Amplify-and-Forward with Full-Duplex Relay Selection

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Abstract—This paper focuses on the relay selection problem in amplify-and-forward (AF) cooperative communication with full-duplex (FD) operation. Different relay selection schemes assuming the availability of different instantaneous information are studied. We consider an optimal relay selection that maximizes the instantaneous FD channel capacity and requires global channel state information (CSI) as well as several sub-optimal relay selection policies that utilize partial CSI knowledge such as a) source-relay and relay-destination links b) loop interference c) source-relay links and loop interference. To facilitate comparison, exact outage probability expressions and asymptotic approximations of these policies that show a zero diversity order are derived. In addition, an optimal relay selection that incorporates an hybrid relaying strategy, which dynamically switches between FD and half-duplexing relaying according to the instantaneous CSI, is also investigated.

I. INTRODUCTION

A large number of existing works on cooperative communication assume half-duplex (HD) relaying, where relays assist the source transmission by retransmitting the source data in orthogonal channels [1]. In order to overcome the associated bandwidth loss and improve system’s spectral efficiency, several protocols and techniques have been investigated in the literature [2]. Recently, there has been a significant research interest in full-duplex (FD) relaying, where relays can receive and transmit in the same frequency band and in the same time slot. This alternative, efficiently uses the channel bandwidth, as it requires only one channel use for the end-to-end transmission, but suffers from a loop interference due to signal leakage between the relay output and input [3]–[5], [7].

Several works have already focused on modeling FD relaying and its performance benefits (from a communication and information theoretic standpoint) against conventional HD approaches as well as on mitigating the related loop interference. See for e.g., [3]–[12]. Assuming binary phase shift keying (BPSK) modulation, the authors of [3] have conducted a bit error rate (BER) analysis for a single relay FD configuration. The outage performance of a decode-and-forward (DF) cooperative scheme with FD relaying is derived in [4] for a Rayleigh fading loop interference. The optimal amplification factor for an amplify-and-forward (AF) cooperative network with FD and an imperfect interference cancellation is proposed in [5], while the associated capacity expressions for both AF and DF protocols are investigated in [7]; this work is extended for a multiple-input multiple-output (MIMO) channel model in [8]. In [9] the authors deal with the investigation of advanced signal processing techniques for loop interference mitigation for a MIMO multiuser configuration and evaluate the achieved performance in terms of the sum rate. An hybrid scheme that incorporates a FD and a HD relaying during the total frame duration is proposed in [10] and an optimization of the duration of each mode is analyzed. Furthermore, the integration of both FD and HD in the same relay node is discussed in [11] and an appropriate scheduling design that handles these two modes from a networking point of view is also discussed. On the other hand, the employment of a relay selection policy for a FD multi-relay network with DF is studied in [12] presenting closed-form expressions for the average capacity and the BER. However, this work assumes a constant loop interference and global channel state information (CSI) knowledge.

In all prior works, relay selection policies for FD relaying have not been adequately addressed and therefore their impact on the system performance remains virtually unknown. This major limitation has motivated our current study as in this paper, the problem of relay selection for an AF cooperative network with FD relaying is considered. We investigate four relay selection policies: an optimal relay selection policy under a global CSI, and three suboptimal relay selection policies that incorporate a partial CSI such as (a) the source-relay and relay-destination links, (b) the instantaneous loop interference and (c) the source-relay links and the instantaneous loop interference. The selection policies are analyzed in terms of the outage probability with loop interference subject to Rayleigh fading. Exact as well as closed-form high signal-to-noise ratio (SNR) outage approximations that enable us to gain key insights are derived. Due to the loop interference at the relay, FD relaying results in a zero diversity order, despite the relay selection process, and the suboptimal policies are suitable for different system configurations. In addition, a hybrid scheme that dynamically switches between FD and HD relaying based on the instantaneous quality of the loop interference is discussed and a related relay selection policy is investigated. This hybrid scheme overcomes the diversity limitations of the FD relaying and outperforms previously reported HD schemes.

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system model. The considered relay selection policies are presented in Section III. Section IV deals with the outage probability performance of the proposed relay selection policies and present exact and high SNR approximations. Simulation and analytical results are presented in Section V, and are followed by our conclusions in Section VI.

II. SYSTEM MODEL AND BASIC ASSUMPTIONS

We assume a clustered network topology consisting of one source $S$, a cluster $C$ of $N$ relays ($R_i$ with $1 \leq i \leq N$) and one destination, $D$. The clustering system configuration is favored for large scale network implementation and in time critical applications since it can provide efficient resource management. A direct link between $S$ and $D$ does not exist (scenario of cell coverage extension) and communication can be established only via the cooperative relays. Each relay employs an AF protocol and is equipped with two antennas (one receive antenna and one transmit antenna) that enable a full-duplex operation at the price of a loop interference. Time is considered to be slotted and in each time slot, only one relay ($R_k \in C$) is selected to assist the source transmission. Power allocation issues are not considered and thus each transmitter (source or relay) is assumed to transmit with a fixed power $P$. In addition the information rate is equal to $R_0$ bits per channel use (BPCU). Fig. 1 schematically presents the system topology considered.

All wireless links in the network exhibit non-selective Rayleigh block fading and additive white Gaussian noise (AWGN). In the considered clustered relay network, the fading channel coefficients $h_{A,B}$ (for the $A \rightarrow B$ link) remain constant during one slot, but change independently from one slot to another according to a circularly symmetric complex Gaussian distribution with zero mean and variance $\sigma^2_{SR}$ and $\sigma^2_{RR}$ for $S \rightarrow R_i$ and $R_i \rightarrow D$ links respectively, $i = 1, \ldots, N$. The variance of the AWGN is assumed normalized with zero mean and unit variance without loss of generality.

Furthermore, in order to reduce the effects of the loop interference on the system performance, an imperfect interference cancellation scheme is used at each relay by following the interference on the system performance, an imperfect interference with zero mean and unit variance without loss of generality.

Finally, in order to facilitate our ensuing mathematical expressions, we define $\lambda_{AB} \triangleq \frac{1}{P\sigma^2_{AB}}$.

III. RELAY SELECTION POLICIES

In this section, we present the details of different relay selection policies for the clustered configuration described in Section II. The proposed selection schemes refer to a centralized architecture where a central unit decides the selected relay based on the available CSI. It is worth noting that the analysis of the considered centralized relay selection policies show the best achievable performance and consists of a guideline for more practical (and distributed) implementations.

A. Optimal Relay Selection

The optimal relay selection policy is based on the capacity expression achieved by FD transmission. Therefore, based on (1), the optimal selection policy activates the relay that satisfies the following condition

$$k = \arg \max_i \{ \gamma_i \},$$

where $\gamma_i \triangleq \frac{\gamma_{S,R_i}}{\gamma_{S,R_i} + \gamma_{R_i,D} + 1}$.

B. max $-$ min Relay Selection

The conventional max $-$ min relay selection policy does not take into account the loop interference and is the optimal relay selection for conventional HD relaying systems [13]; here we examine the performance of this scheme with FD relaying. The max $-$ min relay selection scheme selects the relay with the best end-to-end link without considering the loop interference and can be expressed as

$$k = \arg \max_i \{ \gamma_{S,R_i}, \gamma_{R_i,D} \}.$$  

C. Loop Interference Relay Selection

The loop interference relay selection policy selects the relay with the minimum instantaneous loop interference for FD operation and can be expressed as

$$k = \arg \min_i \left\{ \gamma_{R_i} \right\}.$$
D. Partial Relay Selection

For the optimal AF factor [7], we assume that the relay knows, $\gamma_{S,R_i}$ and $\gamma_{R_i,D}$; so we should investigate a relay selection policy when these two parameters are known. The proposed partial relay selection assumes that the SNR of the first hop as well as the loop interference are known and thus decides about the selected relay based on an appropriate combination of them. More specifically, partial relay selection policy activates the relay that has the maximum ratio between the signal to interference ratio (SINR) of the $S \rightarrow R_i$ link and the loop interference and is written as

$$k = \arg \max_i \left\{ \frac{\gamma_{S,R_i}}{\gamma_{R_i} + 1} \right\}. \quad (5)$$

E. Optimal Relay Selection with Hybrid Relaying

All the above selection schemes suffer from a zero diversity gain due to the nature of the FD transmission and the related loop interference effect. In order to overcome this limitation of FD operation, here, we investigate an optimal relay selection policy based on a dynamic hybrid relaying scheme that switches between FD and HD relaying. More specifically, the proposed scheme allows the relay nodes to switch between FD and HD relaying based on the instantaneous loop interference\(^2\). Given that each relay defines its own (the most efficient) relaying policy, the optimal relay selection policy selects the relay that achieves the best performance (in terms of outage probability). If the instantaneous capacity of HD relaying is defined as [13]

$$C_{R_i}^{(HD)} = \frac{1}{2} \log_2 \left( 1 + \frac{\gamma_{S,R_i} \gamma_{R_i,D}}{\gamma_{S,R_i} + \gamma_{R_i,D} + 1} \right), \quad (6)$$

in the optimal relay selection policy based on an hybrid relaying scheme, the relay that satisfies the following condition is activated:

$$k = \arg \max_i \left\{ C_{R_i}^{(HD)}, C_{R_i}^{(FD)} \right\} \quad (7)$$

where

$$k = \arg \max_i \left\{ \frac{\gamma_{S,R_i} \gamma_{R_i,D}}{\gamma_{S,R_i} + \gamma_{R_i,D} + 1}, \gamma_i \right\}.$$ 

The purpose of this paper is to present the main idea of the optimal relay selection with an hybrid relaying scheme and therefore its theoretical analysis as well as further implementation issues are beyond the scope of this work; numerical results in Section V validate that this scheme outperforms all the previous selection policies without suffering from zero diversity effects.

IV. OUTAGE PROBABILITY

In this Section, we investigate the outage probability of the proposed FD-based relay selection policies. We derive exact as well as simple high SNR outage expressions. By considering the definition of the outage probability, we can write

$$P_\star = P \left\{ \log_2 \left( 1 + \frac{\gamma_{S,R_k} \gamma_{R_k,D}}{\gamma_{S,R_k} + \gamma_{R_k,D} + 1} \right) < R_0 \right\}, \quad (8)$$

where in the cases of Optimal Relay Selection, max $- \min$ Relay Selection, Loop Interference Relay Selection and Partial Relay Selection policies, $*$ refers to OS, MM, LI and PS respectively and $P$ denotes the probability. In the sequel analysis, the reader is cautioned to the fact that the statistical distributions of $\gamma_{S,R_i}$, $\gamma_{R_i,D}$ and $\gamma_{R_i}$ could differ depending on the selection policy. Therefore, any remark made to the distributions of these random variables (RVs) are strictly limited to the referred selection policy.

A. Optimal Relay Selection

Using a simple order statistic result, outage probability of the OS policy, $P_{OS} = P \left\{ C_{R_k}^{(FD)} < R_0 \right\}$, can be written as

$$P_{OS} = P \left\{ \frac{\gamma_{S,R_k} \gamma_{R_k,D}}{\gamma_{S,R_k} + \gamma_{R_k,D} + 1} < T \right\} = \left[ F_{\gamma_1}(T) \right]^N, \quad (9)$$

where $T \triangleq 2R_0 - 1$, $F_{\gamma_1}(-)$ is the cumulative distribution function (cdf) of the RV, $\gamma_1$. The cdf of $\gamma_1$ can be obtained as

$$F_{\gamma_i}(x) = \int_{0}^{\infty} P \left\{ \frac{\gamma_{S,R_i} y}{\gamma_{R_i} + 1} + y + 1 < x \right\} f_{\gamma_{R_i,D}}(y)dy \quad (10)$$

$$= F_{\gamma_{R_i,D}}(x) + \int_{x}^{\infty} F_{X_1} \left( \frac{(y + 1)x}{y - x} \right) f_{\gamma_{R_i,D}}(y)dy, \quad (11)$$

where $F_{\gamma_{R_i,D}}(x) = 1 - e^{-\lambda_{RD} x}$ is the cdf of $\gamma_{R_i,D}$. $f_{\gamma_{R_i,D}}(y)$ is the probability density function (pdf) of $\gamma_{R_i,D}$, and the second equality follows from trivial manipulations. The cdf of the RV defined as $X_1 = \frac{\gamma_{S,R_i}}{\gamma_{R_i} + 1}$ is given by

$$F_{X_1}(x) = \int_{0}^{\infty} F_{\gamma_{S,R_i}}((y + 1)x) f_{\gamma_{R_i}}(y)dy \quad (12)$$

There fore we get

$$F_{\gamma_1}(x) = 1 - \lambda_{RD} e^{-\lambda_{RD} x} \int_{0}^{\infty} e^{-\lambda_{SB}(x+y+1) \frac{x}{y+1}} \lambda_{RR} dy. \quad (12)$$

Now it suffices to substitute (12) into (9) to obtain the exact outage probability of the OS policy. Unfortunately, the best of authors’ knowledge (12) does not have a closed-form solution. However, it can be evaluated numerically using standard software such as Matlab, Maple or Mathematica.

In order to derive an accurate closed-form expression for the outage probability performance applicable in the intermediate to high SNR regime (for $P \rightarrow \infty$), we simplify the outage probability computation as follows:
determined by two mutually exclusive events corresponding
\( \gamma \)
link of
\( \gamma \)
1
an error floor and therefore to a zero diversity gain. Therefore we have
\[ F_{\gamma S,R_k}(x) = N \left\{ \sum_{n=0}^{N-1} \left( \frac{1}{n} \right)^{N-1} \left( 1 - e^{-\frac{1}{n}\lambda_{RD}x} \right) + \frac{\lambda_{SR}}{\lambda_A} \left( \frac{n}{n+1} \right) \frac{e^{-\frac{1}{n+1}\lambda_{RD}x}}{\lambda_{RD}} \right\} \]
Using this result, we can now calculate the required cdf of \( X_2 \) as
\[ F_{X_2}(x) = \frac{N}{\lambda_A} \sum_{n=0}^{N-1} \left( \frac{1}{n} \right)^{N-1} \left( 1 - e^{-\frac{1}{n}\lambda_{RD}x} \right) + \frac{\lambda_{SR}}{\lambda_A} \left( \frac{n}{n+1} \right) \frac{e^{-\frac{1}{n+1}\lambda_{RD}x}}{\lambda_{RD}} \]
The pdf of \( \gamma_{R_k,D} \) required to calculate (14) can be obtained by differentiating (18) and mutually exchanging \( \lambda_{SR} \) and \( \lambda_{RD} \) to yield
\[ f_{\gamma_{R_k,D}}(x) = N \left\{ \sum_{n=0}^{N-1} \left( \frac{1}{n} \right)^{N-1} \left( 1 - e^{-\frac{1}{n}\lambda_{RD}x} \right) + \frac{\lambda_{SR}}{\lambda_A} \left( \frac{n}{n+1} \right) \frac{e^{-\frac{1}{n+1}\lambda_{RD}x}}{\lambda_{RD}} \right\} \]
Finally, recalling that \( \tilde{F}_{X_2}(x) = 1 - F_{X_2}(x) \), we substitute (19) and (20) into (14) and evaluate the exact outage probability using numerical integration.
We now proceed to present a simple high SNR approximation for the outage probability of the MM policy. Let \( \alpha \triangleq \min \{ \gamma_{S,R_k}, \gamma_{R_k,D} \} \) and \( \beta \triangleq \max \{ \gamma_{S,R_k}, \gamma_{R_k,D} \} \) denote the SNR of the max–min branch and the SNR of the other branch for the selected end-to-end path, respectively. The outage probability can be approximated as
\[ P_{\text{MM}} = \mathbb{P}\left\{ C_{\text{MM}} < R_0 \right\} \]
\[ > \frac{\lambda_{SR}}{\lambda_A} \left\{ \min \left\{ \frac{x}{\gamma_{R_k,D}} \right\} + \frac{\lambda_{RD}}{\lambda_A} \left\{ \min \left\{ \frac{x}{\gamma_{R_k}} \right\} \left\{ T \right\} \right\} \]
The constant terms in (21) arise from the probabilities \( \mathbb{P}\{ \gamma_{S,R_k} = \alpha \} = \lambda_{SR} / \lambda_A \) and \( \mathbb{P}\{ \gamma_{R_k,D} = \alpha \} = \lambda_{RD} / \lambda_A \), respectively. In order to simplify the analysis and derive a closed-form expression accurate for low loop interference scenarios\(^3\) (in comparison to the power of the \( S \rightarrow R_i, R_i \rightarrow D \))
\(^3\)For practical scenarios the power of the loop interference is much lower than one of the source-relay and relay-destination links.
links, e.g., $\sigma_{RR}^2 << \sigma_{SR}^2, \sigma_{RD}^2$) the above outage expression can be asymptotically approximated by the first (dominant) term as

$$P_{MM} \rightarrow \frac{\gamma_{SR}}{\lambda_A} P\left\{ \frac{\alpha}{\gamma_{R_k}} < T \right\}$$

$$\rightarrow \frac{\gamma_{SR}}{\lambda_A} \sum_{n=0}^{N} \binom{N}{n} \frac{(-1)^n \lambda_{RR}}{\lambda_{RR} + nT \lambda_A},$$

where to derive (22), we have used the cdf of the RV $\alpha$ given by

$$F_\alpha(x) = \left[ 1 - e^{-\lambda_A x} \right]^N$$

$$= \sum_{n=0}^{N} \binom{N}{n} (-1)^n e^{-n \lambda_A x}.$$  

C. Loop Interference Relay Selection

Since selection of the relay is only based on $\gamma_{R_i}$, note that $\gamma_{SR}, \gamma_{R_k}$ and $\gamma_{RD}$ are exponentially distributed RVs. Therefore, the exact outage probability, $P_{LI} = P\left\{ C_{R_k}^{FD} < R_0 \right\}$, can be written as

$$P_{LI} = F_{R_k,D}(T) + \int_{T}^{\infty} F_{X_2} \left( \frac{(y+1)T}{y-T} \right) f_{\gamma_{R_k,D}}(y) dy,$$

where $X_2 = \frac{\gamma_{SR}}{\gamma_{R_k} + 1}$. The cdf of $X_2$ is given by

$$F_{X_2}(x) = 1 - \frac{N e^{-\lambda_{SR} x}}{N + \lambda_{SR} x}.$$  

Finally, the outage probability can be calculated from

$$P_{LI} = 1 - \lambda_{RD} N e^{-\lambda_{RD} T} \int_{0}^{\infty} e^{-\frac{\gamma_{SR}(y+T+1)T}{y} - \lambda_{RD} y} F_{X_2}(y) dy.$$  

By using similar simplification steps with the above relay selection policies, a tight lower bound for the outage probability at high SNRs (for $P \rightarrow \infty$) can be derived as

$$P_{LI} = P\left\{ \min \left[ \frac{\gamma_{SR}}{\gamma_{R_k}} \right] < T \right\}$$

$$= 1 - \frac{\lambda_{RR} N + \lambda_{SR} T e^{-\lambda_{RD} T}}{\lambda_{RR} N + \lambda_{SR} T}.$$  

D. Partial Relay Selection

The exact outage probability given by $P_{PS} = P\left\{ C_{R_k}^{FD} < R_0 \right\}$, can be easily calculated using the established results in (10) and (11) and is given by

$$P_{PS} = 1 - e^{-\lambda_{RD} T} + \lambda_{RD} e^{-\lambda_{RD} T}$$

$$\times \int_{0}^{\infty} \left[ 1 - e^{-\frac{\gamma_{SR}(y+T+1)T}{\lambda_{SR}}} \right] e^{-\lambda_{RD} y} dy.$$  

By using basic order statistics theory and by following the previous reported simplification steps, the outage probability for the high SNR regime can be approximated by

$$P_{PS} > P\left\{ \min \left[ \frac{\gamma_{SR}}{\gamma_{R_k}} \right] < T \right\}$$

$$= 1 - \frac{\lambda_{SR} T}{\lambda_{SR} T + \lambda_{RR}}$$

$$\times \left[ \frac{\lambda_{SR} T}{\lambda_{SR} T + \lambda_{RR}} \right]^N.$$  

V. SIMULATION RESULTS AND DISCUSSION

In this section, we give numerical examples for the outage probability performance of the proposed relay selection schemes. The simulation system follows the System model of Section II with $R_0 = 2$ BPCU, $\sigma_{SR}^2 = \sigma_{RD}^2 = 1$ and the considered relay selection policies are: OS, MM, PS and the optimal selection with hybrid relaying (HS); the conventional max-min relay selection for HD relaying is also used as a reference selection scheme [13].

Fig. 2 plots the outage probability as a function of $P$ for the different relay selection schemes for a simulation setting with $N = 4$ relays and $\sigma_{SR}^2 = 0.1$. The exact outage probability curves are given by Monte Carlo simulations and perfectly match with the expressions that are obtained by numerically evaluating (9), (14), (26) and (28). The first important observation is that all the FD-based relay selection policies converge to an error floor and thus provide a zero diversity gain at high SNRs. In addition, we can see that the OS policy provides the best outage performance (among the FD-based schemes) for all SNRs and converges to the lowest error floor. As for the PS relay selection scheme, we can see that provides the second best outage performance and converges to the same error floor with the OS scheme. This behavior is justified by (13) which shows that the capacity
of the system is dominated by the ratio \( \gamma_{S,R} / (\gamma_{R,R} + 1) \) at high SNRs and therefore the PS scheme coincides with the OS for the high SNR regime. The LI scheme decides the relay selection only based on the loop interference; given that the considered scenario assumes a low loop interference, the achieved outage probability converges to the worst error floor. The MM relay selection does not take into account the loop interference and thus provides a performance similar to the LI scheme (due to the low loop interference). On the other hand, the HS scheme overcomes the problem of zero diversity and HD scheme outperforms conventional HD-based relay selection schemes at the relays have been proposed. This new scheme outperforms conventional HD-based relay selection schemes and its analysis as well as its implementation can be considered for future work.

VI. CONCLUSION

This paper has dealt with the problem of relay selection for AF cooperative systems with FD relaying. An optimal relay selection scheme which is based on the FD channel capacity expression and requires a global CSI has been investigated. In order to reduce complexity, several suboptimal relay selection schemes which require a partial CSI knowledge have been also proposed. The investigated relay selection schemes have been analyzed in terms of outage probability and exact expressions as well as asymptotic approximations have been derived. We have shown that the FD-based relay selection schemes provide a zero diversity, while a suboptimal scheme that incorporates only the source-relays links and the loop interference coincides with the optimal relay selection scheme for high SNRs. In order to overcome the diversity limitation of the FD-based relay selection, an optimal relay selection policy that elaborates an hybrid relaying strategy (dynamic switching between FD and HD) at the relays have been proposed. This new scheme outperforms conventional HD-based relay selection schemes and its analysis as well as its implementation can be considered for future work.

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