On the survival of strictly dominated strategies in large populations

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Abstract

This paper analyzes a model of social evolution in which (i) successful agents are imitated and (ii) each player observes payo®s realized in a ¯nite number of randomly drawn games (rather than payo®s obtained by randomly drawn players). Considering an in¯nite random matching population, the study shows that if imitation is based on highest average payo®s realized in observed games, then, for any interior initial conditions, a pure strategy strictly dominated by a pure strategy may survive in the long run. Furthermore, for some interior initial conditions, a strictly dominant strategy may be driven to extinction, even if imitation is based on highest absolute payo®s.

JEL Classi¯cation: C72, C79

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1 Introduction

The long run survival of sub-optimal behavior in an evolutionary environment has received a lot of attention in the last decade. In particular, it is now well known that in a ¯nite population in which (i) agents interact in a normal-form game and (ii) success breeds imitation, a strictly dominant strategy may be driven to extinction in the long run. This happens if an agent deviating from the dominant strategy to play a strictly dominated strategy hurts strictly-dominant strategy players more than himself. (See, for example, Scha®er (1988, 1989), Palomino (1996), Rhode and Stegeman (1996) and Vega-Redondo (1997)).

In a large (i.e., in¯nite) population, Dekel and Scotchmer (1992) and Bjornested (1995) show that a pure strategy strictly dominated by a mixed strategy may survive in the long run. BjÄrnerstedt, Dufwenberg, Norman and Weibull (1995) provide an example in which, for a range of initial conditions, a pure strategy strictly dominated by all other pure strategies survives in the long run if the adjustment process satis¯es \weak monotonicity", i.e., the population composition moves in the general direction of increasing payo®s.[1]

In this paper we show that there exist strategic form games and evolution rules based on the imitation of best performing agents such that in a large population, (i) for any interior initial condition, a pure strategy strictly dominated by a pure strategy survives in the long run; and (ii) for some interior initial conditions a strictly dominant strategy is driven to extinction.

We consider an in¯nite random mixing population of assume that agents observe payo®s realized in a ¯nite number of randomly drawn games. For example, in each period t, an agent is randomly matched with opponents, matched players are then randomly located on a circle and observe payo®s realized in their own game and in a few neighboring games[2].

[1]Considering sexual inheritance (i.e., for each population playing the same strategy there are two types of individuals and a player of a type is only matched with a player of the second type), Waldman (1994) shows that a payo® monotonic process may not lead to the survival of only ¯ttest strategies.

[2]Note that since players observe randomly drawn games, sets of observed games overlap. Therefore, evolution in the entire population is not the result of evolutions in an in¯nite number of isolated ¯nite sub-populations (unlike the results of Hansen and Samuelson (1988)).

The strategic form game matched agents play is such that the Nash equilibrium is Pareto optimal and by deviating from the dominant strategy, deviators hurt dominant-strategy players more than themselves. An example of such a game is public-good-contribution game in which contributing is a dominant strategy and the quality of the good produced increases with the number of contributors.

We show that if, at time $t + 1$, a reviewing agent plays the strategy that yielded the largest average payo® in the games observed then, (i) for any interior initial conditions, a pure strategy strictly dominated by a pure strategy may survive in the long run and, (ii) for some interior initial conditions, a strictly dominant strategy may be driven to extinction. If, at time $t + 1$, a reviewing agent plays the strategy that yielded the highest absolute payo® in observed games, we show that, for some interior initial conditions, the strictly dominant strategy may be driven to extinction.

The results are explained by a \dominant strategy" e®ect and a \deviation" e®ect which together determine long-run equilibria. For any strategy played by opponents, the dominant strategy e®ect measures the relative gain of playing the dominant strategy rather than the dominated one.

The deviation e®ect measures the relative di®erence in variation of payo®s between players when one and only one of them deviates from the dominant strategy. If as in the case of the prisoner's dilemma, by deviating from the dominant strategy, \deviators" increase their opponent's payo® and decrease their own, the two e®ects are complementary. Therefore, only strictly dominant strategy players survive in the long run. Conversely, in a public-good contribution game as described above, the deviation e®ect is captured by the contribution cost. By deviating from the dominant strategy, a non-contributor hurts contributors more than himself since he saves the contribution cost. It follows the dominant strategy e®ect and the deviation e®ect act in opposite ways. As a consequence, if the deviation e®ect is strong enough relative to the dominant strategy e®ect, a pure strategy strictly dominated by a pure strategy may survive in the long run and the strictly dominant strategy may be driven to extinction.

In this framework, the outcome of the imitation process strongly depends on both the quality of information (i.e., randomly drawn games vs randomly drawn players) and on the quantity of information (i.e., number of observations). If agents observe payo®s realized by randomly drawn players then the deviation e®ect is never observed. It follows that the aggregate adjustment process is payo® monotonic and a strictly dominated strategy will be driven to extinction in the long run. If agents observe payo®s realized in randomly drawn games, then the deviation e®ect and the dominant-strategy e®ect are both observed. In such a case, the outcome of the adjustment process depends on the quantity of information agents have. If a reviewing agent observes payo®s realized in a small number of games, the deviation e®ect is strong relative to the dominant strategy e®ect, thus a strictly dominant strategy may disappear in the long run. As the number of observation increases, the strength of the dominant-strategy e®ect relative to the deviation e®ect increases. Therefore, there is a number of observations beyond which a strictly dominated strategy is driven to extinction in the long run.

Given our assumption about strategy choice, our model is directly related to the literature on "replication by imitation". These models are based on the assumptions that (i) agents are not able to analyze the game they play and (ii) their strategy choice is driven by either aspiration level or by imitation of successful agents. In the ¯rst type of models (Smallwood and Conlisk (1979), Cabrales (1993), Binmore, Gale and Samuelson (1995), Borgers and Sarin (1997a, 1997b), Palomino and Vega-Redondo (1999)), if an agent receives a payo® exceeding her aspiration level, then she does not change strategy. If she receives a payo® lower than the aspiration level, then she randomly chooses a new strategy. In the second type of models (Schlag (1998), Weibull (1995, section 4.4)), a reviewing agent randomly selects another agent. She adopts his strategy if he has received a higher payo®. Since all these models assume that a reviewing agent observes randomly drawn players[3] they neglect the deviation e®ect. As a consequence, they lead to the extinction of pure strategies strictly dominated by other pure strategies. As far as we know, the consequences of the deviation e®ect in models of \replication by imitation" dealing with a large popula-

[3] As a matter of fact, these models assume that a reviewing agent observes the payo® realized by one other agent at most. However, if a reviewing agent observes payo®s realized by several randomly drawn players, the same results are obtained, the reason being that in an in¯nite population, the probability of observing two matched players is zero. As a consequence, the probability of observing the deviation e®ect is zero.

tion and limited information about payo®s have never been studied. Our model ¯lls in this gap.

Schlag (1998, 1999) derives results on how to imitate. If a reviewing agent observes the payo® realized by one single randomly drawn agent (Schlag (1998)), the optimal imitation rules are such that if the reviewing player has earned a higher payo® than the player she observes, she should keep playing the same strategy. If she has earned a lower payo®, she should switch to the other strategy with a probability proportional to the payo® di®erence. We do not use this imitation rule for the following reason. In Schlag's model, a reviewing agent only observes two payo®: her own and that of the selected agent. Conversely, in our model, a reviewing agent always observes more than three payo®s. Moreover, it is very often the case that the reviewing agent observes several times the same strategy but associated with di®erent payo®s (due to di®erent strategies played by opponents). In this case, the aggregation of the information about the strategies becomes an issue. Schlag (1998)'s imitation rule does not apply if a reviewing agent observes several time the same strategy, the observed strategy having performed better than that of the reviewing agent sometimes and worse some others.

Schlag (1999) addresses partially this issue by considering the case in which agents face a multi-armed bandit and a reviewing agent observes payo®s realized by two randomly drawn agents. If the two selected agents have played the same strategy but realized di®erent payo®s (in particular one has performed better than the reviewing agent and the other has performed worse), the imitation rule proposed is such that the probability of switching to the strategy of the selected agents (i) is proportional to the di®erence in payo®s between the better performing selected agent and the reviewing agent and (ii) is decreasing in the payo® of the worse performing selected agent. However, Schlag (1999) doesn't address the issue of strategic interaction between observed players. Our model does address this issue.

A feature of our model is that, for any number of payo®s observed, the best performing strategy is always imitated. We consider two ways of measuring the performance of a strategy in observed games: the average payo® (see Ellison and Fudenberg (1995)) and the highest absolute payo® (see Axelrod (1984)). Our results still hold if, following Schlag's (1998, 1999) results, we assume that switching probabilities are proportional to di®erences in performances. The reason is that differences in performances are captured by the intensity of the deviation effect relative to the dominant strategy effect. It follows that the stronger the deviation effect, the higher the rate at which dominant strategy players switch to the dominated strategy. Hence, if imitation is based on average payoffs and is proportional to the extent of the success then, for any interior initial condition, a strictly dominated strategy may survive in the long run. If imitation is based on highest absolute payoff, for some interior initial conditions, a strictly dominant strategy may be driven to extinction.

The organization of this paper is as follows. Section 2 assumes that the stage game is a two-player symmetric normal-form game such that there are only two feasible strategies, one of them strictly dominating the other. The main results about the survival and extinction of strictly dominated or dominant strategies are derived. Section 3 extends the analysis to N-player symmetric games. Section 4 discusses the results and concludes.

2 Two-player games

We consider an evolutionary model in which, in each period, a countably infinite set of agents are randomly matched with an opponent and play the following normal-form game (henceforth Game 1) : the set of actions is $S = \{A; B\}$ and payoffs are as follows:

	A	B
A	$(\alpha; \alpha)$	$(\gamma; \beta)$
B	$(\beta; \gamma)$	$(\delta; \delta)$

Game 1

with $\alpha > \beta > \gamma > \delta$.

Such a game can be interpreted as a public-good contribution game in which contribution (i.e., playing A) is a dominant strategy and the quality of the good produced is increasing in the number of contributors. Of course, if only one agent contributes, the payoff of the non-contributor is larger than that of the contributor (i.e., $\beta > \gamma$).

Also assumed here is that the population is divided into J ($J \geq 2$) sub-populations. De¯ne μ_j ($j = 1, \ldots, J$) as the proportion of sub-population-j players in the entire population. In each period, an agent from sub-population j observes strategies played and payo®s realized in his own game and in $(k_j - 1)$ randomly drawn games. Such an assumption means that agents are heterogeneously informed about payo®s realized in the economy. In particular, (i) matched players may not have the same quantity of information (i.e., may not observe the same number of games) and (ii) if two matched players belong to the same sub-population, they do not have the same piece of information (i.e., they do not observe the same set of games almost surely).[4]

After having observed payo®s agents, choose what strategy to play in the following period. We consider two strategy-selection rules. The ¯rst one is based on average payo®s realized in observed games (as in Ellison and Fudenberg (1995)): at time $t + 1$, an agent plays the strategy that has yielded the highest average payo®s in games she has observed at time t. This type of imitation rule is in the spirit of the standard evolutionary game theory: a strategy that, on average (among observed games) performs better attracts new players. The second performance index we consider is based on absolute payo®s: at time $t + 1$, an agent plays the strategy that has yielded the highest absolute payo® among games she has observed at time t.(see Axelrod (1984))

To summarize, in each period t, the "evolutionary game" can be divided into three sub-periods:

t_1: Each player is randomly matched with an opponent.

t_2: Each pair plays Game 1. Each player from population j observes payo®s realized in her own game and in $(k_j - 1)$ randomly drawn games.

t_3: Players choose their strategy for the next period.

[4] Results presented in this paper also hold if $J = 1$ and all players observe the same (¯nite) number of games. However, since it is assumed that agents have a limited access to information about payo®s, it seems natural to consider an economy in which some agents have access to information of a better quality than others.

2.1 Imitation based on higher average payoffs

In order to model strategy choice, we need to define formally what is meant by "average payoff in observed games". This is done as follows.

Let Q be the set of all feasible pairs of strategies (i.e., $Q = \{(A;A);(A;B);(B;B)\}$)[5]. Given any k_j, consider a collection of k_j observations from Q and let $P(k_j)$ be the set of all possible such collections. Therefore, $P(k_j)$ represents the set of all collections of pairs of strategies a player in subpopulation j can observe. For any collection $P_j \in P(k_j)$, define $G(P_j;s)$ as the subcollection of P_j such that, for all pairs of strategies $p \in G(P_j;s)$, strategy s belongs to pair p. For any $p \in G(P_j;s)$, let $\pi(p;s)$ and $n(p;s)$ denote the payoffs obtained by strategy s if pair p is played in Game 1 and the number of times s is in p (i.e., $n(p;s) = 1$ or $n(p;s) = 2$), respectively[6]. For all $P_j \in P(k_j)$ and $s \in \{A;B\}$, if $G(P_j;s) \neq \emptyset$, The average payoff of strategy s in the collection of observations P_j is defined as follows:

$$\mu(P_j;s) = \frac{\sum_{p \in G(P_j;s)} n(p;s)\pi(p;s)}{\sum_{p \in G(P_j;s)} n(p;s)} \tag{1}$$

Having computed average payoffs, we can focus on the strategy choice. Let s^t and P_j denote the strategy played and the collection of games observed by a reviewing agent at time t, respectively. Then, the strategy choice is as follows:

$$\begin{array}{llll} \text{If} & \mu(P_j;A) > \mu(P_j;B); & \text{then} & s^{t+1} = A \\ \text{If} & \mu(P_j;B) > \mu(P_j;A); & \text{then} & s^{t+1} = B \\ \text{If} & \mu(P_j;A) = \mu(P_j;B); & \text{then} & s^{t+1} = s^t \\ \text{If} & \text{for all } p \in P_j \ n(p;s^t) = 2, & \text{then} & s^{t+1} = s^t \end{array} \tag{2}$$

This rule means that players have no memory and use all the information they observe at time t (i.e., the collection P_j) in order to update their strategy. Given that in the k_j

[5] In this model, pairs $(A;B)$ and $(B;A)$ will be equivalent, since they yield the same payoffs have the same probability of being observed.

[6] For example, $n((A;A);A) = 2$, $n((A;B);A) = 1$, and given Game 1, $\pi((A;B);A) = \gamma$ and $\pi((A;B);B) = \beta$.

observed games the number of agents playing each strategy may be different, a reviewing agent computes the average payoff realized by her strategy, compares it with the average payoff realized by the other strategy and, at time $t+1$, plays the strategy that has yielded the higher average payoff at time t. If the two strategies have performed equally well (i.e., $\bar{\pi}(P_j; A) = \bar{\pi}(P_j; B)$) , or if a reviewing agent only observes the strategy s^t she has played at time t (i.e., for all $p \in P_j$, $n(p; s^t) = 2$), she plays again s^t in the next period.

We are interested in the law of motion of the fraction (μ_t) of the population playing the strictly dominant strategy (i.e., A). Given the randomness of both the matching process and the collection of observed games, the process $\{\mu_t\}_{t=0}^{\infty}$ is stochastic. However, as shown by Boylan (1992), there exists a random matching rule such that the law of motion of the stochastic process is (almost surely) the same as the law of motion of the deterministic process in which, in every period, the set of individuals adopting the same strategy is matched with the population average. Hence, in our context, for all t, a fraction μ_t of agents playing a strategy s ($s = A$ or $s = B$) at time t is matched with agents playing strategy A and a fraction $(1 - \mu_t)$ is matched with agents playing B.

As far as the observation of randomly drawn games is concerned, the following randomizing device is used. At each time t, a random variable $\tilde{z}_t$ takes the value 1 with probability μ_t^2, the value 0 with probability $2\mu_t(1 - \mu_t)$ and the value -1 with probability $(1 - \mu_t)^2$. Let z_t denote a realization of $\tilde{z}_t$. If $z_t = 1$ then a game played by 2 strategy-A players is observed, if $z_t = 0$ then a game played by 1 strategy-A player and 1 strategy-B player is observed, and if $z_t = -1$, a game played by 2 strategy-B players is observed. Consider now an agent belonging to sub-population j. The collection of pairs of strategies P_j she observes at time t is given by (i) the pair of strategies used in the game she played at time t and (ii) $(k_j - 1)$ independent realizations of z_t.

In order to derive the law of motion of the system, we define the following functions H_j

from $P(k_j)$ to $[0;1]$.

$$\begin{array}{llll}
\text{If} & \pi(P_j;A) > \pi(P_j;B); & \text{then} & H_j(P_j) = 1 \\
\text{If} & \pi(P_j;A) < \pi(P_j;B); & \text{then} & H_j(P_j) = 0 \\
\text{If} & \pi(P_j;A) = \pi(P_j;B); & \text{then} & H_j(P_j) = \frac{\sum_{p \in G(P_j;A)} n(p;A)}{2k_j} \\
\text{If} & \text{for all } p \in P_j \;\; n(p;A) = 2, & \text{then} & H_j(P_j) = 1 \\
\text{If} & \text{for all } p \in P_j \;\; n(p;B) = 2, & \text{then} & H_j(P_j) = 0
\end{array} \tag{3}$$

The function H_j can be interpreted as follows. If both strategies A and B are present in a collection P_j and A performs better (worse) than B in P_j, then an agent who observes P_j at time t play A (B) at time $t+1$. Consider now the case in which the two strategies perform equally well. Strategy A is represented $\sum_{p \in G(P_j;A)} n(p;A)$ times among the $2k_j$ observed strategies. From the imitation rule (2), we know that in such a case, an agent plays at time $t+1$ the same strategy as at time t. Hence, we deduce that a fraction $[\sum_{p \in G(P_{j;t};A)} n(p;A)]/(2k_j)$ of the agents who observe P_j at time t will play A at time $t+1$ and the remaining fraction $[\sum_{p \in G(P_j;B)} n(p;B)]/(2k_j)$ will plays B. Finally, agents who have observed one single strategy in P_j play this observed strategy in the following period.

Assume that $\mu_0 \in (0;1)$ and denote $F_t(P_j)$ the observation frequency of P_j given μ_t. We derive that the law of motion of the process is:

$$\mu_{t+1} = \sum_{j=1}^{J} \theta_j \left\{ \sum_{P_j \in P(k_j)} F_t(P_j) H(P_j) \right\} \tag{4}$$

Since this paper only deals with the long run survival of the strictly dominated strategy and the extinction of the strictly dominant one, in what follows, we will focus on local properties of the law of motion of the process at $\mu = 0$ and $\mu = 1$.

From the law of motion of the process, we derive the following proposition.

Proposition 1 Assume $\alpha > \beta > \gamma > \delta$ and $\mu_0 \in (0;1)$. All players observe a finite number of games (i.e., for all $j = 1; \ldots; J$, $k_j < \infty$) and the imitation rule is given by (2).

(i) If

$$\beta - \gamma > 2(Max_j(k_j) - 1)(\alpha - \beta) \tag{5}$$

then $\mu^* = 1$ is not an asymptotically stable steady state.

(ii) If

$$\beta - \gamma > 2(Max_j(k_j) - 1)(\gamma - \delta) \qquad (6)$$

then $\mu^* = 0$ is an asymptotically stable steady state.

Proof: See Appendix A.

Imitation based on the observation of a few games stresses the importance of the effect induced by deviating from the Nash equilibrium strategy. If $\alpha > \beta > \gamma > \delta$, then a player, by deviating from the equilibrium strategy, hurts his opponent more than himself. The loss to the deviator is $(\alpha - \beta)$ and the loss to a strategy-A player is $(\alpha - \gamma)$. Therefore, stable steady states can be explained by the combination of two effects: a “dominant strategy” effect and a “deviation” effect. For any strategy played by his opponent, the dominant-strategy effect measures the opportunity cost of deviating from the dominant strategy. Hence, it measures to what extent a deviator has decreased his payoff by playing the dominated strategy instead of playing the dominant strategy. In a game with two strategy-A players, the dominant-strategy effect is given by $(\alpha - \beta)$. In a game with both a strategy-A player and a strategy-B player, the dominant-strategy effect is given by $(\gamma - \delta)$. The deviation effect reflects the relative difference in variation of payoffs between the two players when one and only one of them deviates from the Nash equilibrium strategy. The deviation effect is measured by $(\beta - \gamma)$ and is positive. Therefore, the two effects act in opposite ways and thus, long-run equilibria depend on the relative intensity of the two effects.

Case (i) gives sufficient conditions on the intensity of the deviation effect relative to the dominant-strategy effect to ensure the survival of strictly dominated strategies in the long run. When the share of dominant strategy players in the economy is very large (so that the probability of having more than one dominated-strategy player in the k_j observed games is very small relative to the probability of observing either 0 or 1 dominated-strategy player), inequality (5) ensures that dominated-strategy players are imitated. When k_j $(A; A)$-pairs are observed, no imitation occurs. If $(k_j - 1)$ $(A; A)$-pairs and one $(A; B)$-pair are observed, the average payoff of strategy A is $[2(k_j - 1)\alpha + \gamma]/(2k_j - 1)$ and the average payoff of strategy B is β. If $\beta > [2(k_j - 1)\alpha + \gamma]/(2k_j - 1)$, a strategy-B player will again play B in the following period and a dominant-strategy player will shift to the dominated one. Inequality (5) means that for all j, we have $\beta > [2(k_j - 1)\alpha + \gamma]/(2k_j - 1)$, Therefore, any path starting away from $\mu = 1$ will never converge to 1.

Case (ii) gives a sufficient condition for the imitation of strategy-B players when they represent an overwhelming majority of the population (so that the probability of having more than one dominant-strategy player in the k_j observed games is very small relative to the probability of observing either 0 or 1 dominant-strategy player). If $(k_j - 1)$ $(B; B)$-pairs and one $(A; B)$-pair are observed, the average payoff of strategy B is $[2(k_j - 1)\delta + \beta]/(2k_j - 1)$ and the average payoff of strategy A is γ. Then, if $\gamma < [2(k_j - 1)\delta + \beta]$, a strategy-A player will switch to B, and a strategy-B player will again play B in the following period. Inequality (6) means that for all j, $\gamma < [2(k_j - 1)\delta + \beta]$. Therefore, any path starting arbitrarily close to zero converges to zero.

Results of proposition 1 should be compared to those obtained if the stage game is a prisoner's dilemma ($\gamma > \delta > \alpha > \beta$). In such a case, by deviating from the dominant strategy, a player increases his opponent's payoff and decreases his own. The gain to a strategy-A player is $(\gamma - \alpha)$ and the loss to a deviator is $(\alpha - \beta)$. In the prisoner's dilemma, therefore, deviation can be regarded as an altruistic behavior. It follows that the dominant strategy effect and the deviation effect complement each other and so generate the same type of imitation: a reviewing agent who has played the strictly dominated strategy and observes some dominant-strategy players at time t switches to the dominant strategy at time $t + 1$. As a result, only the dominant strategy is played in the long run.

It should also be noted that, qualitatively, Proposition 1 also holds if it is assumed that agents in sub-population j $(j = 1, \ldots, J)$ observe both payoffs realized in k_j randomly drawn games and payoffs obtained by k'_j randomly drawn players. In such a case, the total payoffs realized by the strictly dominant strategy among randomly drawn players are less than or equal to $k'_j\alpha$ and the total payoffs realized by the strictly dominated strategy among randomly drawn players are greater than or equal to $k'_j\delta$. Hence, if

$$\beta - \gamma > \mathrm{Max}_j[2(k_j - 1) + k_j'](\alpha - \beta) \tag{7}$$

$\mu^* = 1$ is not an asymptotically stable steady state, and if

$$\beta - \gamma > \mathrm{Max}_j[2(k_j - 1) + k_j'](\gamma - \delta) \tag{8}$$

then $\mu^* = 0$ is an asymptotically stable steady state.

The assumption that a reviewing agent also observes payoffs obtained by randomly drawn players (as in standard models of replication by imitation) only modifies the relative strength of the deviation effect sufficient to ensure the survival of the strictly dominated strategy or the extinction of the strictly dominant strategy in the long run.

2.2 Imitation of higher absolute payoffs

In this section, we assume that, at time $t + 1$, an agent from population j plays the strategy that yielded the highest absolute payoff in the k_j she observed at time t. Also in this case, a pure strategy strictly dominated by a pure strategy may survive in the long run. To establish this result, we model the imitation rule as follows: For all $P_j \in P(k_j)$, define $S'(P_j)$ as the subset of S such that, for all $s \in S'(P_j)$, $G(P_j; s) \neq \emptyset$.

For all $s \in S'(P_j)$, let

$$\pi^*(P_j; s) = \mathrm{Max}_{p \in G(P_j;s)} \pi(p; s) \tag{9}$$

Then, $\pi^*(P_j; s)$ represents the highest payoffs realized by strategy s in the k_j observed games. Consider an agent who, at time t, plays s^t and observes $P_{j;t}$. Then, at time $t + 1$, she plays s^{t+1} such that

$$s^{t+1} \in \mathrm{Argmax}_{s \in S'(P_j)} \pi^*(P_j; s) \tag{10}$$

and

$$\text{If } s^t \in \mathrm{Argmax}_{s \in S'(P_j)} \pi^*(P_j; s) \text{ then } s^{t+1} = s^t \tag{11}$$

The imitation rule says that, at time $t+1$, an agent plays one of the strategies that yielded the highest absolute payoff in games observed at time t. If the strategy a reviewing agent has played at time t is one of these strategies, then she plays the same strategy in the following period. Given that $\#S = 2$ and the assumptions about payoffs ($\alpha > \beta > \gamma > \delta$), we always have $\#\mathrm{Argmax}_{s \in S'(P)} \pi^*(P; s) = 1$. Therefore, if s^t is the strategy that yielded the higher payoff at time t, then $s^{t+1} = s^t$. In the opposite case, the reviewing agent switches to the other strategy.

Proposition 2 Assume $\alpha > \beta > \gamma > \delta$ and for all $j = 1; \ldots; J$, $k_j < +\infty$. If the imitation rule is given by (10) and (11), then $\mu^* = 0$ is an asymptotically stable steady state.

Proof: See Appendix A.

Proposition 2 states that, even if at time $t+1$ each agent plays the strategy that yielded the higher absolute payoff in games she has observed at time t, the strictly dominated strategy may be the only strategy played in the long run. The reason is that in a population with an overwhelming majority of dominated-strategy players (such that the probability of observing two dominant-strategy players is very small relative to the probability of observing either zero or one strictly-dominant-strategy player), the strategy that yields the higher absolute payoff is nearly always the dominated strategy since a game played by two dominant-strategy players (so that strategy A yields α) is virtually never observed. Therefore, dominated-strategy players are imitated, and $\mu^* = 0$ is an asymptotically stable steady state.

3 N-player games

This section extends the analysis of the survival of strictly dominated strategies to cases for which the one-shot normal-form game played in each period is a N-player symmetric game. We consider the following game. The set of possible strategies is $S = \{A; B\}$. $\pi(s; n)$ ($s = A; B$ and $n = 0; \ldots; N$) is the payoff of an agent playing s when n agents play B. We make the following assumptions about payoffs.

A1: $\pi(A; n) > \pi(B; n + 1)$.

A2: $\pi(A; 0) > \pi(B; n) > \pi(A; n) > \pi(B; N)$ $(n = 1; \ldots; N - 1)$

Assumption (A1) means that A is a strictly dominant strategy. Assumption (A2) describes the deviation effect in a N-player game. If some players deviate from the equilibrium strategy, they obtain a higher payoff than dominant-strategy players.

As before, the game can be interpreted as a public-good contribution game in which the quality of the good produced increases with the number of contributors and contributing is a dominant strategy.

The main difference between the case in which the stage game is a two-player game and the case in which it is a N-player game is that in the later, if an agent deviates from the dominant strategy, he may hurt up to $N - 1$ agents As a consequence, the deviation effect has now two dimensions: the difference in payoffs $\pi(B; n) - \pi(A; n)$ and the number of agents a deviator hurts.

As in the previous section, we assume that a player in subpopulation j observes payoffs realized in his own game and in $(k_j - 1)$ randomly drawn other games. Given the assumption about the stage game, some additional notations are needed. Define Q^0 as the set of all possible collections of N elements from S. Given any k_j, consider a collection of k_j elements from Q^0 and let $P^0(k_j)$ as the set of all such collections. Hence, the collection of games an agent from subpopulation j observes is an element of $P^0(k_j)$. Other notations do not need to be modified. For all $P_j^0 \in P^0(k_j)$ and if $G(P_j^0; A) \neq \emptyset$, the average payoff realized by strategy A is

$$\mu(P_j^0; A) = \frac{\sum_{p \in G(P_j^0; s)} n(p; A)\pi(A; N - n(p; A))}{\sum_{p \in G(P_j^0; s)} n(p; A)} \tag{12}$$

For all $P_j^0 \in P^0(k_j)$ and if $G(P_j^0; B) \neq \emptyset$, the average payoff realized by strategy B is

$$\mu(P_j^0; B) = \frac{\sum_{p \in G(P_j^0; s)} n(p; B)\pi(B; n(p; B))}{\sum_{p \in G(P_j^0; s)} n(p; B)} \tag{13}$$

Let s^t and $P_{j;t}^0$ be the strategy played and the set of games observed by a reviewing agent at time t, respectively. Imitation is as follows:

$$
\begin{array}{llll}
\text{If} & \bar{\pi}(P'_{j,t}, A) > \bar{\pi}(P'_{j,t}, B) & \text{then} & s^{t+1} = A \\
\text{If} & \bar{\pi}(P'_{j,t}, B) > \bar{\pi}(P'_{j,t}, A) & \text{then} & s^{t+1} = B \\
\text{If} & \bar{\pi}(P'_{j,t}, A) = \bar{\pi}(P'_{j,t}, B) & \text{then} & s^{t+1} = s^t \\
\text{If,} & \text{for all } p \in P'_{j,t},\ n(p, s) = N & \text{then} & s^{t+1} = s
\end{array}
\qquad (14)
$$

If social evolution is based on the imitation of highest absolute payoffs, then imitation rule (10) still holds.

Let η denote the fraction of the population playing A. For both types of imitation rules, we derive the following proposition:

Proposition 3 Assume (A1) and (A2) hold, the imitation rule is given either by (10) or (14) and for all $j = 1, \ldots, J$, $k_j < \infty$. There exists N^* such that for all $N > N^*$, there exists an asymptotically stable steady state $\eta^*(N) < 1$.

Proof: See Appendix B.

Proposition 2 states that no matter how small the difference in payoffs $\pi(B, n) - \pi(A, n)$, if a large enough number of strategy-A players are hurt when an agent plays B, then the fraction of strategy-A players who will switch to B in any period is sufficient to ensure that the strictly dominated strategy is played in the long run.

4 Discussion and conclusion

This article has considered a model of social evolution in which agents observe payoffs realized in a finite number of randomly drawn games. We have shown that in a large population, if imitation is based on highest average payoffs in observed games, then, for any interior initial conditions, a pure strategy strictly dominated by a pure strategy may survive in the long run and a strictly dominant strategy may be driven to extinction. If imitation is based on highest absolute payoffs, then, for some interior initial condition, a strictly dominated strategy may be played in the long run. These results are obtained for games such that the Nash equilibrium is Pareto optimal and by deviating from the dominant strategy, a deviator hurts more dominant-strategy players than himself. An example of such a game is a public-good contribution game in which contributing is a strictly dominant strategy and the quality of the good produced increases with the number of contributors.

Our results stress the importance of the quality and quantity of information agents have about the payoffs realized by feasible strategies, and are explained by a "dominant strategy" effect (contributing) and a "deviation" effect (not contributing). For any strategy played by opponents, the dominant strategy effect measures the gain from contributing given the strategy played by opponents. The deviation effect measures the contribution cost, or in other words, the gain of a non-contributor relative to a contributing opponent.

If a reviewing player observes payoffs realized by randomly drawn players, then he never observes the impact of the action of a non-contributor on his opponents. As a consequence, the deviation effect is never observed and a strictly dominated strategy is driven to extinction in the long run. If players observe payoffs realized in randomly drawn games, then the deviation effect is observed and the long run outcome of the adjustment process depends on the quantity of information a reviewing agent has (i.e., the number of games she observes).

To keep the model simple, some restrictive assumptions were made. Some of them can be relaxed without affecting the results qualitatively. First, our results also hold if it is assumed that either agents have finite memory (about payoffs observed in the previous periods) or agents weight their own game differently than they do other games when computing average payoffs. These two assumptions would only modify the intensity of the deviation effect relative to the dominant-strategy effect necessary to ensure that a strictly dominated strategy survives in the long run or that a strictly dominant strategy is driven to extinction.

Second, a characteristic of our imitation rule is that it says "do not switch if the other strategy performs worse, and always imitate the strategy if it performs better". Schlag (1998, 1999) shows that if a reviewing player observes the payoff realized by either one or two randomly drawn agents, then the reviewing player should keep playing the same strategy if she has earned the higher payoff than observed agents, and if she has performed worse she should switch to the better performing strategy with a probability proportional to the payoff difference. Our results still hold if we assume that switching probabilities are proportional to differences in performances. The reason is that differences in performances are captured by the intensity of the deviation effect relative to the dominant strategy effect. It follows that the stronger the deviation effect, the higher the rate at which dominant strategy players switch to the dominated strategy. Hence, if imitation is based on average payoffs and is proportional to the extent of the success then, for any interior initial condition, a strictly dominated strategy may survive in the long run. If imitation is based on highest absolute payoff, for some interior initial conditions, a strictly dominant strategy may be driven to extinction.(An example is derived in Appendix C.)

Appendix A

Proof of Proposition 1: The dynamics are given by an equation of the following shape.

$$\mu_{t+1} = \sum_{j=1}^{J} \theta_j \left[\mu_t^{2k_j} + \sum_{i=1}^{2k_j-1} a_{j;i}\, \mu_t^i (1-\mu_t)^{2k_j-i} \right] \quad \text{almost surely} \qquad (A.1)$$

(i) The inequality $2(\mathrm{Sup}_j k_j - 1)(\alpha-\beta) < \beta-\gamma$ implies that for all $j = 1;\dots;J$ $a_{j;1} = 2k_j$ and $a_{j;2k_j-1} = 0$. Let $f(\mu_t) = \mu_{t+1} - \mu_t$. Then, $f(1) = 0$ and $f'(1) = 2\sum_{j=1}^{J}\theta_j k_j - 1 > 0$. It follows that $\mu = 1$ is not an asymptotically stable steady state.

(ii) The inequality $\beta-\gamma > 2(\mathrm{Sup}_j k_j - 1)(\gamma-\delta)$ implies that for all j, $a_{j;1} = 0$. It follows that $f(0) = 0$ and $f'(0) = -1$. Therefore, $\mu^* = 0$ is an asymptotically stable steady state.

Proof of Proposition 2: The dynamics are given by an equation of the following shape.

$$\mu_{t+1} = \sum_{j=1}^{J} \theta_j \left(\mu_t^{2k_j} + \sum_{i=1}^{2k_j-1} b_{j;i}\, \mu_t^i (1-\mu_t)^{2k_j-i} \right) \quad \text{almost surely} \qquad (A.2)$$

The assumption about payoffs ($\alpha > \beta > \gamma > \delta$) implies that for all $j = 1;\dots;J$, $b_{j;1} = 0$. Let $f(\mu_t) = \mu_{t+1} - \mu_t$. It follows that $f(0) = 0$ and $f'(0) = -1$. Therefore $\mu^* = 0$ is an asymptotically stable steady state. Q.E.D.

Appendix B

Proof of Proposition 3:

Define η_t as the fraction of dominant strategy (A) players at time t.

(i) Imitation based on average payoffs.

Given assumption (A2), in a game in which the two strategies are played, payoffs to strategy B are higher than payoffs to strategy A. It follows that if, at time t, a player observes k_j games with the same number of agents playing A (and thus the same number of agents playing B), he will play B at time t + 1. Therefore

$$\eta_{t+1} < 1 - \sum_{j=1}^{J} \theta_j \left\{ \sum_{i=1}^{N-1} \binom{N}{i}^{k_j} \eta_t^{(N-i)k_j} (1-\eta_t)^{ik_j} \right\} \tag{B.1}$$

Thus,

$$\eta_{t+1} < 1 - \sum_{j=1}^{J} \theta_j \left(N\eta_t^{(N-1)}(1-\eta_t) \right)^{k_j} \tag{B.2}$$

Define $f_N(\eta_t)$ as the right-hand side of (B.2). Then, a sufficient condition for the existence of an asymptotically stable steady state in $(0;1)$ is that there exists an interval $I \subset (0;1)$ such that for all $\eta \in I$, $f_N(\eta) < \eta$. This is equivalent to

$$\sum_{j=1}^{J} \theta_j \left(\left[N\eta^{(N-1)}(1-\eta) \right]^{k_j} - (1-\eta) \right) > 0 \tag{B.3}$$

Therefore, it is sufficient to show that there exists I such that for all $j = 1;\ldots;J$

$$\left[N\eta^{(N-1)}(1-\eta) \right]^{k_j} - (1-\eta) > 0 \tag{B.4}$$

Let $\eta = (N-1)/N$. Then (B.4) is equivalent to

$$\frac{Ln(N)}{(N-1)[Ln(N) - Ln(N-1)]} > k_j \tag{B.5}$$

The LHS of (B.5) is increasing in N and goes to infinity with N. Therefore, from the continuity of f_N, there exists N^* such that for all $N > N^*$,

$$\frac{Ln(N)}{(N-1)[Ln(N) - Ln(N-1)]} > Max_j k_j \tag{B.6}$$

It follows that there exists an asymptotically stable steady state such that the strictly dominated strategy is played in the long run.

(ii) Imitation based on absolute payoffs.

Any agent who, at time t, observes only games in which the two strategies are played will play B at time $t+1$. Therefore

$$\eta_{t+1} < 1 - \sum_{j=1} \mu_j (1 - \eta_t^N)^{k_j} \tag{B.7}$$

Then, it is sufficient to show that there exists an interval I' such that, for all $\eta \in I'$

$$1 - (1 - \eta^N)^{Max_j k_j} < \eta \tag{B.8}$$

Let $\eta = 1/2$. Then, (B.8) is equivalent to

$$-\frac{Ln(2)}{Ln(1 - 2^{-N})} > Max_j k_j \tag{B.9}$$

The RHS of (B.9) is increasing in N and goes to infinity with N. Therefore, there exists N^{**} such that for all $N > N^{**}$, inequality (B.9) holds. Q.E.D.

Appendix C

Assume that in each period agents play Game 1 and each reviewing player only observes payoffs realized in two games: her own game and one randomly drawn game.

Let payoffs satisfy the following inequalities:

$$3\beta > 2\alpha + \gamma \qquad 3\gamma < 2\delta + \beta \tag{C.1}$$

The first (second) inequality means that if a reviewing agent has played a dominant strategy, been matched with a dominated-strategy player and observed a game played by 2 dominant (dominated) strategy players, then the reviewing agent will switch to the dominated strategy in the following period. Of course, it also means that if the reviewing agent is a dominated-strategy player that has been matched with a dominant strategy player, she will play again the dominated strategy in the following period.

For the sake of tractability, we also assume that payoffs satisfy $\alpha = \beta + \varepsilon$ and $\gamma = \delta + \varepsilon$ with $\varepsilon > 0$. This means that the intensity of the dominant strategy effect is independent of the strategy played by the opponent, i.e., the gain of playing the dominant strategy rather than the dominated strategy is ε, independently of the strategy played by the opponent.

First, consider the case in which imitation is based on average payoffs.

Let $P_1 = \{(A;A);(A;B)\}$, $P_2 = \{(A;B);(A;B)\}$, $P_3 = \{(A;A);(B;B)\}$ and $P_4 = \{(A;B);(B;B)\}$. P_1, P_2, P_3 and P_4 represent the four types of pairs of matched player a reviewing agent can observe. We deduce that

$$\begin{array}{ll} \pi(P_1;A) = (2\alpha + \gamma)/3 & \pi(P_1;B) = \beta \\ \pi(P_2;A) = \gamma & \pi(P_2;B) = \beta \\ \pi(P_3;A) = \alpha & \pi(P_3;B) = \delta \\ \pi(P_4;A) = \gamma & \pi(P_2;B) = (\beta + 2\delta)/3 \end{array}$$

Define $\lambda_i(s_1;s_2)$ $(i = 1;\ldots;4)$ as the probability of switching from s_1 to s_2 ($(s_1;s_2) \in S \times S$) if P_i has been observed and let it be proportional to the difference in performances, i.e.,

$$\lambda_i(s_1;s_2) = \mathrm{Max}\left\{[\pi(P_i;s_2) - \pi(P_i;s_1)]/(\alpha - \delta);0\right\} \tag{C.2}$$

Given the payoffs, $\lambda_1(B;A) = \lambda_2(B;A) = \lambda_4(B;A) = 0$. Hence, it can be shown that, for any $\mu_t \in (0;1)$,

$$\begin{aligned} \mu_{t+1} - \mu_t = {} & \mu_t^4 + 4[1 - \lambda_1(A;B)]\mu_t^3(1-\mu_t) + 2[\lambda_3(B;A) + 2(1 - \lambda_2(A;B))]\mu_t^2(1-\mu_t)^2 \\ & + 4[1 - \lambda_4(A;B)]\mu_t(1-\mu_t)^3 - \mu_t \end{aligned} \tag{C.3}$$

Let $f(\mu_t) = \mu_{t+1} - \mu_t$. Then, $f'(1) = 4\lambda_1(A;B) - 1$. This is equivalent to $\beta - \gamma > 14\varepsilon$. Hence, if the deviation effect is strong enough relative to the dominant strategy effect, then a strictly dominated strategy is never driven to extinction in the long run.

Second, consider the case in which imitation is based on highest absolute payoffs in observed games. One shows that if imitation is proportional to the extent of the success then

$$g(\mu_t) = \mu_{t+1} - \mu_t = \mu_t^4 + 4\mu_t^3(1-\mu_t) + 2\left(3 - 2\frac{\beta - \gamma}{\alpha - \delta}\right)\mu_t^2(1-\mu_t)^2$$

$$+4\left(1-\frac{\beta-\gamma}{\alpha-\delta}\right)\rho_t(1-\rho_t)^3-\rho_t \tag{C.4}$$

$g'(0)<0$ is equivalent to $4(\beta-\gamma)>3(\alpha-\delta)$ which in turn is equivalent to $\beta-\gamma>6\varepsilon$. Hence, if the deviation effect is large enough relative to the dominant strategy effect, there exist some interior initial conditions such that the strictly dominant strategy is driven to extinction in the long run.

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