (0, 1)$ (or $P(f,t_i) \in (1,\infty)$). That is, there exists a $\delta > 0$, such that

$$P(f,t) \in (0, 1) \text{ (or } P(f,t) \in (1,\infty)\text{), for all } t \in T \text{ with } |t - t_i| < \delta$$

Proof A: The ε-δ definition of continuity (e. g. [Forster 80, p. 67] states that for every $\varepsilon := |P(f,t_i) - 1| > 0$ there exists a $\delta > 0$ such that
$|P(f,t) - P(f,t_i)| < \varepsilon$, for all $t \in T$ with $|t - t_i| < \delta$
From this follows that $|P(f,t) - 1| \geq |P(f,t_i) - 1| - |P(f,t) - P(f,t_i)| > 0$, for all $t \in T$ with $|t - t_i| < \delta$, ❑

Therefore, the following transitions are possible:

1. It is required that all parameters are continuously differentiable functions of time.

Point transitions:

$$P(f,t_i) \in (0, 1) \rightarrow P(f,(t_i,t_{i+1})) \in (0, 1) \qquad (1)$$

$$P(f,t_i) = 1 \rightarrow P(f,(t_i,t_{i+1})) \in (0, 1) \vee P(f,(t_i,t_{i+1}))=1 \vee P(f,(t_i,t_{i+1})) \in (1, \infty) \qquad (2)$$

$$P(f,t_i) \in (1, \infty) \rightarrow P(f,(t_i,t_{i+1})) \in (1, \infty) \qquad (3)$$

(2) is ambiguous. Here, information about the derivative of P, especially its sign, can help. But first we have to prove the following lemma:

Lemma B: Let g(t) be a continuously differentiable function of time. If for a time point t_i $g(t_i) = k$ and $g'(t_i) > 0$ ($g'(t_i) < 0$) holds, then there exist an environment of t_i, (t_{i-1},t_{i+1}), $t_{i-1} < t_i < t_{i+1}$, with $g(t_{i-1},t_i) < k$ ($g(t_{i-1},t_i) > k$), and $g(t_i,t_{i+1}) > k$ ($g(t_i,t_{i+1}) > k$).

Proof B: Following proof A, there exist an environment (t_{i-1},t_{i+1}) with $g'(t) > 0$ ($g'(t) < 0$), $t \in (t_{i-1},t_{i+1})$. From this follows that $g(t_{i-1},t_i) < k$ ($g(t_{i-1},t_i) > k$), and $g(t_i,t_{i+1}) > k$ ($g(t_i,t_{i+1}) > k$).

$$P(f,t_i) = 1 \wedge P'(f,t_i) < 0 \rightarrow P(f,(t_i,t_{i+1})) \in (0, 1) \qquad (2a)$$

$$P(f,t_i) = 1 \wedge P'(f,t_i) = 0 \rightarrow P(f,(t_i,t_{i+1})) \in (0, 1) \vee P(f,(t_i,t_{i+1})) = 1 \vee P(f,(t_i,t_{i+1})) \in (1, \infty) \qquad (2b)$$

$$P(f,t_i) = 1 \wedge P'(f,t_i) > 0 \rightarrow P(f,(t_i,t_{i+1})) \in (1, \infty) \qquad (2c)$$

(2a) and (2c) are valid due to lemma B. In the case of (2b), the second derivative would allow further discriminations. However, second derivatives are not considered[1]. Therefore, the ambiguities of (2b) cannot be removed.

Interval transitions:

$$P(f,(t_{i-1},t_i)) \in (0, 1) \rightarrow P(f,t_i) \in (0, 1) \vee P(f,t_i) = 1 \qquad (4)$$

$$P(f,(t_{i-1},t_i)) = 1 \rightarrow P(f,t_i) = 1 \qquad (5)$$

$$P(f,(t_{i-1},t_i)) \in (1, \infty) \rightarrow P(f,t_i) = 1 \vee P(f,t_i) \in (1, \infty) \qquad (6)$$

Under certain conditions the first derivative of P can also help to remove ambiguities of interval transitions. If during time interval (t_{i-1}, t_i) the distance to landmark 1 is increasing or remaining constant, landmark, 1, will not have been reached at time point t_i. Whether the distance is increasing or decreasing can easily be deduced from the signs of $P(f,(t_{i-1},t_i))$ and $P'(f,(t_{i-1},t_i))$. The different variants shall not be listed. Instead the two classes "distance is decreasing" and "distance is not decreasing" are used.

1. In fact, the first derivative, P´(f,t), is not modeled either. But information about P´(f,t) can be gained from PQ(f´,t), as we will see later.

$P(t_{i-1},t_i) \in (0, 1) \wedge$ "distance is decreasing" $\rightarrow P(f,t_i) \in (0, 1) \vee P(f,t_i) = 1$ (4a)

$P(t_{i-1},t_i) \in (0, 1) \wedge$ "distance is not decreasing" $\rightarrow P(f,t_i) \in (0, 1)$ (4b)

$P(t_{i-1},t_i) \in (1, \infty) \wedge$ "distance is decreasing" $\rightarrow P(f,t_i) = 1 \vee P(f,t_i) \in (1, \infty)$ (6a)

$P(t_{i-1},t_i) \in (1, \infty) \wedge$ "distance is not decreasing" $\rightarrow P(f,t_i) \in (1, \infty)$ (6b)

We still have to deal with the situation that $f(t)-l_a = f_{ref}(t)-l_a = 0$. For a time point t_i with $f(t_i) = fref(t_i) = l_a$, restrictions for following or preceding time intervals can be made if $f'(t_i) > f_{ref}'(t_i) > 0$ or $f_{ref}'(t_i) > f'(t_i) > 0$ or $f'(t_i) < f_{ref}'(t_i) < 0$ or $f_{ref}'(t_i) < f'(t_i) < 0$. In the following, only the first case will be treated. The others can be handled in an analogous way.

Lemma C: If for a time point t_i $f'(t_i) > f_{ref}'(t_i) > 0$ holds, then there exists an environment of t_i, (t_{i-1},t_{i+1}), $t_{i-1} < t_i < t_{i+1}$, with $f(t_{i-1},t_i) < f_{ref}(t_{i-1},t_i) < 0$, and $f(t_i,t_{i+1}) > f_{ref}(t_i,t_{i+1}) > 0$.

Proof C: Let x be some value in $(f_{ref}'(t_i),f'(t_i))$, i.e. $f_{ref}'(t_i) < x < f'(t_i)$. Proof A states, that there exists an environment (t_{i-1},t_{i+1}) of t_i with $f_{ref}'(t) < x < f'(t)$, $t \in (t_{i-1},t_{i+1})$. That means, $f(t)$ increases faster than $f_{ref}(t)$ in the time interval (t_{i-1},t_{i+1}). Therefore, $f(t_{i-1},t_i) < f_{ref}(t_{i-1},t_i) < 0$, and $f(t_i,t_{i+1}) > f_{ref}(t_i,t_{i+1}) > 0$. ❑

Remark: If $f'(t_i) = f_{ref}'(t_i) = 0$, all possibilities for $f(t)$ and $f_{ref}(t)$ in the intervals (t_{i-1},t_i) and (t_i,t_{i+1}) have to be taken into account. These are: $f(t) < f_{ref}(t) < 0$, $f(t) > f_{ref}(t) > 0$, $f_{ref}(t) < f(t) < 0$, $f_{ref}(t) > f(t) > 0$, and $f(t) = f_{ref}(t) = 0$. Discontinuous deviations are not considered.

Now the basic proofs for PQ transitions have been given. Below, we state for each transition from which propositions the proof follows. PQ transitions refer to the parameter descriptions PQ(f,t), PQ(f´,t), sign(f,t), and sign(f´,t). Since our considerations above refer to P(f,t), P´(f,t), f(t), and $f_{ref}(t)$, one has to map from PQ values and signs to P values. For P´(f,t) this is not as obvious as for P(f,t). Therefore Table 9 gives some help.

Table 9: Mapping from PQ values and signs to the derivative of the corresponding P value

PQ(f,t)	PQ(f´,t)	sign(f(t))	sign(f´(t))	P´(f,t)
L	L	$- \vee +$	$- \vee +$	?
L	$N \vee H$	-	-	> 0
L	$N \vee H$	-	+	< 0
L	$N \vee H$	+	-	< 0
L	$N \vee H$	+	+	> 0

Table 9: Mapping from PQ values and signs to the derivative of the corresponding P value

PQ(f,t)	PQ(f´,t)	sign(f(t))	sign(f´(t))	P´(f,t)
N	L	-	-	< 0
N	L	+	-	> 0
N	L	-	+	> 0
N	L	+	+	< 0
L ∨ H	N	- ∨ +	0	= 0
N	L ∨ N ∨ H	0	- ∨ +	not defined
N	N	0	0	not defined
N	N	- ∨ +	- ∨ 0 ∨ +	= 0
N	H	-	-	> 0
N	H	+	-	< 0
N	H	-	+	< 0
N	H	+	+	> 0
H	L ∨ N	-	-	< 0
H	L ∨ N	-	+	> 0
H	L ∨ N	+	-	> 0
H	L ∨ N	+	+	< 0
H	H	- ∨ +	- ∨ +	?

Point Transitions

Point transition PDL follows from (1).

Point transition PDH follows from (3).

Point transition PALHDN follows from (2).

Point transition PAL follows from (1).

Point transition PAH follows from (3).

Point transition PANDLH1 follows for PQ(f´(t_i) = L and sign(f(t_i)) ≠ 0 from (2a),
for sign(f(t_i)) = 0 from Lemma C,
for PQ(f´(t_i) = H from (2c).

Point transition PANDLH2 follows for $PQ(f'(t_i) = H$ and $sign(f(t_i)) \neq 0$ from (2a),
for $sign(f(t_i)) = 0$ from Lemma C,
for $PQ(f'(t_i) = L$ from (2c).

Point transitions PANDN* describe the case, where a continuously differentiable function has a landmark value at a certain time point and it is not known in which direction it will move. Therefore, all possibilities must be taken into account. However, some constraints between the future values of the function and its derivative exist. The PANDN* transitions constitute an interface to discontinuous deviations and have to be extended for an integration of them.

Interval Transitions

Interval transition IDL follows from (4).

Interval transition IDN follows from (5).

Interval transition IDH follows from (6).

Interval transitions IAL, IAN and IAH refer to a constant function and constant reference function. Therefore, a deviation will be constant too.

The remaining interval transitions distinguish between an ending of the interval with or without a synchronization:

1. $Duration(t_i,t_j) = L$ means that due to the synchronization, the deviating value is compared to a reference value that corresponds to a later time point than normally.
2. $Duration(t_i,t_j) = N$ means that no synchronization takes place.
3. $Duration(t_i,t_j) = H$ means that due to the synchronization, the deviating value is compared to a reference value that corresponds to an earlier time point than normally.

If no synchronization takes place, the interval transitions directly follow from propositions (5), (4a), (4b), (6a), or (6b). Otherwise considerations of the following kind are necessary:

If $sign(f,(t_i,t_j]) = +$, and $sign(f',(t_i,t_j]) = +$, then
$P^*(f,t_j) = (f(t_j) - l_a) / (f_{ref}(t_j - \Delta t) - l_a) > P(f,t_j) = (f(t_j) - l_a) / (f_{ref}(t_j - \Delta t) - l_a)$.

That is, a synchronization with too high duration will increase the P value of an increasing, positive parameter. So, if its P value is N or H without synchronization, it will be H. But if its P value is L, the effect of the synchronization is ambiguous. Analogous considerations can be made for the other configurations of signs and synchronization type. In the following, we state on which propositions the remaining transitions are grounded, if no synchronization takes place.

Interval transitions IANDN1 and IANDN2 follow from (5).

Interval transitions IALDLN follows from (4b).

Interval transitions IALDNH follows from (4b).

Interval transitions IAHDLN follows from (6b).

Interval transitions IAHDNH follows from (6b).

Interval transitions IALDH follows from (4a).

Interval transitions IAHDL follows from (6a).

Interval transitions IALDL follows from (4a).

Interval transitions IAHDH follows from (6a).

B: Proofs of P Transition Rules

In Appendix B, the P transition rules, P== and I= are proven to be sound. While the proof of P== involves the solution of a differential equation, I= can be proven analogously to (5) of Appendix A.

Proof of P==

P== states that from

$$(P(f,t_i) = P(f',t_i) = P(f'',t_i) \wedge \text{sign}(f',t_i) \in \{-,+\}) \vee (P(f,t_i) = P(f'',t_i) \wedge \text{sign}(f',t_i) = 0) \quad (1)$$

follows

$$P(f,(t_i,t_j)) = P(f',(t_i,t_j)) = P(f'',(t_i,t_j)) \vee P(f',(t_i,t_j)) \neq P(f,(t_i,t_j)) \neq P(f'',(t_i,t_j)) \quad (2)$$

<u>Proof</u>:

In accordance with RSIM, the proof assumes a basis landmark of 0.

Since from $(a \Leftrightarrow b)$ follows $((a \wedge b) \vee (\neg a \wedge \neg b))$, it suffices to show that from (1) follows

$$P(f,(t_i,t_j)) = P(f',(t_i,t_j)) \Leftrightarrow P(f,(t_i,t_j)) = P(f'',(t_i,t_j)).$$

<u>First direction</u>: We have to show that from (1) and $P(f,(t_i,t_j)) = P(f',(t_i,t_j))$ follows $P(f,(t_i,t_j)) = P(f'',(t_i,t_j))$. For this direction can renounce (1).

$$P(f,t) = P(f',t), \quad t \in (t_i,t_j)$$

$$\Leftrightarrow \; f(t) / f_{ref}(t) = f'(t) / f'_{ref}(t), \quad t \in (t_i,t_j)$$

$$P'(f,t) = (f'(t) * f_{ref}(t) - f(t) * f'_{ref}(t)) / (f_{ref}(t))^2, \quad t \in (t_i,t_j)$$

$$\Rightarrow \; P'(f,t) = 0, \quad t \in (t_i,t_j) \quad (3)$$

$$f(t) = P(f,t) * f_{ref}(t)$$

$$\Rightarrow \; f'(t) = P'(f,t) * f_{ref}(t) + P(f,t) * f_{ref}'(t)$$

Because of (3), the following holds

$$f'(t) = P(f,t) * f_{ref}'(t), \quad t \in (t_i,t_j)$$

$$\Rightarrow \; f''(t) = P'(f,t) * f_{ref}'(t) + P(f,t) * f_{ref}''(t) = P(f,t) * f_{ref}''(t), \quad t \in (t_i,t_j)$$

$$\Rightarrow \; P(f,t) = P''(f,t), \quad t \in (t_i,t_j)$$ ❑

Second direction: We have to show that from (1) and $P(f,(t_i,t_j)) = P(f'',(t_i,t_j))$ follows $P(f,(t_i,t_j)) = P(f',(t_i,t_j))$.

$$P(f,t) = f(t) / f_{ref}(t)$$
$$\Leftrightarrow \quad f(t) = P(f,t) * f_{ref}(t)$$
$$\Rightarrow \quad f'(t) = P'(f,t) * f_{ref}(t) + P(f,t) * f_{ref}'(t) \qquad (4)$$
$$\Rightarrow \quad f''(t) = P''(f,t) * f_{ref}(t) + P'(f,t) * f_{ref}'(t) + P'(f,t) * f_{ref}'(t) + P(f,t) * f_{ref}''(t) \qquad (5)$$

$$P(f,(t_i,t_j)) = P''(f,(t_i,t_j)), \quad t \in [t_i,t_j)$$
$$\Rightarrow \quad f''(t) = P(f,t) * f_{ref}''(t), \quad t \in [t_i,t_j) \qquad (6)$$

From (5) and (6) follows

$$0 = P''(f,t) * f_{ref}(t) + 2* P'(f,t) * f_{ref}'(t), \quad t \in [t_i,t_j)$$

Let $P'(f,t) = k(t)$:

$$0 = k'(t) * f_{ref}(t) + 2* k(t) * f_{ref}'(t), \quad t \in [t_i,t_j)$$
$$\Rightarrow \quad k'(t) + 2*f_{ref}'(t)/ f_{ref}(t) * k(t) = 0, \quad t \in [t_i,t_j)$$

The solution of this differential equation is (cf. Boyce):

$$k(t) = c / \mu(t), \quad \mu(t) = \exp \int^{t} 2\, f_{ref}'(s) / f_{ref}(s)\, ds \qquad (\text{ c is a constant})$$

$$\Rightarrow \quad \mu(t = \exp (2 \ln (f_{ref}(t))) = 1 / f_{ref}(t)^2, \quad t \in [t_i,t_j)$$
$$\Rightarrow \quad k(t) = c / f_{ref}(t)^2 = P'(f,t), \quad t \in [t_i,t_j) \qquad (7)$$

(1) states that either $P(f,t_i) = P(f',t_i)$ or $sign(f',t_i) = 0$ holds. Above we have seen that $P'(f,t_i) = 0$ follows from $P(f,t_i) = P(f',t_i)$. On the other hand, $sign(f',t_i) = 0$ implies that $sign(f_{ref}',t_i) = 0$, since we don't allow discontinuous deviations. From this follows $P'(f,t_i) = 0$ too, i.e. $P'(f,t_i) = 0$ holds in both cases. Thus from (7) follows:

$$c / f_{ref}(t_i)^2 = 0, \quad f_{ref}(t_i) \neq 0$$
$$\Rightarrow \quad c = 0$$
$$\Rightarrow \quad P'(f,t) = 0, \quad t \in [t_i,t_j) \qquad (8)$$

From (4) and (8) follows:

$$f'(t) = P(f,t) * f_{ref}'(t), \quad t \in [t_i,t_j)$$
$$\Rightarrow \quad P(f,(t_i,t_j)) = P'(f,(t_i,t_j)).\square$$

Proof of I=

I= only constrains the relationship between $P(f,t_j)$ and $P(f',t_j)$, if $f(t)$, $f_{ref}(t)$, $f'(t)$, and $f_{ref}'(t)$ all are unequal to 0. But for this case $P(f,t)$ and $P(f',t)$ are continuously differentiable. Therefore I= is an analogy to (5) of appendix A.❑

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A Simulator for Relative Descriptions[1]

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Abstract

This report deals with the qualitative simulation of physical systems based on descriptions relative to normal behavior. Relative descriptions are important because some kinds of system behavior cannot adequately be described by absolute descriptions. In particular, this is true for faulty behavior that is often viewed relative to the normal case. Most of the existing approaches use relative descriptions only to analyse static systems. In addition, a comparison of deviations in dynamic systems is not possible. In this report, a simulator, called RSIM, is presented, that predicts the effects of system deviations with a “less than normal” or “greater than normal” character. Aside from absolute descriptions, the deviations themselves and the resulting behavior are described with respect to the normal case, i.e. to a reference system and its reference behavior. The simulation of absolute behavior is carried out by a QSIM-like simulator, since the concepts of RSIM are oriented towards QSIM. RSIM can be viewed as an extension to QSIM.

In RSIM, deviations cannot only be described by “less than normal” or “greater than normal”, but additionally they can be compared with each other. In this way, a refined system description is achieved. Furthermore, some spurious behaviors are prevented on the relative description layer, and a more accurate prediction of behavior is possible.

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Contents

1. Introduction

The obvious attractiveness of qualitative simulations, their effectiveness, efficiency, and naturalness (compare [Struss 89]) has initiated a large amount of work in this field. The most influencing approaches are ENVISION [de Kleer, Brown 84], QPT [Forbus 84], and QSIM [Kuipers 86]. While relative descriptions have widely been used ([de Kleer 79], [Raiman 86], [Dague, Devès, Raiman 87], [Weld 87,88a,88b,90], [Downing 87], [Mavrovouniotis, Stephanopoulos 88], [Gallanti, Stefanini, Tomada 89], [Kockskämper, Neumann, et al. 93]), only few approaches deal with relative simulation of dynamical systems (e.g. [Weld 90])[1]. A relative simulation is valuable for system analysis and fault diagnosis. In this paper, concepts of relative simulation are described, and a relative simulator, called RSIM, is presented. RSIM's input and output, and some essential inference features are oriented towards QSIM. In fact, RSIM can be viewed as an extension to QSIM. Extending QSIM by RSIM leads to a refinement of input and output. In order to exploit existing QSIM techniques, RSIM is integrated into the system SLOD, which allows **s**imulation on different **l**ayers **o**f **d**escription. SLOD contains four description layers: sign and landmark descriptions, as QSIM, and two relative description layers belonging to RSIM.

The development of RSIM was initiated by the observation that there are kinds of faulty system behavior that cannot adequately be described by absolute descriptions. And even worse, a natural modeling of a faulty system and the corresponding correct system in some cases led to identical descriptions. Instead, terms like "less than normal" or "too high" characterized this sort of faulty behavior. Additionally, comparisons of too low or too high deviations turned out to be necessary. Therefore, we formalized these terms and developed an inference engine for them, the simulator RSIM. RSIM's modeling language offers a set of constraints that - due to relative descriptions - facilitates a more specialized system description than in usual simulators. Inferences about the relative duration of time intervals are carried out by RSIM's transition rules. Thus, the problems that were identified at the beginning of our work, are solved by the RSIM simulator.

The report is organized as follows: First we explain how qualitative simulation works and why relative descriptions are useful and necessary. Afterwards specific properties of simulation by relative values are discussed. In Chapter 3, the main part of the report, the relative simulator, RSIM, is presented. Finally some notes regarding future work are given. The appendix contains proofs of RSIM's transition rules.

1. [Neitzke 92a] gives an overview about relative descriptions in qualitative simulation.

1.1. How Does Qualitative Simulation Work?[1]

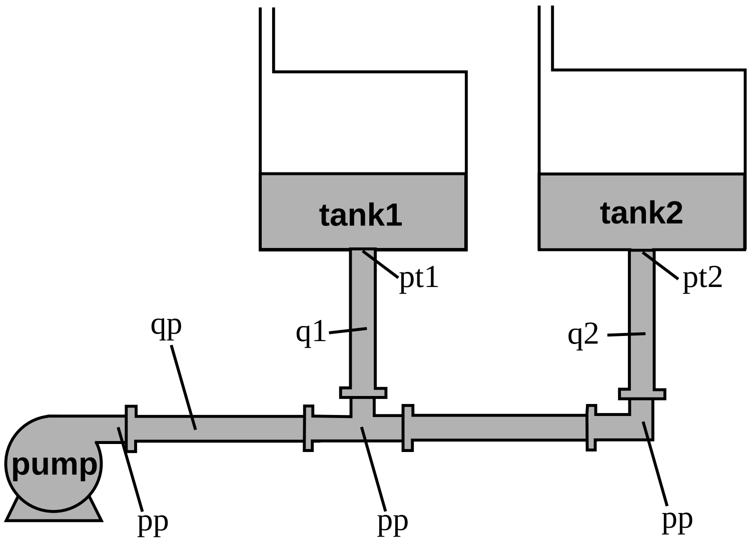
Qualitative simulation, as conventional numerical simulation, has the purpose to predict the behavior of a system. System behavior, however, is not described by exact numerical values but by qualitative descriptions. Qualitative descriptions have two characteristics: they are more general than quantitative descriptions and they are more imprecise. A qualitative system description does not concern a single system but a whole class of systems. Figure 1.1 shows a simple model for tank systems of two tanks and a pump. This and other tank systems will be used as examples throughout the report.

In a qualitative simulation, ambiguities arise in the prediction of behavior, so that in general not exactly one behavior is predicted, but a set of possible behaviors. One reason for ambiguities is that not all members of the simulated class of systems behave in the same way. In the 2-tank system above, for example, it cannot be decided which tank is full first. Another reason lies in the inexactness of qualitative descriptions. As a consequence of the latter not all of the predicted behaviors correspond to physically possible behavior. These impossible behaviors are called spurios behaviors. However, it can be guaranteed that every possible behavior is among the predicted behaviors.

While in a numerical simulation the state of a system is calculated at fixed equidistant time points, a qualitative simulation is guided by the occurrence of “interesting” events. In the kind of simulation we are concerned with, an interesting event occurs whenever a parameter reaches an interesting value (a so-called ***landmark value***). At each interesting event the system state, i.e. the qualitative values of all parameters, is recorded. Additionally, the system state is recorded for the time intervals lying between two events. So, the generation of behavior is a generation of a sequence, or - because of ambiguities - a tree of system states.

As input a simulator needs a system description and some information about its initial state, i.e. the values of some parameters. There are different ways to describe a system. We follow QSIM and describe a system by a set of constraints (compare Fig. 1.1a), i.e. a set of relations between the system parameters. The output of a simulation, the tree of system states, is generated in a cycle of two phases, called intrastate analysis and interstate analysis. Simulation starts with an intrastate analysis. In intrastate analysis, a set of completely described system states is derived from an incompletely specified system state. In QSIM-like simulators this is done by using the constraints of the system description (in a process called constraint propagation). For our example, an incomplete description of the initial state can be seen in Fig. 1.1b.

1. Excellent introductions to qualitative simulation are given by [Forbus 88] and [Struss 89]



(DERIV q1 vol1)	Flow *q1* is the derivative of volume of tank1 *vol1*.
(DERIV q2 vol2)	Flow *q2* is the derivative of volume of tank2 *vol2*.
(M+ pt1 vol1)	Pressure of tank1 *pt1* is an increasing function of *vol1*.
(M+ pt2 vol2)	Pressure of tank2 *pt2* is an increasing function of *vol2*.
(PRODUCT pf1 k1 q1)	"Friction pressure" of tank1 *pf1* is the product of friction coefficient *k1* and *q1*.
(PRODUCT pf2 k2 q2)	"Friction pressure" of tank2 *pf2* is the product of friction coefficient *k*2 and *q2*.
(SUM pp pt1 pf1)	The pressure of the pump *pp* is the sum of *pt*1 and *pf1*.
(SUM pp pt2 pf2)	The pressure of the pump *pp* is the sum of *pt2* and *pf2*.

Fig. 1.1a A simple model of a system of two tanks and a pump. For the sake of simplicity and reasons of demonstration, friction has only been modelled for vertical pipes and linearly dependent from flow.

vol1	**: 0**	Tank1 is empty.
vol2	**: 0**	Tank2 is empty.
pp	**: +**	Pressure of pump is positive.
k1	**: +**	Friction coefficient k1 is positive
k2	**: +**	Friction coefficient k2 is positive

Fig. 1.1b Initial information about the system's state.

In interstate analysis, an incomplete successor state is inferred from a complete system state. Interstate analysis works with continuity information about continuously differentiable functions. It is required that every system parameter is a continuously differentiable function of time. To determine the future course of a parameter, qualitative information about its derivative is needed. Therefore, a parameter is described by two numbers: its amount and its derivative. Figure 1.2 shows the tree of behaviors when the filling of the 2-tank system is simulated. The behavior tree is the result of a sign simulation, the simplest qualitative simula-

tion, working with just one landmark value, 0. Therefore, the only interesting event is reaching the landmark value 0.

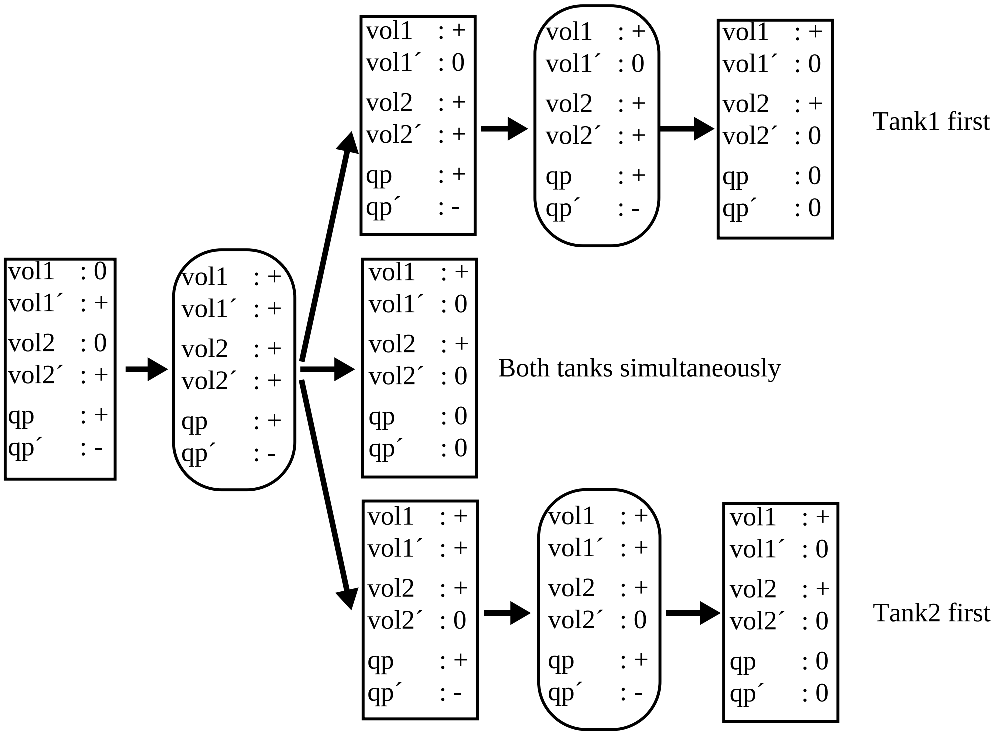


Fig. 1.2 Behavior tree for the 2-tanks system. For simplicity only three parameters have been listed. Time points are indicated by rectangles, time intervals by ovals.

1.2. Relative Descriptions and their Use[1]

1.2.1. Why Relative Descriptions?

There are kinds of system behavior that cannot adequately be described by absolute descriptions. Take for example a leaky water pipe. An adequate description of its behavior says that the output flow is less than the input flow. Or consider a pump that generates less pressure than in the normal case. The general meaning of “less” cannot be described by assigning absolute values, like signs or intervals between landmark values, to the parameters involved.

1.2.2. What are Relative Descriptions Used for?

Relative descriptions can be used for many different purposes in qualitative simulation. Our aim in using relative descriptions is to describe deviations with respect to a reference system and reference behavior. The deviations we are interested in are those that can be described as less than normal or greater than normal. Such descriptions can be used to characterize the

1. Section 1.2 overlaps with [Neitzke 92b] and [Neitzke 92c].

values of system parameters in a certain system state. Furthermore, they can be used to characterize the duration of a process taking place in a system. Before going into details, we have to take a closer look at the different types of deviations.

1.2.3. Types of Deviations

What are the characteristic features of deviations that can be described as less or greater than normal? Of course, not all deviations can be described in this way. Consider, for example, water in a pipe, that is flowing in the wrong direction, or an electrical charge that is positive, but normally negative. One property of less/greater-than-normal (LGTN) deviations is that they can be arbitrarily small. Weld calls arbitrarily small changes in the value of a parameter ***differential*** and more drastic changes ***non-differential***. The essential property of a differential change is that it could be arbitrarily smaller without falling into a qualitatively different area, i.e. no landmark value may lie between the deviating value and the normal value. Following Weld we call deviations that lie in the same qualitative area, i.e. have the same absolute description, as the normal value ***differential*** and deviations that lie outside this area ***non-differential***.

Unfortunately, the definition of differential does not capture all deviations that are called less or greater than normal. The voltage between the ports of a diode, for example, could be characterized as less than normal even if it is less than the diode's threshold voltage. Instead, expressions like less or greater describe deviations in the distance to a reference point, which we want to call ***basis landmark***[1]. If the deviating value lies on or beyond the basis landmark, as in the examples above, one cannot use the terms less or greater than normal. In most of all cases, the suitable basis landmark is given by the real number 0. But as the *quantity space* of {-, 0, +} in general corresponds to the regions $x < a$, $x = a$, and $x > a$, (where a is an arbitrary landmark value in the range of the parameter x (compare for example de Kleer)) the basis landmark may in principal be different from 0. We only require that basis landmark and landmark value, a, must be identical.

Definition: The deviation of a value v_1 from a value v_2 is called a ***continuous*** deviation with regard to a basis landmark l_a if v_1 is on the same side of l_a as v_2, that is $(v_1 < l_a \wedge v_2 < l_a) \vee (v_1 > l_a \wedge v_2 > l_a)$. Otherwise it is called a ***discontinuous*** deviation wrt l_a (Fig. 1.3).

1. The basis landmark may not be confused with the normal value, which we often call reference value.

Differential and continuous deviations are very similar. For quantity spaces with only one landmark like {-, 0, +} both classes are identical. In general, each differential deviation is a continuous deviation and each discontinuous deviation is a non-differential deviation.

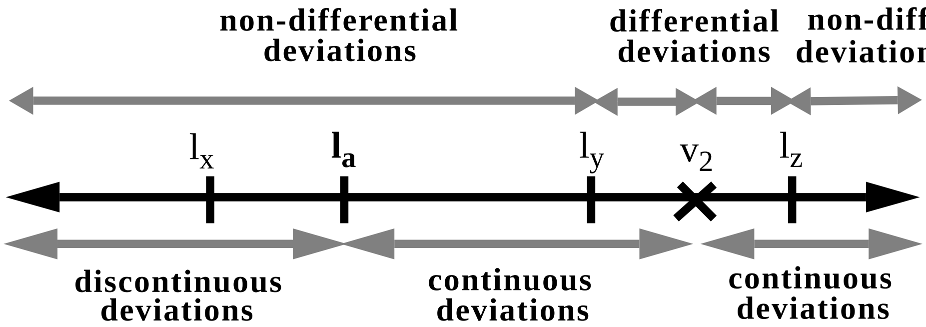


Fig. 1.3 Differential, non-differential, continuous and discontinuous deviations wrt l_a from a value v_2.

1.2.4. A Relative Description that allows Comparison of Deviations

The way and the extent a value v_1 differs from a normal value v_2 can be described by the quotient of the distances of v_1 and v_2 from the basis landmark. We call this quotient P-value.

Definition: $$P(f,t) = \frac{f(t) - l_a}{f_{ref}(t) - l_a}$$

f is a continuously differentiable function of time, f_{ref} is the corresponding reference function, l_a is f’s basis landmark in the range of f. Deviations from a normal value that lie beyond l_a cannot be described as too-low or too-high.

We call the values of the function P ***P values***. Note, that P values are relative, but not qualitative. However, we are only interested in qualitative properties of P values. Therefore, we don’t need exact values for the function P. The following qualitative areas of P values are relevant for us:

$P(f,t) > 1$ The distance of the deviating value to the basis landmark is greater than the normal value’s distance. Usually, such deviations are called “too high”.

$P(f,t) = 1$ There is no deviation.

$0 < P(f,t) < 1$ The distance of the deviating value to the basis landmark is less than the normal value’s distance. Usually, such deviations are called “too low”.

$P(f,t) = 0$ The deviating value lies on the basis landmark. This deviation is discontinuous. It can be expressed by absolute descriptions.

P(f,t) < 0 — Deviating value and normal value lie on different sides of the basis landmark. This deviation is discontinuous. It can be expressed by absolute descriptions.

Now we want to define, what we exactly mean by “too low”, “normal” and “too high” by the function PQ. We call the values of PQ ***PQ values***.

PQ(f,t) = *too-low* ::= $0 < P(f,t) < 1$

PQ(f,t) = *normal* ::= $P(f,t) = 1 \vee f(t) - l_a = f_{ref}(t) - l_a = 0$

PQ(f,t) = *too-high* ::= $P(f,t) > 1$

As an important extension of the above mentioned qualitative descriptions that are based on fixed intervals, we introduce comparisons of P values. For example, the P value of the output flow of a leaky pipe is always less than the P value of the input flow (provided that there is a flow). Besides the relations <, =, > the reciprocal relation can be useful too because if the product of a *too-low* value and a *too-high* value results in a *normal* value, the P value of the first factor is the reciprocal of the second. Comparison of P values gives us a handle to refine qualitative system descriptions and behaviors, to distinguish more specific classes of constraints between system parameters, and to diminish some unwanted spurious behaviors.

These two qualitative descriptions, PQ values and relations between P values, shall be the basis for our relative simulation. Before we come to the corresponding inference machine, the relative simulator RSIM, we want to examine the properties of a relative simulation in general.

2. Simulating with Relative Descriptions

As we have seen, a qualitative simulator produces a behavior description from a system description and information about the systems initial state. A relative simulator, as a special qualitative simulator, works the same way: As input it receives a system description that contains the deviations from a reference system, and as output it predicts the possible system behaviors, using descriptions that contain the deviations from the reference behavior. Of course, the input of our relative simulator must have this LGTN character we are interested in. But does it always follow from such input that an output has this character too?

2.1. Describing Deviations from a Reference System

The kind of qualitative simulators we deal with (the QSIM-type) gets as input a system description consisting of constraints between the system parameters (Fig. 1.1a) as well as information about the initial state of the system in form of some parameter values (Fig. 1.1b). We can distinguish three kinds of deviations from the reference system with a LGTN character:

1. Differential deviations of initial parameter values
2. Continuous deviations of initial parameter values
3. Deviations in the system description

2.1.1. Differential Deviations of Initial Parameter Values

Differential deviations cannot be expressed with absolute descriptions by definition, i.e. the absolute description of deviating parameters does not change. But, if there is no difference in the input concerning absolute descriptions, there cannot be any difference in the output. That is, the set of predicted behaviors is identical for the deviating system and for the reference system as long as one focusses on absolute descriptions. This does not mean that a differential deviation in the initial state cannot lead to changes of the absolute behavior of a system. But both, the reference behavior and the deviating behavior must be elements of the common set of absolute behaviors. Consider the 2-tank system of Fig. 1.1. If the friction coefficient, *k2*, of tank2 is higher than normal, then filling tank2 takes more time, and therefore, the order of interesting events may change. How are the three possible behaviors of the reference system (Fig. 1.2) affected, if the friction coefficient of tank2 is too high?

First behavior (tank1 is full at first): The volume of tank2 will be too low when tank1 is full. However, the absolute description does not change: At first tank1 will be full and afterwards tank2.

Second behavior (both tanks are full simultaneously): Tank1 will be the first. The deviating behavior is characterized by the first behavior. The absolute description of

behavior has changed. Weld calls this a change in the ***behavioral topology*** [Weld 87,88b,90].

Third behavior (tank2 is full at first): It cannot be decided whether the deviation of the friction coefficient is so heavy that now tank1 is the first to be full, or whether both tanks are full simultaneously, or whether tank2 still is the first. Thus each behavior of the tree may be a consequence of the deviation. That is, there exist solutions with and without a change in the behavioral topology.

2.1.2. Continuous Deviations of Initial Parameter Values

In contrast to differential deviations, continuous deviations allow changes of absolute descriptions. Therefore, the resulting absolute behavior can change too. Continuous deviations imply that the basis landmark of the deviating parameter is not reached. Since we require the identity of basis landmark and the landmark value defining the sign quantity space, continuous deviations cannot take place on signs. So a projection of absolute descriptions onto signs leads to identical behavior sets for the deviating and the reference system. That is, in principle one can transform continuous deviations into differential deviations by leaving out some landmark. However, there are situations where it is not adequate to do this. Suppose, we want to model the value of tank pressure, at the moment when the valve of a tank closes, as a special landmark value in the tank pressure quantity space. There are situations, where a pressure above, at, and below this landmark value must be considered as too high. Fig. 2.1 shows another example of a continuous deviation. The deviating behavior of Fig. 2.1a and the corresponding reference behavior of Fig. 2.1b describe the emptying of a tank. The quantity space of the tank level contains three landmark values, *0*, *half-full* and *full*. In the reference behavior, the initial value of the tank level is *half-full*, while in the deviating behavior it is *full*, i.e. the tank level is too high. The topology of the behaviors is different, but the deviations are continuous, because all amounts and derivatives keep their signs. (The problems concerning the different duration of the behaviors are discussed below.)

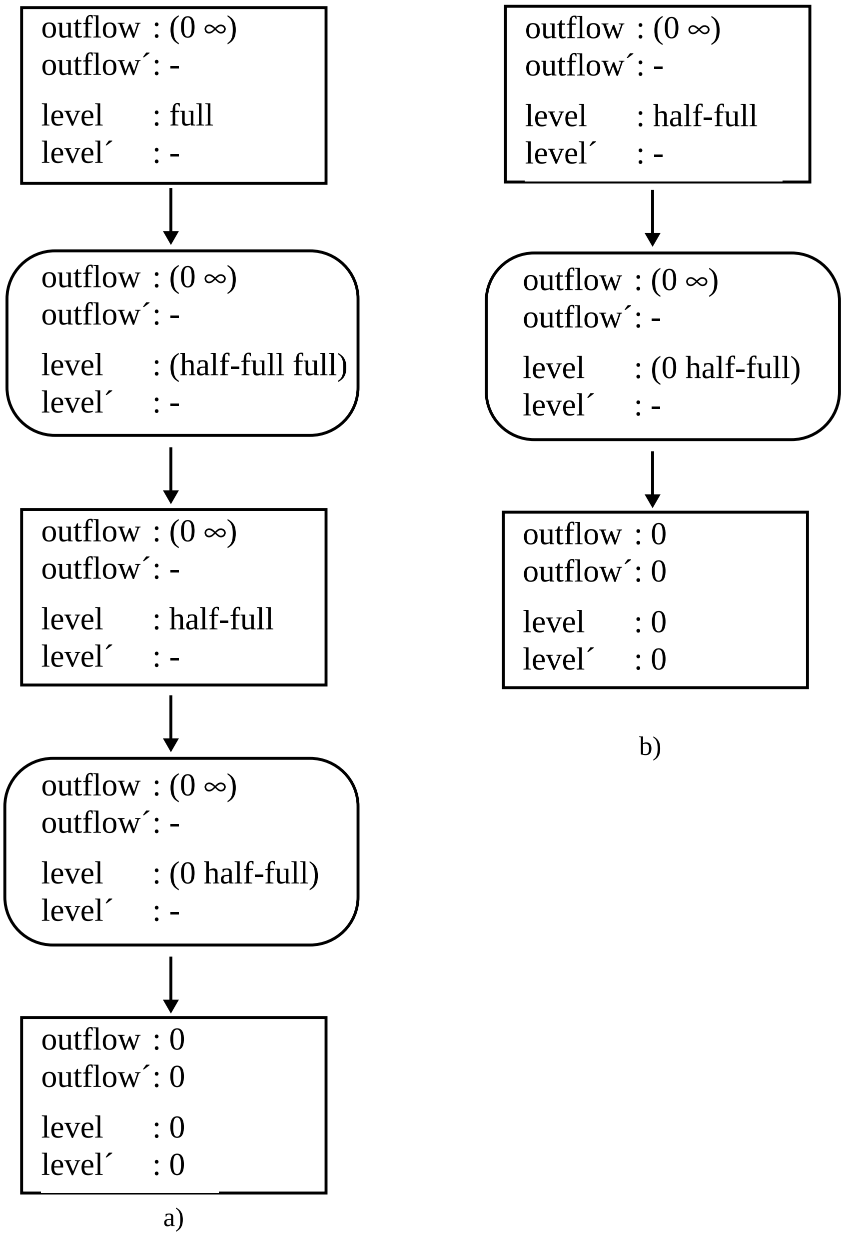


Fig. 2.1 Behaviors for emptying a tank. a) Deviating behavior
b) Reference behavior

2.1.3. Deviations in the System Descriptions

In general, changes in the system descriptions lead to plainly different behaviors, so that a comparison via less/greater descriptions is not possible. However, there exist some changes that have no significance on absolute descriptions but on relative descriptions. Fig. 2.2 shows the constraints used by the original QSIM. While ADD, MULT, MINUS and DERIV are "exact" constraints, M^+, M_0^+, M^-, and M_0^- have a qualitative character. They each refer to a

whole class of functional relationships. It would bring no advantages for absolute descriptions to distinguish between subclasses of M*, because the qualitative definitions of the corresponding constraints would be identical. The definition of P values, however, makes it possible to differentiate between some subclasses. On the one hand one can distinguish

ADD(f,g,h)	$:\Leftrightarrow f(t) + g(t) = h(t)$
MULT(f,g,h)	$:\Leftrightarrow f(t) * g(t) = h(t)$
MINUS(f,g)	$:\Leftrightarrow f(t) = - g(t)$
DERIV(f,g)	$:\Leftrightarrow f'(t) = g(t)$
$\mathbf{M^+(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0$
$\mathbf{M_0^+(f,g)}$	$:\Leftrightarrow M^+(f,g) \wedge H(0) = 0$
$\mathbf{M^-(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0$
$\mathbf{M_0^-(f,g)}$	$:\Leftrightarrow M^-(f,g) \wedge H(0) = 0$

Fig. 2.2 Constraints of QSIM. The description has been adopted from [Struss 89]

between linear, overlinear, and underlinear relationships. This feature will be discussed below. On the other hand one can express faulty relationships, for example a M_0^+ relationship with a greater-than-normal gradient. Fig. 2.3 shows the sign and PQ tupels of the standard M_0^+

M_0^+

Sign tupels: (- -)
(0 0)
(+ +)

PQ tupels:

If sign tupel is (- -) or (+ +):
(too-low too-low)
(normal normal)
(too-high too-high)

If sign tupel is (0 0):
(normal normal)

M_0^+TOO-STEEP

Sign tupels: (- -)
(0 0)
(+ +)

PQ tupels:

If sign tupel is (- -) or (+ +):
(too-low too-low)
(normal too-low)
(too-high too-low)
(too-high normal)
(too-high too-high)

If sign tupel is (0 0):
(normal normal)

Fig. 2.3 Sign and PQ tupels of the standard M_0^+ constraint and the M_0^+TOO-STEEP constraint. The sign tupels of both are identical.

constraint and a M_0^+ constraint with too-high gradient, called M_0^+TOO-STEEP. What are the consequences of such relative deviations in the system description? First, the set of absolute behaviors does not change, since the absolute system description does not change. And second, a change of the behavioral topology is possible. If, for example, two parameters *x* and *y* are related by a M_0^+TOO-STEEP relationship, (M_0^+TOO-STEEP *x y*), and normally reach

certain landmark values at the same time, then in the deviating system, *x* will reach its landmark value before *y*.

2.2. Comparison of Behaviors

In the last chapter, we have seen that under deviations with a LGTN character in the system description or the initial conditions, the set of possible absolute behaviors, or at least the set of possible sign behaviors, does not change. Additionally, we have seen that changes of the behavioral topology are possible. So, when a deviating behavior is compared to a reference behavior, two kinds of comparisons can be distinguished: comparisons with and without a change in the behavioral topology. Let us first consider the easier case:

2.2.1. Comparisons without a Change in the Behavioral Topology

No change in the behavioral topology means no change in the absolute behavior of the system. That is, for every state of the path of deviating behavior there exists a corresponding state in the reference behavior with identical absolute descriptions, and vice versa. All possible deviations are differential: parameter values of certain states may be too low or too high (or normal), and the duration of an interval state may be too low or too high (or normal) too[1]. The comparison of behaviors is a comparison of each state of the deviating behavior with the corresponding state of the reference behavior.

The comparison of point states is simple. For each parameter it must be stated, whether its amount and derivative are too low, normal or too high with respect to the reference state. The comparison of interval states is more complicated. Since the duration of an interval may change, either the deviating interval or the reference interval will end before the other. So a comparison with respect to time cannot concern both intervals in full length, unless the rest of the longer interval would be compared to states following the shorter interval. To cope with this difficulty, Weld has introduced the concept of ***perspective*** [Weld 87,88b,90]. A comparison of the values of a parameter x under the perspective of a parameter y means that those values of x are compared with each other that belong to the same values of y. Using appropriate parameters as perspectives often leads to simpler descriptions than using the standard perspective of time. Weld shows that relevant statements about the duration of an interval and the magnitude of values compared to reference values are possible through working with different perspectives. The corresponding method, Weld's ***DQ analysis***, determines analytically and not by simulation the consequences of differential deviations. The concept of perspective facilitates elegant and effective descriptions of differential deviations. In this paper, we show that a simulation assuming the standard case of time as perspective yields these results too, without having the problem of incompleteness of DQ analysis (see [Weld 90]). Additionally, due to higher exactness that is gained by relations between P values, some ambiguities can be avoided and more problems can be solved.

1. Deviations of the duration of an interval must be differential, because the duration is always positive and further absolute information is not given.

2.2.2. Comparisons with Changes in the Behavioral Topology

If the behavioral topology changes, the deviations become non-differential. However, LGTN descriptions still can be used, if the deviations are continuous. So one has to distinguish between continuous and discontinuous changes of the behavioral topology. An example of continuous, non-differential changes is given by Fig. 2.1a. Continuous changes of the behavioral topology can be handled in a similar way as differential changes of behavior (dealed with in the previous section) because the sign behavior does not change. Differences are related to the fact, that the mapping between the states of the deviating and the reference behavior in contrast to differential changes is not bijective (because of their different number). Further details will be explained in Section 3.5. In the following we will focus on discontinuous changes.

Discontinuous Changes of the Behavioral Topology

Discontinuous changes of the behavioral topology occur if the deviating and the reference behavior correspond to different paths of the behavior tree. The point where deviating and reference behavior split is a point of ambiguities. In principle at each branching of the behavior tree the deviating and the reference behavior can split. There are two types of branchings and correspondingly two types of topology changes. One reason for a branching is that it cannot be decided which event out of a number of possible ones will occur first. Every order of events must be taken into account. This type of branching is called ***occurrence branching*** (see e.g. [Fouché,Kuipers 91]). The other reason for a branching is that it cannot be decided in which direction the amount or derivative of a parameter moves. In this case, all possible developments, i.e. increase, decrease, or constancy, must be considered.

In the case of discontinuous topology changes, a sensible mapping between deviating and reference system states in general is not possible. But if the topology changes because of occurrence branching, parameter histories[1] still can be compared. In occurrence branching, the order of parameter-specific events changes, but the absolute descriptions of parameter histories themselves do not. The behavior tree of Fig. 1.2 is an example for occurrence branching.

But if the reason for branching is, that the further course of a parameter cannot be determined, the parameter history itself can change. For example, this type of branching happens, if in a system of coupled tanks it cannot be decided in which direction the water will flow. Now, the last chance of comparisons via LGTN descriptions is restricted to parameter histories that have not changed. But the essential characteristic of the deviation, the change of some parameter histories, cannot be described by this means.

1. A parameter history is the qualitative course of a parameter value.

2.3. Summary

If a deviating system shall be simulated, the deviations from the reference system can be expressed by changing the description of the reference system, i.e. the model, or by changing the initial parameter values of the reference system. Model changes with a LGTN character are restricted to the M* constraints. Deviations of the initial parameter values are subdivided into differential and continuous deviations. Differential deviations of the initial parameter values may cause changes of the behavioral topology, but the set of predicted behaviors does not change with respect to absolute descriptions. Continuous deviations, on the other hand, may cause changes of the set of absolute behaviors.

If a deviating behavior has to be compared to a reference behavior, the comparison may or may not have to deal with changes of the behavioral topology. If there is no change in the behavioral topology, a state-wise comparison is possible. The comparison has to determine for each state whether the parameter values are too low, normal, or too high, and it has to determine for each interval state whether the duration of the interval is too low, normal, or too high. Changes of the behavioral topology are subdivided into continuous and discontinuous changes. Continuous changes of the behavioral topology can be handled in a similar way as if no changes of the topology exist. Concerning discontinuous changes of topology, one has to distinguish between changes because of occurrence branching and changes because of ambiguities in the further course of a parameter. If the topology changes because of occurrence branching, LGTN description have to be restricted to parameter histories. For the other type of topology changes, no sensible comparison via LGTN descriptions is possible.

3. The Relative Simulator RSIM

In this chapter the relative simulator RSIM is presented. RSIM's input and output, and some essential inference features are oriented towards QSIM. In fact, RSIM can be viewed as an extension to QSIM. Extending QSIM by RSIM leads to a refinement of input and output. In order to exploit existing QSIM techniques, RSIM is integrated into a system, SLOD, which allows **s**imulation on different **l**ayers **o**f **d**escription. SLOD contains four description layers: sign and landmark descriptions, as QSIM, and two relative description layers belonging to RSIM.

In the following, the concepts of RSIM are explained. First, we will deal with system descriptions and state descriptions in RSIM. After that the two essential parts of a qualitative simulation, intrastate and interstate analysis, are explained. Finally we give some examples of output. But before we come to the concepts of RSIM, some words about SLOD have to be said.

3.1. Simulating on Different Layers of Description

In qualitative simulation, a parameter is usually described by two numbers: its amount and its derivative. The derivative carries dynamical information so that the future course of the amount can be computed. In SLOD, the different ways of describing a number qualitatively are represented in description layers. SLOD has four description layers: a coarse and a fine absolute layer, and a coarse and a fine relative layer.[1] SLOD's description layers are listed below.

<u>Absolute layers</u>

Sign layer: The simplest qualitative information about a real number is its sign. The sign layer is the basic description layer for the other layers.

Qval layer: On the qval layer the real number line or a part of it is divided into regions of interest by so-called landmark values. The landmark values themselves and the open intervals between them form a quantity space. The sign layer can be seen as a special qval layer with landmark value 0. In contrast to the other description layers, on the qval layer only the amount of a parameter is described, but not its derivative.[2]

1. In principal, more description layers can be added.
2. The name *qval* has been taken from QSIM since the qvals in SLOD have the same properties as in QSIM (totally ordered set of landmark values, dynamical generation of landmarks possible).

Relative description layers (belonging to RSIM):

PQ-layer: The PQ layer works with three qualitative values: *too-low, normal*, *too-high*. The semantics of PQ values has been explained in Section 1.2.4.

P-layer: On the P layer, relations between P values are collected. So far, RSIM deals with the relations *less*, *greater* and *equal*. In some few cases the reciprocal relation would be a helpful extension.

A description layer can be activated or deactivated. Therefore, depending on the status of description layers, simulation can produce different outputs. In the simplest case a pure sign simulation takes place. However, some dependencies between the description layers have to be taken into account (Fig. 3.1). The sign layer is required by all the other description layers, so it must not be deactivated. Besides that, the P layer needs the PQ layer.

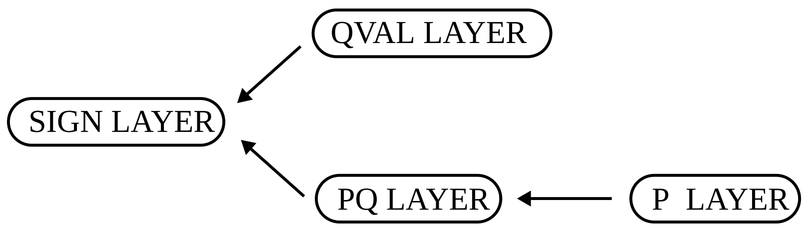


Fig. 3.1 Dependencies between description layers.

Each description layer performs its own intrastate and interstate analysis. The techniques of sign and qval layer are similar to existing simulators, in particular to QSIM, and therefore are not discussed further on.

3.2. System Description in RSIM

As in QSIM, a system is described in RSIM by a number of constraints. Aside from QSIM's constraint classes (Fig. 2.2) some specializations of the M* constraints are possible (Fig. 3.2). Two types of specializations can be distinguished. On one hand a faulty M* relationship can be expressed by the M*TOO-FLAT and M*TOO-STEEP constraints. They concern an M* relationship that is too flat or too steep on every point. For example, if the breather tube of a tank is clogged (the tanks of Fig. 1.1a and Fig. 3.3 have breather tubes), the air in the tank is compressed when filling the tank. From this follows, that the static pressure corresponding to a certain volume of water is greater than normal. The relationship between static pressure and volume level is M^{+}_{0}TOO-STEEP. A model of a tank with a clogged breather tube can be seen in Fig. 3.3.

On the other hand, an M* relationship can be stated more precisely by using the LINEAR*, UNDERLINEAR* or OVERLINEAR* constraints. The specializations of M* constraints have different definitions only on relative description layers. On absolute layers they have identical definitions. Under the assumption of straight tank sides, the relationship between static pressure and volume in a correct tank is $LINEAR^{+}_{0}$.

ADD(f,g,h)	$:\Leftrightarrow f(t) + g(t) = h(t)$
MULT(f,g,h)	$:\Leftrightarrow f(t) * g(t) = h(t)$
MINUS(f,g)	$:\Leftrightarrow f(t) = - g(t)$
DERIV(f,g)	$:\Leftrightarrow f'(t) = g(t)$
$\mathbf{M^{+}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge PQ(H'(x)) = \text{normal}$
$\mathbf{M^{+}TOO\text{-}FLAT(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge PQ(H'(x)) = \text{too-low}$
$\mathbf{M^{+}TOO\text{-}STEEP(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge PQ(H'(x)) = \text{too-high}$
$\mathbf{M_0^{+}(f,g)}$	$:\Leftrightarrow M^{+}(f,g) \wedge H(0) = 0$
$\mathbf{M_0^{+}TOO\text{-}FLAT(f,g)}$	$:\Leftrightarrow M^{+}TOO\text{-}FLAT(f,g) \wedge H(0) = 0$
$\mathbf{M_0^{+}TOO\text{-}STEEP(f,g)}$	$:\Leftrightarrow M^{+}TOO\text{-}STEEP(f,g) \wedge H(0) = 0$
$\mathbf{M^{-}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge PQ(H'(x)) = \text{normal}$
$\mathbf{M^{-}TOO\text{-}FLAT(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge PQ(H'(x)) = \text{too-low}$
$\mathbf{M^{-}TOO\text{-}STEEP(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge PQ(H'(x)) = \text{too-high}$
$\mathbf{M_0^{-}(f,g)}$	$:\Leftrightarrow M^{-}(f,g) \wedge H(0) = 0$
$\mathbf{M_0^{-}TOO\text{-}FLAT(f,g)}$	$:\Leftrightarrow M^{-}TOO\text{-}FLAT(f,g) \wedge H(0) = 0$
$\mathbf{M_0^{-}TOO\text{-}STEEP(f,g)}$	$:\Leftrightarrow M^{-}TOO\text{-}STEEP(f,g) \wedge H(0) = 0$
$\mathbf{LINEAR^{+}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge H''(x) = 0$
$\mathbf{LINEAR_0^{+}(f,g)}$	$:\Leftrightarrow LINEAR^{+}(f,g) \wedge H(0) = 0$
$\mathbf{LINEAR^{-}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge H''(x) = 0$
$\mathbf{LINEAR_0^{-}(f,g)}$	$:\Leftrightarrow LINEAR^{-}(f,g) \wedge H(0) = 0$
$\mathbf{UNDERLINEAR^{+}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge H''(x) < 0$
$\mathbf{UNDERLINEAR_0^{+}(f,g)}$	$:\Leftrightarrow UNDERLINEAR^{+}(f,g) \wedge H(0) = 0$
$\mathbf{UNDERLINEAR^{-}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge H''(x) > 0$
$\mathbf{UNDERLINEAR_0^{-}(f,g)}$	$:\Leftrightarrow UNDERLINEAR^{-}(f,g) \wedge H(0) = 0$
$\mathbf{OVERLINEAR^{+}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge H''(x) > 0$
$\mathbf{OVERLINEAR_0^{+}(f,g)}$	$:\Leftrightarrow OVERLINEAR^{+}(f,g) \wedge H(0) = 0$
$\mathbf{OVERLINEAR^{-}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge H''(x) < 0$
$\mathbf{OVERLINEAR_0^{-}(f,g)}$	$:\Leftrightarrow OVERLINEAR^{-}(f,g) \wedge H(0) = 0$

Fig. 3.2 Constraints of RSIM.

```
(DERIV inflow volume)
(SIGNED-SQUARE inflow2 inflow)
(PRODUCT p-pressure c-frict inflow2)
(M+0TOO-STEEP p-static volume)
(SUM p p-pressure p-static)
```

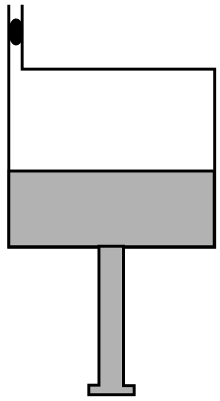


Fig. 3.3 A model for a tank with clogged breather tube. C-frict is a friction coefficient. The SIGNED-SQUARE constraint-class has been added to the basic constraints of Fig. 3.2. It is defined as follows:
SIGNED-SQUARE(f,g) : ⇔ f(t) = g(t) * |g(t)|

3.3. State Descriptions in RSIM

A system state is characterized by all parameter values of a certain time point or time interval. Compared with QSIM, state descriptions in RSIM additionally contain PQ values of the parameter amounts and derivatives, and relations between their P values. In addition the duration for reaching a certain point state is described by a PQ value. Fig. 3.4 gives an example for a state that describes an interval during the filling of the clogged tank. While the parameters on the sign, qval and PQ layers are described by qualitative values, qualitative information on the P layer is expressed by relations.

sign(volume) = +
sign(volume´) = +
qval(volume) = (0 half-full)
pq(volume) = normal
pq(volume´) = normal

sign(inflow) = +
sign(inflow´) = 0
qval(inflow) = (0 ∞)
pq(inflow) = normal
pq(inflow´) = normal

sign(inflow2) = +
sign(inflow2´) = 0
qval(inflow2) = (0 ∞)
pq(inflow2) = normal
pq(inflow2´) = normal

sign(c-frict) = +
sign(c-frict´) = 0
qval(c-frict) = c-frict*
pq(c-frict) = normal
pq(c-frict´) = normal

sign(p-pressure) = +
sign(p-pressure´) = +
qval(p-pressure) = (0 ∞)
pq(p-pressure) = normal
pq(p-pressure´) = normal

sign(p-static) = +
sign(p-static´) = +
qval(p-static) = (p-half-full p-full)
pq(p-static) = too-high
pq(p-static´) = too-high

sign(p) = +
sign(p) = +
qval(p) = (0 ∞)
pq(p) = too-high
pq(p) = too-high

p(p) < p(p-static)
p(p´) < p(p-static´)

Fig. 3.4 An interval state of the process of filling a tank with clogged breather tube. All parameters are described with sign, qval, and PQ values. Additionally some relations between P values can be stated.

3.4. Intrastate Analysis of RSIM

In intrastate analysis, constraint propagation takes place. As on the sign layer, the constraints on the pq layer are described by listing the possible tupels of values. Tables 1 and 2 show the definitions of the SUM and PRODUCT constraints. The left column contains the respective

PQ tupels and the right column assertions about relationships between the corresponding P values. The relationships between P values refine the information given by a PQ tupel. The tupels of the SUM-PQ-relation and the PRODUCT-PQ-relation are identical, but there are differences on the P layer. For example, if a too high value is added to or multiplied with a normal value, sum and product are both too high. But while the sum is less too high than the too high summand, the product is exactly as high as the too high factor.

If a certain PQ tupel is established during constraint propagation, the corresponding P assertions are recorded. On the P layer, the main task is to collect relationships between P values and to generate a consistent graph of P values. In this process, transitivity and symmetry have to be taken into account. Since RSIM uses the relations *less*, *equal* and *greater*, the graph represents a partial order of sets of P values. Each set consists of P values of the same magnitude. Constraining on the P layer means increasing the degree of order between the P values of all parameters. It is not the aim to reach a total order. Generating a total order would still entail ambiguities. The additional information on the P layer facilitates to avoid spurious behavior on the PQ layer. That is, if a system state is completely described on the PQ layer, it may happen that the corresponding P assertions are not consistent, for example because of cycles in a path of *less* relationships.

Table 1: Part of the definition of the SUM constraint

SUM-PQ-Relation ⊂ PQ(f) × PQ(g) × PQ(h) (SUM(f,g,h) ⇔ f(t)=g(t)+h(t))	Corresponding P assertions
(too-low, too-low, too-low)	
(too-low, too-low, normal)	P(g) < P(f)
(too-low, too-low, too-high)	P(g) < P(f)
(normal, too-low, too-high)	
(too-high, too-low, too-high)	P(f) < P(h)
(too-low, normal, too-low)	P(h) < P(f)
(normal, normal, normal)	
(too-high, normal, too-high)	P(f) < P(h)
(too-low, too-high, too-low)	P(h) < P(f)
(normal, too-high, too-low)	
(too-high, too-high, too-low)	P(f) < P(g)
(too-high, too-high, normal)	P(f) < P(g)
(too-high, too-high, too-high)	

Table 2: Part of the definition of the PRODUCT constraint

PRODUCT-PQ-Relation ⊂ PQ(f) × PQ(g) × PQ(h) (PRODUCT(f,g,h) ⇔ f(t)=g(t)*h(t))			Corresponding P assertions
(too-low,	too-low,	too-low)	P(f) < P(g) ∧ P(f) < P(h)
(too-low,	too-low,	normal)	P(f) = P(g)
(too-low,	too-low,	too-high)	P(g) < P(f)
(normal,	too-low,	too-high)	
(too-high,	too-low,	too-high)	P(f) < P(h)
(too-low,	normal,	too-low)	P(f) = P(h)
(normal,	normal,	normal)	
(too-high,	normal,	too-high)	P(f) < P(h)
(too-low,	too-high,	too-low)	P(h) < P(f)
(normal,	too-high,	too-low)	
(too-high,	too-high,	too-low)	P(f) < P(g)
(too-high,	too-high,	normal)	P(f) = P(g)
(too-high,	too-high,	too-high)	P(g) < P(f)∧ P(h) < P(f)

3.5. Interstate Analysis in RSIM

In interstate analysis, transition rules are applied. A transition deduces information about the parameter values of successor states. Usually continuity information about continuously differentiable functions is used for this. Two types of transitions are distinguished: point transitions and interval transitions. A point transition is applied on point states and infers information about the following interval state. An interval transition works in the corresponding way. In contrast to absolute transitions, a relative transition can infer information about the duration of the interval.

3.5.1. Transitions on the PQ Layer

In the current version of RSIM, PQ transitions do not consider discontinuous changes of the behavioral topology (see Section 2.1.1). Therefore, they just use the values *too low*, *normal*, and *too high*.[1] PQ transitions, as sign transitions or qval transitions, like those defined in

[Kuiper 86], refer to the values of exactly one parameter. Although there are analogies between sign transitions and PQ transitions, the following differences have to be mentioned:

- In PQ transitions, information about the duration of a time interval can be deduced. This information is of relative nature and expressed by the values *too low*, *normal*, *too high*.
- In sign transitions (and qval transitions), qualitative information about the derivative is used to determine the further course of the amount. In PQ transitions, as they are formulated below, the PQ value of the derivative is used instead of qualitative information about the derivative of a P value. That is (for a basis landmark of 0):

$$PQ(f',t) = \frac{f'(t)}{f_{ref}'(t)} \quad \text{instead of} \quad P'(f,t) = \left(\frac{f(t)}{f_{ref}(t)}\right)'$$

- Because of the definition of P values, a PQ transition depends not only on the PQ values of amount and derivative of a parameter, but additionally on their signs. For example, if a parameter's amount is too low and its derivative is too high, the amount would come closer to normal if both, amount and derivative, are positive (or negative). But if the derivative is negative and the amount positive (or vice versa) the amount would become more and more too low.

Point Transitions

Table 3 shows the point transitions of the PQ layer. Information about the parameter values at a time point ti is used to determine the values in the following time interval. The first three transitions determine the derivative of the following interval, the next four transitions the amount. The remaining transitions handle the ambiguous case of normal amount and normal derivative at time point ti. For example the transition PANDN2 can be read in the following way:

> *If at some time point ti, the PQ value of a parameter's amount is normal and the PQ value of its derivative is normal too, and if the sign of its amount is positive and of its derivative negative or vice versa then in the following time interval, either the PQ value of the parameter's amount will be too high and the PQ value of its derivative too low, or the amount's PQ value will be too low and the derivative's PQ value too high, or both PQ values will be normal.*

1. An extension is suggested in Chapter 4.

Table 3: Point transitions on the PQ layer

Name of transition	$PQ(f,t_i)$	$PQ(f',t_i)$	$sign(f(t_i))$	$sign(f'(t_i))$	$PQ(f,(t_i,t_j))$	$PQ(f',(t_i,t_j))$	$sign(f',(t_i,t_j))$
PDL		L				L	
PDH		H				H	
PALHDN	L H	N N				unconstrained	
PAL	L				L		
PAH	H				H		
PANDLH1	N N N N N N	L L L L H H	- 0 0 + + -	- - + + - +	L		
PANDLH2	N N N N N N	L L H H H H	+ - - 0 0 +	- + - - + +	H		
PANDN1	N N N N N	N N N N N	- 0 0 0 +	- - 0 + +	L N H	L N H	
PANDN2	N N	N N	+ -	- +	H N L	L N H	
PANDN3	N	N	-	0	L L N H H	L H N L H	- + unconstrained + -
PANDN4	N	N	+	0	L L N H H	L H N L H	+ - unconstrained - +

Interval Transitions

Interval transitions use information about the parameter values of a time interval to determine the events that may finish the qualitative state described in the interval. The reason for a change of the qualitative state is, that a landmark is reached by a parameter's amount or derivative. On the PQ layer, the corresponding event is reaching *normal*, or more exactly the landmark 1 in the range of P values. Therefore, a qualitative behavior description can be refined by additional events on relative description layers. Or in other words: An interval on absolute descriptions layers can have a structure on relative description layers. In Fig. 3.5 deviating and reference behavior of a parameter *f* can be seen. On absolute layers the time intervals (t_a, t_p) or (t_a, t_s) describe the state of decreasing *f* . On relative layers, however, there is an event at time point t_i where the normal value is reached by *f*. So in a complete description

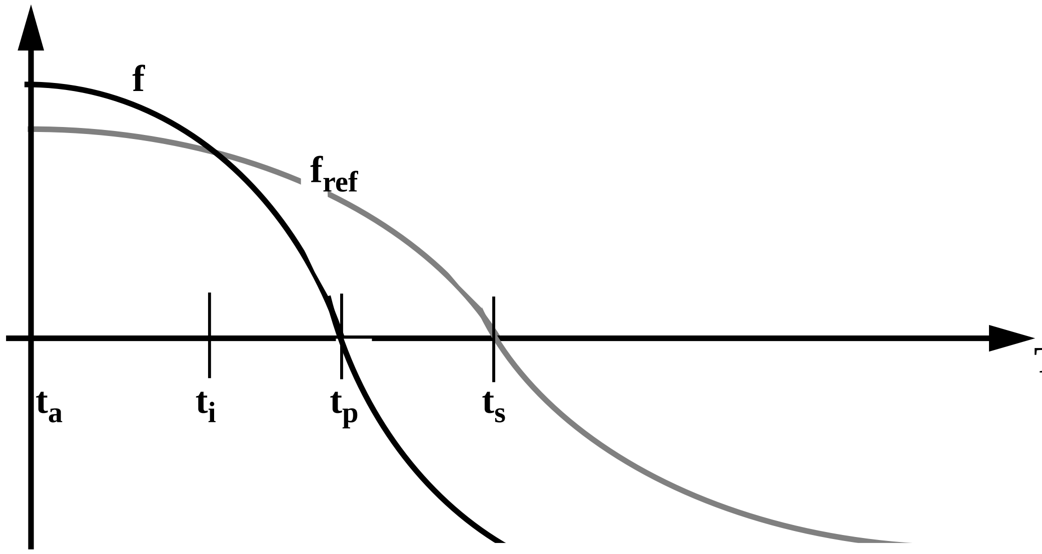


Fig. 3.5 A deviating parameter f and its reference course f_{ref}.

we have the following sequence of states:

Time point t_a:	*f* is positive and *too-high*.
Time interval (t_a, t_i):	*f* is positive and *too-high*.
Time point t_i:	*f* is positive and *normal*.
Time inteval (t_i, t_p):	*f* is positive and *too-low*.
Time point t_p:	*f* reaches 0...

The event at time point t_p entails a difficulty. Here, the deviation of *f* is discontinuous. And, in the time interval beginning at t_p, it remains discontinuous. In general it makes no sense, to compare *f* with f_{ref} during interval (t_p, t_s). Instead, the parts of *f* and f_{ref} where both are negative should be compared with each other. And *f* at time point t_p should be compared with f_{ref} at time point t_s. That means, a synchronization step is necessary. Fig. 3.6 shows the

synchronized behaviors. For f_{ref} no comparison takes place during (t_p, t_s). Now the list of qualitative states can be continued:

Time point t_{p*}: *f* is 0 and *normal*.

Time interval (t_{p*}, $t_?$) *f* is negative and *too high*[1].

In addition, it can be stated that the duration for reaching 0 is less than normal, i.e. *too-low*.

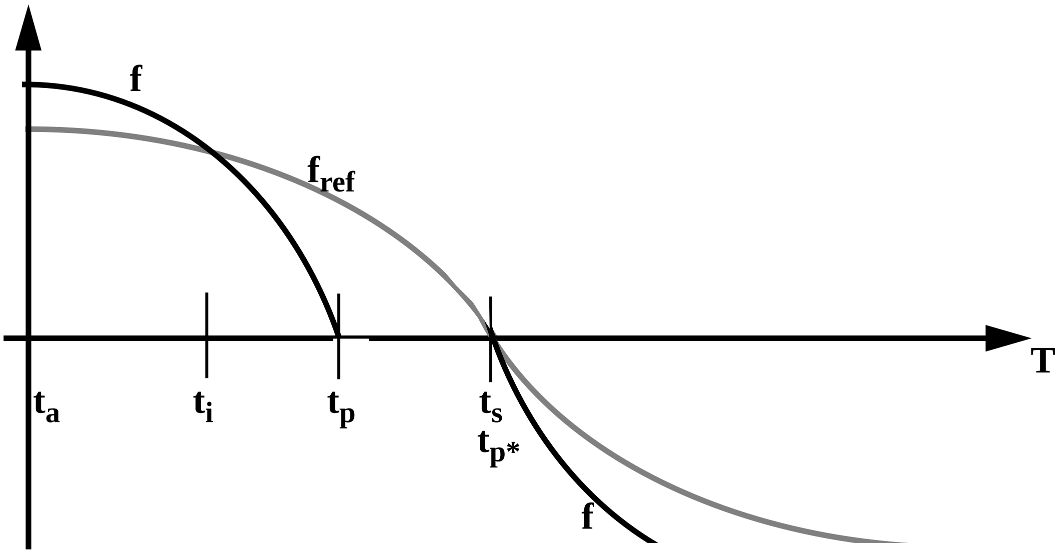


Fig. 3.6 Synchronized behaviors of Fig.3.5.

So, the event of reaching 0 requires a synchronization because otherwise the deviations would become discontinuous and sensible comparisons would not be possible. Reaching other landmarks than 0 does only requires a synchronization, if comparisons of behaviors shall be restricted to differential deviations. (If a landmark different from 0 is reached and no synchronization takes place, a deviation would become non-differential, but it would still remain continuous).

In Fig. 3.7. a parameter *g* first reaches a landmark l_k and afterwards a landmark l_m, both faster than normal. At the same time when *g* reaches l_m, parameter *f* reaches 0 (time point t_p). Therefore, a synchronization is necessary. Before that, at time point t_b, *g* reaches landmark l_k. And no other parameter reaches 0 at that time. The deviations of *g* at and after t_b are non-differential, but continuous. So, only if one wants to restrict oneself to differential deviations, a synchronization becomes necessary. Both qualitative behaviors, with and without synchronization, are listed below.

1. Note, a PQ value describes the magnitude of a value, so a negative value that is too low in a mathematical sense has the PQ value *too-high*.

Without synchronization at t_b (see Fig. 3.7):

Time point t_a:	*f* is positive and *too-high*.	g is 0 and normal
Time interval (t_a, t_b):	*f* is positive and *too-high*.	*g* is positive and *too-high*
Time point t_b:	is positive and *too-high*.	*g* is positive at l_k and ***too-high***
Time inteval (t_b, t_i):	*f* is positive and *too-high*.	*g* is positive in (l_k, l_m) and *too-high*
Time point t_i:	*f* is positive and *normal*.	*g* is positive in (l_k, l_m) and *too-high*
Time inteval (t_i, t_p):	*f* is positive and *too-low*.	*g* is positive in (l_k, l_m) and *too-high*
Time point t_p:	*f* is 0 and *normal*.	*g* is positive at l_k and *normal*[1]

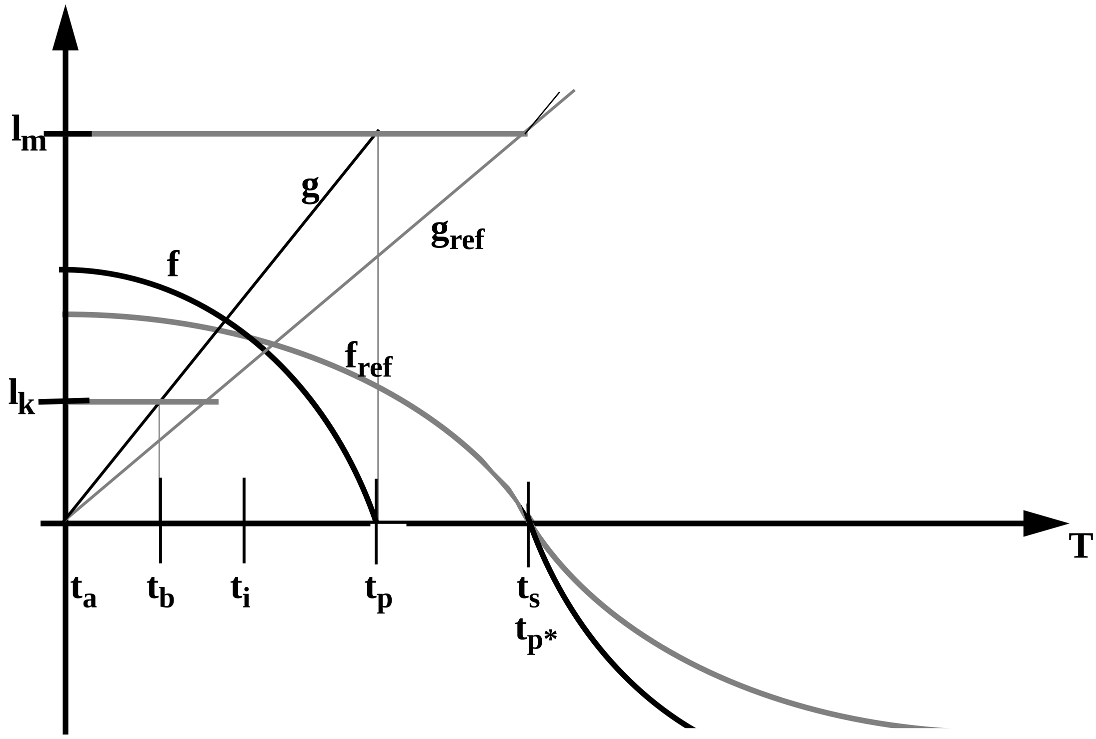


Fig. 3.7 Additional events at a parameter *g*.

With synchronization at t_b (see Fig.3.8):

Time point t_a:	*f* is positive and *too-high*.	g is 0 and normal
Time interval (t_a, t_b):	*f* is positive and *too-high*.	*g* is positive and *too-high*
Time point t_{b*}:	is positive and *too-high*.	*g* is positive at l_k and ***normal***
Time inteval (t_{b*}, t_{i*}):	*f* is positive and *too-high*.	*g* is positive in (l_k, l_m) and *too-high*

1. *G* is normal at t_p because g_{ref} and f_{ref} reach their landmarks at the same time.

Time point t_{i*}:	*f* is positive and *normal*.	*g* is positive in (l_k, l_m) and *too-high*
Time inteval (t_{i*}, t_{p*}):	*f* is positive and *too-low.*	*g* is positive in (l_k, l_m) and *too-high*
Time point t_{p**}:	*f* is 0 and *normal*.	*g* is positive at l_k and *normal*[1]

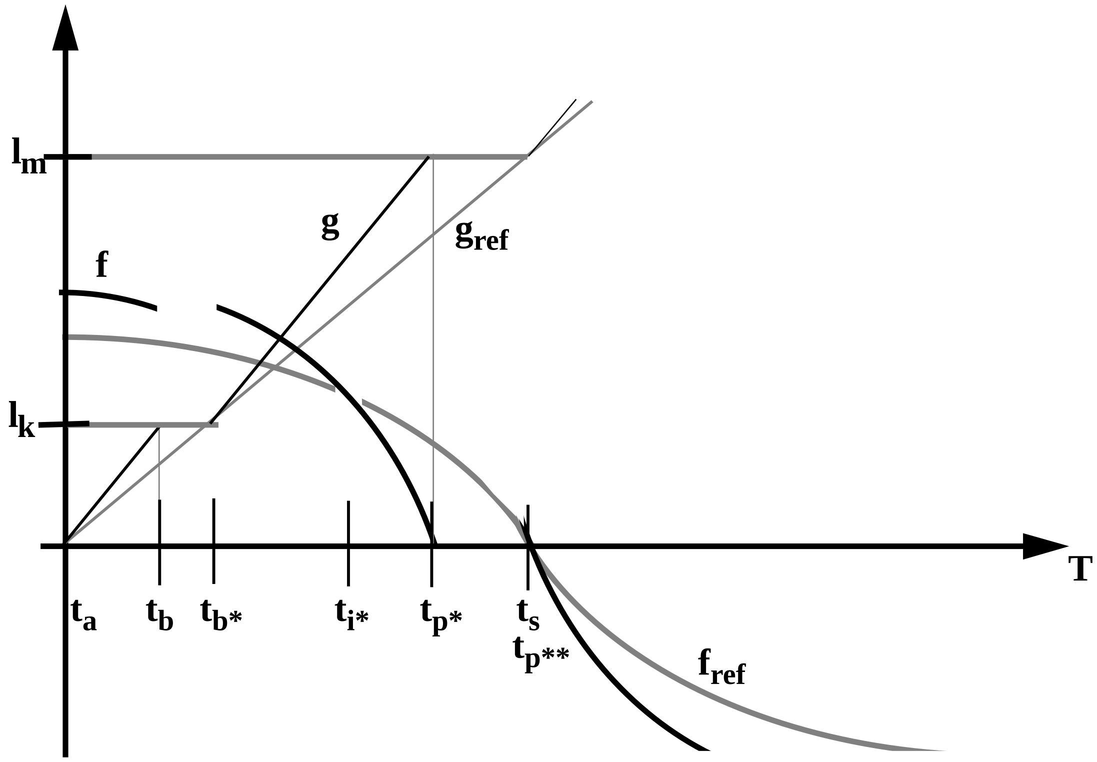


Fig. 3.8 Additional synchronization at t_b

In the example above the duration of the deviating behavior has been too low. A too high duration has similar consequences. Fig. 3.9 shows the difference between a too low and a too high duration of the deviating behavior. Here, synchronization takes place when a landmark value l_k is reached. In the deviating behavior b_l, l_k is reached earlier than normal, in deviating behavior b_h later than normal. The qualitative state describing the phase of increase refers to the full length of b_l, but for b_h it ends at t_n. So, for the interval (t_n, t_h) of b_h no description exists. However, the relevant information that the duration to reach l_k is *too-high*, is included.

1. *G* is normal at t_p because g_{ref} and f_{ref} at the same time reach their landmarks.

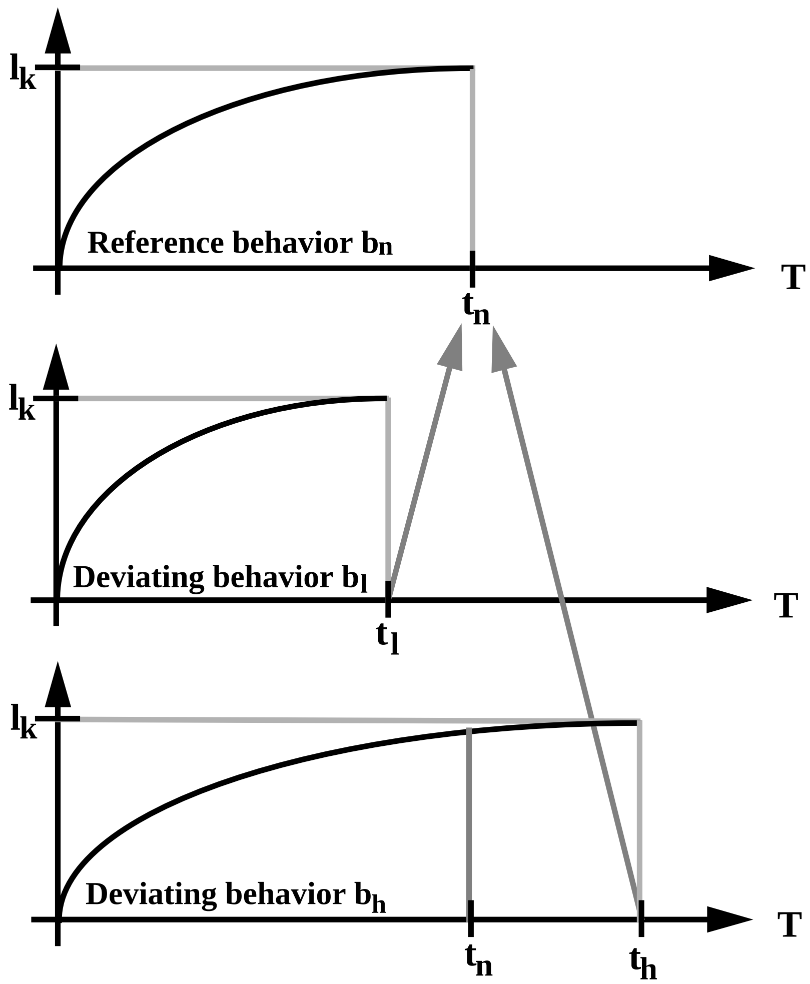


Fig. 3.9 Synchronization for a *too-low* and a *too-high* duration.

We have seen that a synchronization is necessary when landmark 0 or any landmark[1] respectively is reached, depending on whether continuous or merely differential deviations are allowed, . We want to call a landmark that requires a synchronization a ***synchronization landmark***. Whenever a synchronization landmark is reached by a parameter's amount or derivative, its PQ value becomes *normal* because of the synchronization. So, there are two reasons for reaching *normal*:

1. The amount or derivative of a parameter in the deviating system reaches a synchronization landmark.
2. The amount or derivative of a parameter in the deviating system reaches the normal value.

In qualitative simulation, in general, it cannot be determined which event occurs first, i.e. which parameter is involved in the next event. Therefore, transition rules (which are specific to a parameter) addionally have to supply the values of a parameter's amount and derivative

1. on an absolute description layer

for the case that the parameter is not involved in the next event, i.e. the next event concerns a different parameter. This “foreign” event may entail a synchronization. Therefore, an interval transition on the PQ layer has to cover five cases:

1. The parameter’s amount or derivative reaches a synchronization landmark.
2. The parameter’s amount or derivative reaches the normal value.
3. The next event occurs outside with normal duration.
4. The next event occurs outside with too low duration.
5. The next event occurs outside with too high duration.

Tables 4 and 5 show the PQ interval transitions[1]. The five situations listed above cannot be directly identified in the transition tables, because a situation can have different consequences as a result of ambiguities, or different situations can have the same consequences. In transition IAHDH, for example, the five situations have the following consequences:

Case 1: duration(t_i,t_j) = *normal*, PQ(f,(t_j) = *normal* or
duration(t_i,t_j) = *too-high*, PQ(f,(t_j) = *normal*

Case 2: duration(t_i,t_j) = *normal*, PQ(f,(t_j) = *normal*

Case 3: duration(t_i,t_j) = *normal*, PQ(f,(t_j) = *too-high*

Case 4: duration(t_i,t_j) = *too-low*, PQ(f,(t_j) = *too-high*

Case 5: duration(t_i,t_j) = *too-high*,
PQ(f,(t_j) = *too-low* or *normal* or *too-high* (= unconstrained)

Table 4: Interval transitions on the PQ layer (1)

Name of transition	PQ(f´,(t_i,t_j))	duration(t_i,t_j)	PQ(f´,(t_j)
IDL	L	L	unconstrained
		N	L
		N	N
		H	unconstrained
IDN	N	L	unconstrained
		N	N
		H	unconstrained
IDH	H	L	unconstrained
		N	N
		N	H
		H	unconstrained

1. PQ values are abbreviated by L, N, and H.

Table 5: Interval transitions on the PQ layer (2)

Name of transition	PQ(f,(t_i,t_j))	PQ(f´,(t_i,t_j))	sign(f,(t_i,t_j))	sign(f´,(t_i,t_j))	duration(t_i,t_j)	PQ(f,(t_j))
IAL	L			0		L
IAN	N			0		N
IAH	H			0		H
IANDN1	N N	N N	- +	- +	L N H	L N H
IANDN2	N N	N N	+ -	- +	L N H	H N L
IALDLN	L L L L	L L N N	- + - +	- + - +	L N H	L L unconstrained
IALDNH	L L L L	N N H H	+ - + -	- + - +	L N H	unconstrained L L
IAHDLN	H H H H	L L N N	+ - + -	- + - +	L N H	H H unconstrained
IAHDNH	H H H H	N N H H	- + - +	- + - +	L N H	unconstrained H H
IALDH	L L	H H	- +	- +	L N N H	L L N unconstrained
IAHDL	H H	L L	- +	- +	L N N H	unconstrained N H H
IALDL	L L	L L	+ -	- +	L N N H	unconstrained L N L
IAHDH	H H	H H	+ -	- +	L N N H	H N H unconstrained

3.5.2. Transitions on the P Layer

As explained above, an incomplete description is generated on the P layer, but one that supplies additional information and helps to avoid spurious behavior on the PQ layer. Accordingly it is not necessary for transitions on the P layer to cover all possible configurations of values. Instead, some special but very useful transitions are formulated. When required, more transitions can be added.

While sign transitions, qval transitions, or PQ transitions work with qualitative values of amounts and derivatives, transitions on the P layer refer to relations between P values. Relations between P values can be used in different ways: They can help to further constrain PQ transitions. They can deduce relationships between P values in the following time interval or time point. And because the relations used on the P layer are binary, they can concern different parameters. As a consequence, P transitions are not as homogeneous as the other types of transitions.

Point Transitions on the P Layer

One obvious point transition is given by the continuity rule, that if two P values are different at a time point t_i, there must exist an interval around t_i where they are different too. While the same consequences mostly can be derived from the P layer's intrastate analysis and from PQ transitions, the case of equality between P values is a more important one. In the transition P==[1] (Table 6), an interesting correlation between the P values of a parameter's amount and its first and second derivatives is formulated.[2] P== says:

> *If at a time point t_i on the one hand the P values of a parameter's amount and its second derivative are equal and on the other hand the P values of amount and first derivative are equal too, or the sign of the first derivative is 0, then in the following time interval (t_i, t_j) the P values of amount and derivatives must be equal, or the P values of amount and first derivative and amount and second derivative must be unequal.*

In the current version of RSIM, P== is the only point transition on the P layer. Further point transitions may follow.

Table 6: Point transition P==

Name of transition	relation $(P(f,t_i), P(f',t_i))$	relation $(P(f,t_i), P(f'',t_i))$	$sign(f',t_i)$	relation $(P(f,(t_i,t_j)), P(f',(t_i,t_j)))$	relation $(P(f,(t_i,t_j)), P(f'',(t_i,t_j)))$
P= =	= undefined =	= = =	- 0 +	= ≠	= ≠

1. P== is proven for a basis landmark of 0 (see appendix B).
2. Since only the first derivative of a parameter is modeled, the rule can only be applied to parameters that are connected to another parameter via the DERIV constraint.

Interval Transitions on the P layer

The first group of interval transitions on the P layer treats the situation when the amount of a parameter moves to the landmark 0 and both amount and derivative of the parameter are too low or too high. The PQ transitions describing this case are IALDL and IAHDH. On the P layer, it can be expressed that one value is more too high or too low than another value. The more detailed information on the P layer facilitates a more constrained transition consequence. Table 7 shows the transitions IALDL and IAHDH augmented by P information. For each of them there are six transitions on the P layer.

Table 7: Interval transitions IALDL* and IAHDH*

Name of transition	PQ $(f,(t_i,t_j))$	PQ $(f',(t_i,t_j))$	relation $(P(f,(t_i,t_j)), P(f',(t_i,t_j)))$	sign $(f,(t_i,t_j))$	sign $(f',(t_i,t_j))$	duration (t_i,t_j)	PQ$(f,(t_j)$	sign $(f,(t_j)$
IALDL<-	L	L	<	-	+	L L N H	N unconstrained L L	0 - - -
IALDL<+	L	L	<	+	-	L L N H	N unconstrained L L	0 + + +
IALDL=-	L	L	=	-	+	L N N H	unconstrained L N L	- - 0 -
IALDL=+	L	L	=	+	-	L N N H	unconstrained L N L	+ + 0 +
IALDL>-	L	L	>	-	+	L N N H	unconstrained L N L	- - - -
IALDL>+	L	L	>	+	-	L N N H	unconstrained L N L	+ + + +
IAHDH<-	H	H	<	-	+	L N N H	H N H unconstrained	- - - -
IAHDH<+	H	H	<	+	-	L N N H	H N H unconstrained	+ + + +

Table 7: Interval transitions IALDL* and IAHDH*

Name of transition	PQ $(f,(t_i,t_j))$	PQ $(f',(t_i,t_j))$	relation $(P(f,(t_i,t_j)),$ $P(f',(t_i,t_j)))$	sign $(f,(t_i,t_j))$	sign $(f',(t_i,t_j))$	duration (t_i,t_j)	PQ(f,(t_j)	sign (f,(t_j)
IAHDH=-	H	H	=	-	+	L N N H	H N H unconstrained	- 0 - -
IAHDH=+	H	H	=	+	-	L N N H	H N H unconstrained	+ 0 + +
IAHDH>-	H	H	>	-	+	L N H H	H H N unconstrained	- - 0 -
IAHDH>+	H	H	>	+	-	L N N H	H N H unconstrained	+ + 0 +

As an example, the P transition IAHDH=+ shall be explained.The transition IAHDH=+ (and the transition IAHDH=-) can be interpreted in the following way:

If at every time point of a time interval (t_i, t_j) the distance that has to be traveled is too high by the same factor as the velocity then it costs as much time as normally to travel the whole distance.

Transition IAHDH=+ is illustrated in Figure 3.10. The reference behavior is represented by the black line and the deviating behavior by the dotted line. Let’s examine how the five possible endings (Section 3.5.1) of the time interval (t_i, t_j) look like.

1. A landmark is reached: One has to distinguish between a positive landmark l_k and the landmark 0. Every positive landmark l_k will first be reached by the reference function. Therefore, the duration to reach a positive landmark is too high. This case is covered by line 4 of the transition’s description in Table 7. On the other hand, the duration to reach the landmark 0 is the same as in the reference behavior (line 2 in transition table).

2. The normal value is reached: This happens at time point t_n. Transition IAHDH=+ handles the special case where landmark 0 is reached in normal duration. Reaching the normal value is covered by line 2.

3. The next event occurs outside with normal duration: In Figure 3.10 this case is represented by the vertical arrow. At every time point in the interval (t_i, t_j), the deviating value is *too-high* (line 3 in transition table).

4. The next event occurs outside with too low duration: In Figure 3.10 this case is represented by the arrow pointing to the right. At every time point in the interval (t_i, t_j), the deviating value is *too-high* (line 1 in transition table).
5. The next event occurs outside with too high duration: In Figure 3.10 this case is represented by three arrows pointing to the left. The deviating value may be *too-low*, *normal*, or *too-high*. (Line 4 in transition table.)

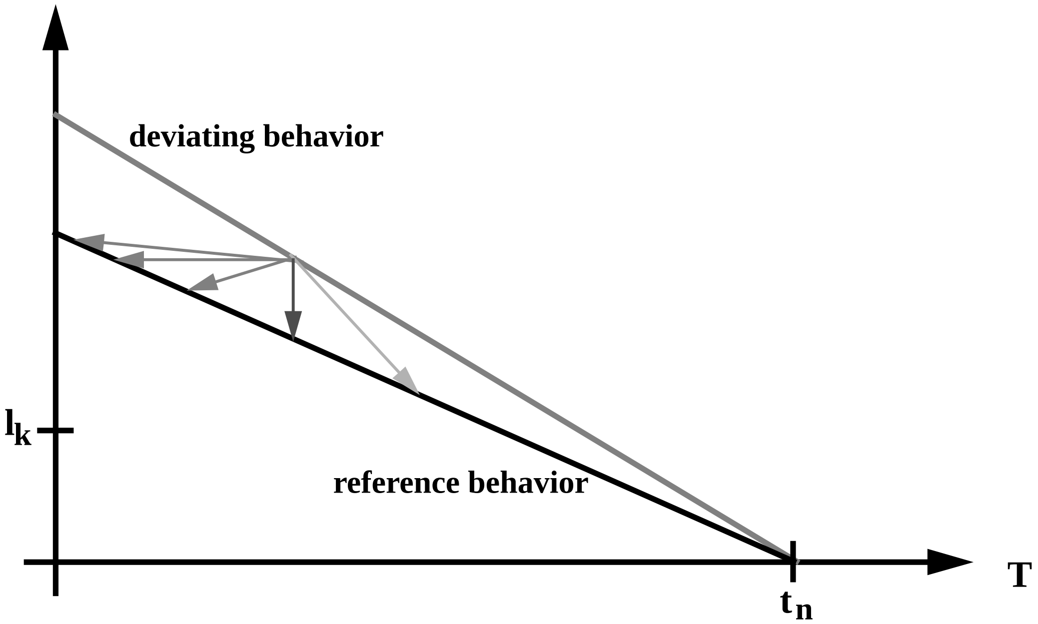


Fig. 3.10 An illustration of the interval transition IAHDH=+. Possible cases of comparisons are indicated by arrows.

Another type of transition makes predictions about the relations between P values at time

Table 8:

Name of transition	relation (P(f,(t_i,t_j)), P(f´,(t_i,t_j)))	duration (t_i,t_j)	relation (P(f,t_j), P(f´,t_j))	sign(f,t_j)	sign(f´,t_j)
I=	=	L	unconstrained	unconstrained	unconstrained
		N	=	{-, +}	{-, +}
		N	undefined	unconstrained	0
		N	undefined	0	unconstrained
		H	unconstrained	unconstrained	unconstrained

point t_j. Transition I= handles the case of equality between amount and derivative of a parameter. In words, I= expresses the following:

If during a time interval (t_i, t_j) the P values of amount and derivative of a parameter are equal then if the duration of (t_i, t_j) is normal and the signs of amount and derivative are unequal to 0, the P values of amount and derivative are equal at time point t_j.

3.6. The Output of RSIM

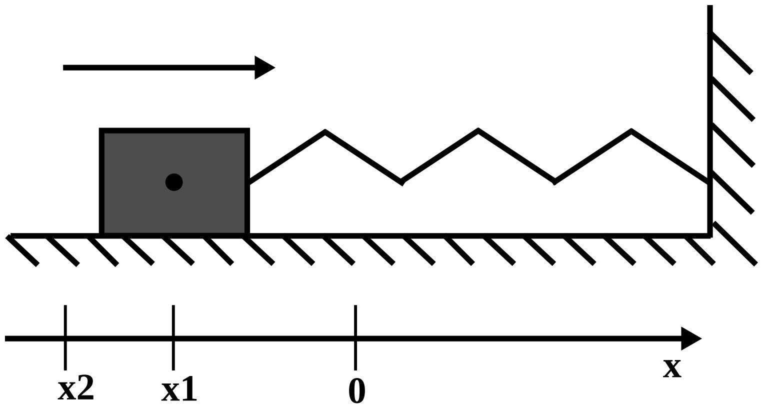
RSIM's output is a tree of behaviors. An RSIM behavior is more specific than a QSIM behavior, i.e. the description of behavior is more detailed. This means that

1. the description of a parameter and thus a state description is more detailed,
2. a behavior consists of more states, because additional events on relative description layers have to be integrated in a description of behavior,
3. the behavior tree contains more branchings, because the additional descriptions are not free of ambiguities either.

Especially the last point leads to time consuming computations and to extensive and sometimes confusing output. To cope with this problem, some methods have been added, that help to prevent irrelevant distinctions. It is possible, for example, to deactivate a description layer just for a certain amount or derivative. Another very effective method prevents distinctions for the PQ values of derivatives. It first generates fully described system states and then merges them afterwards.

We want to illustrate RSIM's output for the popular example system of a mass on a spring. Fig. 3.11 shows a model of this system and two different initializations. On the one hand, we want to examine a model with a too high mass. On the other hand we want to know what happens, if the amplitude is higher than normal. In Fig. 3.12 RSIM's output is listed for the position of the block, its velocity, and its acceleration, until the block reaches the point of no spring tension ($x = 0$).

Both behaviors of Fig. 3.12 consist of 5 states and have no ambiguities. In fact, there are ambiguities for the PQ values of some derivatives. But the simplifying strategies, mentioned above, merge these states. For a too high mass, it is derived that the duration of a period is too high, that the velocity is too low for $x = 0$, and that the acceleration at first is too low, then becomes normal, and afterwards is too high. For a too high amplitude it is unambiguously derived that the duration of a period is normal. This is true, because the deviations of x and v have the same strength, i.e. x and v have equal P values.




```
(create-model *mass-on-a-spring*
    :quantity-spaces ((k-qs (k* 0))                   ;Definition of quantity spaces:
                      (m-qs (0 m*))                   ;(name-of-quantity-space list-of-landmarks)
                      (energy-qs (0 te*))
                      (x-qs (x2* x1* 0)))
    :variables ((x x-qs) v vv a (f f-qs) (ke energy-qs) (pe energy-qs))
    :constants ((m m-qs) (k k-qs) (te energy-qs))
    :non-negatives (pe ke te m)
    :constraints ((deriv v x)          ;The velocity of the block is the derivative of its position.
                  (deriv a v)          ;The acceleration of the block is the derivative of its velocity.
                  (product f m a)      ;The force of the spring is the product of spring constant and position
                                        (Hooke's law).
                  (product f k x)      ;Force is the product of mass and acceleration (Newton's second law of
                                        motion).
                  (square vv v)        ;Vv is the square of v.
                  (product ke m vv)    ;Kinetic energy depends on the product of mass and the square of velocity.
                  (square pe f)        ;Weld calls it a "cheating definition of potential energy" [Weld 90, p.159].
                  (sum te pe ke)))     ;Total energy is the sum of kinetic and potential energy.
```

```
(initialize-model *mass-on-a-spring*
  (v  :sign-of-amount 0
      :pq-of-amount normal)
  (x  :qval-of-amount x1*
      :pq-of-amount normal)
  (k  :qval-of-amount k*
      :pq-of-amount normal)
  (m  :qval-of-amount m*
      :pq-of-amount too-high)
  (te :qval-of-amount te*))
```

```
(initialize-model *mass-on-a-spring*
  (v  :sign-of-amount 0
      :pq-of-amount normal)
  (x  :qval-of-amount x2*
      :pq-of-amount too-high)
  (k  :qval-of-amount k*
      :pq-of-amount normal)
  (m  :qval-of-amount m*
      :pq-of-amount normal)
  (te :qval-of-amount te*))
```

Fig. 3.11 A model for a mass on a spring and two different initializations in RSIM's language. In the left initialization the mass is two high, in the right initialization the amplitude is greater than normal.

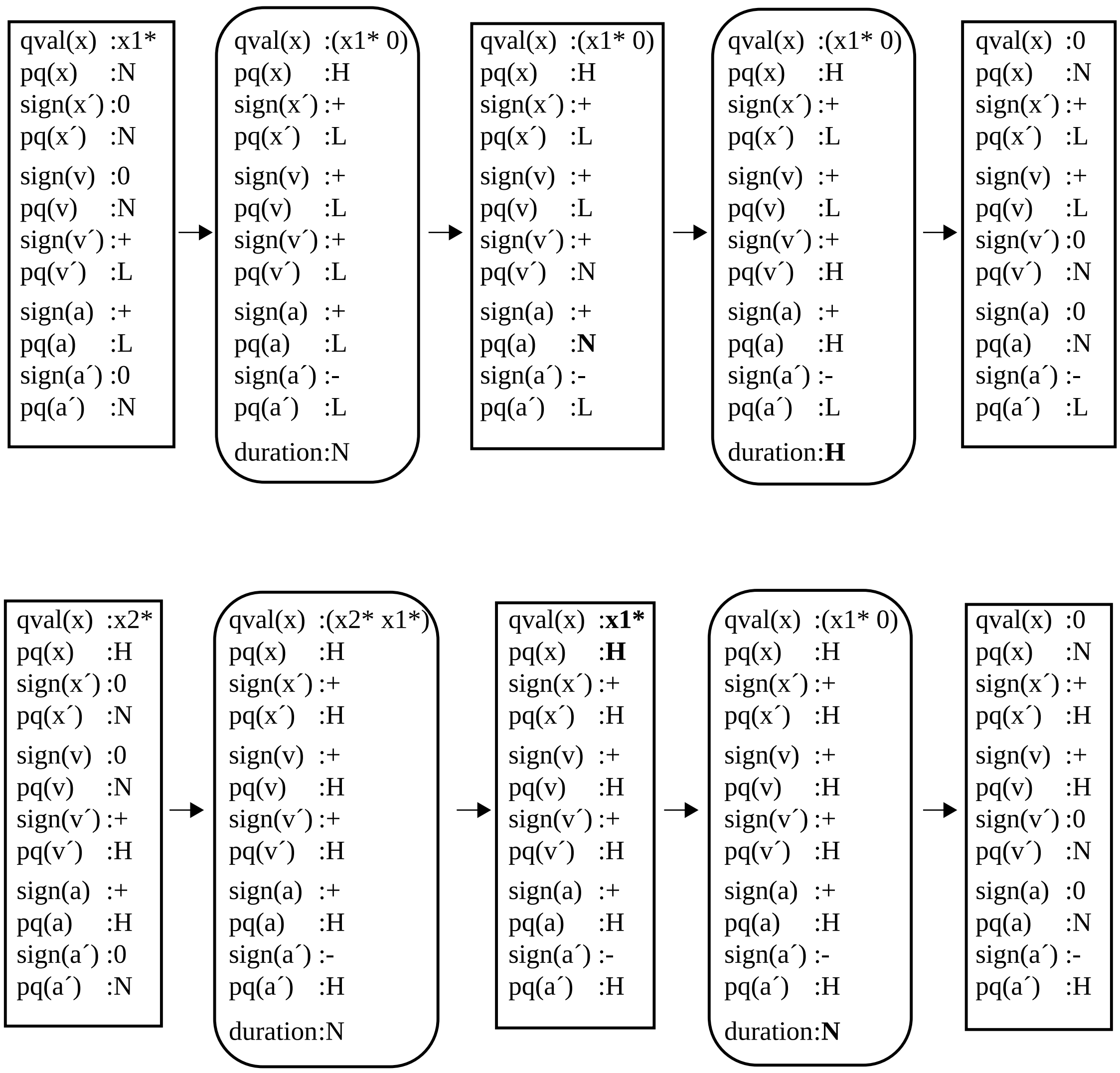


Fig. 3.12 The deviating behaviors for the mass-on-a-spring system, produced by RSIM. At the top the behavior for a too high mass, at the bottom the behavior for a too high amplitude.

4. Related Work

Relative descriptions are used in various approaches. [Raiman 86] reasons about orders of magnitude, in [Dague,Devès,Raiman 87] these concepts are used for a fault diagnosis. The *IQ analysis* of [de Kleer 79] is a qualitative sensitivity analysis of a system's steady state. Similar techniques that aim at a fault diagnosis can be found in [Downing 87], [Gallanti, Stefanini, Tomada 89] and [Kockskämper, Neumann, et al. 93]. [D'Ambrosio 89] describes a qualitative perturbation analysis that also is based on IQ analysis.

Dynamic systems, on the other hand, are analysed in Weld's approaches *DQ analysis* and *exaggeration* [Weld 87,88a,88b,90]. Both techniques predict the effects of differential changes as RSIM does. Therefore DQ analysis and exaggeration are the most relevant works for a comparison with RSIM. Since RSIM is oriented towards QSIM, we additionally will outline some differences to QSIM concerning the simulation techniques[1].

QSIM

RSIM's (or SLOD's) simulation techniques are similar to those of QSIM. Both simulators have an identical form of input and output: A system is described by constraints and a behavior is described by a sequence of states. However, RSIM's inferences have a more constructive character than QSIM's inferences. This becomes apparent in form and processing of transition rules. In QSIM, a single situation generally matches to several alternate transition rules. Constraint propagation in QSIM means filtering out the unsuitable transitions or the unsuitable combinations of transitions, which corresponds to filtering out impossible combinations of parameter values. That is, the set of possible values for a parameter is gradually reduced. In RSIM, on the other hand, transition rules are non-overlapping. A transition rule that can be applied will be applied. In general, the consequence of an RSIM transition rule contains a disjunction of value combinations. In order to fulfill the allowed combinations of an applicable transition rule, temporary *transition constraints* are generated. Constraint propagation in RSIM directly leads to single parameter values instead of a reduced set of values.

Weld's DQ Analysis

While the input/output behavior of RSIM and DQ analysis are similar, the underlying techniques are very different. DQ analysis uses a set of inference rules that analytically, and not by simulation, determine the effects of differential deviations. These inference rules refer to a model, expressed by QSIM constraints, and to a QSIM behavior. To facilitate a comparison of intervals of different length, DQ analysis introduces the concept of *perspective* (see Section 2.2.1). In RSIM, this problem is solved by interval transitions that perform a synchronization (see Section 3.5.1). DQ analysis sometimes produces no output, that is DQ analysis is incomplete. RSIM, on the other hand, always produces an output. But the current version of RSIM

1. Differences resulting from a refinement by means of relative descriptions have extensively been explained in the previous chapters.

does not capture changes in the behavioral topology.[1] That is, RSIM only is sound if topology changes can be excluded.

RSIM can answer some questions that DQ analysis cannot answer because DQ analysis has problems to predict the resulting effect of contrary deviations. If, for example, a longer distance has to be covered with a higher velocity, DQ analysis cannot decide, whether the duration increases, decreases or does not change. RSIM's relations between P values provide the means to answer such questions.

Exaggeration

In contrast to DQ analysis, exaggeration is based on simulation. Exaggeration's input and output, however, are different from RSIM. Exaggeration answers concrete questions about the effects of a certain differential deviation on a target parameter. For this purpose it first exaggerates the given deviation, e.g. a value higher than normal is considered as infinite. Then two simulations take place: a standard QSIM simulation and a simulation of the exaggerated system. The latter simulation works with hyperreal values, like *infinite* or *negligible*, and is carried out by a simulator called HR-QSIM. Finally, the exaggerated output is rescaled by a comparison of both simulation results. A negligible value, e.g., may be rescaled to "less than normal". In general, a QSIM simulation as well as a HR-QSIM simulation produces a set of behaviors because of ambiguities. As it is implemented, exaggeration compares the HR-QSIM behaviors to a single QSIM behavior and not to all behaviors that result from a QSIM simulation. The decision which behavior to choose is obtained from the input question. A general prediction of the effects of a deviation, as it is done by RSIM, would require a pairwise comparison of all exaggerated and all normal behaviors. Exaggeration is complete, but unfortunately only sound if all relationships between parameters are monotonous. The idea of an exaggeration conflicts with a comparison of deviations. Therefore, an important advantage of RSIM again lies in the comparison of deviations.

1. DQ analysis also has problems with topology changes. See, for example, Section 4 in [Weld 88b].

5. Future Work

At the moment, simulation by RSIM predicts all behaviors that entail no discontinuous changes of topology. Future work will investigate this topic. Occurrence branching shall be captured by making parameter-specific statements about duration instead of state-specific statements. The other type of branchings corresponds to discontinuous deviations in parameter histories. We want to integrate these changes of behavior by extending the set of PQ values, so that discontinuous deviations of parameter values can be described. In addition, the definitions of constraints have to be extended by the new PQ values.

RSIM takes all behaviors that result from the system description as reference behavior. In practice, often the correct behavior of a system is well known, such that this knowledge can be used to constrain the prediction of faulty behavior [Neitzke 91]. To realize this, RSIM needs additional input about the reference behavior. In the extreme, this can be the path of the behavior tree that corresponds to the reference behavior. (This is done in Weld's DQ analysis [Weld 87,88b,90].) During generation of behavior, RSIM then has to decide which behavior can result from the correct behavior.

6. Summary

Relative descriptions are necessary to characterize certain kinds of system deviations and the resulting behavior. These deviations are classified as differential or continuous. RSIM is a simulator that works with relative descriptions and is able to compare deviations with each other. RSIM cannot only deal with differential deviations but additionally accepts continuous deviations. In RSIM, the special properties of linear, overlinear and underlinear relationships between parameters are exploited to gain more accuracy in the prediction of system behavior. Additionally, faulty M^* relationships can be expressed. Due to these refined description facilities, some system behaviors can be distinguished that are identical under usual qualitative descriptions. The class of physical systems that can be described by RSIM corresponds to that of QSIM. In the current version of RSIM, a prediction of behaviors with discontinuous changes of topology is not possible. But RSIM's concepts allow an extension in this direction.

Acknowledgements

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A: Proofs of PQ Transition Rules

In Appendix A, it will be proven that the PQ transition rules are mathematically sound. The proofs are based on continuity conditions of continuously differentiable functions of time.

PQ values refer to the function P. (See the definition of P in Section 1.2.4.) Since $P(f,t) = (f(t) - l_a) / (f_{ref}(t) - l_a)$ is a continuously differentiable function of time for $(f_{ref}(t) - l_a) \neq 0$[1], PQ transitions have a similar character as sign transitions or qval transitions.

According to the definition of PQ values, we have to deal with the following situations:

1. $0 < P(f,t) < 1 \quad \Leftrightarrow \quad PQ(f,t) =$ *too-low*
2. $P(f,t) = 1 \quad \Rightarrow \quad PQ(f,t) =$ *normal*
3. $P(f,t) > 1 \quad \Leftrightarrow \quad PQ(f,t) =$ *too-high*
4. $f(t)-l_a = f_{ref}(t)-l_a = 0 \quad \Rightarrow \quad PQ(f,t) =$ *normal*

For the moment, the last situation will be left out. For the function P, interesting transitions are from the intervals (0, 1) or $(1, \infty)$ to a value of 1 and vice versa. Discontinuous transitions are not considered at this place. We want to see, which transitions between time points and time intervals are possible. At a time point t_i, either

1. $P(f,t_i) \in (0, 1)$, or
2. $P(f,t_i) = 1$, or
3. $P(f,t_i) \in (1, \infty)$

If $P(f,t_i) = 1$, nothing can be said about time intervals (t_{i-1}, t_i) or (t_i, t_{i+1}) without additional information. In these intervals, P(f,t) may be less, equal or greater than 1. But,

Lemma A: If $P(f,t_i) \in (0, 1)$ (or $P(f,t_i) \in (1,\infty)$), then there exists an environment of t_i with $P(f,t) \in (0, 1)$ (or $P(f,t_i) \in (1,\infty)$). That is, there exists a $\delta > 0$, such that

$P(f,t) \in (0, 1)$ (or $P(f,t) \in (1,\infty)$), for all $t \in T$ with $|t - t_i| < \delta$

Proof A: The ε-δ definition of continuity (e. g. [Forster 80, p. 67] states that for every $\varepsilon := |P(f,t_i) - 1| > 0$ there exists a $\delta > 0$ such that
$|P(f,t) - P(f,t_i)| < \varepsilon$, for all $t \in T$ with $|t - t_i| < \delta$
From this follows that $|P(f,t) - 1| \geq |P(f,t_i) - 1| - |P(f,t) - P(f,t_i)| > 0$, for all $t \in T$ with $|t - t_i| < \delta$, ❑

Therefore, the following transitions are possible:

1. It is required that all parameters are continuously differentiable functions of time.

Point transitions:

$P(f,t_i) \in (0, 1) \rightarrow P(f,(t_i,t_{i+1})) \in (0, 1)$ (1)

$P(f,t_i) = 1 \rightarrow P(f,(t_i,t_{i+1})) \in (0, 1) \vee P(f,(t_i,t_{i+1}))=1 \vee P(f,(t_i,t_{i+1})) \in (1, \infty)$ (2)

$P(f,t_i) \in (1, \infty) \rightarrow P(f,(t_i,t_{i+1})) \in (1, \infty)$ (3)

(2) is ambiguous. Here, information about the derivative of P, especially its sign, can help. But first we have to prove the following lemma:

Lemma B: Let g(t) be a continuously differentiable function of time. If for a time point t_i $g(t_i) = k$ and $g'(t_i) > 0$ ($g'(t_i) < 0$) holds, then there exist an environment of t_i, (t_{i-1},t_{i+1}), $t_{i-1} < t_i < t_{i+1}$, with $g(t_{i-1},t_i) < k$ ($g(t_{i-1},t_i) > k$), and $g(t_i,t_{i+1}) > k$ ($g(t_i,t_{i+1}) > k$).

Proof B: Following proof A, there exist an environment (t_{i-1},t_{i+1}) with $g'(t) > 0$ ($g'(t) < 0$), $t \in (t_{i-1},t_{i+1})$. From this follows that $g(t_{i-1},t_i) < k$ ($g(t_{i-1},t_i) > k$), and $g(t_i,t_{i+1}) > k$ ($g(t_i,t_{i+1}) > k$).

$P(f,t_i) = 1 \wedge P'(f,t_i) < 0 \rightarrow P(f,(t_i,t_{i+1})) \in (0, 1)$ (2a)

$P(f,t_i) = 1 \wedge P'(f,t_i) = 0 \rightarrow P(f,(t_i,t_{i+1})) \in (0, 1) \vee P(f,(t_i,t_{i+1})) = 1$
$\vee P(f,(t_i,t_{i+1})) \in (1, \infty)$ (2b)

$P(f,t_i) = 1 \wedge P'(f,t_i) > 0 \rightarrow P(f,(t_i,t_{i+1})) \in (1, \infty)$ (2c)

(2a) and (2c) are valid due to lemma B. In the case of (2b), the second derivative would allow further discriminations. However, second derivatives are not considered[1]. Therefore, the ambiguities of (2b) cannot be removed.

Interval transitions:

$P(f,(t_{i-1},t_i)) \in (0, 1) \rightarrow P(f,t_i) \in (0, 1) \vee P(f,t_i) = 1$ (4)

$P(f,(t_{i-1},t_i)) = 1 \rightarrow P(f,t_i) = 1$ (5)

$P(f,(t_{i-1},t_i)) \in (1, \infty) \rightarrow P(f,t_i) = 1 \vee P(f,t_i) \in (1, \infty)$ (6)

Under certain conditions the first derivative of P can also help to remove ambiguities of interval transitions. If during time interval (t_{i-1}, t_i) the distance to landmark 1 is increasing or remaining constant, landmark, 1, will not have been reached at time point t_i. Whether the distance is increasing or decreasing can easily be deduced from the signs of $P(f,(t_{i-1},t_i))$ and $P'(f,(t_{i-1},t_i))$. The different variants shall not be listed. Instead the two classes “distance is decreasing” and “distance is not decreasing” are used.

1. In fact, the first derivative, P´(f,t), is not modeled either. But information about P´(f,t) can be gained from PQ(f´,t), as we will see later.

$P(t_{i-1},t_i) \in (0, 1) \wedge$ "distance is decreasing" $\rightarrow P(f,t_i) \in (0, 1) \vee P(f,t_i) = 1$ (4a)

$P(t_{i-1},t_i) \in (0, 1) \wedge$ "distance is not decreasing" $\rightarrow P(f,t_i) \in (0, 1)$ (4b)

$P(t_{i-1},t_i) \in (1, \infty) \wedge$ "distance is decreasing" $\rightarrow P(f,t_i) = 1 \vee P(f,t_i) \in (1, \infty)$ (6a)

$P(t_{i-1},t_i) \in (1, \infty) \wedge$ "distance is not decreasing" $\rightarrow P(f,t_i) \in (1, \infty)$ (6b)

We still have to deal with the situation that $f(t)-l_a = f_{ref}(t)-l_a = 0$. For a time point t_i with $f(t_i) = fref(t_i) = l_a$, restrictions for following or preceding time intervals can be made if $f'(t_i) > f_{ref}'(t_i) > 0$ or $f_{ref}'(t_i) > f'(t_i) > 0$ or $f'(t_i) < f_{ref}'(t_i) < 0$ or $f_{ref}'(t_i) < f'(t_i) < 0$. In the following, only the first case will be treated. The others can be handled in an analogous way.

<u>Lemma C</u>: If for a time point t_i $f'(t_i) > f_{ref}'(t_i) > 0$ holds, then there exists an environment of t_i, (t_{i-1},t_{i+1}), $t_{i-1} < t_i < t_{i+1}$, with $f(t_{i-1},t_i) < f_{ref}(t_{i-1},t_i) < 0$, and $f(t_i,t_{i+1}) > f_{ref}(t_i,t_{i+1}) > 0$.

<u>Proof C</u>: Let x be some value in $(f_{ref}'(t_i),f'(t_i))$, i.e. $f_{ref}'(t_i) < x < f'(t_i)$. Proof A states, that there exists an environment (t_{i-1},t_{i+1}) of t_i with $f_{ref}'(t) < x < f'(t)$, $t \in (t_{i-1},t_{i+1})$. That means, $f(t)$ increases faster than $f_{ref}(t)$ in the time interval (t_{i-1},t_{i+1}). Therefore, $f(t_{i-1},t_i) < f_{ref}(t_{i-1},t_i) < 0$, and $f(t_i,t_{i+1}) > f_{ref}(t_i,t_{i+1}) > 0$. ❑

<u>Remark</u>: If $f'(t_i) = f_{ref}'(t_i) = 0$, all possibilities for $f(t)$ and $f_{ref}(t)$ in the intervals (t_{i-1},t_i) and (t_i,t_{i+1}) have to be taken into account. These are: $f(t) < f_{ref}(t) < 0$, $f(t) > f_{ref}(t) > 0$, $f_{ref}(t) < f(t) < 0$, $f_{ref}(t) > f(t) > 0$, and $f(t) = f_{ref}(t) = 0$. Discontinuous deviations are not considered.

Now the basic proofs for PQ transitions have been given. Below, we state for each transition from which propositions the proof follows. PQ transitions refer to the parameter descriptions PQ(f,t), PQ(f´,t), sign(f,t), and sign(f´,t). Since our considerations above refer to P(f,t), P´(f,t), f(t), and $f_{ref}(t)$, one has to map from PQ values and signs to P values. For P´(f,t) this is not as obvious as for P(f,t). Therefore Table 9 gives some help.

Table 9: Mapping from PQ values and signs to the derivative of the corresponding P value

PQ(f,t)	PQ(f´,t)	sign(f(t))	sign(f´(t))	P´(f,t)
L	L	$- \vee +$	$- \vee +$	?
L	$N \vee H$	-	-	> 0
L	$N \vee H$	-	+	< 0
L	$N \vee H$	+	-	< 0
L	$N \vee H$	+	+	> 0

Table 9: Mapping from PQ values and signs to the derivative of the corresponding P value

PQ(f,t)	PQ(f´,t)	sign(f(t))	sign(f´(t))	P´(f,t)
N	L	-	-	< 0
N	L	+	-	> 0
N	L	-	+	> 0
N	L	+	+	< 0
L ∨ H	N	- ∨ +	0	= 0
N	L ∨ N ∨ H	0	- ∨ +	not defined
N	N	0	0	not defined
N	N	- ∨ +	- ∨ 0 ∨ +	= 0
N	H	-	-	> 0
N	H	+	-	< 0
N	H	-	+	< 0
N	H	+	+	> 0
H	L ∨ N	-	-	< 0
H	L ∨ N	-	+	> 0
H	L ∨ N	+	-	> 0
H	L ∨ N	+	+	< 0
H	H	- ∨ +	- ∨ +	?

Point Transitions

Point transition PDL follows from (1).

Point transition PDH follows from (3).

Point transition PALHDN follows from (2).

Point transition PAL follows from (1).

Point transition PAH follows from (3).

Point transition PANDLH1 follows for PQ(f´(t_i) = L and sign(f(t_i)) ≠ 0 from (2a),
for sign(f(t_i)) = 0 from Lemma C,
for PQ(f´(t_i) = H from (2c).

Point transition PANDLH2 follows for $PQ(f'(t_i) = H$ and $sign(f(t_i)) \neq 0$ from (2a),
for $sign(f(t_i)) = 0$ from Lemma C,
for $PQ(f'(t_i) = L$ from (2c).

Point transitions PANDN* describe the case, where a continuously differentiable function has a landmark value at a certain time point and it is not known in which direction it will move. Therefore, all possibilities must be taken into account. However, some constraints between the future values of the function and its derivative exist. The PANDN* transitions constitute an interface to discontinuous deviations and have to be extended for an integration of them.

Interval Transitions

Interval transition IDL follows from (4).

Interval transition IDN follows from (5).

Interval transition IDH follows from (6).

Interval transitions IAL, IAN and IAH refer to a constant function and constant reference function. Therefore, a deviation will be constant too.

The remaining interval transitions distinguish between an ending of the interval with or without a synchronization:

1. $Duration(t_i,t_j) = L$ means that due to the synchronization, the deviating value is compared to a reference value that corresponds to a later time point than normally.
2. $Duration(t_i,t_j) = N$ means that no synchronization takes place.
3. $Duration(t_i,t_j) = H$ means that due to the synchronization, the deviating value is compared to a reference value that corresponds to an earlier time point than normally.

If no synchronization takes place, the interval transitions directly follow from propositions (5), (4a), (4b), (6a), or (6b). Otherwise considerations of the following kind are necessary:

If $sign(f,(t_i,t_j]) = +$, and $sign(f',(t_i,t_j]) = +$, then
$P^*(f,t_j) = (f(t_j) - l_a) / (f_{ref}(t_j - \Delta t) - l_a) > P(f,t_j) = (f(t_j) - l_a) / (f_{ref}(t_j - \Delta t) - l_a)$.

That is, a synchronization with too high duration will increase the P value of an increasing, positive parameter. So, if its P value is N or H without synchronization, it will be H. But if its P value is L, the effect of the synchronization is ambiguous. Analogous considerations can be made for the other configurations of signs and synchronization type. In the following, we state on which propositions the remaining transitions are grounded, if no synchronization takes place.

Interval transitions IANDN1 and IANDN2 follow from (5).

Interval transitions IALDLN follows from (4b).

Interval transitions IALDNH follows from (4b).

Interval transitions IAHDLN follows from (6b).

Interval transitions IAHDNH follows from (6b).

Interval transitions IALDH follows from (4a).

Interval transitions IAHDL follows from (6a).

Interval transitions IALDL follows from (4a).

Interval transitions IAHDH follows from (6a).

B: Proofs of P Transition Rules

In Appendix B, the P transition rules, P== and I= are proven to be sound. While the proof of P== involves the solution of a differential equation, I= can be proven analogously to (5) of Appendix A.

Proof of P==

P== states that from

$$(P(f,t_i) = P(f',t_i) = P(f'',t_i) \wedge sign(f',t_i) \in \{-, +\}) \vee (P(f,t_i) = P(f'',t_i) \wedge sign(f',t_i) = 0) \quad (1)$$

follows

$$P(f,(t_i,t_j)) = P(f',(t_i,t_j)) = P(f'',(t_i,t_j)) \vee P(f',(t_i,t_j)) \neq P(f,(t_i,t_j)) \neq P(f'',(t_i,t_j)) \quad (2)$$

<u>Proof</u>:

In accordance with RSIM, the proof assumes a basis landmark of 0.

Since from $(a \Leftrightarrow b)$ follows $((a \wedge b) \vee (\neg a \wedge \neg b))$, it suffices to show that from (1) follows

$$P(f,(t_i,t_j)) = P(f',(t_i,t_j)) \Leftrightarrow P(f,(t_i,t_j)) = P(f'',(t_i,t_j)).$$

<u>First direction</u>: We have to show that from (1) and $P(f,(t_i,t_j)) = P(f',(t_i,t_j))$ follows $P(f,(t_i,t_j)) = P(f'',(t_i,t_j))$. For this direction can renounce (1).

$$P(f,t) = P(f',t), \quad t \in (t_i,t_j)$$

$$\Leftrightarrow \quad f(t) / f_{ref}(t) = f'(t) / f'_{ref}(t), \quad t \in (t_i,t_j)$$

$$P'(f,t) = (f'(t) * f_{ref}(t) - f(t) * f'_{ref}(t)) / (f_{ref}(t))^2, \quad t \in (t_i,t_j)$$

$$\Rightarrow \quad P'(f,t) = 0, \quad t \in (t_i,t_j) \quad (3)$$

$$f(t) = P(f,t) * f_{ref}(t)$$

$$\Rightarrow \quad f'(t) = P'(f,t) * f_{ref}(t) + P(f,t) * f_{ref}'(t)$$

Because of (3), the following holds

$$f'(t) = P(f,t) * f_{ref}'(t), \quad t \in (t_i,t_j)$$

$$\Rightarrow \quad f''(t) = P'(f,t) * f_{ref}'(t) + P(f,t) * f_{ref}''(t) = P(f,t) * f_{ref}''(t), \quad t \in (t_i,t_j)$$

$$\Rightarrow \quad P(f,t) = P''(f,t), \quad t \in (t_i,t_j) \quad \square$$

Second direction: We have to show that from (1) and $P(f,(t_i,t_j)) = P(f'',(t_i,t_j))$ follows $P(f,(t_i,t_j)) = P(f',(t_i,t_j))$.

$$P(f,t) = f(t) / f_{ref}(t)$$
$$\Leftrightarrow \quad f(t) = P(f,t) * f_{ref}(t)$$
$$\Rightarrow \quad f'(t) = P'(f,t) * f_{ref}(t) + P(f,t) * f_{ref}'(t) \qquad (4)$$
$$\Rightarrow \quad f''(t) = P''(f,t) * f_{ref}(t) + P'(f,t) * f_{ref}'(t) + P'(f,t) * f_{ref}'(t) + P(f,t) * f_{ref}''(t) \qquad (5)$$

$$P(f,(t_i,t_j)) = P''(f,(t_i,t_j)), \quad t \in [t_i,t_j)$$
$$\Rightarrow \quad f''(t) = P(f,t) * f_{ref}''(t), \quad t \in [t_i,t_j) \qquad (6)$$

From (5) and (6) follows

$$0 = P''(f,t) * f_{ref}(t) + 2 * P'(f,t) * f_{ref}'(t), \quad t \in [t_i,t_j)$$

Let $P'(f,t) = k(t)$:

$$0 = k'(t) * f_{ref}(t) + 2 * k(t) * f_{ref}'(t), \quad t \in [t_i,t_j)$$
$$\Rightarrow \quad k'(t) + 2 * f_{ref}'(t) / f_{ref}(t) * k(t) = 0, \quad t \in [t_i,t_j)$$

The solution of this differential equation is (cf. Boyce):

$$k(t) = c / \mu(t), \quad \mu(t) = \exp \int^{t} 2\, f_{ref}'(s) / f_{ref}(s)\, ds \qquad \text{(c is a constant)}$$

$$\Rightarrow \quad \mu(t = \exp (2 \ln (f_{ref}(t))) = 1 / f_{ref}(t)^2, \quad t \in [t_i,t_j)$$
$$\Rightarrow \quad k(t) = c / f_{ref}(t)^2 = P'(f,t), \quad t \in [t_i,t_j) \qquad (7)$$

(1) states that either $P(f,t_i) = P(f',t_i)$ or $sign(f',t_i) = 0$ holds. Above we have seen that $P'(f,t_i) = 0$ follows from $P(f,t_i) = P(f',t_i)$. On the other hand, $sign(f',t_i) = 0$ implies that $sign(f_{ref}',t_i) = 0$, since we don't allow discontinuous deviations. From this follows $P'(f,t_i) = 0$ too, i.e. $P'(f,t_i) = 0$ holds in both cases. Thus from (7) follows:

$$c / f_{ref}(t_i)^2 = 0, \quad f_{ref}(t_i) \neq 0$$
$$\Rightarrow \quad c = 0$$
$$\Rightarrow \quad P'(f,t) = 0, \quad t \in [t_i,t_j) \qquad (8)$$

From (4) and (8) follows:

$$f'(t) = P(f,t) * f_{ref}'(t), \quad t \in [t_i,t_j)$$
$$\Rightarrow \quad P(f,(t_i,t_j)) = P'(f,(t_i,t_j)). \square$$

Proof of I=

I= only constrains the relationship between $P(f,t_j)$ and $P(f',t_j)$, if f(t), $f_{ref}(t)$, f´(t), and $f_{ref}'(t)$ all are unequal to 0. But for this case P(f,t) and P(f´,t) are continuously differentiable. Therefore I= is an analogy to (5) of appendix A.❑

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A Simulator for Relative Descriptions[1]

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Abstract

This report deals with the qualitative simulation of physical systems based on descriptions relative to normal behavior. Relative descriptions are important because some kinds of system behavior cannot adequately be described by absolute descriptions. In particular, this is true for faulty behavior that is often viewed relative to the normal case. Most of the existing approaches use relative descriptions only to analyse static systems. In addition, a comparison of deviations in dynamic systems is not possible. In this report, a simulator, called RSIM, is presented, that predicts the effects of system deviations with a “less than normal” or “greater than normal” character. Aside from absolute descriptions, the deviations themselves and the resulting behavior are described with respect to the normal case, i.e. to a reference system and its reference behavior. The simulation of absolute behavior is carried out by a QSIM-like simulator, since the concepts of RSIM are oriented towards QSIM. RSIM can be viewed as an extension to QSIM.

In RSIM, deviations cannot only be described by “less than normal” or “greater than normal”, but additionally they can be compared with each other. In this way, a refined system description is achieved. Furthermore, some spurious behaviors are prevented on the relative description layer, and a more accurate prediction of behavior is possible.

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Contents

1. Introduction

The obvious attractiveness of qualitative simulations, their effectiveness, efficiency, and naturalness (compare [Struss 89]) has initiated a large amount of work in this field. The most influencing approaches are ENVISION [de Kleer, Brown 84], QPT [Forbus 84], and QSIM [Kuipers 86]. While relative descriptions have widely been used ([de Kleer 79], [Raiman 86], [Dague, Devès, Raiman 87], [Weld 87,88a,88b,90], [Downing 87], [Mavrovouniotis, Stephanopoulos 88], [Gallanti, Stefanini, Tomada 89], [Kockskämper, Neumann, et al. 93]), only few approaches deal with relative simulation of dynamical systems (e.g. [Weld 90])[1]. A relative simulation is valuable for system analysis and fault diagnosis. In this paper, concepts of relative simulation are described, and a relative simulator, called RSIM, is presented. RSIM's input and output, and some essential inference features are oriented towards QSIM. In fact, RSIM can be viewed as an extension to QSIM. Extending QSIM by RSIM leads to a refinement of input and output. In order to exploit existing QSIM techniques, RSIM is integrated into the system SLOD, which allows **s**imulation on different **l**ayers **o**f **d**escription. SLOD contains four description layers: sign and landmark descriptions, as QSIM, and two relative description layers belonging to RSIM.

The development of RSIM was initiated by the observation that there are kinds of faulty system behavior that cannot adequately be described by absolute descriptions. And even worse, a natural modeling of a faulty system and the corresponding correct system in some cases led to identical descriptions. Instead, terms like “less than normal” or “too high” characterized this sort of faulty behavior. Additionally, comparisons of too low or too high deviations turned out to be necessary. Therefore, we formalized these terms and developed an inference engine for them, the simulator RSIM. RSIM's modeling language offers a set of constraints that - due to relative descriptions - facilitates a more specialized system description than in usual simulators. Inferences about the relative duration of time intervals are carried out by RSIM's transition rules. Thus, the problems that were identified at the beginning of our work, are solved by the RSIM simulator.

The report is organized as follows: First we explain how qualitative simulation works and why relative descriptions are useful and necessary. Afterwards specific properties of simulation by relative values are discussed. In Chapter 3, the main part of the report, the relative simulator, RSIM, is presented. Finally some notes regarding future work are given. The appendix contains proofs of RSIM's transition rules.

1. [Neitzke 92a] gives an overview about relative descriptions in qualitative simulation.

1.1. How Does Qualitative Simulation Work?[1]

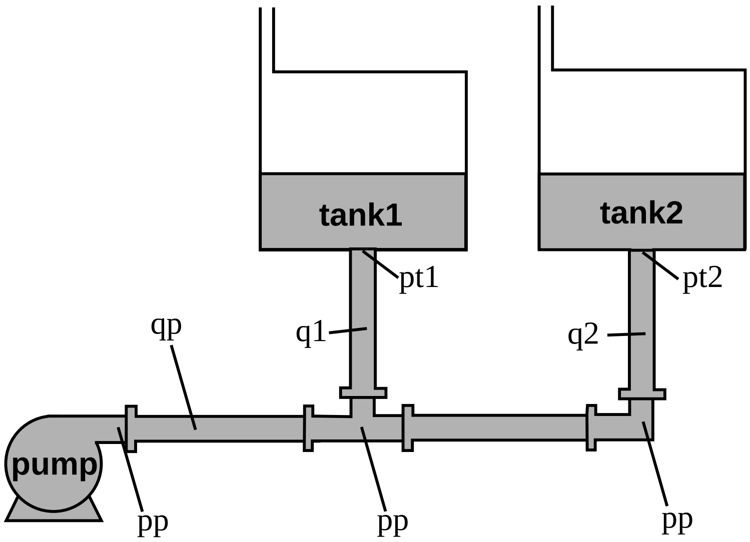
Qualitative simulation, as conventional numerical simulation, has the purpose to predict the behavior of a system. System behavior, however, is not described by exact numerical values but by qualitative descriptions. Qualitative descriptions have two characteristics: they are more general than quantitative descriptions and they are more imprecise. A qualitative system description does not concern a single system but a whole class of systems. Figure 1.1 shows a simple model for tank systems of two tanks and a pump. This and other tank systems will be used as examples throughout the report.

In a qualitative simulation, ambiguities arise in the prediction of behavior, so that in general not exactly one behavior is predicted, but a set of possible behaviors. One reason for ambiguities is that not all members of the simulated class of systems behave in the same way. In the 2-tank system above, for example, it cannot be decided which tank is full first. Another reason lies in the inexactness of qualitative descriptions. As a consequence of the latter not all of the predicted behaviors correspond to physically possible behavior. These impossible behaviors are called spurios behaviors. However, it can be guaranteed that every possible behavior is among the predicted behaviors.

While in a numerical simulation the state of a system is calculated at fixed equidistant time points, a qualitative simulation is guided by the occurrence of “interesting” events. In the kind of simulation we are concerned with, an interesting event occurs whenever a parameter reaches an interesting value (a so-called ***landmark value***). At each interesting event the system state, i.e. the qualitative values of all parameters, is recorded. Additionally, the system state is recorded for the time intervals lying between two events. So, the generation of behavior is a generation of a sequence, or - because of ambiguities - a tree of system states.

As input a simulator needs a system description and some information about its initial state, i.e. the values of some parameters. There are different ways to describe a system. We follow QSIM and describe a system by a set of constraints (compare Fig. 1.1a), i.e. a set of relations between the system parameters. The output of a simulation, the tree of system states, is generated in a cycle of two phases, called intrastate analysis and interstate analysis. Simulation starts with an intrastate analysis. In intrastate analysis, a set of completely described system states is derived from an incompletely specified system state. In QSIM-like simulators this is done by using the constraints of the system description (in a process called constraint propagation). For our example, an incomplete description of the initial state can be seen in Fig. 1.1b.

1. Excellent introductions to qualitative simulation are given by [Forbus 88] and [Struss 89]



(DERIV q1 vol1)	Flow *q1* is the derivative of volume of tank1 *vol1*.
(DERIV q2 vol2)	Flow *q2* is the derivative of volume of tank2 *vol2*.
(M+ pt1 vol1)	Pressure of tank1 *pt1* is an increasing function of *vol1*.
(M+ pt2 vol2)	Pressure of tank2 *pt2* is an increasing function of *vol2*.
(PRODUCT pf1 k1 q1)	"Friction pressure" of tank1 *pf1* is the product of friction coefficient *k1* and *q1*.
(PRODUCT pf2 k2 q2)	"Friction pressure" of tank2 *pf2* is the product of friction coefficient *k*2 and *q2*.
(SUM pp pt1 pf1)	The pressure of the pump *pp* is the sum of *pt*1 and *pf1*.
(SUM pp pt2 pf2)	The pressure of the pump *pp* is the sum of *pt2* and *pf2*.

Fig. 1.1a A simple model of a system of two tanks and a pump. For the sake of simplicity and reasons of demonstration, friction has only been modelled for vertical pipes and linearly dependent from flow.

vol1	**: 0**	Tank1 is empty.
vol2	**: 0**	Tank2 is empty.
pp	**: +**	Pressure of pump is positive.
k1	**: +**	Friction coefficient k1 is positive
k2	**: +**	Friction coefficient k2 is positive

Fig. 1.1b Initial information about the system's state.

In interstate analysis, an incomplete successor state is inferred from a complete system state. Interstate analysis works with continuity information about continuously differentiable functions. It is required that every system parameter is a continuously differentiable function of time. To determine the future course of a parameter, qualitative information about its derivative is needed. Therefore, a parameter is described by two numbers: its amount and its derivative. Figure 1.2 shows the tree of behaviors when the filling of the 2-tank system is simulated. The behavior tree is the result of a sign simulation, the simplest qualitative simula-

tion, working with just one landmark value, 0. Therefore, the only interesting event is reaching the landmark value 0.

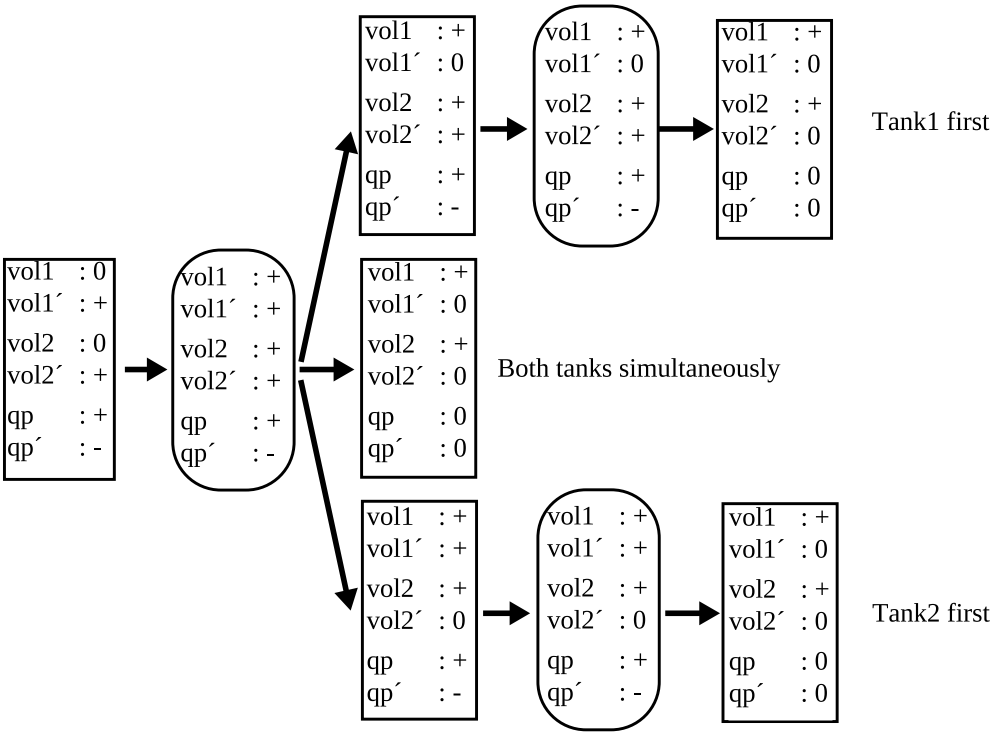


Fig. 1.2 Behavior tree for the 2-tanks system. For simplicity only three parameters have been listed. Time points are indicated by rectangles, time intervals by ovals.

1.2. Relative Descriptions and their Use[1]

1.2.1. Why Relative Descriptions?

There are kinds of system behavior that cannot adequately be described by absolute descriptions. Take for example a leaky water pipe. An adequate description of its behavior says that the output flow is less than the input flow. Or consider a pump that generates less pressure than in the normal case. The general meaning of “less” cannot be described by assigning absolute values, like signs or intervals between landmark values, to the parameters involved.

1.2.2. What are Relative Descriptions Used for?

Relative descriptions can be used for many different purposes in qualitative simulation. Our aim in using relative descriptions is to describe deviations with respect to a reference system and reference behavior. The deviations we are interested in are those that can be described as less than normal or greater than normal. Such descriptions can be used to characterize the

1. Section 1.2 overlaps with [Neitzke 92b] and [Neitzke 92c].

values of system parameters in a certain system state. Furthermore, they can be used to characterize the duration of a process taking place in a system. Before going into details, we have to take a closer look at the different types of deviations.

1.2.3. Types of Deviations

What are the characteristic features of deviations that can be described as less or greater than normal? Of course, not all deviations can be described in this way. Consider, for example, water in a pipe, that is flowing in the wrong direction, or an electrical charge that is positive, but normally negative. One property of less/greater-than-normal (LGTN) deviations is that they can be arbitrarily small. Weld calls arbitrarily small changes in the value of a parameter ***differential*** and more drastic changes ***non-differential***. The essential property of a differential change is that it could be arbitrarily smaller without falling into a qualitatively different area, i.e. no landmark value may lie between the deviating value and the normal value. Following Weld we call deviations that lie in the same qualitative area, i.e. have the same absolute description, as the normal value ***differential*** and deviations that lie outside this area ***non-differential***.

Unfortunately, the definition of differential does not capture all deviations that are called less or greater than normal. The voltage between the ports of a diode, for example, could be characterized as less than normal even if it is less than the diode's threshold voltage. Instead, expressions like less or greater describe deviations in the distance to a reference point, which we want to call ***basis landmark***[1]. If the deviating value lies on or beyond the basis landmark, as in the examples above, one cannot use the terms less or greater than normal. In most of all cases, the suitable basis landmark is given by the real number 0. But as the *quantity space* of {-, 0, +} in general corresponds to the regions $x < a$, $x = a$, and $x > a$, (where a is an arbitrary landmark value in the range of the parameter x (compare for example de Kleer)) the basis landmark may in principal be different from 0. We only require that basis landmark and landmark value, a, must be identical.

Definition: The deviation of a value v_1 from a value v_2 is called a ***continuous*** deviation with regard to a basis landmark l_a if v_1 is on the same side of l_a as v_2, that is $(v_1 < l_a \wedge v_2 < l_a) \vee (v_1 > l_a \wedge v_2 > l_a)$. Otherwise it is called a ***discontinuous*** deviation wrt l_a (Fig. 1.3).

1. The basis landmark may not be confused with the normal value, which we often call reference value.

Differential and continuous deviations are very similar. For quantity spaces with only one landmark like {-, 0, +} both classes are identical. In general, each differential deviation is a continuous deviation and each discontinuous deviation is a non-differential deviation.

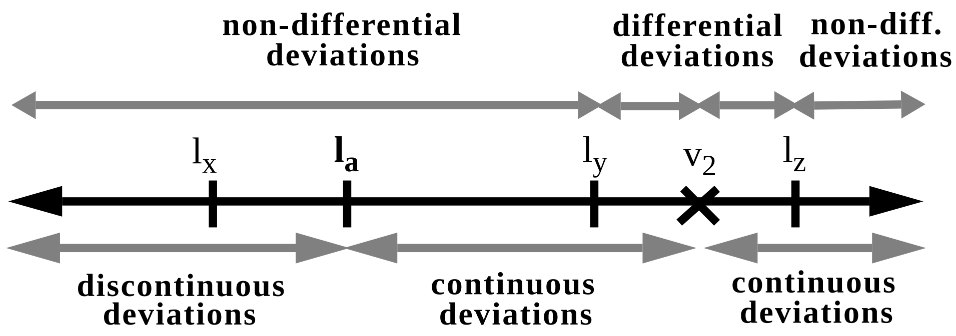


Fig. 1.3 Differential, non-differential, continuous and discontinuous deviations wrt l_a from a value v_2.

1.2.4. A Relative Description that allows Comparison of Deviations

The way and the extent a value v_1 differs from a normal value v_2 can be described by the quotient of the distances of v_1 and v_2 from the basis landmark. We call this quotient P-value.

Definition: $$P(f,t) = \frac{f(t) - l_a}{f_{ref}(t) - l_a}$$

f is a continuously differentiable function of time, f_{ref} is the corresponding reference function, l_a is f's basis landmark in the range of f. Deviations from a normal value that lie beyond l_a cannot be described as too-low or too-high.

We call the values of the function P ***P values***. Note, that P values are relative, but not qualitative. However, we are only interested in qualitative properties of P values. Therefore, we don't need exact values for the function P. The following qualitative areas of P values are relevant for us:

$P(f,t) > 1$	The distance of the deviating value to the basis landmark is greater than the normal value's distance. Usually, such deviations are called "too high".
$P(f,t) = 1$	There is no deviation.
$0 < P(f,t) < 1$	The distance of the deviating value to the basis landmark is less than the normal value's distance. Usually, such deviations are called "too low".
$P(f,t) = 0$	The deviating value lies on the basis landmark. This deviation is discontinuous. It can be expressed by absolute descriptions.

$P(f,t) < 0$	Deviating value and normal value lie on different sides of the basis landmark. This deviation is discontinuous. It can be expressed by absolute descriptions.

Now we want to define, what we exactly mean by “too low”, “normal” and “too high” by the function PQ. We call the values of PQ ***PQ values***.

$PQ(f,t) = \textit{too-low}$ ::= $0 < P(f,t) < 1$

$PQ(f,t) = \textit{normal}$ ::= $P(f,t) = 1 \vee f(t) - l_a = f_{ref}(t) - l_a = 0$

$PQ(f,t) = \textit{too-high}$ ::= $P(f,t) > 1$

As an important extension of the above mentioned qualitative descriptions that are based on fixed intervals, we introduce comparisons of P values. For example, the P value of the output flow of a leaky pipe is always less than the P value of the input flow (provided that there is a flow). Besides the relations <, =, > the reciprocal relation can be useful too because if the product of a *too-low* value and a *too-high* value results in a *normal* value, the P value of the first factor is the reciprocal of the second. Comparison of P values gives us a handle to refine qualitative system descriptions and behaviors, to distinguish more specific classes of constraints between system parameters, and to diminish some unwanted spurious behaviors.

These two qualitative descriptions, PQ values and relations between P values, shall be the basis for our relative simulation. Before we come to the corresponding inference machine, the relative simulator RSIM, we want to examine the properties of a relative simulation in general.

2. Simulating with Relative Descriptions

As we have seen, a qualitative simulator produces a behavior description from a system description and information about the systems initial state. A relative simulator, as a special qualitative simulator, works the same way: As input it receives a system description that contains the deviations from a reference system, and as output it predicts the possible system behaviors, using descriptions that contain the deviations from the reference behavior. Of course, the input of our relative simulator must have this LGTN character we are interested in. But does it always follow from such input that an output has this character too?

2.1. Describing Deviations from a Reference System

The kind of qualitative simulators we deal with (the QSIM-type) gets as input a system description consisting of constraints between the system parameters (Fig. 1.1a) as well as information about the initial state of the system in form of some parameter values (Fig. 1.1b). We can distinguish three kinds of deviations from the reference system with a LGTN character:

1. Differential deviations of initial parameter values
2. Continuous deviations of initial parameter values
3. Deviations in the system description

2.1.1. Differential Deviations of Initial Parameter Values

Differential deviations cannot be expressed with absolute descriptions by definition, i.e. the absolute description of deviating parameters does not change. But, if there is no difference in the input concerning absolute descriptions, there cannot be any difference in the output. That is, the set of predicted behaviors is identical for the deviating system and for the reference system as long as one focusses on absolute descriptions. This does not mean that a differential deviation in the initial state cannot lead to changes of the absolute behavior of a system. But both, the reference behavior and the deviating behavior must be elements of the common set of absolute behaviors. Consider the 2-tank system of Fig. 1.1. If the friction coefficient, *k2*, of tank2 is higher than normal, then filling tank2 takes more time, and therefore, the order of interesting events may change. How are the three possible behaviors of the reference system (Fig. 1.2) affected, if the friction coefficient of tank2 is too high?

> First behavior (tank1 is full at first): The volume of tank2 will be too low when tank1 is full. However, the absolute description does not change: At first tank1 will be full and afterwards tank2.
>
> Second behavior (both tanks are full simultaneously): Tank1 will be the first. The deviating behavior is characterized by the first behavior. The absolute description of

behavior has changed. Weld calls this a change in the ***behavioral topology*** [Weld 87,88b,90].

Third behavior (tank2 is full at first): It cannot be decided whether the deviation of the friction coefficient is so heavy that now tank1 is the first to be full, or whether both tanks are full simultaneously, or whether tank2 still is the first. Thus each behavior of the tree may be a consequence of the deviation. That is, there exist solutions with and without a change in the behavioral topology.

2.1.2. Continuous Deviations of Initial Parameter Values

In contrast to differential deviations, continuous deviations allow changes of absolute descriptions. Therefore, the resulting absolute behavior can change too. Continuous deviations imply that the basis landmark of the deviating parameter is not reached. Since we require the identity of basis landmark and the landmark value defining the sign quantity space, continuous deviations cannot take place on signs. So a projection of absolute descriptions onto signs leads to identical behavior sets for the deviating and the reference system. That is, in principle one can transform continuous deviations into differential deviations by leaving out some landmark. However, there are situations where it is not adequate to do this. Suppose, we want to model the value of tank pressure, at the moment when the valve of a tank closes, as a special landmark value in the tank pressure quantity space. There are situations, where a pressure above, at, and below this landmark value must be considered as too high. Fig. 2.1 shows another example of a continuous deviation. The deviating behavior of Fig. 2.1a and the corresponding reference behavior of Fig. 2.1b describe the emptying of a tank. The quantity space of the tank level contains three landmark values, *0*, *half-full* and *full*. In the reference behavior, the initial value of the tank level is *half-full*, while in the deviating behavior it is *full*, i.e. the tank level is too high. The topology of the behaviors is different, but the deviations are continuous, because all amounts and derivatives keep their signs. (The problems concerning the different duration of the behaviors are discussed below.)

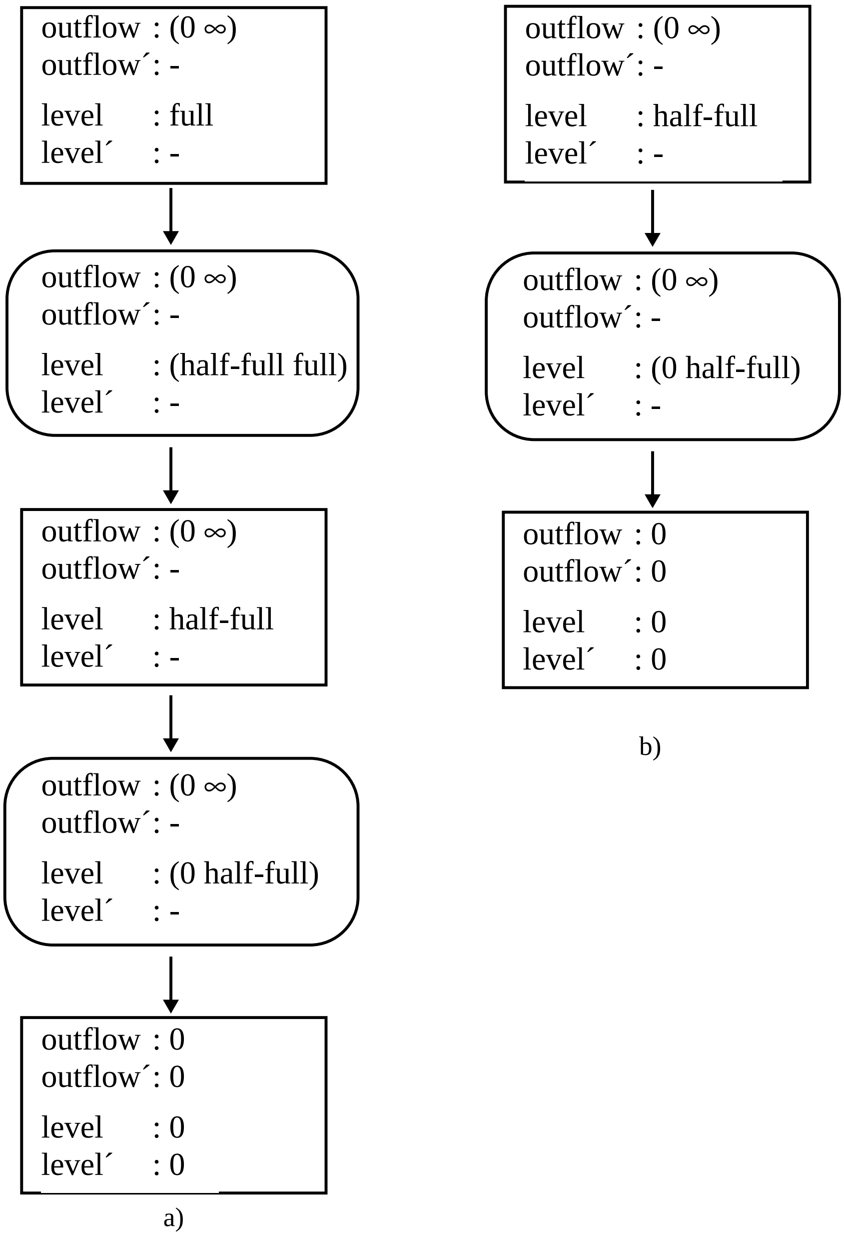


Fig. 2.1 Behaviors for emptying a tank. a) Deviating behavior
b) Reference behavior

2.1.3. Deviations in the System Descriptions

In general, changes in the system descriptions lead to plainly different behaviors, so that a comparison via less/greater descriptions is not possible. However, there exist some changes that have no significance on absolute descriptions but on relative descriptions. Fig. 2.2 shows the constraints used by the original QSIM. While ADD, MULT, MINUS and DERIV are "exact" constraints, M^+, M_0^+, M^-, and M_0^- have a qualitative character. They each refer to a

whole class of functional relationships. It would bring no advantages for absolute descriptions to distinguish between subclasses of M*, because the qualitative definitions of the corresponding constraints would be identical. The definition of P values, however, makes it possible to differentiate between some subclasses. On the one hand one can distinguish

ADD(f,g,h)	$:\Leftrightarrow f(t) + g(t) = h(t)$
MULT(f,g,h)	$:\Leftrightarrow f(t) * g(t) = h(t)$
MINUS(f,g)	$:\Leftrightarrow f(t) = - g(t)$
DERIV(f,g)	$:\Leftrightarrow f'(t) = g(t)$
$\mathbf{M^+(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0$
$\mathbf{M_0^+(f,g)}$	$:\Leftrightarrow M^+(f,g) \wedge H(0) = 0$
$\mathbf{M^-(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0$
$\mathbf{M_0^-(f,g)}$	$:\Leftrightarrow M^-(f,g) \wedge H(0) = 0$

Fig. 2.2 Constraints of QSIM. The description has been adopted from [Struss 89]

between linear, overlinear, and underlinear relationships. This feature will be discussed below. On the other hand one can express faulty relationships, for example a M_0^+ relationship with a greater-than-normal gradient. Fig. 2.3 shows the sign and PQ tupels of the standard M_0^+

M_0^+

Sign tupels: (- -)
(0 0)
(+ +)

PQ tupels:

If sign tupel is (- -) or (+ +):
(too-low too-low)
(normal normal)
(too-high too-high)

If sign tupel is (0 0):
(normal normal)

M_0^+TOO-STEEP

Sign tupels: (- -)
(0 0)
(+ +)

PQ tupels:

If sign tupel is (- -) or (+ +):
(too-low too-low)
(normal too-low)
(too-high too-low)
(too-high normal)
(too-high too-high)

If sign tupel is (0 0):
(normal normal)

Fig. 2.3 Sign and PQ tupels of the standard M_0^+ constraint and the M_0^+TOO-STEEP constraint. The sign tupels of both are identical.

constraint and a M_0^+ constraint with too-high gradient, called M_0^+TOO-STEEP. What are the consequences of such relative deviations in the system description? First, the set of absolute behaviors does not change, since the absolute system description does not change. And second, a change of the behavioral topology is possible. If, for example, two parameters *x* and *y* are related by a M_0^+TOO-STEEP relationship, (M_0^+TOO-STEEP *x y*), and normally reach

certain landmark values at the same time, then in the deviating system, *x* will reach its landmark value before *y*.

2.2. Comparison of Behaviors

In the last chapter, we have seen that under deviations with a LGTN character in the system description or the initial conditions, the set of possible absolute behaviors, or at least the set of possible sign behaviors, does not change. Additionally, we have seen that changes of the behavioral topology are possible. So, when a deviating behavior is compared to a reference behavior, two kinds of comparisons can be distinguished: comparisons with and without a change in the behavioral topology. Let us first consider the easier case:

2.2.1. Comparisons without a Change in the Behavioral Topology

No change in the behavioral topology means no change in the absolute behavior of the system. That is, for every state of the path of deviating behavior there exists a corresponding state in the reference behavior with identical absolute descriptions, and vice versa. All possible deviations are differential: parameter values of certain states may be too low or too high (or normal), and the duration of an interval state may be too low or too high (or normal) too[1]. The comparison of behaviors is a comparison of each state of the deviating behavior with the corresponding state of the reference behavior.

The comparison of point states is simple. For each parameter it must be stated, whether its amount and derivative are too low, normal or too high with respect to the reference state. The comparison of interval states is more complicated. Since the duration of an interval may change, either the deviating interval or the reference interval will end before the other. So a comparison with respect to time cannot concern both intervals in full length, unless the rest of the longer interval would be compared to states following the shorter interval. To cope with this difficulty, Weld has introduced the concept of ***perspective*** [Weld 87,88b,90]. A comparison of the values of a parameter x under the perspective of a parameter y means that those values of x are compared with each other that belong to the same values of y. Using appropriate parameters as perspectives often leads to simpler descriptions than using the standard perspective of time. Weld shows that relevant statements about the duration of an interval and the magnitude of values compared to reference values are possible through working with different perspectives. The corresponding method, Weld's ***DQ analysis***, determines analytically and not by simulation the consequences of differential deviations. The concept of perspective facilitates elegant and effective descriptions of differential deviations. In this paper, we show that a simulation assuming the standard case of time as perspective yields these results too, without having the problem of incompleteness of DQ analysis (see [Weld 90]). Additionally, due to higher exactness that is gained by relations between P values, some ambiguities can be avoided and more problems can be solved.

1. Deviations of the duration of an interval must be differential, because the duration is always positive and further absolute information is not given.

2.2.2. Comparisons with Changes in the Behavioral Topology

If the behavioral topology changes, the deviations become non-differential. However, LGTN descriptions still can be used, if the deviations are continuous. So one has to distinguish between continuous and discontinuous changes of the behavioral topology. An example of continuous, non-differential changes is given by Fig. 2.1a. Continuous changes of the behavioral topology can be handled in a similar way as differential changes of behavior (dealed with in the previous section) because the sign behavior does not change. Differences are related to the fact, that the mapping between the states of the deviating and the reference behavior in contrast to differential changes is not bijective (because of their different number). Further details will be explained in Section 3.5. In the following we will focus on discontinuous changes.

Discontinuous Changes of the Behavioral Topology

Discontinuous changes of the behavioral topology occur if the deviating and the reference behavior correspond to different paths of the behavior tree. The point where deviating and reference behavior split is a point of ambiguities. In principle at each branching of the behavior tree the deviating and the reference behavior can split. There are two types of branchings and correspondingly two types of topology changes. One reason for a branching is that it cannot be decided which event out of a number of possible ones will occur first. Every order of events must be taken into account. This type of branching is called ***occurrence branching*** (see e.g. [Fouché,Kuipers 91]). The other reason for a branching is that it cannot be decided in which direction the amount or derivative of a parameter moves. In this case, all possible developments, i.e. increase, decrease, or constancy, must be considered.

In the case of discontinuous topology changes, a sensible mapping between deviating and reference system states in general is not possible. But if the topology changes because of occurrence branching, parameter histories[1] still can be compared. In occurrence branching, the order of parameter-specific events changes, but the absolute descriptions of parameter histories themselves do not. The behavior tree of Fig. 1.2 is an example for occurrence branching.

But if the reason for branching is, that the further course of a parameter cannot be determined, the parameter history itself can change. For example, this type of branching happens, if in a system of coupled tanks it cannot be decided in which direction the water will flow. Now, the last chance of comparisons via LGTN descriptions is restricted to parameter histories that have not changed. But the essential characteristic of the deviation, the change of some parameter histories, cannot be described by this means.

1. A parameter history is the qualitative course of a parameter value.

2.3. Summary

If a deviating system shall be simulated, the deviations from the reference system can be expressed by changing the description of the reference system, i.e. the model, or by changing the initial parameter values of the reference system. Model changes with a LGTN character are restricted to the M* constraints. Deviations of the initial parameter values are subdivided into differential and continuous deviations. Differential deviations of the initial parameter values may cause changes of the behavioral topology, but the set of predicted behaviors does not change with respect to absolute descriptions. Continuous deviations, on the other hand, may cause changes of the set of absolute behaviors.

If a deviating behavior has to be compared to a reference behavior, the comparison may or may not have to deal with changes of the behavioral topology. If there is no change in the behavioral topology, a state-wise comparison is possible. The comparison has to determine for each state whether the parameter values are too low, normal, or too high, and it has to determine for each interval state whether the duration of the interval is too low, normal, or too high. Changes of the behavioral topology are subdivided into continuous and discontinuous changes. Continuous changes of the behavioral topology can be handled in a similar way as if no changes of the topology exist. Concerning discontinuous changes of topology, one has to distinguish between changes because of occurrence branching and changes because of ambiguities in the further course of a parameter. If the topology changes because of occurrence branching, LGTN description have to be restricted to parameter histories. For the other type of topology changes, no sensible comparison via LGTN descriptions is possible.

3. The Relative Simulator RSIM

In this chapter the relative simulator RSIM is presented. RSIM's input and output, and some essential inference features are oriented towards QSIM. In fact, RSIM can be viewed as an extension to QSIM. Extending QSIM by RSIM leads to a refinement of input and output. In order to exploit existing QSIM techniques, RSIM is integrated into a system, SLOD, which allows **s**imulation on different **l**ayers **o**f **d**escription. SLOD contains four description layers: sign and landmark descriptions, as QSIM, and two relative description layers belonging to RSIM.

In the following, the concepts of RSIM are explained. First, we will deal with system descriptions and state descriptions in RSIM. After that the two essential parts of a qualitative simulation, intrastate and interstate analysis, are explained. Finally we give some examples of output. But before we come to the concepts of RSIM, some words about SLOD have to be said.

3.1. Simulating on Different Layers of Description

In qualitative simulation, a parameter is usually described by two numbers: its amount and its derivative. The derivative carries dynamical information so that the future course of the amount can be computed. In SLOD, the different ways of describing a number qualitatively are represented in description layers. SLOD has four description layers: a coarse and a fine absolute layer, and a coarse and a fine relative layer.[1] SLOD's description layers are listed below.

Absolute layers

Sign layer: The simplest qualitative information about a real number is its sign. The sign layer is the basic description layer for the other layers.

Qval layer: On the qval layer the real number line or a part of it is divided into regions of interest by so-called landmark values. The landmark values themselves and the open intervals between them form a quantity space. The sign layer can be seen as a special qval layer with landmark value 0. In contrast to the other description layers, on the qval layer only the amount of a parameter is described, but not its derivative.[2]

1. In principal, more description layers can be added.

2. The name *qval* has been taken from QSIM since the qvals in SLOD have the same properties as in QSIM (totally ordered set of landmark values, dynamical generation of landmarks possible).

Relative description layers (belonging to RSIM):

PQ-layer: The PQ layer works with three qualitative values: *too-low, normal*, *too-high*. The semantics of PQ values has been explained in Section 1.2.4.

P-layer: On the P layer, relations between P values are collected. So far, RSIM deals with the relations *less*, *greater* and *equal*. In some few cases the reciprocal relation would be a helpful extension.

A description layer can be activated or deactivated. Therefore, depending on the status of description layers, simulation can produce different outputs. In the simplest case a pure sign simulation takes place. However, some dependencies between the description layers have to be taken into account (Fig. 3.1). The sign layer is required by all the other description layers, so it must not be deactivated. Besides that, the P layer needs the PQ layer.

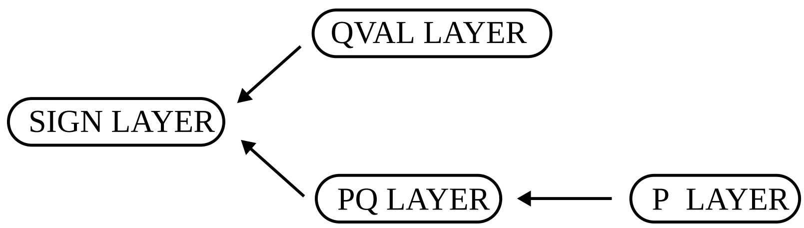


Fig. 3.1 Dependencies between description layers.

Each description layer performs its own intrastate and interstate analysis. The techniques of sign and qval layer are similar to existing simulators, in particular to QSIM, and therefore are not discussed further on.

3.2. System Description in RSIM

As in QSIM, a system is described in RSIM by a number of constraints. Aside from QSIM's constraint classes (Fig. 2.2) some specializations of the M* constraints are possible (Fig. 3.2). Two types of specializations can be distinguished. On one hand a faulty M* relationship can be expressed by the M*TOO-FLAT and M*TOO-STEEP constraints. They concern an M* relationship that is too flat or too steep on every point. For example, if the breather tube of a tank is clogged (the tanks of Fig. 1.1a and Fig. 3.3 have breather tubes), the air in the tank is compressed when filling the tank. From this follows, that the static pressure corresponding to a certain volume of water is greater than normal. The relationship between static pressure and volume level is M^+_0TOO-STEEP. A model of a tank with a clogged breather tube can be seen in Fig. 3.3.

On the other hand, an M* relationship can be stated more precisely by using the LINEAR*, UNDERLINEAR* or OVERLINEAR* constraints. The specializations of M* constraints have different definitions only on relative description layers. On absolute layers they have identical definitions. Under the assumption of straight tank sides, the relationship between static pressure and volume in a correct tank is $\text{LINEAR}^{+}{}_{0}$.

ADD(f,g,h)	$:\Leftrightarrow f(t) + g(t) = h(t)$
MULT(f,g,h)	$:\Leftrightarrow f(t) * g(t) = h(t)$
MINUS(f,g)	$:\Leftrightarrow f(t) = - g(t)$
DERIV(f,g)	$:\Leftrightarrow f'(t) = g(t)$
$\mathbf{M^{+}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge PQ(H'(x)) = \text{normal}$
$\mathbf{M^{+}TOO\text{-}FLAT(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge PQ(H'(x)) = \text{too-low}$
$\mathbf{M^{+}TOO\text{-}STEEP(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge PQ(H'(x)) = \text{too-high}$
$\mathbf{M_0{}^{+}(f,g)}$	$:\Leftrightarrow M^{+}(f,g) \wedge H(0) = 0$
$\mathbf{M_0{}^{+}TOO\text{-}FLAT(f,g)}$	$:\Leftrightarrow M^{+}\text{TOO-FLAT}(f,g) \wedge H(0) = 0$
$\mathbf{M_0{}^{+}TOO\text{-}STEEP(f,g)}$	$:\Leftrightarrow M^{+}\text{TOO-STEEP}(f,g) \wedge H(0) = 0$
$\mathbf{M^{-}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge PQ(H'(x)) = \text{normal}$
$\mathbf{M^{-}TOO\text{-}FLAT(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge PQ(H'(x)) = \text{too-low}$
$\mathbf{M^{-}TOO\text{-}STEEP(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge PQ(H'(x)) = \text{too-high}$
$\mathbf{M_0{}^{-}(f,g)}$	$:\Leftrightarrow M^{-}(f,g) \wedge H(0) = 0$
$\mathbf{M_0{}^{-}TOO\text{-}FLAT(f,g)}$	$:\Leftrightarrow M^{-}\text{TOO-FLAT}(f,g) \wedge H(0) = 0$
$\mathbf{M_0{}^{-}TOO\text{-}STEEP(f,g)}$	$:\Leftrightarrow M^{-}\text{TOO-STEEP}(f,g) \wedge H(0) = 0$
$\mathbf{LINEAR^{+}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge H''(x) = 0$
$\mathbf{LINEAR_0{}^{+}(f,g)}$	$:\Leftrightarrow \text{LINEAR}^{+}(f,g) \wedge H(0) = 0$
$\mathbf{LINEAR^{-}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge H''(x) = 0$
$\mathbf{LINEAR_0{}^{-}(f,g)}$	$:\Leftrightarrow \text{LINEAR}^{-}(f,g) \wedge H(0) = 0$
$\mathbf{UNDERLINEAR^{+}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge H''(x) < 0$
$\mathbf{UNDERLINEAR_0{}^{+}(f,g)}$	$:\Leftrightarrow \text{UNDERLINEAR}^{+}(f,g) \wedge H(0) = 0$
$\mathbf{UNDERLINEAR^{-}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge H''(x) > 0$
$\mathbf{UNDERLINEAR_0{}^{-}(f,g)}$	$:\Leftrightarrow \text{UNDERLINEAR}^{-}(f,g) \wedge H(0) = 0$
$\mathbf{OVERLINEAR^{+}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) > 0 \wedge H''(x) > 0$
$\mathbf{OVERLINEAR_0{}^{+}(f,g)}$	$:\Leftrightarrow \text{OVERLINEAR}^{+}(f,g) \wedge H(0) = 0$
$\mathbf{OVERLINEAR^{-}(f,g)}$	$:\Leftrightarrow f(t) = H(g(t)) \wedge H'(x) < 0 \wedge H''(x) < 0$
$\mathbf{OVERLINEAR_0{}^{-}(f,g)}$	$:\Leftrightarrow \text{OVERLINEAR}^{-}(f,g) \wedge H(0) = 0$

Fig. 3.2 Constraints of RSIM.

```
(DERIV inflow volume)
(SIGNED-SQUARE inflow2 inflow)
(PRODUCT p-pressure c-frict inflow2)
(M+0TOO-STEEP p-static volume)
(SUM p p-pressure p-static)
```

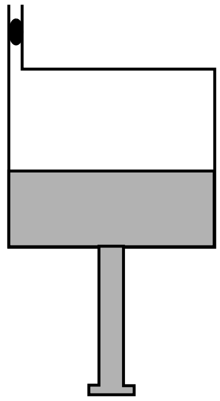


Fig. 3.3 A model for a tank with clogged breather tube. C-frict is a friction coefficient. The SIGNED-SQUARE constraint-class has been added to the basic constraints of Fig. 3.2. It is defined as follows:
SIGNED-SQUARE(f,g) : ⇔ f(t) = g(t) * |g(t)|

3.3. State Descriptions in RSIM

A system state is characterized by all parameter values of a certain time point or time interval. Compared with QSIM, state descriptions in RSIM additionally contain PQ values of the parameter amounts and derivatives, and relations between their P values. In addition the duration for reaching a certain point state is described by a PQ value. Fig. 3.4 gives an example for a state that describes an interval during the filling of the clogged tank. While the parameters on the sign, qval and PQ layers are described by qualitative values, qualitative information on the P layer is expressed by relations.

sign(volume) = +
sign(volume´) = +
qval(volume) = (0 half-full)
pq(volume) = normal
pq(volume´) = normal

sign(inflow) = +
sign(inflow´) = 0
qval(inflow) = (0 ∞)
pq(inflow) = normal
pq(inflow´) = normal

sign(inflow2) = +
sign(inflow2´) = 0
qval(inflow2) = (0 ∞)
pq(inflow2) = normal
pq(inflow2´) = normal

sign(c-frict) = +
sign(c-frict´) = 0
qval(c-frict) = c-frict*
pq(c-frict) = normal
pq(c-frict´) = normal

sign(p-pressure) = +
sign(p-pressure´) = +
qval(p-pressure) = (0 ∞)
pq(p-pressure) = normal
pq(p-pressure´) = normal

sign(p-static) = +
sign(p-static´) = +
qval(p-static) = (p-half-full p-full)
pq(p-static) = too-high
pq(p-static´) = too-high

sign(p) = +
sign(p) = +
qval(p) = (0 ∞)
pq(p) = too-high
pq(p) = too-high

p(p) < p(p-static)
p(p´) < p(p-static´)

Fig. 3.4 An interval state of the process of filling a tank with clogged breather tube. All parameters are described with sign, qval, and PQ values. Additionally some relations between P values can be stated.

3.4. Intrastate Analysis of RSIM

In intrastate analysis, constraint propagation takes place. As on the sign layer, the constraints on the pq layer are described by listing the possible tupels of values. Tables 1 and 2 show the definitions of the SUM and PRODUCT constraints. The left column contains the respective

PQ tupels and the right column assertions about relationships between the corresponding P values. The relationships between P values refine the information given by a PQ tupel. The tupels of the SUM-PQ-relation and the PRODUCT-PQ-relation are identical, but there are differences on the P layer. For example, if a too high value is added to or multiplied with a normal value, sum and product are both too high. But while the sum is less too high than the too high summand, the product is exactly as high as the too high factor.

If a certain PQ tupel is established during constraint propagation, the corresponding P assertions are recorded. On the P layer, the main task is to collect relationships between P values and to generate a consistent graph of P values. In this process, transitivity and symmetry have to be taken into account. Since RSIM uses the relations *less*, *equal* and *greater*, the graph represents a partial order of sets of P values. Each set consists of P values of the same magnitude. Constraining on the P layer means increasing the degree of order between the P values of all parameters. It is not the aim to reach a total order. Generating a total order would still entail ambiguities. The additional information on the P layer facilitates to avoid spurious behavior on the PQ layer. That is, if a system state is completely described on the PQ layer, it may happen that the corresponding P assertions are not consistent, for example because of cycles in a path of *less* relationships.

Table 1: Part of the definition of the SUM constraint

SUM-PQ-Relation ⊂ PQ(f) × PQ(g) × PQ(h) (SUM(f,g,h) ⇔ f(t)=g(t)+h(t))	Corresponding P assertions
(too-low, too-low, too-low)	
(too-low, too-low, normal)	P(g) < P(f)
(too-low, too-low, too-high)	P(g) < P(f)
(normal, too-low, too-high)	
(too-high, too-low, too-high)	P(f) < P(h)
(too-low, normal, too-low)	P(h) < P(f)
(normal, normal, normal)	
(too-high, normal, too-high)	P(f) < P(h)
(too-low, too-high, too-low)	P(h) < P(f)
(normal, too-high, too-low)	
(too-high, too-high, too-low)	P(f) < P(g)
(too-high, too-high, normal)	P(f) < P(g)
(too-high, too-high, too-high)	

Table 2: Part of the definition of the PRODUCT constraint

PRODUCT-PQ-Relation $\subset$ PQ(f) $\times$ PQ(g) $\times$ PQ(h) (PRODUCT(f,g,h) $\Leftrightarrow$ f(t)=g(t)*h(t))			Corresponding P assertions
(too-low,	too-low,	too-low)	P(f) < P(g) $\wedge$ P(f) < P(h)
(too-low,	too-low,	normal)	P(f) = P(g)
(too-low,	too-low,	too-high)	P(g) < P(f)
(normal,	too-low,	too-high)	
(too-high,	too-low,	too-high)	P(f) < P(h)
(too-low,	normal,	too-low)	P(f) = P(h)
(normal,	normal,	normal)	
(too-high,	normal,	too-high)	P(f) < P(h)
(too-low,	too-high,	too-low)	P(h) < P(f)
(normal,	too-high,	too-low)	
(too-high,	too-high,	too-low)	P(f) < P(g)
(too-high,	too-high,	normal)	P(f) = P(g)
(too-high,	too-high,	too-high)	P(g) < P(f)$\wedge$ P(h) < P(f)

3.5. Interstate Analysis in RSIM

In interstate analysis, transition rules are applied. A transition deduces information about the parameter values of successor states. Usually continuity information about continuously differentiable functions is used for this. Two types of transitions are distinguished: point transitions and interval transitions. A point transition is applied on point states and infers information about the following interval state. An interval transition works in the corresponding way. In contrast to absolute transitions, a relative transition can infer information about the duration of the interval.

3.5.1. Transitions on the PQ Layer

In the current version of RSIM, PQ transitions do not consider discontinuous changes of the behavioral topology (see Section 2.1.1). Therefore, they just use the values *too low*, *normal*, and *too high*.[1] PQ transitions, as sign transitions or qval transitions, like those defined in

[Kuiper 86], refer to the values of exactly one parameter. Although there are analogies between sign transitions and PQ transitions, the following differences have to be mentioned:

- In PQ transitions, information about the duration of a time interval can be deduced. This information is of relative nature and expressed by the values *too low*, *normal*, *too high*.
- In sign transitions (and qval transitions), qualitative information about the derivative is used to determine the further course of the amount. In PQ transitions, as they are formulated below, the PQ value of the derivative is used instead of qualitative information about the derivative of a P value. That is (for a basis landmark of 0):

$$PQ(f',t) = \frac{f'(t)}{f_{ref}'(t)} \quad \text{instead of} \quad P'(f,t) = \left(\frac{f(t)}{f_{ref}(t)}\right)'$$

- Because of the definition of P values, a PQ transition depends not only on the PQ values of amount and derivative of a parameter, but additionally on their signs. For example, if a parameter's amount is too low and its derivative is too high, the amount would come closer to normal if both, amount and derivative, are positive (or negative). But if the derivative is negative and the amount positive (or vice versa) the amount would become more and more too low.

Point Transitions

Table 3 shows the point transitions of the PQ layer. Information about the parameter values at a time point ti is used to determine the values in the following time interval. The first three transitions determine the derivative of the following interval, the next four transitions the amount. The remaining transitions handle the ambiguous case of normal amount and normal derivative at time point ti. For example the transition PANDN2 can be read in the following way:

> *If at some time point ti, the PQ value of a parameter's amount is normal and the PQ value of its derivative is normal too, and if the sign of its amount is positive and of its derivative negative or vice versa then in the following time interval, either the PQ value of the parameter's amount will be too high and the PQ value of its derivative too low, or the amount's PQ value will be too low and the derivative's PQ value too high, or both PQ values will be normal.*

1. An extension is suggested in Chapter 4.

Table 3: Point transitions on the PQ layer

Name of transition	PQ(f,t_i)	PQ($f´,t_i$)	sign($f(t_i)$)	sign($f´(t_i)$)	PQ($f,(t_i,t_j)$)	PQ($f´,(t_i,t_j)$)	sign($f´,(t_i,t_j)$)
PDL		L				L	
PDH		H				H	
PALHDN	L H	N N				unconstrained	
PAL	L				L		
PAH	H				H		
PANDLH1	N N N N N N	L L L L H H	- 0 0 + + -	- - + + - +	L		
PANDLH2	N N N N N N	L L H H H H	+ - - 0 0 +	- + - - + +	H		
PANDN1	N N N N N	N N N N N	- 0 0 0 +	- - 0 + +	L N H	L N H	
PANDN2	N N	N N	+ -	- +	H N L	L N H	
PANDN3	N	N	-	0	L L N H H	L H N L H	- + unconstrained + -
PANDN4	N	N	+	0	L L N H H	L H N L H	+ - unconstrained - +

Interval Transitions

Interval transitions use information about the parameter values of a time interval to determine the events that may finish the qualitative state described in the interval. The reason for a change of the qualitative state is, that a landmark is reached by a parameter's amount or derivative. On the PQ layer, the corresponding event is reaching *normal*, or more exactly the landmark 1 in the range of P values. Therefore, a qualitative behavior description can be refined by additional events on relative description layers. Or in other words: An interval on absolute descriptions layers can have a structure on relative description layers. In Fig. 3.5 deviating and reference behavior of a parameter *f* can be seen. On absolute layers the time intervals (t_a, t_p) or (t_a, t_s) describe the state of decreasing *f* . On relative layers, however, there is an event at time point t_i where the normal value is reached by *f*. So in a complete description

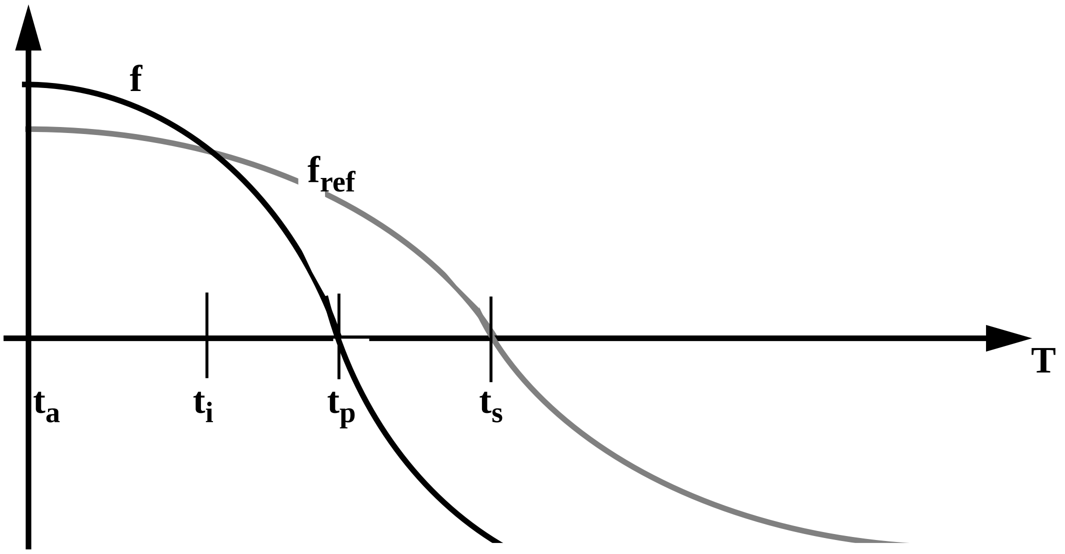


Fig. 3.5 A deviating parameter f and its reference course f_{ref}.

we have the following sequence of states:

Time point t_a:	*f* is positive and *too-high*.
Time interval (t_a, t_i):	*f* is positive and *too-high*.
Time point t_i:	*f* is positive and *normal*.
Time inteval (t_i, t_p):	*f* is positive and *too-low*.
Time point t_p:	*f* reaches 0...

The event at time point t_p entails a difficulty. Here, the deviation of *f* is discontinuous. And, in the time interval beginning at t_p, it remains discontinuous. In general it makes no sense, to compare *f* with f_{ref} during interval (t_p, t_s). Instead, the parts of *f* and f_{ref} where both are negative should be compared with each other. And *f* at time point t_p should be compared with f_{ref} at time point t_s. That means, a synchronization step is necessary. Fig. 3.6 shows the

synchronized behaviors. For f_{ref} no comparison takes place during (t_p, t_s). Now the list of qualitative states can be continued:

Time point t_{p*}: f is 0 and *normal*.

Time interval (t_{p*}, $t_?$) f is negative and *too high*[1].

In addition, it can be stated that the duration for reaching 0 is less than normal, i.e. *too-low*.

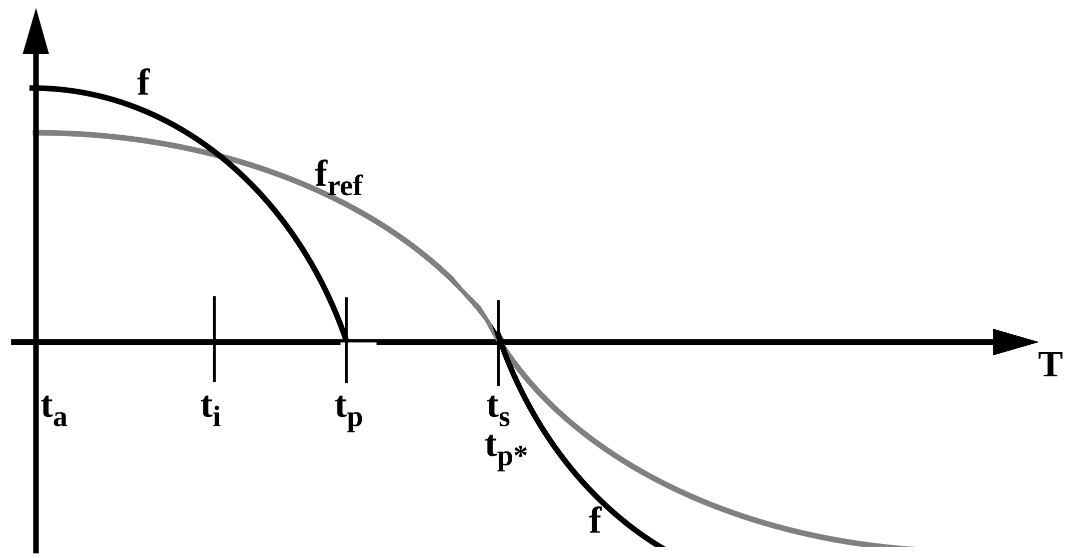


Fig. 3.6 Synchronized behaviors of Fig.3.5.

So, the event of reaching 0 requires a synchronization because otherwise the deviations would become discontinuous and sensible comparisons would not be possible. Reaching other landmarks than 0 does only requires a synchronization, if comparisons of behaviors shall be restricted to differential deviations. (If a landmark different from 0 is reached and no synchronization takes place, a deviation would become non-differential, but it would still remain continuous).

In Fig. 3.7. a parameter g first reaches a landmark l_k and afterwards a landmark l_m, both faster than normal. At the same time when g reaches l_m, parameter f reaches 0 (time point t_p). Therefore, a synchronization is necessary. Before that, at time point t_b, g reaches landmark l_k. And no other parameter reaches 0 at that time. The deviations of g at and after t_b are non-differential, but continuous. So, only if one wants to restrict oneself to differential deviations, a synchronization becomes necessary. Both qualitative behaviors, with and without synchronization, are listed below.

1. Note, a PQ value describes the magnitude of a value, so a negative value that is too low in a mathematical sense has the PQ value *too-high*.

Without synchronization at t_b (see Fig. 3.7):

Time point t_a: *f* is positive and *too-high*. g is 0 and normal

Time interval (t_a, t_b): *f* is positive and *too-high*. *g* is positive and *too-high*

Time point t_b: is positive and *too-high*. *g* is positive at l_k and ***too-high***

Time inteval (t_b, t_i): *f* is positive and *too-high*. *g* is positive in (l_k, l_m) and *too-high*

Time point t_i: *f* is positive and *normal*. *g* is positive in (l_k, l_m) and *too-high*

Time inteval (t_i, t_p): *f* is positive and *too-low.* *g* is positive in (l_k, l_m) and *too-high*

Time point t_p: *f* is 0 and *normal*. *g* is positive at l_k and *normal*[1]

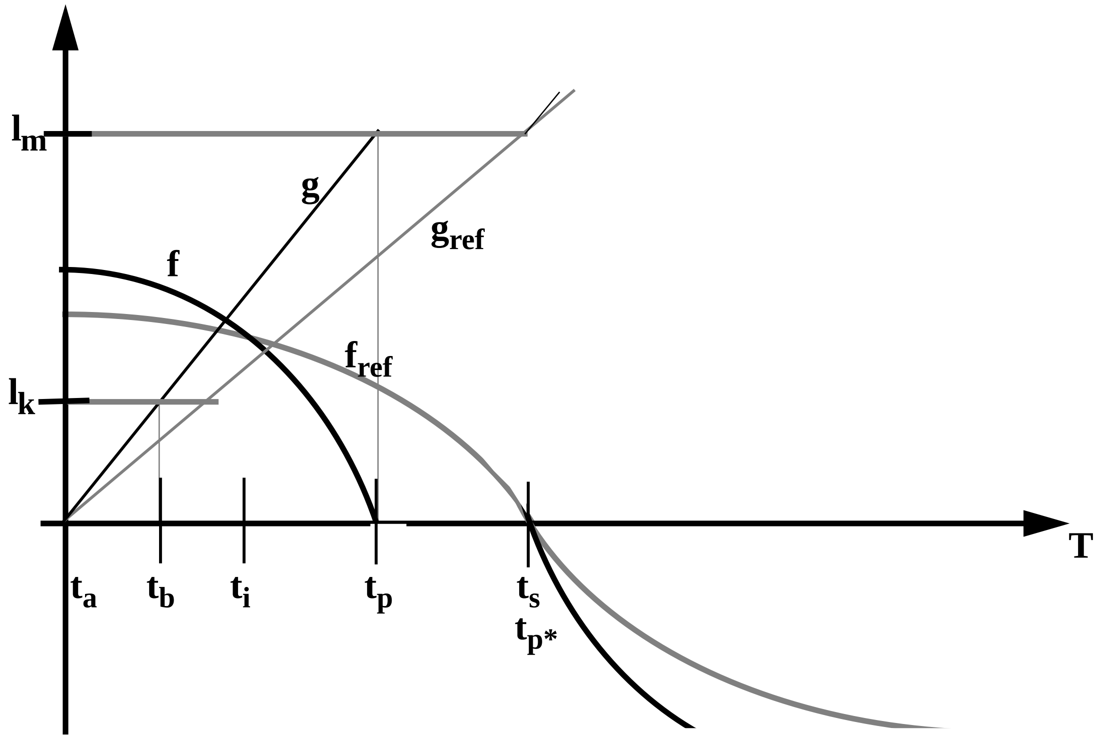


Fig. 3.7 Additional events at a parameter *g*.

With synchronization at t_b (see Fig.3.8):

Time point t_a: *f* is positive and *too-high*. g is 0 and normal

Time interval (t_a, t_b): *f* is positive and *too-high*. *g* is positive and *too-high*

Time point t_{b*}: is positive and *too-high*. *g* is positive at l_k and ***normal***

Time inteval (t_{b*}, t_{i*}): *f* is positive and *too-high*. *g* is positive in (l_k, l_m) and *too-high*

1. *G* is normal at t_p because g_{ref} and f_{ref} reach their landmarks at the same time.

Time point t_{i*}:	*f* is positive and *normal*.	*g* is positive in (l_k, l_m) and *too-high*
Time inteval (t_{i*}, t_{p*}):	*f* is positive and *too-low*.	*g* is positive in (l_k, l_m) and *too-high*
Time point t_{p**}:	*f* is 0 and *normal*.	*g* is positive at l_k and *normal*[1]

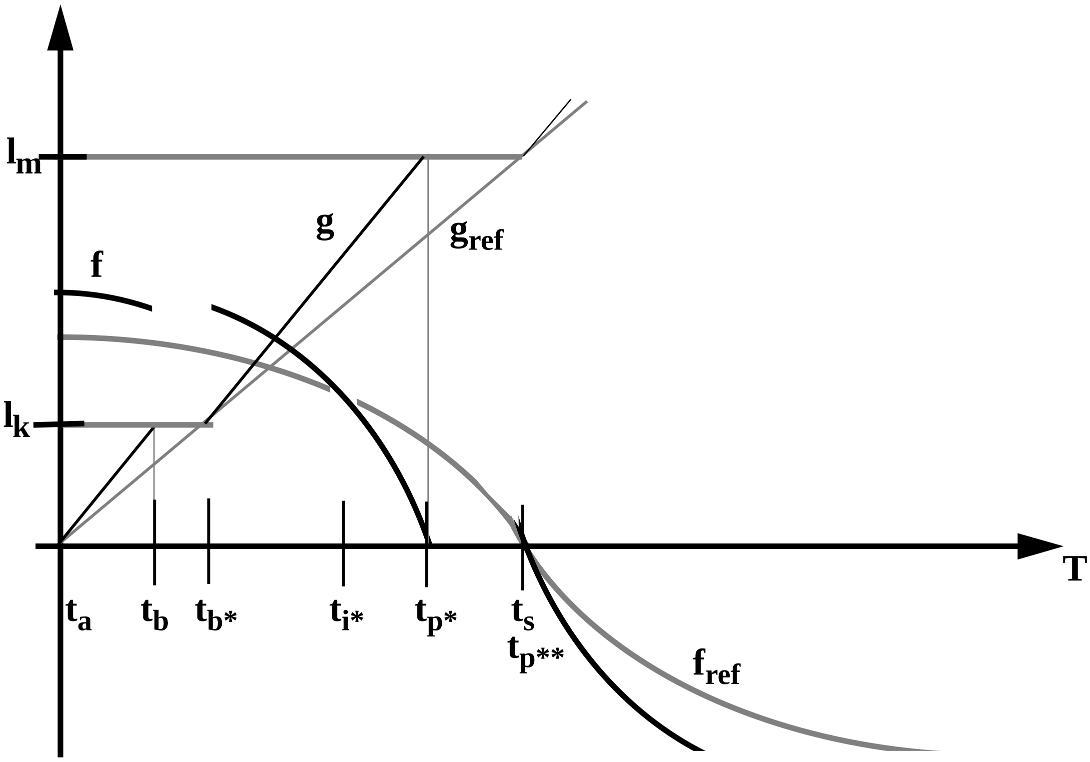


Fig. 3.8 Additional synchronization at t_b

In the example above the duration of the deviating behavior has been too low. A too high duration has similar consequences. Fig. 3.9 shows the difference between a too low and a too high duration of the deviating behavior. Here, synchronization takes place when a landmark value l_k is reached. In the deviating behavior b_l, l_k is reached earlier than normal, in deviating behavior b_h later than normal. The qualitative state describing the phase of increase refers to the full length of b_l, but for b_h it ends at t_n. So, for the interval (t_n, t_h) of b_h no description exists. However, the relevant information that the duration to reach l_k is *too-high*, is included.

1. *G* is normal at t_p because g_{ref} and f_{ref} at the same time reach their landmarks.

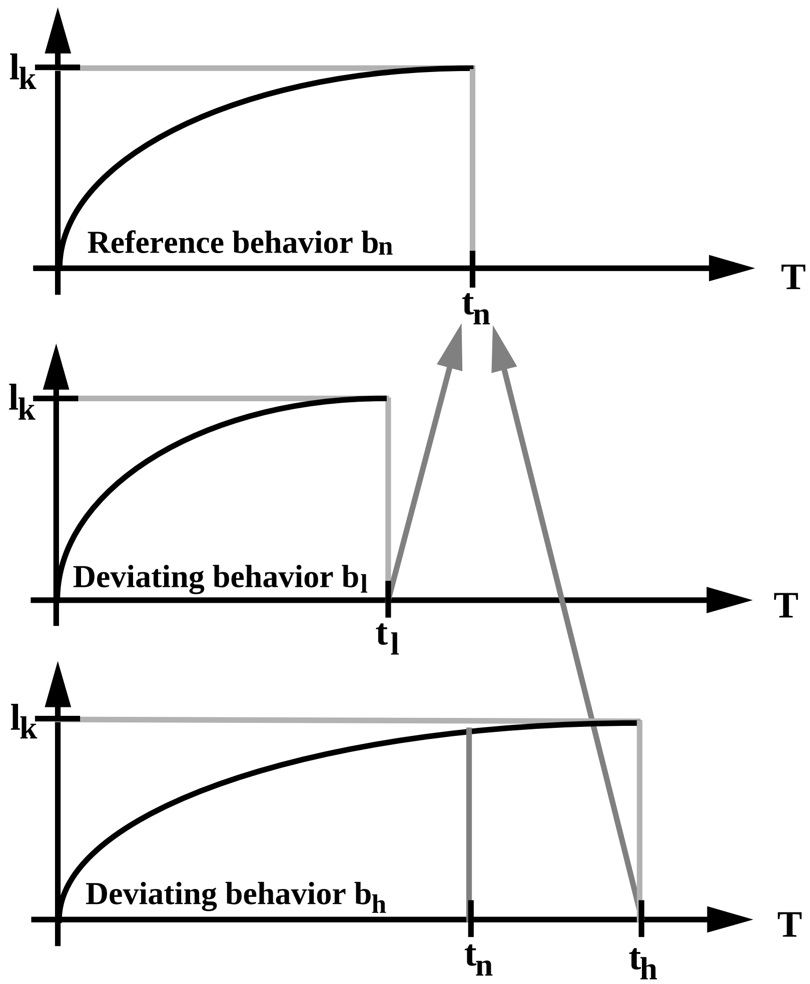


Fig. 3.9 Synchronization for a *too-low* and a *too-high* duration.

We have seen that a synchronization is necessary when landmark 0 or any landmark[1] respectively is reached, depending on whether continuous or merely differential deviations are allowed, . We want to call a landmark that requires a synchronization a ***synchronization landmark***. Whenever a synchronization landmark is reached by a parameter's amount or derivative, its PQ value becomes *normal* because of the synchronization. So, there are two reasons for reaching *normal*:

1. The amount or derivative of a parameter in the deviating system reaches a synchronization landmark.
2. The amount or derivative of a parameter in the deviating system reaches the normal value.

In qualitative simulation, in general, it cannot be determined which event occurs first, i.e. which parameter is involved in the next event. Therefore, transition rules (which are specific to a parameter) addionally have to supply the values of a parameter's amount and derivative

1. on an absolute description layer

for the case that the parameter is not involved in the next event, i.e. the next event concerns a different parameter. This “foreign” event may entail a synchronization. Therefore, an interval transition on the PQ layer has to cover five cases:

1. The parameter’s amount or derivative reaches a synchronization landmark.
2. The parameter’s amount or derivative reaches the normal value.
3. The next event occurs outside with normal duration.
4. The next event occurs outside with too low duration.
5. The next event occurs outside with too high duration.

Tables 4 and 5 show the PQ interval transitions[1]. The five situations listed above cannot be directly identified in the transition tables, because a situation can have different consequences as a result of ambiguities, or different situations can have the same consequences. In transition IAHDH, for example, the five situations have the following consequences:

Case 1: duration(t_i,t_j) = *normal*, PQ(f,(t_j) = *normal* or
duration(t_i,t_j) = *too-high*, PQ(f,(t_j) = *normal*

Case 2: duration(t_i,t_j) = *normal*, PQ(f,(t_j) = *normal*

Case 3: duration(t_i,t_j) = *normal*, PQ(f,(t_j) = *too-high*

Case 4: duration(t_i,t_j) = *too-low*, PQ(f,(t_j) = *too-high*

Case 5: duration(t_i,t_j) = *too-high*,
PQ(f,(t_j) = *too-low* or *normal* or *too-high* (= unconstrained)

Table 4: Interval transitions on the PQ layer (1)

Name of transition	PQ(f´,(t_i,t_j))	duration(t_i,t_j)	PQ(f´,(t_j)
IDL	L	L N N H	unconstrained L N unconstrained
IDN	N	L N H	unconstrained N unconstrained
IDH	H	L N N H	unconstrained N H unconstrained

1. PQ values are abbreviated by L, N, and H.

Table 5: Interval transitions on the PQ layer (2)

Name of transition	$PQ(f,(t_i,t_j))$	$PQ(f',(t_i,t_j))$	$sign(f,(t_i,t_j))$	$sign(f',(t_i,t_j))$	$duration(t_i,t_j)$	$PQ(f,(t_j))$
IAL	L			0		L
IAN	N			0		N
IAH	H			0		H
IANDN1	N N	N N	- +	- +	L N H	L N H
IANDN2	N N	N N	+ -	- +	L N H	H N L
IALDLN	L L L L	L L N N	- + - +	- + - +	L N H	L L unconstrained
IALDNH	L L L L	N N H H	+ - + -	- + - +	L N H	unconstrained L L
IAHDLN	H H H H	L L N N	+ - + -	- + - +	L N H	H H unconstrained
IAHDNH	H H H H	N N H H	- + - +	- + - +	L N H	unconstrained H H
IALDH	L L	H H	- +	- +	L N N H	L L N unconstrained
IAHDL	H H	L L	- +	- +	L N N H	unconstrained N H H
IALDL	L L	L L	+ -	- +	L N N H	unconstrained L N L
IAHDH	H H	H H	+ -	- +	L N N H	H N H unconstrained

3.5.2. Transitions on the P Layer

As explained above, an incomplete description is generated on the P layer, but one that supplies additional information and helps to avoid spurious behavior on the PQ layer. Accordingly it is not necessary for transitions on the P layer to cover all possible configurations of values. Instead, some special but very useful transitions are formulated. When required, more transitions can be added.

While sign transitions, qval transitions, or PQ transitions work with qualitative values of amounts and derivatives, transitions on the P layer refer to relations between P values. Relations between P values can be used in different ways: They can help to further constrain PQ transitions. They can deduce relationships between P values in the following time interval or time point. And because the relations used on the P layer are binary, they can concern different parameters. As a consequence, P transitions are not as homogeneous as the other types of transitions.

Point Transitions on the P Layer

One obvious point transition is given by the continuity rule, that if two P values are different at a time point t_i, there must exist an interval around t_i where they are different too. While the same consequences mostly can be derived from the P layer's intrastate analysis and from PQ transitions, the case of equality between P values is a more important one. In the transition P==[1] (Table 6), an interesting correlation between the P values of a parameter's amount and its first and second derivatives is formulated.[2] P== says:

> *If at a time point t_i on the one hand the P values of a parameter's amount and its second derivative are equal and on the other hand the P values of amount and first derivative are equal too, or the sign of the first derivative is 0, then in the following time interval (t_i, t_j) the P values of amount and derivatives must be equal, or the P values of amount and first derivative and amount and second derivative must be unequal.*

In the current version of RSIM, P== is the only point transition on the P layer. Further point transitions may follow.

Table 6: Point transition P==

Name of transition	relation (P(f,t_i), P(f´,t_i))	relation (P(f,t_i), P(f´´,t_i))	sign(f´,t_i)	relation (P(f,(t_i,t_j)), P(f´,(t_i,t_j)))	relation (P(f,(t_i,t_j)), P(f´´,(t_i,t_j)))
P= =	= undefined =	= = =	- 0 +	= ≠	= ≠

1. P== is proven for a basis landmark of 0 (see appendix B).
2. Since only the first derivative of a parameter is modeled, the rule can only be applied to parameters that are connected to another parameter via the DERIV constraint.

Interval Transitions on the P layer

The first group of interval transitions on the P layer treats the situation when the amount of a parameter moves to the landmark 0 and both amount and derivative of the parameter are too low or too high. The PQ transitions describing this case are IALDL and IAHDH. On the P layer, it can be expressed that one value is more too high or too low than another value. The more detailed information on the P layer facilitates a more constrained transition consequence. Table 7 shows the transitions IALDL and IAHDH augmented by P information. For each of them there are six transitions on the P layer.

Table 7: Interval transitions IALDL* and IAHDH*

Name of transition	PQ $(f,(t_i,t_j))$	PQ $(f',(t_i,t_j))$	relation $(P(f,(t_i,t_j)), P(f',(t_i,t_j)))$	sign $(f,(t_i,t_j))$	sign $(f',(t_i,t_j))$	duration (t_i,t_j)	PQ $(f,(t_j)$	sign $(f,(t_j)$
IALDL<-	L	L	<	-	+	L L N H	N unconstrained L L	0 - - -
IALDL<+	L	L	<	+	-	L L N H	N unconstrained L L	0 + + +
IALDL=-	L	L	=	-	+	L N N H	unconstrained L N L	- - 0 -
IALDL=+	L	L	=	+	-	L N N H	unconstrained L N L	+ + 0 +
IALDL>-	L	L	>	-	+	L N N H	unconstrained L N L	- - - -
IALDL>+	L	L	>	+	-	L N N H	unconstrained L N L	+ + + +
IAHDH<-	H	H	<	-	+	L N N H	H N H unconstrained	- - - -
IAHDH<+	H	H	<	+	-	L N N H	H N H unconstrained	+ + + +

Table 7: Interval transitions IALDL* and IAHDH*

Name of transition	PQ $(f,(t_i,t_j))$	PQ $(f',(t_i,t_j))$	relation $(P(f,(t_i,t_j)), P(f',(t_i,t_j)))$	sign $(f,(t_i,t_j))$	sign $(f',(t_i,t_j))$	duration (t_i,t_j)	PQ$(f,(t_j)$	sign $(f,(t_j)$
IAHDH=-	H	H	=	-	+	L N N H	H N H unconstrained	- 0 - -
IAHDH=+	H	H	=	+	-	L N N H	H N H unconstrained	+ 0 + +
IAHDH>-	H	H	>	-	+	L N H H	H H N unconstrained	- - 0 -
IAHDH>+	H	H	>	+	-	L N N H	H N H unconstrained	+ + 0 +

As an example, the P transition IAHDH=+ shall be explained.The transition IAHDH=+ (and the transition IAHDH=-) can be interpreted in the following way:

> *If at every time point of a time interval (t_i, t_j) the distance that has to be traveled is too high by the same factor as the velocity then it costs as much time as normally to travel the whole distance.*

Transition IAHDH=+ is illustrated in Figure 3.10. The reference behavior is represented by the black line and the deviating behavior by the dotted line. Let’s examine how the five possible endings (Section 3.5.1) of the time interval (t_i, t_j) look like.

1. A landmark is reached: One has to distinguish between a positive landmark l_k and the landmark 0. Every positive landmark l_k will first be reached by the reference function. Therefore, the duration to reach a positive landmark is too high. This case is covered by line 4 of the transition’s description in Table 7. On the other hand, the duration to reach the landmark 0 is the same as in the reference behavior (line 2 in transition table).
2. The normal value is reached: This happens at time point t_n. Transition IAHDH=+ handles the special case where landmark 0 is reached in normal duration. Reaching the normal value is covered by line 2.
3. The next event occurs outside with normal duration: In Figure 3.10 this case is represented by the vertical arrow. At every time point in the interval (t_i, t_j), the deviating value is *too-high* (line 3 in transition table).

4. The next event occurs outside with too low duration: In Figure 3.10 this case is represented by the arrow pointing to the right. At every time point in the interval (t_i, t_j), the deviating value is *too-high* (line 1 in transition table).
5. The next event occurs outside with too high duration: In Figure 3.10 this case is represented by three arrows pointing to the left. The deviating value may be *too-low*, *normal*, or *too-high*. (Line 4 in transition table.)

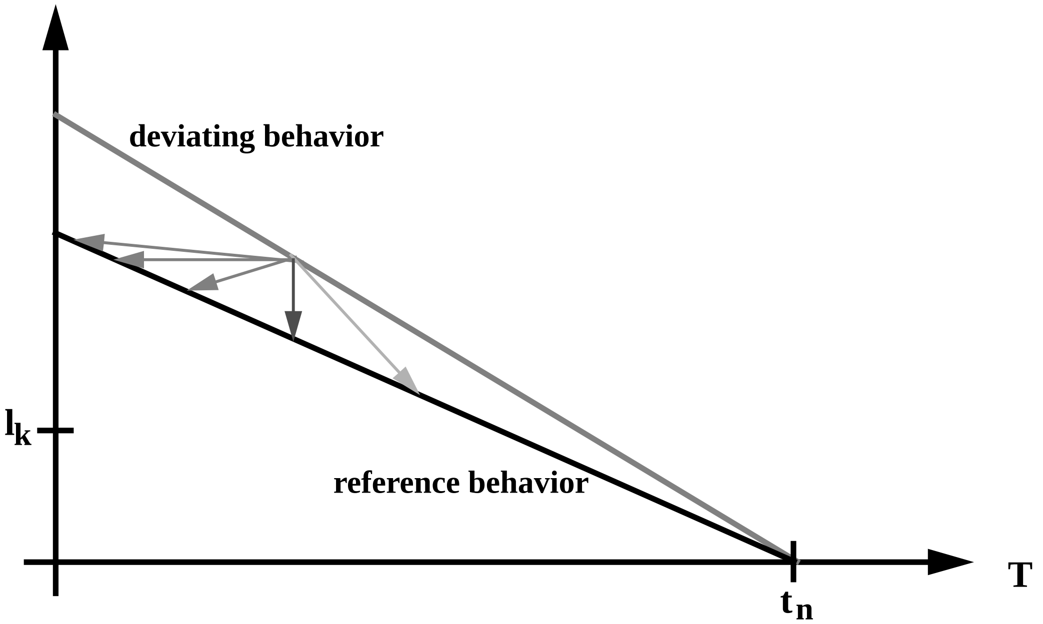


Fig. 3.10 An illustration of the interval transition IAHDH=+. Possible cases of comparisons are indicated by arrows.

Another type of transition makes predictions about the relations between P values at time

Table 8:

Name of transition	relation (P(f,(t_i,t_j)), P(f´,(t_i,t_j)))	duration (t_i,t_j)	relation (P(f,t_j), P(f´,t_j))	sign(f,t_j)	sign(f´,t_j)
I=	=	L	unconstrained	unconstrained	unconstrained
		N	=	{-, +}	{-, +}
		N	undefined	unconstrained	0
		N	undefined	0	unconstrained
		H	unconstrained	unconstrained	unconstrained

point t_j. Transition I= handles the case of equality between amount and derivative of a parameter. In words, I= expresses the following:

If during a time interval (t_i, t_j) the P values of amount and derivative of a parameter are equal then if the duration of (t_i, t_j) is normal and the signs of amount and derivative are unequal to 0, the P values of amount and derivative are equal at time point t_j.

3.6. The Output of RSIM

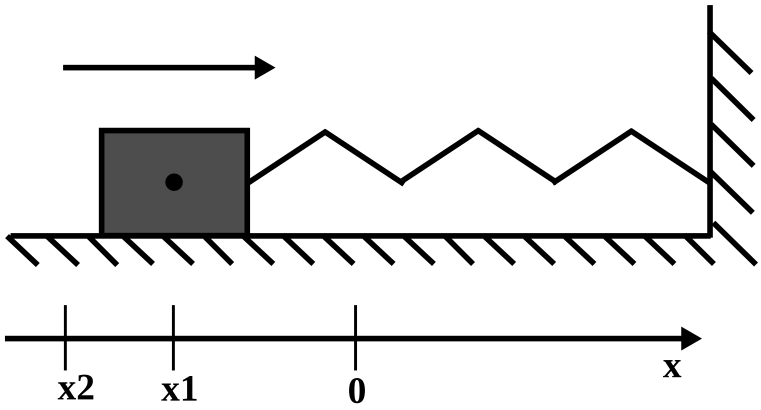
RSIM's output is a tree of behaviors. An RSIM behavior is more specific than a QSIM behavior, i.e. the description of behavior is more detailed. This means that

1. the description of a parameter and thus a state description is more detailed,
2. a behavior consists of more states, because additional events on relative description layers have to be integrated in a description of behavior,
3. the behavior tree contains more branchings, because the additional descriptions are not free of ambiguities either.

Especially the last point leads to time consuming computations and to extensive and sometimes confusing output. To cope with this problem, some methods have been added, that help to prevent irrelevant distinctions. It is possible, for example, to deactivate a description layer just for a certain amount or derivative. Another very effective method prevents distinctions for the PQ values of derivatives. It first generates fully described system states and then merges them afterwards.

We want to illustrate RSIM's output for the popular example system of a mass on a spring. Fig. 3.11 shows a model of this system and two different initializations. On the one hand, we want to examine a model with a too high mass. On the other hand we want to know what happens, if the amplitude is higher than normal. In Fig. 3.12 RSIM's output is listed for the position of the block, its velocity, and its acceleration, until the block reaches the point of no spring tension ($x = 0$).

Both behaviors of Fig. 3.12 consist of 5 states and have no ambiguities. In fact, there are ambiguities for the PQ values of some derivatives. But the simplifying strategies, mentioned above, merge these states. For a too high mass, it is derived that the duration of a period is too high, that the velocity is too low for $x = 0$, and that the acceleration at first is too low, then becomes normal, and afterwards is too high. For a too high amplitude it is unambiguously derived that the duration of a period is normal. This is true, because the deviations of x and v have the same strength, i.e. x and v have equal P values.




```
(create-model *mass-on-a-spring*
    :quantity-spaces ((k-qs (k* 0))                ;Definition of quantity spaces:
                      (m-qs (0 m*))                ;(name-of-quantity-space list-of-landmarks)
                      (energy-qs (0 te*))
                      (x-qs (x2* x1* 0)))
    :variables ((x x-qs) v vv a (f f-qs) (ke energy-qs) (pe energy-qs))
    :constants ((m m-qs) (k k-qs) (te energy-qs))
    :non-negatives (pe ke te m)
    :constraints ((deriv v x)        ;The velocity of the block is the derivative of its position.
                  (deriv a v)        ;The acceleration of the block is the derivative of its velocity.
                  (product f m a)    ;The force of the spring is the product of spring constant and position
                                      (Hooke's law).
                  (product f k x)    ;Force is the product of mass and acceleration (Newton's second law of
                                      motion).
                  (square vv v)      ;Vv is the square of v.
                  (product ke m vv) ;Kinetic energy depends on the product of mass and the square of velocity.
                  (square pe f)      ;Weld calls it a "cheating definition of potential energy" [Weld 90, p.159].
                  (sum te pe ke)))   ;Total energy is the sum of kinetic and potential energy.
```

```
(initialize-model *mass-on-a-spring*
  (v  :sign-of-amount 0
      :pq-of-amount normal)
  (x  :qval-of-amount x1*
      :pq-of-amount normal)
  (k  :qval-of-amount k*
      :pq-of-amount normal)
  (m  :qval-of-amount m*
      :pq-of-amount too-high)
  (te :qval-of-amount te*))
```

```
(initialize-model *mass-on-a-spring*
  (v  :sign-of-amount 0
      :pq-of-amount normal)
  (x  :qval-of-amount x2*
      :pq-of-amount too-high)
  (k  :qval-of-amount k*
      :pq-of-amount normal)
  (m  :qval-of-amount m*
      :pq-of-amount normal)
  (te :qval-of-amount te*))
```

Fig. 3.11 A model for a mass on a spring and two different initializations in RSIM's language. In the left initialization the mass is two high, in the right initialization the amplitude is greater than normal.

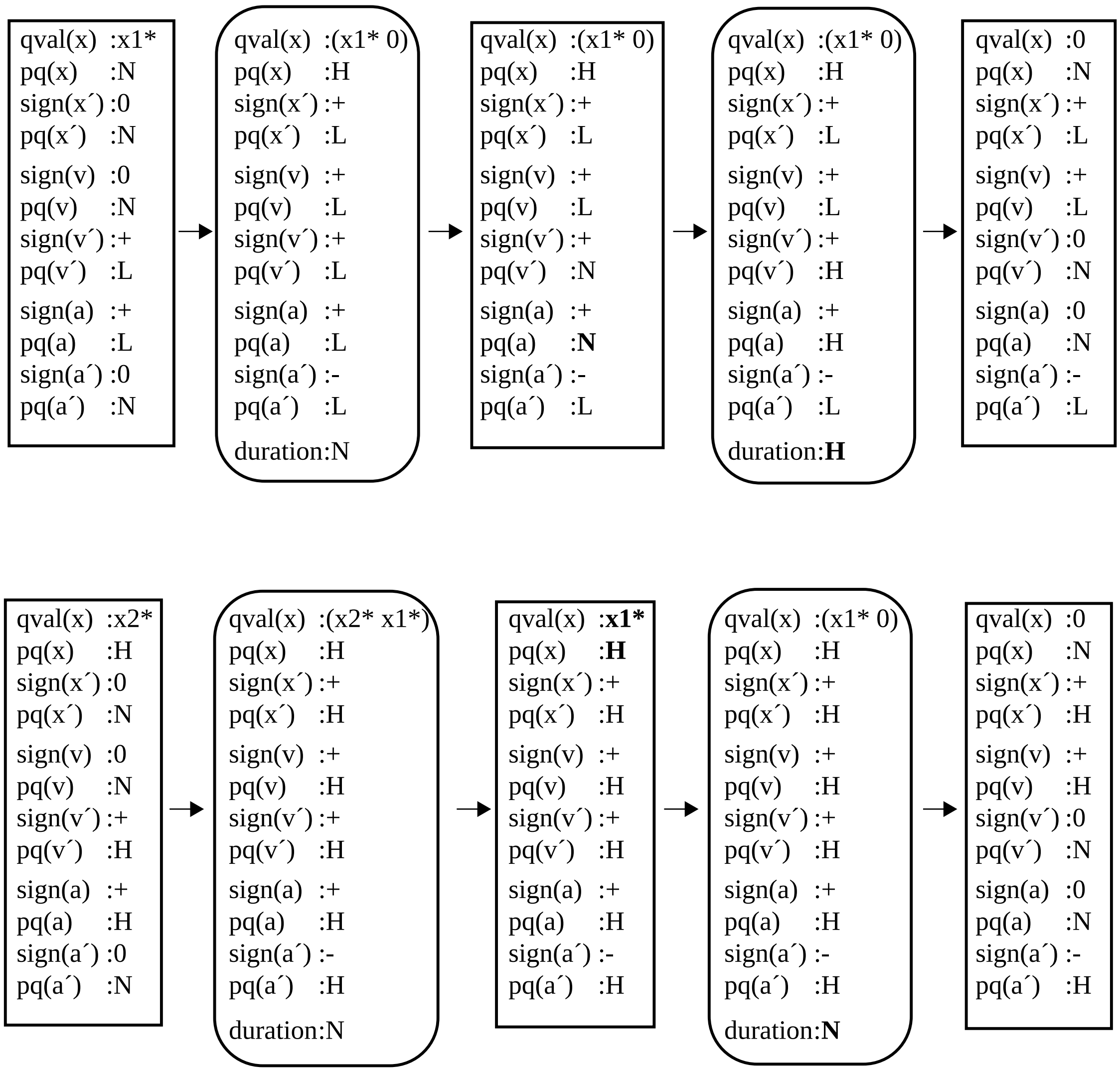


Fig. 3.12 The deviating behaviors for the mass-on-a-spring system, produced by RSIM. At the top the behavior for a too high mass, at the bottom the behavior for a too high amplitude.

4. Related Work

Relative descriptions are used in various approaches. [Raiman 86] reasons about orders of magnitude, in [Dague,Devès,Raiman 87] these concepts are used for a fault diagnosis. The *IQ analysis* of [de Kleer 79] is a qualitative sensitivity analysis of a system's steady state. Similar techniques that aim at a fault diagnosis can be found in [Downing 87], [Gallanti, Stefanini, Tomada 89] and [Kockskämper, Neumann, et al. 93]. [D'Ambrosio 89] describes a qualitative perturbation analysis that also is based on IQ analysis.

Dynamic systems, on the other hand, are analysed in Weld's approaches *DQ analysis* and *exaggeration* [Weld 87,88a,88b,90]. Both techniques predict the effects of differential changes as RSIM does. Therefore DQ analysis and exaggeration are the most relevant works for a comparison with RSIM. Since RSIM is oriented towards QSIM, we additionally will outline some differences to QSIM concerning the simulation techniques[1].

QSIM

RSIM's (or SLOD's) simulation techniques are similar to those of QSIM. Both simulators have an identical form of input and output: A system is described by constraints and a behavior is described by a sequence of states. However, RSIM's inferences have a more constructive character than QSIM's inferences. This becomes apparent in form and processing of transition rules. In QSIM, a single situation generally matches to several alternate transition rules. Constraint propagation in QSIM means filtering out the unsuitable transitions or the unsuitable combinations of transitions, which corresponds to filtering out impossible combinations of parameter values. That is, the set of possible values for a parameter is gradually reduced. In RSIM, on the other hand, transition rules are non-overlapping. A transition rule that can be applied will be applied. In general, the consequence of an RSIM transition rule contains a disjunction of value combinations. In order to fulfill the allowed combinations of an applicable transition rule, temporary *transition constraints* are generated. Constraint propagation in RSIM directly leads to single parameter values instead of a reduced set of values.

Weld's DQ Analysis

While the input/output behavior of RSIM and DQ analysis are similar, the underlying techniques are very different. DQ analysis uses a set of inference rules that analytically, and not by simulation, determine the effects of differential deviations. These inference rules refer to a model, expressed by QSIM constraints, and to a QSIM behavior. To facilitate a comparison of intervals of different length, DQ analysis introduces the concept of *perspective* (see Section 2.2.1). In RSIM, this problem is solved by interval transitions that perform a synchronization (see Section 3.5.1). DQ analysis sometimes produces no output, that is DQ analysis is incomplete. RSIM, on the other hand, always produces an output. But the current version of RSIM

1. Differences resulting from a refinement by means of relative descriptions have extensively been explained in the previous chapters.

does not capture changes in the behavioral topology.[1] That is, RSIM only is sound if topology changes can be excluded.

RSIM can answer some questions that DQ analysis cannot answer because DQ analysis has problems to predict the resulting effect of contrary deviations. If, for example, a longer distance has to be covered with a higher velocity, DQ analysis cannot decide, whether the duration increases, decreases or does not change. RSIM's relations between P values provide the means to answer such questions.

Exaggeration

In contrast to DQ analysis, exaggeration is based on simulation. Exaggeration's input and output, however, are different from RSIM. Exaggeration answers concrete questions about the effects of a certain differential deviation on a target parameter. For this purpose it first exaggerates the given deviation, e.g. a value higher than normal is considered as infinite. Then two simulations take place: a standard QSIM simulation and a simulation of the exaggerated system. The latter simulation works with hyperreal values, like *infinite* or *negligible*, and is carried out by a simulator called HR-QSIM. Finally, the exaggerated output is rescaled by a comparison of both simulation results. A negligible value, e.g., may be rescaled to "less than normal". In general, a QSIM simulation as well as a HR-QSIM simulation produces a set of behaviors because of ambiguities. As it is implemented, exaggeration compares the HR-QSIM behaviors to a single QSIM behavior and not to all behaviors that result from a QSIM simulation. The decision which behavior to choose is obtained from the input question. A general prediction of the effects of a deviation, as it is done by RSIM, would require a pairwise comparison of all exaggerated and all normal behaviors. Exaggeration is complete, but unfortunately only sound if all relationships between parameters are monotonous. The idea of an exaggeration conflicts with a comparison of deviations. Therefore, an important advantage of RSIM again lies in the comparison of deviations.

1. DQ analysis also has problems with topology changes. See, for example, Section 4 in [Weld 88b].

5. Future Work

At the moment, simulation by RSIM predicts all behaviors that entail no discontinuous changes of topology. Future work will investigate this topic. Occurrence branching shall be captured by making parameter-specific statements about duration instead of state-specific statements. The other type of branchings corresponds to discontinuous deviations in parameter histories. We want to integrate these changes of behavior by extending the set of PQ values, so that discontinuous deviations of parameter values can be described. In addition, the definitions of constraints have to be extended by the new PQ values.

RSIM takes all behaviors that result from the system description as reference behavior. In practice, often the correct behavior of a system is well known, such that this knowledge can be used to constrain the prediction of faulty behavior [Neitzke 91]. To realize this, RSIM needs additional input about the reference behavior. In the extreme, this can be the path of the behavior tree that corresponds to the reference behavior. (This is done in Weld's DQ analysis [Weld 87,88b,90].) During generation of behavior, RSIM then has to decide which behavior can result from the correct behavior.

6. Summary

Relative descriptions are necessary to characterize certain kinds of system deviations and the resulting behavior. These deviations are classified as differential or continuous. RSIM is a simulator that works with relative descriptions and is able to compare deviations with each other. RSIM cannot only deal with differential deviations but additionally accepts continuous deviations. In RSIM, the special properties of linear, overlinear and underlinear relationships between parameters are exploited to gain more accuracy in the prediction of system behavior. Additionally, faulty M^* relationships can be expressed. Due to these refined description facilities, some system behaviors can be distinguished that are identical under usual qualitative descriptions. The class of physical systems that can be described by RSIM corresponds to that of QSIM. In the current version of RSIM, a prediction of behaviors with discontinuous changes of topology is not possible. But RSIM's concepts allow an extension in this direction.

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A: Proofs of PQ Transition Rules

In Appendix A, it will be proven that the PQ transition rules are mathematically sound. The proofs are based on continuity conditions of continuously differentiable functions of time.

PQ values refer to the function P. (See the definition of P in Section 1.2.4.) Since $P(f,t) = (f(t) - l_a) / (f_{ref}(t) - l_a)$ is a continuously differentiable function of time for $(f_{ref}(t) - l_a) \neq 0$[1], PQ transitions have a similar character as sign transitions or qval transitions.

According to the definition of PQ values, we have to deal with the following situations:

1. $0 < P(f,t) < 1$ $\Leftrightarrow$ PQ(f,t) = *too-low*
2. $P(f,t) = 1$ $\Rightarrow$ PQ(f,t) = *normal*
3. $P(f,t) > 1$ $\Leftrightarrow$ PQ(f,t) = *too-high*
4. $f(t)-l_a = f_{ref}(t)-l_a = 0$ $\Rightarrow$ PQ(f,t) = *normal*

For the moment, the last situation will be left out. For the function P, interesting transitions are from the intervals (0, 1) or $(1, \infty)$ to a value of 1 and vice versa. Discontinuous transitions are not considered at this place. We want to see, which transitions between time points and time intervals are possible. At a time point t_i, either

1. $P(f,t_i) \in (0, 1)$, or
2. $P(f,t_i) = 1$, or
3. $P(f,t_i) \in (1, \infty)$

If $P(f,t_i) = 1$, nothing can be said about time intervals (t_{i-1}, t_i) or (t_i, t_{i+1}) without additional information. In these intervals, P(f,t) may be less, equal or greater than 1. But,

Lemma A: If $P(f,t_i) \in (0, 1)$ (or $P(f,t_i) \in (1,\infty)$), then there exists an environment of t_i with $P(f,t) \in (0, 1)$ (or $P(f,t_i) \in (1,\infty)$). That is, there exists a $\delta > 0$, such that

$P(f,t) \in (0, 1)$ (or $P(f,t) \in (1,\infty)$), for all $t \in T$ with $|t - t_i| < \delta$

Proof A: The ε-δ definition of continuity (e. g. [Forster 80, p. 67] states that for every $\varepsilon := |P(f,t_i) - 1| > 0$ there exists a $\delta > 0$ such that
$|P(f,t) - P(f,t_i)| < \varepsilon$, for all $t \in T$ with $|t - t_i| < \delta$
From this follows that $|P(f,t) - 1| \geq |P(f,t_i) - 1| - |P(f,t) - P(f,t_i)| > 0$, for all $t \in T$ with $|t - t_i| < \delta$, ❑

Therefore, the following transitions are possible:

1. It is required that all parameters are continuously differentiable functions of time.

Point transitions:

$P(f,t_i) \in (0, 1) \;\rightarrow\; P(f,(t_i,t_{i+1})) \in (0, 1)$ (1)

$P(f,t_i) = 1 \;\rightarrow\; P(f,(t_i,t_{i+1})) \in (0, 1) \vee P(f,(t_i,t_{i+1}))=1 \vee P(f,(t_i,t_{i+1})) \in (1, \infty)$ (2)

$P(f,t_i) \in (1, \infty) \;\rightarrow\; P(f,(t_i,t_{i+1})) \in (1, \infty)$ (3)

(2) is ambiguous. Here, information about the derivative of P, especially its sign, can help. But first we have to prove the following lemma:

Lemma B: Let g(t) be a continuously differentiable function of time. If for a time point t_i $g(t_i) = k$ and $g'(t_i) > 0$ ($g'(t_i) < 0$) holds, then there exist an environment of t_i, (t_{i-1},t_{i+1}), $t_{i-1} < t_i < t_{i+1}$, with $g(t_{i-1},t_i) < k$ ($g(t_{i-1},t_i) > k$), and $g(t_i,t_{i+1}) > k$ ($g(t_i,t_{i+1}) > k$).

Proof B: Following proof A, there exist an environment (t_{i-1},t_{i+1}) with $g'(t) > 0$ ($g'(t) < 0$), $t \in (t_{i-1},t_{i+1})$. From this follows that $g(t_{i-1},t_i) < k$ ($g(t_{i-1},t_i) > k$), and $g(t_i,t_{i+1}) > k$ ($g(t_i,t_{i+1}) > k$).

$P(f,t_i) = 1 \wedge P'(f,t_i) < 0 \;\rightarrow\; P(f,(t_i,t_{i+1})) \in (0, 1)$ (2a)

$P(f,t_i) = 1 \wedge P'(f,t_i) = 0 \;\rightarrow\; P(f,(t_i,t_{i+1})) \in (0, 1) \vee P(f,(t_i,t_{i+1})) = 1 \vee P(f,(t_i,t_{i+1})) \in (1, \infty)$ (2b)

$P(f,t_i) = 1 \wedge P'(f,t_i) > 0 \;\rightarrow\; P(f,(t_i,t_{i+1})) \in (1, \infty)$ (2c)

(2a) and (2c) are valid due to lemma B. In the case of (2b), the second derivative would allow further discriminations. However, second derivatives are not considered[1]. Therefore, the ambiguities of (2b) cannot be removed.

Interval transitions:

$P(f,(t_{i-1},t_i)) \in (0, 1) \;\rightarrow\; P(f,t_i) \in (0, 1) \vee P(f,t_i) = 1$ (4)

$P(f,(t_{i-1},t_i)) = 1 \;\rightarrow\; P(f,t_i) = 1$ (5)

$P(f,(t_{i-1},t_i)) \in (1, \infty) \;\rightarrow\; P(f,t_i) = 1 \vee P(f,t_i) \in (1, \infty)$ (6)

Under certain conditions the first derivative of P can also help to remove ambiguities of interval transitions. If during time interval (t_{i-1}, t_i) the distance to landmark 1 is increasing or remaining constant, landmark, 1, will not have been reached at time point t_i. Whether the distance is increasing or decreasing can easily be deduced from the signs of $P(f,(t_{i-1},t_i))$ and $P'(f,(t_{i-1},t_i))$. The different variants shall not be listed. Instead the two classes “distance is decreasing” and “distance is not decreasing” are used.

1. In fact, the first derivative, P´(f,t), is not modeled either. But information about P´(f,t) can be gained from PQ(f´,t), as we will see later.

$P(t_{i-1},t_i) \in (0, 1) \wedge$ “distance is decreasing” $\rightarrow P(f,t_i) \in (0, 1) \vee P(f,t_i) = 1$ (4a)

$P(t_{i-1},t_i) \in (0, 1) \wedge$ “distance is not decreasing” $\rightarrow P(f,t_i) \in (0, 1)$ (4b)

$P(t_{i-1},t_i) \in (1, \infty) \wedge$ “distance is decreasing” $\rightarrow P(f,t_i) = 1 \vee P(f,t_i) \in (1, \infty)$ (6a)

$P(t_{i-1},t_i) \in (1, \infty) \wedge$ “distance is not decreasing” $\rightarrow P(f,t_i) \in (1, \infty)$ (6b)

We still have to deal with the situation that $f(t)\text{-}l_a = f_{ref}(t)\text{-}l_a = 0$. For a time point t_i with $f(t_i) = fref(t_i) = l_a$, restrictions for following or preceding time intervals can be made if $f'(t_i) > f_{ref}'(t_i) > 0$ or $f_{ref}'(t_i) > f'(t_i) > 0$ or $f'(t_i) < f_{ref}'(t_i) < 0$ or $f_{ref}'(t_i) < f'(t_i) < 0$. In the following, only the first case will be treated. The others can be handled in an analogous way.

<u>Lemma C</u>: If for a time point t_i $f'(t_i) > f_{ref}'(t_i) > 0$ holds, then there exists an environment of t_i, (t_{i-1},t_{i+1}), $t_{i-1} < t_i < t_{i+1}$, with $f(t_{i-1},t_i) < f_{ref}(t_{i-1},t_i) < 0$, and $f(t_i,t_{i+1}) > f_{ref}(t_i,t_{i+1}) > 0$.

<u>Proof C</u>: Let x be some value in $(f_{ref}'(t_i),f'(t_i))$, i.e. $f_{ref}'(t_i) < x < f'(t_i)$. Proof A states, that there exists an environment (t_{i-1},t_{i+1}) of t_i with $f_{ref}'(t) < x < f'(t)$, $t \in (t_{i-1},t_{i+1})$. That means, $f(t)$ increases faster than $f_{ref}(t)$ in the time interval (t_{i-1},t_{i+1}). Therefore, $f(t_{i-1},t_i) < f_{ref}(t_{i-1},t_i) < 0$, and $f(t_i,t_{i+1}) > f_{ref}(t_i,t_{i+1}) > 0$. ❑

<u>Remark</u>: If $f'(t_i) = f_{ref}'(t_i) = 0$, all possibilities for $f(t)$ and $f_{ref}(t)$ in the intervals (t_{i-1},t_i) and (t_i,t_{i+1}) have to be taken into account. These are: $f(t) < f_{ref}(t) < 0$, $f(t) > f_{ref}(t) > 0$, $f_{ref}(t) < f(t) < 0$, $f_{ref}(t) > f(t) > 0$, and $f(t) = f_{ref}(t) = 0$. Discontinuous deviations are not considered.

Now the basic proofs for PQ transitions have been given. Below, we state for each transition from which propositions the proof follows. PQ transitions refer to the parameter descriptions PQ(f,t), PQ(f´,t), sign(f,t), and sign(f´,t). Since our considerations above refer to P(f,t), P´(f,t), f(t), and $f_{ref}(t)$, one has to map from PQ values and signs to P values. For P´(f,t) this is not as obvious as for P(f,t). Therefore Table 9 gives some help.

Table 9: Mapping from PQ values and signs to the derivative of the corresponding P value

PQ(f,t)	PQ(f´,t)	sign(f(t))	sign(f´(t))	P´(f,t)
L	L	- ∨ +	- ∨ +	?
L	N ∨ H	-	-	> 0
L	N ∨ H	-	+	< 0
L	N ∨ H	+	-	< 0
L	N ∨ H	+	+	> 0

Table 9: Mapping from PQ values and signs to the derivative of the corresponding P value

PQ(f,t)	PQ(f´,t)	sign(f(t))	sign(f´(t))	P´(f,t)
N	L	-	-	< 0
N	L	+	-	> 0
N	L	-	+	> 0
N	L	+	+	< 0
L ∨ H	N	- ∨ +	0	= 0
N	L ∨ N ∨ H	0	- ∨ +	not defined
N	N	0	0	not defined
N	N	- ∨ +	- ∨ 0 ∨ +	= 0
N	H	-	-	> 0
N	H	+	-	< 0
N	H	-	+	< 0
N	H	+	+	> 0
H	L ∨ N	-	-	< 0
H	L ∨ N	-	+	> 0
H	L ∨ N	+	-	> 0
H	L ∨ N	+	+	< 0
H	H	- ∨ +	- ∨ +	?

Point Transitions

Point transition PDL follows from (1).

Point transition PDH follows from (3).

Point transition PALHDN follows from (2).

Point transition PAL follows from (1).

Point transition PAH follows from (3).

Point transition PANDLH1 follows for PQ(f´(t_i) = L and sign(f(t_i)) ≠ 0 from (2a),
for sign(f(t_i)) = 0 from Lemma C,
for PQ(f´(t_i) = H from (2c).

Point transition PANDLH2 follows for $PQ(f'(t_i) = H$ and $sign(f(t_i)) \neq 0$ from (2a),
for $sign(f(t_i)) = 0$ from Lemma C,
for $PQ(f'(t_i) = L$ from (2c).

Point transitions PANDN* describe the case, where a continuously differentiable function has a landmark value at a certain time point and it is not known in which direction it will move. Therefore, all possibilities must be taken into account. However, some constraints between the future values of the function and its derivative exist. The PANDN* transitions constitute an interface to discontinuous deviations and have to be extended for an integration of them.

Interval Transitions

Interval transition IDL follows from (4).

Interval transition IDN follows from (5).

Interval transition IDH follows from (6).

Interval transitions IAL, IAN and IAH refer to a constant function and constant reference function. Therefore, a deviation will be constant too.

The remaining interval transitions distinguish between an ending of the interval with or without a synchronization:

1. $Duration(t_i,t_j) = L$ means that due to the synchronization, the deviating value is compared to a reference value that corresponds to a later time point than normally.
2. $Duration(t_i,t_j) = N$ means that no synchronization takes place.
3. $Duration(t_i,t_j) = H$ means that due to the synchronization, the deviating value is compared to a reference value that corresponds to an earlier time point than normally.

If no synchronization takes place, the interval transitions directly follow from propositions (5), (4a), (4b), (6a), or (6b). Otherwise considerations of the following kind are necessary:

If $sign(f,(t_i,t_j]) = +$, and $sign(f',(t_i,t_j]) = +$, then
$P^*(f,t_j) = (f(t_j) - l_a) / (f_{ref}(t_j - \Delta t) - l_a) > P(f,t_j) = (f(t_j) - l_a) / (f_{ref}(t_j - \Delta t) - l_a)$.

That is, a synchronization with too high duration will increase the P value of an increasing, positive parameter. So, if its P value is N or H without synchronization, it will be H. But if its P value is L, the effect of the synchronization is ambiguous. Analogous considerations can be made for the other configurations of signs and synchronization type. In the following, we state on which propositions the remaining transitions are grounded, if no synchronization takes place.

Interval transitions IANDN1 and IANDN2 follow from (5).

Interval transitions IALDLN follows from (4b).

Interval transitions IALDNH follows from (4b).

Interval transitions IAHDLN follows from (6b).

Interval transitions IAHDNH follows from (6b).

Interval transitions IALDH follows from (4a).

Interval transitions IAHDL follows from (6a).

Interval transitions IALDL follows from (4a).

Interval transitions IAHDH follows from (6a).

B: Proofs of P Transition Rules

In Appendix B, the P transition rules, P== and I= are proven to be sound. While the proof of P== involves the solution of a differential equation, I= can be proven analogously to (5) of Appendix A.

Proof of P==

P== states that from

$$(P(f,t_i) = P(f',t_i) = P(f'',t_i) \wedge sign(f',t_i) \in \{-, +\}) \vee (P(f,t_i) = P(f'',t_i) \wedge sign(f',t_i) = 0) \qquad (1)$$

follows

$$P(f,(t_i,t_j)) = P(f',(t_i,t_j)) = P(f'',(t_i,t_j)) \vee P(f',(t_i,t_j)) \neq P(f,(t_i,t_j)) \neq P(f'',(t_i,t_j)) \qquad (2)$$

<u>Proof</u>:

In accordance with RSIM, the proof assumes a basis landmark of 0.

Since from $(a \Leftrightarrow b)$ follows $((a \wedge b) \vee (\neg a \wedge \neg b))$, it suffices to show that from (1) follows

$$P(f,(t_i,t_j)) = P(f',(t_i,t_j)) \Leftrightarrow P(f,(t_i,t_j)) = P(f'',(t_i,t_j)).$$

<u>First direction</u>: We have to show that from (1) and $P(f,(t_i,t_j)) = P(f',(t_i,t_j))$ follows $P(f,(t_i,t_j)) = P(f'',(t_i,t_j))$. For this direction can renounce (1).

$$P(f,t) = P(f',t), \quad t \in (t_i,t_j)$$
$$\Leftrightarrow \; f(t) / f_{ref}(t) = f'(t) / f'_{ref}(t), \quad t \in (t_i,t_j)$$
$$P'(f,t) = (f'(t) * f_{ref}(t) - f(t) * f'_{ref}(t)) / (f_{ref}(t))^2, \quad t \in (t_i,t_j)$$
$$\Rightarrow \; P'(f,t) = 0, \quad t \in (t_i,t_j) \qquad (3)$$

$$f(t) = P(f,t) * f_{ref}(t)$$
$$\Rightarrow \; f'(t) = P'(f,t) * f_{ref}(t) + P(f,t) * f_{ref}'(t)$$

Because of (3), the following holds

$$f'(t) = P(f,t) * f_{ref}'(t), \quad t \in (t_i,t_j)$$
$$\Rightarrow \; f''(t) = P'(f,t) * f_{ref}'(t) + P(f,t) * f_{ref}''(t) = P(f,t) * f_{ref}''(t), \quad t \in (t_i,t_j)$$
$$\Rightarrow \; P(f,t) = P''(f,t), \quad t \in (t_i,t_j) \; \square$$

Second direction: We have to show that from (1) and $P(f,(t_i,t_j)) = P(f'',(t_i,t_j))$ follows $P(f,(t_i,t_j)) = P(f',(t_i,t_j))$.

$$P(f,t) = f(t) / f_{ref}(t)$$

$$\Leftrightarrow \quad f(t) = P(f,t) * f_{ref}(t)$$

$$\Rightarrow \quad f'(t) = P'(f,t) * f_{ref}(t) + P(f,t) * f_{ref}'(t) \qquad (4)$$

$$\Rightarrow \quad f''(t) = P''(f,t) * f_{ref}(t) + P'(f,t) * f_{ref}'(t) + P'(f,t) * f_{ref}'(t) + P(f,t) * f_{ref}''(t) \qquad (5)$$

$$P(f,(t_i,t_j)) = P''(f,(t_i,t_j)), \quad t \in [t_i,t_j)$$

$$\Rightarrow \quad f''(t) = P(f,t) * f_{ref}''(t), \quad t \in [t_i,t_j) \qquad (6)$$

From (5) and (6) follows

$$0 = P''(f,t) * f_{ref}(t) + 2* P'(f,t) * f_{ref}'(t), \quad t \in [t_i,t_j)$$

Let $P'(f,t) = k(t)$:

$$0 = k'(t) * f_{ref}(t) + 2* k(t) * f_{ref}'(t), \quad t \in [t_i,t_j)$$

$$\Rightarrow \quad k'(t) + 2*f_{ref}'(t)/ f_{ref}(t) * k(t) = 0, \quad t \in [t_i,t_j)$$

The solution of this differential equation is (cf. Boyce):

$$k(t) = c / \mu(t), \quad \mu(t) = \exp \int^{t} 2 f_{ref}'(s) / f_{ref}(s)\, ds \qquad (\text{ c is a constant})$$

$$\Rightarrow \quad \mu(t = \exp (2 \ln (f_{ref}(t))) = 1 / f_{ref}(t)^2, \quad t \in [t_i,t_j)$$

$$\Rightarrow \quad k(t) = c / f_{ref}(t)^2 = P'(f,t), \quad t \in [t_i,t_j) \qquad (7)$$

(1) states that either $P(f,t_i) = P(f',t_i)$ or $sign(f',t_i) = 0$ holds. Above we have seen that $P'(f,t_i) = 0$ follows from $P(f,t_i) = P(f',t_i)$. On the other hand, $sign(f',t_i) = 0$ implies that $sign(f_{ref}',t_i) = 0$, since we don't allow discontinuous deviations. From this follows $P'(f,t_i) = 0$ too, i.e. $P'(f,t_i) = 0$ holds in both cases. Thus from (7) follows:

$$c / f_{ref}(t_i)^2 = 0, \quad f_{ref}(t_i) \neq 0$$

$$\Rightarrow \quad c = 0$$

$$\Rightarrow \quad P'(f,t) = 0, \quad t \in [t_i,t_j) \qquad (8)$$

From (4) and (8) follows:

$$f'(t) = P(f,t) * f_{ref}'(t), \quad t \in [t_i,t_j)$$

$$\Rightarrow \quad P(f,(t_i,t_j)) = P'(f,(t_i,t_j)).\square$$

Proof of I=

I= only constrains the relationship between $P(f,t_j)$ and $P(f´,t_j)$, if f(t), $f_{ref}(t)$, f´(t), and $f_{ref}´(t)$ all are unequal to 0. But for this case P(f,t) and P(f´,t) are continuously differentiable. Therefore I= is an analogy to (5) of appendix A.❑

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