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The modified simple equation method and its applications to (2+1)-dimensional systems of nonlinear evolution equations

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The modified simple equation method is employed to find the exact traveling wave solutions involving parameters for nonlinear systems of evolution equations via the (2+1)-dimensional Konopelchneko-Dubrovsky equations and the (2+1)-dimensional Nizhnik-Novikov-Vesselov equations in two dimensions. When these parameters are taken to be special values, the solitary wave solutions are derived from the exact traveling wave solutions. It is shown that the modified simple equation method provides an effective and a more powerful mathematical tool for solving nonlinear evolution equations in mathematical physics. Comparison between our results and the well-known results will be presented.

Key words: Modified simple equation method, Konopelchneko-Dubrovsky equations, Nizhnik-Novikov-Vesselov equations, exact traveling solutions, solitary wave solutions.

INTRODUCTION

In recent years, nonlinear partial differential equations (PDEs) are widely used to describe many important phenomena and dynamic processes in physics, mechanics chemistry, biology, etc. As mathematical models of the phenomena, the investigation of exact traveling wave solutions of these equations will help one to understand these phenomena better. Many powerful methods for obtaining the exact traveling wave solutions have been presented, such as the inverse scattering method (Ablowitz and Clarkson, 1991) the Hirota's bilinear method (Hirota, 1971; Ma, 2011), the Backlund transform method (Miura, 1978), the Painleve expansions methods (Weiss et al., 1983), the sine-cosine method (Yan, 1996), the homotopy perturbation method (He, 2005a, b; El-Shahed, 2005), the Adomian Pade approximation method (Abassy et al., 2004), the homogeneous balance method (Wang, 1996), the variational iteration method (He, 2004, 2005a, b, c; Liu, 2004; Liu, 2005, Liu et al., 2013), the algebraic method (Hu, 2005), the tanh-function method (Malfliet, 1992; Parkes and Duffy, 1997; Fan, 2000; Yan and Zhang, 2001; Zayed et al., 2004; Abdusalam, 2005; Xie et al., 2005; Wang and Wei, 2010), the exp-function method (He and Wu, 2006; Yusufoglu, 2008; Zhang, 2008; Bekir, 2009, 2010), the Jacobi-elliptic function method (Liu et al., 2001, 2004; Fu et al., 2001; Parkes et al., 2002), the F-expansion method (Wang and Li, 2005; Liu and Yang, 2004; Wang and Zhang, 2005; Chen et al., 2005; Zhang and Xia, 2006), the (G'/G)-expansion method (Wang et al., 2008; Zhang, 2008; Zayed and Gepreel, 2009; Zayed, 2009; Bekir, 2008; Ayhan and Bekir, 2012; Kudryashov, 2010a, b; Aslan, 2010; Li et al., 2010; Zayed and Abdelaziz, 2012), the modified simple equation method (Jawad et al., 2010; Zayed, 2011; Zayed and Hoda Ibrahim, 2012; Zayed and Arnous, 2012), the multiple exp-function algorithm (Ma and Zhu, 2012; Ma et al., 2010), the transformed rational function method (Ma and Lee, 2009; Ma et al., 2007; Ma and Fuchssteliner, 1996),

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local fractional variation iteration method (Yang and Baleanu, 2013), local fractional series expansion method (Yang et al., 2013; Hu et al., 2012) and so on. Based on the observation that it has been a successful idea to generate exact solutions of nonlinear wave equations by reducing PDEs into ordinary differential equations (ODEs), Ma and Lee (2009) proposed the transformed rational function method for constructing these solutions by using the rational function transformations. Ma and Lee's method is more general and will be applied in forthcoming articles for some nonlinear wave equations with both integer and fractal orders.

In the present article, we will apply the modified simple equation method (Jawad et al., 2010; Zayed, 2011; Zayed and Hoda Ibrahim, 2012; Zayed and Arnous, 2012) to find the exact solutions for two coupled systems of nonlinear evolution equations via the (2+1)-dimensional nonlinear Konopelchneko-Dubrovsky equations (Wang and Zhang, 2005; Zhang and Xia, 2006; Xia et al., 2004; Zhang, 2007; Wang and Wei, 2010) and the (2+1)- dimensional nonlinear Nizhnik-Novikov-Vesselov equations (Ren and Zhang, 2006; Xia et al., 2001; Zayed and Abdel Rahman, 2011) which play an important role in mathematical physics. The main idea of this method is that we choose a suitable wave transformation to reduce the nonlinear partial differential equations (PDEs) to nonlinear ordinary differential equations (ODEs). Then, we assume that the formal solutions of these nonlinear equations can be expressed by polynomials in $[\psi'(\xi)/\psi(\xi)]$ where $\psi(\xi)$ is an unknown function such that $\psi'(\xi)\neq 0$ and the dash denotes the derivative with respect to $\xi = x + y - ct$ where c is a constant. The degrees of these polynomials can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in the given nonlinear equations. The coefficients of these polynomials as well as the unknown function $\psi(\xi)$ can be obtained as follows: Substituting the formal solutions into the given ODEs, we have other polynomials in $\psi^{-j}, (j = 0,1,2,...)$. Equating with zero all the coefficients of these polynomials, we get a system of equations which can be solved easily without using the Maple or Mathematica to find the required unknowns and hence we derive the exact solutions.

The rest of this article is organized as follows: First is a description of the modified simple equation method. This is followed by an application of this method to solve the nonlinear Konopelchneko-Dubrovsky equations and the nonlinear Nizhnik-Novikov-Vesselov equations. Thereafter some conclusions are presented.

DESCRIPTION OF THE MODIFIED SIMPLE EQUATION METHOD

Suppose we have a nonlinear evolution equation in the form

$$F(u, u_t, u_x, u_y, u_{xx}, u_{yy}, ...) = 0, \tag{1}$$

where F is a polynomial in $u(x, y, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method (Jawad et al., 2010; Zayed, 2011; Zayed and Hoda Ibrahim, 2012; Zayed and Arnous, 2012) as follows:

Step 1: We use the wave transformation

$$u(x, y, t) = u(\xi), \quad \xi = x + y - ct, \tag{2}$$

where c is a constant, to reduce Equation (1) to the following ODE:

$$P(u, u', u'', ...) = 0, \tag{3}$$

where P is a polynomial in u and its total derivatives, while the primes denote the derivatives with respect to ξ.

Step 2: We suppose that Equation (3) has the formal solution

$$u(\xi) = \sum_{k=0}^{N} A_k \left[\frac{\psi'(\xi)}{\psi(\xi)}\right]^k, \tag{4}$$

where A_k are constants to be determined, such that $A_N \neq 0$. The function $\psi(\xi)$ is an unknown function to be determined later, such that $\psi'(\xi)\neq 0$.

Step 3: We determine the positive integer N in (4) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Equation (3).

Step 4: We substitute (4) into (3), we calculate all the necessary derivatives $u', u'', u''', ...$ of the function $u(\xi)$ and we account the function $\psi(\xi)$. As a result of this substitution, we get a polynomial of $\psi^{-j}, (j = 0,1,...)$. In this polynomial, we gather all the terms of the same power of $\psi^{-j}, (j = 0,1,...)$, and we equate with zero all the coefficients of this polynomial. This operation yields a system of equations which can be solved to find A_k and $\psi(\xi)$. Consequently, we can get the exact solutions of Equation (1).

APPLICATIONS

Here, we will apply the modified simple equation method to find the exact traveling wave solutions and then the solitary wave solutions for the following nonlinear systems of evolution equations in two-dimensions:

Example 1: The (2+1)-dimensional nonlinear Konopelchneko-Dubrovsky equations

These equations are well known and have the following forms:

$$u_t - u_{xxx} - 6\alpha u u_x + \frac{3}{2}\beta^2 u^2 u_x - 3v_y + 3\beta u_x v = 0, \quad (5)$$

$$v_x = u_y, \quad (6)$$

where α and β are real nonzero parameters. Many methods were used to find the exact traveling wave solutions of Equations (5) and (6) (Wang and Zhang, 2005; Zhang and Xia, 2006; Xia et al., 2004; Zhang, 2007; Wang and Wei, 2010). Here, we solve Equations (5) and (6) using the proposed method. To this end, we use the wave transformation (2) to reduce Equations (5) and (6) to the following ODE:

$$-cu' - u''' - 6\alpha u u' + \frac{3}{2}\beta^2 u^2 u' - 3v' + 3\beta u' v = 0, \quad (7)$$

$$u' = v'. \quad (8)$$

From (7) and (8) we deduce that

$$[3\beta k_1 - (c+3)]u' - u''' - 6\alpha u u' + \frac{3}{2}\beta^2 u^2 u' + 3\beta u u' = 0, \quad (9)$$

where $v = u + k_1$, and k_1 is an arbitrary constant of integration. Integrating (9) once with respect to ξ and vanishing the constant of integration, we have the equation

$$[3\beta k_1 - (c+3)]u - u'' + \left(\frac{3}{2}\beta - 3\alpha\right)u^2 + \frac{1}{2}\beta^2 u^3 = 0. \quad (10)$$

Balancing u'' with u^3 yields $N = 1$. Consequently, we have the formal solution

$$u(\xi) = A_0 + A_1\left[\frac{\psi'(\xi)}{\psi(\xi)}\right], \quad (11)$$

where A_0 and A_1 are constants to be determined such that $A_1 \neq 0$. Consequently, it is easy to see that

$$u' = A_1\left(\frac{\psi''}{\psi} - \frac{\psi'^2}{\psi^2}\right), \quad (12)$$

$$u'' = A_1\left(\frac{\psi'''}{\psi} - 3\frac{\psi'\psi''}{\psi^2} + 2\frac{\psi'^3}{\psi^3}\right). \quad (13)$$

Substituting (11)-(13) into (10) and equating all the coefficients of $\psi^0, \psi^{-1}, \psi^{-2}, \psi^{-3}$ to zero, we respectively obtain

$$\psi^0 : [3\beta k_1 - (c+3)]A_0 + \left(\frac{3}{2}\beta - 3\alpha\right)A_0^2 + \frac{\beta^2}{2}A_0^3 = 0, \quad (14)$$

$$\psi^{-1} : [3\beta k_1 - (c+3)]A_1\psi' + (3\beta - 6\alpha)A_0 A_1 \psi' + \frac{3\beta^2}{2}A_0^2 A_1 \psi' - A_1\psi''' = 0, \quad (15)$$

$$\psi^{-2} : A_1\psi'[(\beta - 2\alpha)A_1\psi' + \beta^2 A_0 A_1 \psi' + 2\psi''] = 0, \quad (16)$$

$$\psi^{-3} : A_1\psi'^3(\beta^2 A_1^2 - 4) = 0. \quad (17)$$

From Equations (14) and (17) we deduce that

$$A_0 = 0, \quad \frac{1}{2}\beta^2 A_0^2 + \left(\frac{3}{2}\beta - 3\alpha\right)A_0 + [3\beta k_1 - (c+3)] = 0, \quad A_1 = \pm\frac{2}{\beta}. \quad (18)$$

Let us now discuss the following cases:

Case 1

If $A_0 = 0,\ A_1 \neq 0,\ \psi' \neq 0$ then we deduce from Equations (15) and (16) that

$$[3\beta k_1 - (c+3)]\psi' - \psi''' = 0, \quad (19)$$

$$(\beta - 2\alpha)A_1\psi' + 2\psi'' = 0. \quad (20)$$

Consequently, Equations (19) and (20) yield

$$\psi''' / \psi'' = \frac{2(c+3) - 6\beta k_1}{(\beta - 2\alpha)A_1}, \quad (21)$$

where $\beta \neq 2\alpha,\ c + 3 \neq 3\beta k_1$.

Integrating (21) and using (20) we have

$$\psi' = \frac{-2c_1}{(\beta - 2\alpha)A_1}\exp\left\{\frac{2(c+3) - 6\beta k_1}{(\beta - 2\alpha)A_1}\xi\right\}, \quad (22)$$

and then we have

$$\psi = c_2 - \frac{c_1}{(c+3) - 3\beta k_1}\exp\left\{\frac{2(c+3) - 6\beta k_1}{(\beta - 2\alpha)A_1}\xi\right\}, \quad (23)$$

where c_1 and c_2 are arbitrary constants of integration. From Equations (11), (22) and (23) we have the exact solution

$$u(\xi)=\frac{-\left(\frac{2(c+3)-6\beta k_1}{\beta-2\alpha}\right)\exp\left\{\frac{2(c+3)-6\beta k_1}{(\beta-2\alpha)A_1}(\xi+\xi_0)\right\}}{c_2-\exp\left\{\frac{2(c+3)-6\beta k_1}{(\beta-2\alpha)A_1}(\xi+\xi_0)\right\}}, \quad (24)$$

where $c_1=[(c+3)-3\beta k_1]\exp\left\{\frac{2(c+3)-6\beta k_1}{(\beta-2\alpha)A_1}\xi_0\right\}$ and ξ_0 is a constant.

From (24) we deduce respectively the following kink-shaped solitary wave solutions:

(i) If $c_2=\mp1$ and $\frac{(c+3)-3\beta k_1}{(\beta-2\alpha)A_1}>0,$ we obtain

$$u_1(\xi)=\frac{(c+3)-3\beta k_1}{\beta-2\alpha}\left\{1+\tanh\left[\frac{(c+3)-3\beta k_1}{(\beta-2\alpha)A_1}\right](\xi+\xi_0)\right\}, \quad (25)$$

$$u_2(\xi)=\frac{(c+3)-3\beta k_1}{\beta-2\alpha}\left\{1+\coth\left[\frac{(c+3)-3\beta k_1}{(\beta-2\alpha)A_1}\right](\xi+\xi_0)\right\}, \quad (26)$$

(ii) If $c_2=\mp1$ and $\frac{(c+3)-3\beta k_1}{(\beta-2\alpha)A_1}<0,$ we obtain

$$u_3(\xi)=\frac{(c+3)-3\beta k_1}{\beta-2\alpha}\left\{1-\tanh\left[\frac{(c+3)-3\beta k_1}{(\beta-2\alpha)A_1}(\xi+\xi_0)\right]\right\}, \quad (27)$$

$$u_4(\xi)=\frac{(c+3)-3\beta k_1}{\beta-2\alpha}\left\{1-\coth\left[\frac{(c+3)-3\beta k_1}{(\beta-2\alpha)A_1}(\xi+\xi_0)\right]\right\}, \quad (28)$$

Case 2

If $A_0\neq0, A_1\neq0, \psi'\neq0$ then we deduce from Equations (15) and (16) that

$$\left[\frac{3}{2}\beta^2A_0^2+(3\beta-6\alpha)A_0+3\beta k_1-(c+3)\right]\psi'=\psi''', \quad (29)$$

$$\left[\beta-2\alpha+\beta^2A_0\right]A_1\psi'+2\psi''=0. \quad (30)$$

With the help of (18), we can simplify Equation (29) to take the form

$$\left[\beta^2A_0^2+\left(\frac{3}{2}\beta-3\alpha\right)A_0\right]\psi'=\psi'''. \quad (31)$$

From (30) and (31) we have

$$\psi'''/\psi''=-A/B, \quad (32)$$

where, $A=2\beta^2A_0^2+(3\beta-6\alpha)A_0$ and $B=(\beta-2\alpha)A_1+\beta^2A_0A_1.$ Integrating (32) and using (30) we deduce that

$$\psi'=-\frac{2c_1}{B}\exp\left(-\frac{A}{B}\xi\right), \quad (33)$$

and then

$$\psi=c_2+\frac{2c_1}{A}\exp\left(-\frac{A}{B}\xi\right), \quad (34)$$

where c_1 and c_2 are constants of integration. From (11), (33) and (34), we have the exact solution

$$u(\xi)=A_0-\frac{\left(\frac{AA_1}{B}\right)\exp\left[-\frac{A}{B}(\xi+\xi_0)\right]}{c_2+\exp\left[-\frac{A}{B}(\xi+\xi_0)\right]}, \quad (35)$$

where $c_1=\frac{A}{2}\exp\left[-\frac{A}{B}\xi_0\right]$ and $A_0=\frac{-(3\beta-6\alpha)\pm\sqrt{(3\beta-6\alpha)^2+8\beta^2[(c+3)-3\beta k_1]}}{2\beta^2}.$

From (35) we deduce the following kink-shaped solitary wave solutions:

(i) If $c_2=\pm1$ and $A/B>0,$ we get

$$u_1(\xi)=A_0-\frac{AA_1}{2B}\left\{1-\tanh\left[\frac{A}{2B}(\xi+\xi_0)\right]\right\}, \quad (36)$$

$$u_2(\xi)=A_0-\frac{AA_1}{2B}\left\{1-\coth\left[\frac{A}{2B}(\xi+\xi_0)\right]\right\}, \quad (37)$$

respectively.

(ii) If $c_2=\pm1$ and $A/B<0,$ we get

$$u_3(\xi)=A_0-\frac{AA_1}{2B}\left\{1+\tanh\left[\frac{A}{2B}(\xi+\xi_0)\right]\right\}, \quad (38)$$

$$u_4(\xi)=A_0-\frac{AA_1}{2B}\left\{1+\coth\left[\frac{A}{2B}(\xi+\xi_0)\right]\right\}, \quad (39)$$

respectively.

Example 2: The (2+1)-dimensional nonlinear Nizhnik-Novikov-Vesselov equations

These equations are well known and have the following forms:

$$u_t + ku_{xxx} + ru_{yyy} + su_x + qu_y = 3k(uv)_x + 3r(uw)_y, \quad (40)$$

$$u_x = v_y, \quad u_y = w_x, \quad (41)$$

where k, r, s and q are real parameters. Many methods were used to find the exact traveling wave solutions of Equations (40) and (41) (Ren and Zhang, 2006; Xia et al., 2001; Zayed and Abdel Rahman, 2011). Here, we solve these equations using the modified simple equation method previously described. To this end, we use the wave transformation (2) to reduce these equations to the following ODEs:

$$(q + s - c)u' + (k + r)u''' - 3k(uv)' - 3r(uw)' = 0, \quad (42)$$

$$u' = v', \quad u' = w'. \quad (43)$$

Integrating (43), we get

$$v = u + k_1 \text{ and } w = u + k_2, \quad (44)$$

where k_1 and k_2 are arbitrary constants of integration. From (42) and (44) we get the equation

$$[(q + s - c) - 3(kk_1 + rk_2)]u' + (k + r)u''' - 3(k + r)(u^2)' = 0. \quad (45)$$

Integrating Equation (45) once with respect to ξ and vanishing the constant of integration, we have

$$[(q + s - c) - 3(kk_1 + rk_2)]u + (k + r)u'' - 3(k + r)u^2 = 0. \quad 46)$$

Balancing u'' with u^2 yields $N = 2$. Consequently, we get the formal solution

$$u(\xi) = A_0 + A_1\left[\frac{\psi'(\xi)}{\psi(\xi)}\right] + A_2\left[\frac{\psi'(\xi)}{\psi(\xi)}\right]^2, \quad (47)$$

where A_0, A_1 and A_2 are constants to be determined, such that $A_2 \neq 0$. It is easy to see that

$$u' = A_1\left[\frac{\psi''}{\psi} - \frac{\psi'^2}{\psi^2}\right] + A_2\left[\frac{2\psi'\psi''}{\psi^2} - \frac{2\psi'^3}{\psi^3}\right], \quad (48)$$

$$u'' = A_1\left[\frac{\psi'''}{\psi} - \frac{3\psi'\psi''}{\psi^2} + \frac{2\psi'^3}{\psi^3}\right] + 2A_2\left[\frac{\psi'\psi'''}{\psi^2} + \frac{\psi''^2}{\psi^2} - \frac{5\psi'^2\psi''}{\psi^3} + \frac{3\psi'^4}{\psi^4}\right]. \quad (49)$$

Substituting (47) and (49) into (46) and equating all the coefficients of $\psi^0, \psi^{-1}, \psi^{-2}, \psi^{-3}, \psi^{-4}$ to zero, we deduce respectively that

$$\psi^0 : [(q + s - c) - 3(kk_1 + rk_2)]A_0 - 3(k + r)A_0^2 = 0, \quad (50)$$

$$\psi^{-1} : [(q + s - c) - 3(kk_1 + rk_2)]A_1\psi' - 6(k + r)A_0A_1\psi' + (k + r)A_1\psi''' = 0, \quad (51)$$

$$\psi^{-2} : [(q + s - c) - 3(kk_1 + rk_2)]A_2\psi'^2 - 3(k + r)A_1^2\psi'^2 - 6(k + r)A_0A_2\psi'^2 - 3(k + r)A_1\psi'\psi'' + 2(k + r)A_2(\psi'\psi''' + \psi''^2) = 0, \quad (52)$$

$$\psi^{-3} : -6(k + r)A_1A_2\psi'^3 + 2(k + r)A_1\psi'^3 - 10(k + r)A_2\psi'^2\psi'' = 0, \quad (53)$$

$$\psi^{-4} : -3(k + r)A_2^2\psi'^4 + 6(k + r)A_2\psi'^4 = 0. \quad (54)$$

From (50) and (54), we have the following results:

$$A_0 = 0, \quad A_0 = \frac{(q + s - c) - 3(kk_1 + rk_2)}{3(k + r)}, \quad A_2 = 2, \quad (55)$$

provided $k + r \neq 0$ and $q + s - c \neq 3(kk_1 + rk_2)$. Let us now discuss the following cases:

Case 1

If $A_0 = 0$, $A_1 \neq 0$ and $\psi' \neq 0$, then we deduce from Equations (51), (52) and (53) that

$$[(q + s - c) - 3(kk_1 + rk_2)]\psi' + (k + r)\psi''' = 0, \quad (56)$$

$$\left\{2(q + s - c) - 6(kk_1 + rk_2) - 3A_1^2(k + r)\right\}\psi'^2 - 3(k + r)A_1\psi'\psi'' + 4(k + r)[\psi'\psi''' + \psi''^2] = 0, \quad (57)$$

$$A_1\psi' + 2\psi'' = 0. \quad (58)$$

From (56) and (58) we have

$$\psi' = \frac{-(k + r)\psi'''}{(q + s - c) - 3(kk_1 + rk_2)} = \frac{-2\psi''}{A_1}, \quad (59)$$

and consequently we get

$$\psi''' / \psi'' = \frac{2(q + s - c) - 6(kk_1 + rk_2)}{A_1(k + r)}. \quad (60)$$

Integrating (60), we get

$$\psi'' = c_1 \exp\left[\frac{2(q + s - c) - 6(kk_1 + rk_2)}{A_1(k + r)}\xi\right], \quad (61)$$

and substituting from (61) into (59), we have

$$\psi' = \frac{-2c_1}{A_1}\exp\left[\frac{2(q+s-c)-6(kk_1+rk_2)}{A_1(k+r)}\xi\right]. \qquad (62)$$

Integrating (62), we have

$$\psi = c_2 - \frac{c_1(k+r)}{(q+s-c)-3(kk_1+rk_2]}\exp\left[\frac{2(q+s-c)-6(kk_1+rk_2)}{A_1(k+r)}\xi\right], \qquad (63)$$

where c_1 and c_2 are arbitrary constants of integration. Substituting (59) into (57), we have

$$A_1 = \pm 2\sqrt{\frac{3(kk_1+rk_2)-(q+s-c)}{k+r}}, \qquad (64)$$

provided

$$\frac{3(kk_1+rk_2)-(q+s-c)}{k+r} > 0.$$

Now, the exact solution of the system (40) and (41) in this case has the form

$$u(\xi) = \frac{6(kk_1+rk_2)-2(q+s-c)}{k+r}\left\{\frac{\exp\left[\pm(\xi+\xi_0)\sqrt{\frac{3(kk_1+rk_2)-(q+s-c)}{k+r}}\right]}{c_2+\exp\left[\pm(\xi+\xi_0)\sqrt{\frac{3(kk_1+rk_2)-(q+s-c)}{k+r}}\right]}\right\}^2 - \frac{6(kk_1+rk_2)-2(q+s-c)}{k+r}\left\{\frac{\exp\left[\pm(\xi+\xi_0)\sqrt{\frac{3(kk_1+rk_2)-(q+s-c)}{k+r}}\right]}{c_2+\exp\left[\pm(\xi+\xi_0)\sqrt{\frac{3(kk_1+rk_2)-(q+s-c)}{k+r}}\right]}\right\}, \qquad (65)$$

where $c_1 = \frac{3(kk_1+rk_2)-(q+s-c)}{k+r}\exp\left[\pm\xi_0\sqrt{\frac{3(kk_1+rk_2)-(q+s-c)}{k+r}}\right].$

If we set $c_2 = \pm 1$ we have respectively the bell-shaped solitary wave solutions:

$$u_1(\xi) = \frac{-[3(kk_1+rk_2)-(q+s-c)]}{2(k+r)}\operatorname{sech}^2\left[\frac{1}{2}\sqrt{\frac{3(kk_1+rk_2)-(q+s-c)}{k+r}}(\xi+\xi_0)\right], \qquad (66)$$

$$u_2(\xi) = \frac{[3(kk_1+rk_2)-(q+s-c)]}{2(k+r)}\operatorname{cosech}^2\left[\frac{1}{2}\sqrt{\frac{3(kk_1+rk_2)-(q+s-c)}{k+r}}(\xi+\xi_0)\right]. \qquad (67)$$

Case 2

If $A_0 \neq 0$ $A_1 \neq 0,$ and $\psi' \neq 0,$ then we deduce from Equations (51) to (53) that

$$[3(kk_1+rk_2)-(q+s-c)]\psi' + (k+r)\psi''' = 0, \qquad (68)$$

$$\begin{aligned}&-\{6(kk_1+rk_2)-2(q+s-c)+3(k+r)A_1^2\}\psi'^2 \\ &-3A_1(k+r)\psi'\psi'' + 4(k+r)[\psi'\psi''' + \psi''^2] = 0,\end{aligned} \qquad (69)$$

$$A_1\psi' + 2\psi'' = 0. \qquad (70)$$

Consequently, we deduce from Equations (68) and (70) that

$$\psi' = \frac{-(k+r)\psi'''}{3(kk_1+rk_2)-(q+s-c)} = \frac{-2\psi''}{A_1}, \qquad (71)$$

and thus, we get

$$\psi'''/\psi'' = \frac{6(kk_1+rk_2)-2(q+s-c)}{A_1(k+r)}. \qquad (72)$$

Integrating (72), we have

$$\psi'' = c_1\exp\left[\frac{6(kk_1+rk_2)-2(q+s-c)}{A_1(k+r)}\xi\right], \qquad (73)$$

From (71) and (73), we get

$$\psi' = \frac{-2c_1}{A_1}\exp\left\{\frac{6(kk_1+rk_2)-2(q+s-c)}{A_1(k+r)}\xi\right\}, \qquad (74)$$

Integrating (74), we have

$$\psi = c_2 - \frac{c_1(k+r)}{3(kk_1+rk_2)-(q+s-c)}\exp\left\{\frac{6(kk_1+rk_2)-2(q+s-c)}{A_1(k+r)}\xi\right\}, \qquad (75)$$

where c_1 and c_2 are arbitrary constants of integration and $k+r \neq 0,$ $q+s-c \neq 3(kk_1+rk_2).$ Consequently, we conclude from Equation (69) that

$$A_1 = \pm 2\sqrt{\frac{3(q+s-c)-9(kk_1+rk_2)}{k+r}}, \qquad (76)$$

provided that

$$\frac{(q+s-c)-3(kk_1+rk_2)}{k+r} > 0.$$

Now, the exact solution of Equations (40) and (41) in this case has the form:

$$u(\xi)=\frac{2[(q+s-c)-3(kk_1+rk_2)]}{3(k+r)}\left\{\frac{\exp\left[\pm(\xi+\xi_0)\sqrt{\frac{(q+s-c)-3(kk_1+rk_2)}{3(k+r)}}\right]}{c_2+\exp\left[\pm(\xi+\xi_0)\sqrt{\frac{(q+s-c)-3(kk_1+rk_2)}{3(k+r)}}\right]}\right\}^2$$
$$-\frac{2[(q+s-c)-3(kk_1+rk_2)]}{k+r}\left\{\frac{\exp\left[\pm(\xi+\xi_0)\sqrt{\frac{(q+s-c)-3(kk_1+rk_2)}{3(k+r)}}\right]}{c_2+\exp\left[\pm(\xi+\xi_0)\sqrt{\frac{(q+s-c)-3(kk_1+rk_2)}{3(k+r)}}\right]}\right\}+A_0. \quad (77)$$

where $c_1=\frac{(q+s-c)-3(kk_1+rk_2)}{k+r}\exp\left[\pm\xi_0\sqrt{\frac{(q+s-c)-3(kk_1+rk_2)}{3(k+r)}}\right].$

If we set $c_2=1$ in (77) we have the following solitary wave solutions:

$$u_1(\xi)=\frac{-[(q+s-c)-3(kk_1+rk_2)]}{6(k+r)}\left\{3\pm4\tanh\left[\frac{1}{2}\sqrt{\frac{(q+s-c)-3(kk_1+rk_2)}{3(k+r)}}(\xi+\xi_0)\right]\right\}$$
$$+\frac{(q+s-c)-3(kk_1+rk_2)}{6(k+r)}\tanh^2\left[\frac{1}{2}\sqrt{\frac{(q+s-c)-3(kk_1+rk_2)}{3(k+r)}}(\xi+\xi_0)\right], \quad (78)$$

while, if $c_2=-1,$ we have the following solitary wave solutions:

$$u_2(\xi)=\frac{-[(q+s-c)-3(kk_1+rk_2)]}{6(k+r)}\left\{3\pm4\coth\left[\frac{1}{2}\sqrt{\frac{(q+s-c)-3(kk_1+rk_2)}{3(k+r)}}(\xi+\xi_0)\right]\right\}$$
$$+\frac{(q+s-c)-3(kk_1+rk_2)}{6(k+r)}\coth^2\left[\frac{1}{2}\sqrt{\frac{(q+s-c)-3(kk_1+rk_2)}{3(k+r)}}(\xi+\xi_0)\right]. \quad (79)$$

Let us now examine Figures 1 to 4 as it illustrates some of our results obtained in this article. To this end, we select some special values of the parameters obtained, for example, in some of the solutions (36) and (37) of the system (5), (6) and the solutions (66) and (67) of the system (40), (41) to get the following diagrams:

Conclusions

In this paper, we have applied the modified simple equation method to find some new exact solutions and solitary wave solutions of the (2+1)-dimensional Konopelchneko-Dubrovsky Equations (5), (6) and the (2+1)-dimensional Nizhnik-Novikov-Vesselov Equations (40), (41). Let us now compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results (24) and (35) to (39) of the system of Equations (5), (6) are new and different from those obtained in Wang and Zhang (2005), Zhang and Xia (2006), Xia et al. (2004), Zhang (2007), and Wang and Wei (2010) using different methods, while after some simple calculations our results (25), (26), (27) and (28) are in agreement with the results (27) and (28) obtained in Wang and Wei (2010) using the extended tanh-function method. Also, our results (65), (66) and (77) to (79) of the system of Equations (40) and (41) are new and different from the results obtained in Zhang (2007), Ren and Zhang (2006), Xia et al. (2001), and Zayed and Abdel Rahman (2011) using other methods, while after some simple calculations our result (67) is in agreement with the results $u_2(x,y,t)$ obtained in Xia et al. (2001, p. 141) using the hyperbola function method. From these observations, we deduce that the proposed method in the present article is simple, effective and can be applied to

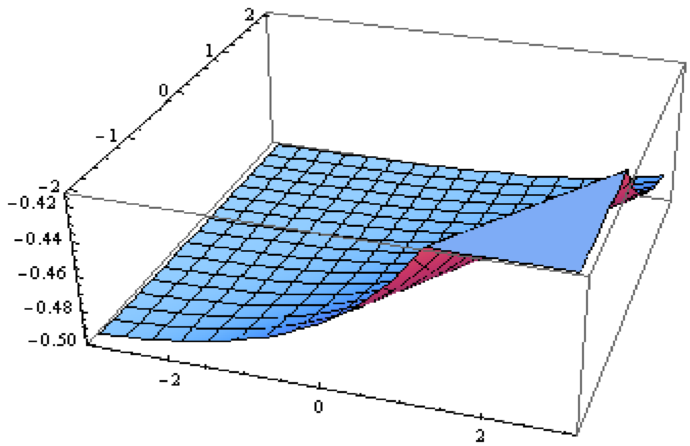

Figure 1. The plot of the solution (36) when $A_0 = 1,\ A_1 = 2,\ A = 3,\ B = 4,\ \xi_0 = -7,\ c = 1,\ y = 0.$

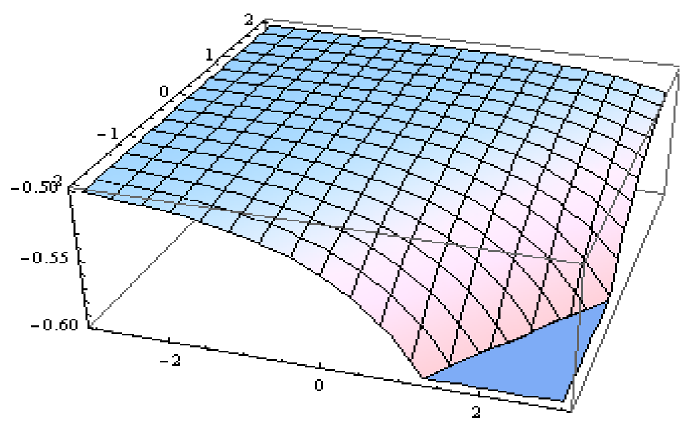

Figure 2. The plot of the solution (37) when $A_0 = 1,\ A_1 = 2,\ A = 3,\ B = 4,\ \xi_0 = -7,\ c = 1,\ y = 0.$

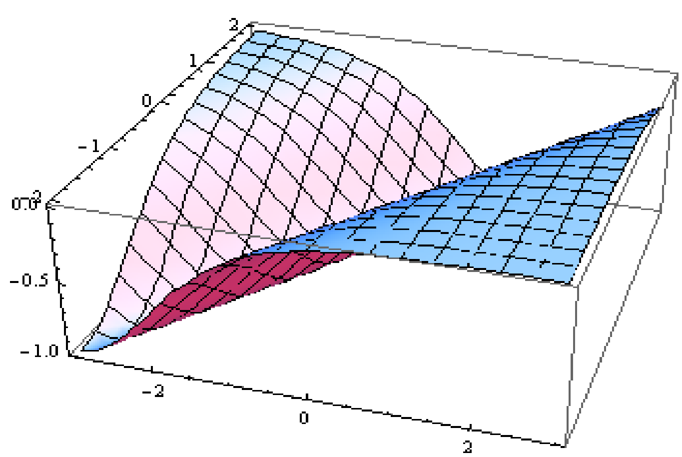

Figure 3. The plot of the solution (66) when $k_1 = 1,\ k_2 = 2,\ k = 3,\ r = -2,\ q = -3,\ s = -1,\ c = 1,\ \xi_0 = 1,\ y = 0.$

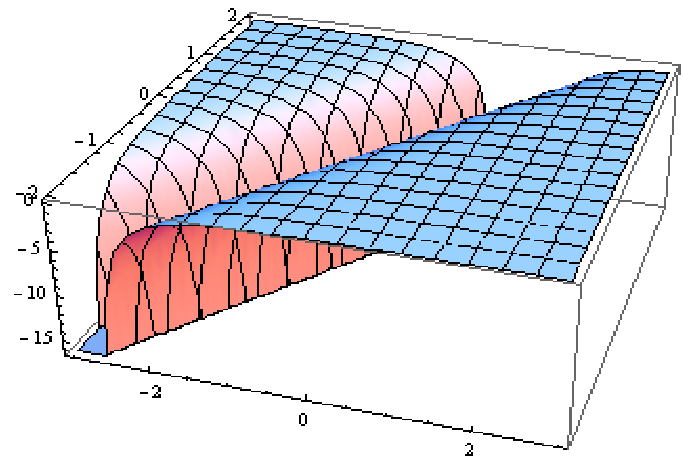

Figure 4. The plot of the solution (67) when $k_1 = 1,\ k_2 = 2,\ k = 3,\ r = -2,\ q = -3,\ s = -1,\ c = 1,\ \xi_0 = 1,\ y = 0.$

many other nonlinear partial differential equations in the mathematical physics. With the aid of the Maple or Mathematica, we have assured the correctness of our solutions by putting them back into the original equations.

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