Abstract—Pneumatic actuators convert pneumatic energy into mechanical motion. This motion can be linear or rotary. Linear motion is feasible with pneumatic cylinders (e.g., single-acting cylinder, double-acting cylinder, rodless cylinder) and pneumatic artificial muscles (PAMs). Pneumatic artificial muscle is the newest and most promising type of pneumatic actuators. PAM is a membrane that expands radially and contracts axially when inflated, while generating high pulling forces along the longitudinal axis. The force and motion produced by PAM are linear and unidirectional. Different designs of PAM have already been developed. Recently Fluidic Muscle manufactured by Festo Company and Shadow Air Muscle manufactured by Shadow Robot Company are the most popular and commercially available. The most often mentioned characteristic of PAMs is the force as a function of pressure and contraction. In this paper our newest function approximation for the force generated by Fluidic Muscles is shown that can be generally used for different muscles made by Festo Company.

I. INTRODUCTION

Electric, hydraulic and pneumatic systems are commonly used in industrial environment, robotics and education [1], [2], [3], [4]. Pneumatic artificial muscles have a wide range of applications, too, e.g. for tab punching, vibratory hopper, lifting device and walking robot [5], [6]. Many important daily activities, such as eating, drinking, dressing and walking depend on two-handed or/and two-legged functions. Rehabilitation and prosthetic devices driven by PAMs can help such people who have difficulties in these areas [7], [8].

There are a lot of advantages of PAMs like the high strength, good power/weight ratio, good power/volume ratio, low price, little maintenance needed, great compliance, compactness, flexibility, inherent safety and usage under rough environments, but their dynamic behaviour is highly nonlinear, therefore a nonlinear robust control technique is needed for accurate positioning [9], [10].

The pneumatic artificial muscle is a one-way acting device. Therefore, two ones are needed to generate bidirectional motion: one of them moves the load, the other one will act as a brake to stop the load at its wanted position and the muscles have to change function to move the load in the opposite direction. This specific connection of the muscles to the load is generally named as an antagonistic set-up: the driving muscle is called the flexor or agonist, while the brake muscle is called the extensor or antagonist. The antagonistic configuration of the actuators causes the active muscle to pull against the stiffness of the passive muscle. Different investigations of PAMs in antagonistic connection are well described in [11] and [12]. Bharadwaj et al. in [13] presented the possibility of bidirectional motion with spring over muscle (SOM).

Many researchers have investigated the relationship of the force, length and pressure to find a good theoretical approach for the equation of force produced by pneumatic artificial muscles. Some of them report several static and dynamic models [11], [14], [15], [16], [17], [18], [19]. Our goal was to develop a precise approximation algorithm with minimum number of parameters for the force of different Fluidic Muscles.

The layout of this paper is as follows. Section II (Static Force Model of Pneumatic Artificial Muscles) is devoted to describe several force equations on the basis of professional literature. Section III (Experimental Results) compares the measured and theoretical data. Finally, Section IV (Conclusion and Future Work) gives the investigations we plan.

Fluidic Muscles type DMSP-10-100N-RM-RM (with inner diameter of 10 mm and initial length of 100 mm) produced by Festo Company was selected for this study.

II. STATIC FORCE MODEL OF PNEUMATIC ARTIFICIAL MUSCLES

The general behaviour of PAMs with regard to shape, contraction and tensile force when inflated depends on the geometry of the inner elastic part and of the braid at rest (Fig. 1), and on the materials used [20]. Typical materials used for the membrane construction are latex and silicone rubber, while nylon is normally used in the fibres.

The load carrying structure of Fluidic Muscles is embedded helically in its membrane. The membrane is made from chloroprene and the load carrying structure is made from aramid (Fig. 2).
With the help of [11] and [14] the input and output (virtual) work can be calculated:

\[ dW_{\text{in}} = p \cdot dV \]  \hspace{1cm} (1)

\[ dW_{\text{in}} \text{ can be divided into a radial and an axial component:} \]

\[ dW_{\text{in}} = 2 \cdot r \cdot \pi \cdot p \cdot (+dr) - r^2 \cdot \pi \cdot p \cdot (-dl) \]  \hspace{1cm} (2)

The output work:

\[ dW_{\text{out}} = -F \cdot dl \]  \hspace{1cm} (3)

By equating the virtual work components:

\[ dW_{\text{in}} = dW_{\text{out}} \]  \hspace{1cm} (4)

Using (1) and (3):

\[ F = p \cdot \frac{dV}{dl} \]  \hspace{1cm} (5)

Using (2) and (3):

\[ F = -2 \cdot r \cdot \pi \cdot p \cdot \frac{dr}{dl} - r^2 \cdot \pi \cdot p \]  \hspace{1cm} (6)

On the basis of Fig. 1:

\[ \cos \alpha_0 = \frac{l_0}{h} \text{ and } \cos \alpha = \frac{1}{h} \]  \hspace{1cm} (7)

\[ \sin \alpha_0 = \frac{2 \cdot \pi \cdot r_0 \cdot n}{h} \text{ and } \sin \alpha = \frac{2 \cdot \pi \cdot r \cdot n}{h} \]  \hspace{1cm} (8)

\[ \frac{1}{l_0} = \frac{\cos \alpha_0}{\cos \alpha} \text{ and } r = \frac{\sin \alpha}{\sin \alpha_0} \]  \hspace{1cm} (9)

\[ r = \frac{r_0}{\sqrt{1 - \cos^2 \alpha_0}} = \frac{r_0}{\sqrt{1 - \left(\frac{1}{l_0} \cdot \cos \alpha_0\right)^2}} \]  \hspace{1cm} (10)

\[ \frac{dr}{dl} = \frac{r_0 \cdot 1 - \cos^2 \alpha_0}{l_0 \cdot \sin \alpha_0} \cdot \frac{1}{\sqrt{1 - \left(\frac{1}{l_0} \cdot \cos \alpha_0\right)^2}} \]  \hspace{1cm} (11)

By using (10) and (11) with (6) the force equation is found:

\[ F(p, \kappa) = r_0^2 \cdot \pi \cdot p \cdot (a \cdot (1 - \kappa)^2 - b) \]  \hspace{1cm} (12)

Where \( a = \frac{3}{l_0^2 a_0} \), \( b = \frac{1}{\sin^2 \alpha_0} \), \( \kappa = \frac{l_0}{l_0} - 1 \) (contraction, relative displacement), \( 0 \leq \kappa \leq \kappa_{\text{max}} \), \( V \) the muscle volume, \( F \) the pulling force, \( p \) the applied pressure, \( r_0, l_0, \alpha_0 \) the initial inner radius and length of the PAM and the initial angle between the thread and the muscle long axis, \( r, l, \alpha \) the inner radius and length of the PAM and angle between the thread and the muscle long axis when the muscle is contracted, \( h \) the constant thread length, \( n \) the number of turns of thread and \( \kappa \) the contraction.

Consequently:

\[ F_{\text{max}} = r_0^2 \cdot \pi \cdot (a - b), \text{if } \kappa = 0 \]  \hspace{1cm} (13)

and

\[ \kappa_{\text{max}} = 1 - \sqrt{\frac{b}{a}}, \text{if } F = 0 \]  \hspace{1cm} (14)

Equation (12) is based on the admittance of a continuously cylindrical-shaped muscle. The fact is that the shape of the muscle is not cylindrical on the end, but rather is flattened, accordingly, the more the muscle contracts, the more its active part decreases, so the actual maximum contraction ration is smaller than expected [10].

Tondu and Lopez in [11] consider improving (12) with a correction factor \( \varepsilon \), because it predicts for various pressures the same maximal contraction. This new equation is relatively good for higher pressure \((p \geq 200 \text{ kPa})\). Kerscher et al. in [16] suggest achieving similar approximation for smaller pressure another correction factor \( \mu \) is needed, so the modified equation is:

\[ F(p, \kappa) = \mu \cdot r_0^2 \cdot \pi \cdot p \cdot (a \cdot (1 - \varepsilon \cdot \kappa)^2 - b) \]  \hspace{1cm} (15)

Where \( \varepsilon = a_k \cdot e^{-p} - b_k \) and \( \mu = a_k \cdot e^{-p r_0^2} - b_k \).
Significant differences between the theoretical and experimental results using (12) and (15) have been proven in [21] and [22]. To eliminate the differences a new approximation algorithm with six unknown parameters has been introduced for the force generated by Fluidic Muscles:

\[
F(p, \kappa) = (a_1 \cdot p + a_2) \cdot \exp \left( a_4 \cdot p + a_5 \cdot \kappa + a_6 \right)
\]  

(16)

The unknown parameters of (16) were found using least squares method with Microsoft Excel Solver.

III. EXPERIMENTAL RESULTS

The analyses were carried out in MS Excel environment. Firstly, tensile force of Fluidic Muscle under a pressure of 600 kPa was determined as shown in Fig. 3.

To approximate the measured force generated by Fluidic Muscles type DMSP-100N-RM-RM (16) was used. Values of the unknown parameters of (16) are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
</tr>
<tr>
<td>(a_1)</td>
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<td>(a_2)</td>
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<td>(a_3)</td>
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<tr>
<td>(a_4)</td>
</tr>
<tr>
<td>(a_5)</td>
</tr>
<tr>
<td>(a_6)</td>
</tr>
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</table>

Fig. 4 presents the experimental and theoretical results on the same graph. To describe the nature and strength of the relationship between the experimental and calculated results, regression and correlation analysis were used. \(R^2 = 0.9999 \rightarrow R = 0.99995\) coefficient approaches the maximum (strongest, \(R = 1\)) correlation (Fig. 5).

Secondly, the investigation was repeated under different pressure values (0-600 kPa). The force always drops from its highest value at full muscle length to zero at full inflation and position (Fig. 6).
The accurate fitting of (16) can be seen in Fig. 7 and Fig. 8 illustrates the relationship between the measured force and calculated force. The $R^2 = 0.9989 \rightarrow R = 0.9994$ coefficient proves the tight relationship between them, i.e. the correlation coefficient slightly reduced if the pressure is extended to the range of 0-600 kPa.

**TABLE II.**
VALUES OF UNKNOWN PARAMETERS OF (16) (0-600 kPa)

<table>
<thead>
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<tr>
<td>$a_1$</td>
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<tr>
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</tr>
</tbody>
</table>

To approximate the hysteresis loops using (16), besides the parameters in Table I. and Table II., new parameters had to be specified for lower curves (Table III. and Table IV.).

**TABLE III.**
VALUES OF UNKNOWN PARAMETERS OF (16) FOR LOWER CURVE (600 kPa)

<table>
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<tr>
<td>$a_6$</td>
<td>-193.56</td>
</tr>
</tbody>
</table>

Next, the hysteresis in the force-length (contraction) loop was analysed. Chou and Hannaford in [14] report hysteresis to be substantially due to the friction, which is caused by the contact between the bladder and the shell, between the braided threads and each other, and the shape changing of the bladder. Fig. 9 illustrates the hysteresis loop under a pressure of 600 kPa, while Fig. 10 demonstrates it under different pressure values (0-600 kPa).
TABLE IV.
VALUES OF UNKNOWN PARAMETERS OF (16) FOR LOWER CURVES (0-600 kPa)

<table>
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</table>

The accurate fitting of (16) for the hysteresis loops can be seen in Fig. 11 and Fig. 12. Finally, Fig. 13 and Fig. 14 prove the accuracy of static force model. Consequently, (16) is capable of making accurate and reliable predictions of static force.

Figure 11. Approximation of hysteresis loop using (16) under a pressure of 600 kPa

Figure 12. Approximation of hysteresis loops using (16) under different pressure values (0-600 kPa)

Figure 13. Result of regression and correlation analysis for lower curve under a pressure of 600 kPa

Figure 14. Result of regression and correlation analysis for lower curves under different pressure values (0-600 kPa)

IV. CONCLUSION AND FUTURE WORK

This paper presents a static force model for Fluidic Muscles. It was proven that the six-parameter function can be used for accurate prediction of static force. The regression and correlation analysis were carried out in MS Excel environment.

The behaviour of pneumatic artificial muscles under operation can be described by dynamic models. On the basis of this static force model a new dynamic model has been developed. With the help of dynamic model the stiffness and damping coefficient of PAMs can be determined and the whole system containing PAM can be described.
REFERENCES


