The role of short-term synaptic depression (STSD) [Tsodyks and Markram, PNAS 94, 719 (1997)] and other forms of synaptic plasticity in brain dynamics is a topical problem in neuroscience. Here we study the role of STSD in the formation of complex patterns of network oscillations and chaotic activity. We use a cortical circuit model of neural networks composed of irregular spiking excitatory and inhibitory neurons having type 1 and 2 excitability and stochastic dynamics [Goltsev et al, Phys. Rev. E. 81, 061921 (2010)]. Neurons form a sparsely connected network. In the model, spontaneous activity of neurons is driven by random spikes representing synaptic noise. Using analytical calculations and simulations, we find that if the STSD is absent, then depending on the noise level, the neural network shows either asynchronous behavior or regular network oscillations. In networks with STSD, changing parameters of synaptic plasticity and the noise level, we observe transitions to complex patterns of collective activities: mixed-mode and spindle oscillations, bursts of collective activity, and chaotic behavior (see Figure). Interestingly, these patterns are stable in a certain range of the parameters and separated by critical boundaries. Thus, the parameters of synaptic plasticity can play a role of control parameters or switchers between different network states. We analyze the chaotic neural activity by use of the 0-1 test for chaos (Gottwald and Melbourne (2005)) and show that it has a collective nature. We compare temporal behavior of neural activity, synaptic efficacies, the average firing rate, and power spectral density for different dynamical regimes with measurements of brain activity.