Exact Modeling and Performance Analysis of Distance-Based Registration Considering the Implicit Registration Effect of Outgoing Calls

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SUMMARY We consider distance-based registration (DBR). DBR causes a mobile station (MS) to reregister when the distance between the current base station (BS) and the BS with which it last registered exceeds a distance threshold. The addition of implicit registration to DBR (DBIR) was proposed to improve the performance of DBR, and its performance has also been presented using a continuous-time Markov chain. In this study, we point out some problems of the previous DBIR performance analysis, and we propose a new model of the DBIR to analyze its exact performance. Using the new method, we show that DBIR is always superior to DBR, and the extent of the improvement is generally greater than what is currently known.

key words: distance-based registration, implicit registration

1. Introduction

In this study, we consider distance-based registration (DBR). In general, zone-based registration (ZBR) has been adopted in most mobile communication systems because of good performance. But, one of main problems of the ZBR is that excessive location registrations occur when a mobile station (MS) zigzags between two neighboring location areas. On the other hand, this ping-pong phenomenon does not occur in the DBR and the registration cost of the DBR distributes equally to all cells in the location area. So, the DBR may need less registration cost than the ZBR [7]. However, in general, DBR cause an MS to register more frequently than ZBR. In order to overcome this weakness, the DBR with implicit registration (DBIR) was proposed, and its performance was also evaluated using a continuous-time Markov chain (CTMC) [1]. In this study, we point out some problems of the previous analytical model of the DBIR and propose a new modeling of the DBIR in order to analyze its exact performance. Using our exact analytical model, we show that DBIR is always superior to DBR, and the extent of the improvement is generally greater than that shown in the previous study.

2. DBR and DBIR

2.1 Distance-Based Registration (DBR)

Distance-based registration causes an MS to register when the distance between the current base station (BS) and the BS with which it last registered exceeds a distance threshold, \( D \). The MS stores the latitude and longitude \((X_r, Y_r)\) of the BS whose access channel was used for the MS’s last registration and the latitude and longitude \((X_c, Y_c)\) of the current BS. The MS computes the current BS’s distance from the last registration point \(\text{DISTANCE}\) as:

\[
\text{DISTANCE} = \sqrt{\Delta \text{lat}^2 + \Delta \text{long}^2} / 16,
\]

where \(\Delta \text{lat} = X_c - X_r\) and \(\Delta \text{long} = (Y_c - Y_r) \times \cos(\pi/180 \times X_r/14400)\) [8]. The above is calculated considering the curvature of the earth’s surface. For convenience, this study defines the \(\text{DISTANCE}\) between two cells as the minimum number of cells between two cells. In Fig. 1 for example, \(\text{DISTANCE}\) between two neighboring cells is 1 and \(\text{DISTANCE}\) between a ring-0 cell and one of the ring-2 cells is 2. A location area is a set of cells in which MS’s movement causes no location registration. Figure 1 shows the location area in the hexagonal configuration when the MS registers in the ring-0 cell and the distance threshold, \(D = 3\).

2.2 DBR with Implicit Registration (DBIR)

According to the CDMA technical requirements [8], when
an MS successfully sends an *Origination Message* or *Page Response Message*, the BS can infer the MS's location. This is called an *implicit registration*. In other words, when an outgoing call from an MS occurs successfully or an incoming call to an MS occurs successfully, the BS can infer the MS's location from the *Origination Message* or a *Page Response Message* without a separate registration message.

Hence, if a mobile cellular network adopting a DBR also adopts an implicit registration, then the network can determine the MS's location whenever an outgoing call from or incoming call to the MS occurs without a real registration process. The network can therefore set up a new location area in which the MS's cell is the center cell (ring-0 cell) in order to reduce the number of registrations.

In this way, the DBR performance can be improved using an implicit registration, and the performance of this combined scheme is enhanced as the call generation of the MS increases. This enhanced scheme that combines a DBR with the implicit registration by incoming and outgoing calls was proposed in [1], and we call it DBIR (DBR with IR).

### 3. Registration Cost of the DBIR

It is assumed that the mobile communication network is composed of hexagonal cells of the same size, as shown in Fig. 1. In order to analyze the performance, the following are also assumed:

- When the MS leaves a cell, there is an equal probability that any one of six neighboring cells will be selected as the destination.
- The cell sojourn time follows a general distribution with a mean, $1/\lambda_m$, and the incoming and outgoing call generations to/from each MS follow Poisson processes with rates of $\lambda_c$ and $\lambda_{oc}$, respectively.

A location area is composed of $D$ rings (ring 0, 1, $\cdots$, $D-1$). An analytical model was proposed to analyze the registration cost using CTMC [1]. Let us examine the previous analytical model to determine its problems in order to develop a new analytical model.

#### 3.1 Approximate Registration Cost of the DBIR

To obtain the registration cost of the DBIR scheme, the MS should be observed at each cell crossing. Each MS resides in a cell for a time period and then moves to one of its neighbors with an equal probability, i.e., $1/6$. The mobility of the MS is therefore a random walk in a 2-dimensional hexagonal plan, and this 2-dimensional random walk can be reduced to a simple random walk as shown in Fig. 2.

In this CTMC, an MS is in state $i$ if it is currently residing in a ring-$i$ cell. In Fig. 2, for example, an MS in state 0 can transit i) to state 0 with transition rate, $\lambda_m/6$; ii) back to state 1 with transition rate, $\lambda_m/3$; iii) to state 2 with transition rate, $\lambda_m/2$; or iv) to state 0 with transition rate, $\lambda_c + \lambda_{oc}$. The $D \times D$ transition rate matrix $Q$ for the DBIR is

![Fig. 2 State transition diagram for the DBIR.](image)

An element $Q_{D-1,0}$ in $Q$ is the rate at which an MS in a ring-$i$ cell moves to a ring-0 cell, which is composed of and $\lambda_c$. Note that the former produces real registration, but the latter produces only an implicit registration.

Letting $\pi_i$ be the steady-state probability of state $i$ each $\pi_i$ can be obtained by using the following equations [5].

$$ \Pi \cdot Q = 0, \quad \sum_i \pi_i = 1 \quad (1) $$

Letting $C_{dir}^{\text{dir}}$ denote the registration cost of DBIR, the following equation can be derived:

$$ C_{dir}^{\text{dir}} = U \pi_{D-1} q_{D-1,0} = U \frac{2(D-1)+1}{6(D-1)} \lambda_m \pi_{D-1} \quad (2) $$

where $U$ is the unit registration cost required for one registration and $q_{D-1,0} = \frac{2(D-1)+1}{6(D-1)} \lambda_m$.

Note that the registration cost for the DBR is slightly different from that of the DBIR. The main difference is that an outgoing call in the DBIR causes the MS to be in the ring-0 cell as shown in Fig. 2, but an outgoing call in the DBIR does not. Deletion of every $\lambda_{oc}$ in Fig. 2 removes the effect of outgoing calls and results in the transition rate diagram of the DBR.

#### 3.2 Exact Registration Cost of the DBIR

When the registration cost of the DBIR is calculated using the method described above, the results may differ from the exact values. When the DBIR is analyzed using the CTMC model with the transition rate diagram from Fig. 2, there are two types of problems.

First, the CTMC model with the transition rate diagram from Fig. 2 is problematic since it assumes that the sojourn time of each state follows an exponential distribution [5]. Therefore, when the sojourn time of each state does not follow an exponential distribution, the CTMC model cannot offer a correct analytical result but gives an approximated
result. Since, in general, the sojourn time of each state cannot be assumed to follow an exponential distribution, one can easily see that the CTMC model for DBIR is problematic.

Second, even when the sojourn time of each state follows an exponential distribution, the CTMC model with the transition rate diagram from Fig. 2 still has a drawback. Unlike the case in which the MS of state 0 in Fig. 2 transits to state 0 if an incoming call or an outgoing call occurs, CTMC does not allow such a self-loop. In a similar manner, an MS in ring 1 can easily see that the CTMC model for DBIR is problematic. In this section, we propose a new analytical model for DBIR in order to obtain the exact performance. We analyze the exact registration cost of the DBIR by using an embedded Markov chain (MC) [5], which can reflect the general cell sojourn time and self-looping.

The state transition probability diagram of the embedded MC for the DBIR is shown in Fig. 3. First, let us examine the state transition probability. In Fig. 3, an MS is in state i if it is currently residing in a ring-i cell. To get the probability, let’s define three random variables \( T_c \), \( T_m \), and \( T_{oc} \) which represent the time between two incoming calls, the time between two outgoing calls, and the sojourn time of an MS at a cell, respectively. For example, an MS in ring 1 (state 1) transits to i) a cell of ring 0 (state 0) with a probability, \( P[T_c \geq T_m][T_{oc} \geq T_m]/6 \); ii) other cells of ring 1 (state 1) with a probability, \( P[T_c \geq T_m][T_{oc} \geq T_m]/3 \); iii) cells of ring 2 (state 2) with a probability, \( P[T_c \geq T_m][T_{oc} \geq T_m]/2 \); or iv) a cell of ring 0 (state 0) with a probability, \( 1 - P[T_c \geq T_m][T_{oc} \geq T_m] \).

Letting \( P \) be the transition probability matrix of the embedded MC for DBIR and \( \pi_i \) be the steady-state probability of state \( i \), each \( \pi_i \) can be obtained using the following equations.

\[
\Pi \cdot P = \Pi, \quad \sum_i \pi_i = 1 \quad (3)
\]

The registration cost of DBIR, \( C_{dbir}^U \), can be derived using Eq. (2).

In DBR, the state transition probability can be obtained more easily because the outgoing call is not considered. The state transition probability diagram of the embedded MC for the DBR is as shown in Fig. 4.

Letting \( \pi'_i \) be the steady-state probability of state \( i \) and \( C_{dbir}^u \) be the registration cost of DBR, the following equation can be derived:

\[
C_{dbir}^u = U \frac{2(D - 1) + 1}{6(D - 1)} \pi'_D \lambda_m,
\]

where \( U \) is the unit registration cost required for one registration.

3.3 Paging Cost and Total Costs

Even though some efficient paging strategies have been proposed, most mobile cellular systems still adopt a simultaneous paging strategy [1], [3]. Assuming that all of the cells in a location area are paged simultaneously whenever an incoming call arrives, the paging cost is the same when both schemes are applied. Hence, the expected paging cost per unit time \( C_p \) is:

\[
C_p = V \left[ 1 + \sum_{i=1}^{M-1} 6i \right] = V[1 + 3D(D - 1)],
\]

where \( V \) is the unit paging cost required for one cell.

Finally, the total expected cost for registration and paging per unit time will be:

\[
C_T = C_U + C_p.
\]

4. Performance Evaluation

Proposition 1. For the given threshold \( D \), the registration cost of the DBIR, \( C_{dbir}^U \), is equal to or less than that of the DBR, \( C_{dbir}^u \), i.e.,

\[
C_{dbir}^U = U \frac{2(D - 1) + 1}{6(D - 1)} \pi'_D \lambda_m \geq U \frac{2(D - 1) + 1}{6(D - 1)} \pi_D \lambda_m = C_{dbir}^u
\]

Proof. It is sufficient to show that \( \pi'_D \geq \pi_D \) is true for every \( D \). The left side is the probability that an MS is in ring \( D-1 \) in the DBR, and the right side is the probability that an MS is in ring \( D-1 \) in the DBIR. Comparing the state transition probability diagram of the DBR in Fig. 4 with that of the DBIR in Fig. 3, it is apparent that the state transition probability diagram of the DBIR in Fig. 3 is obtained.
Table 1  Registration cost for various cell sojourn times.

<table>
<thead>
<tr>
<th>Modeling &amp; Distribution</th>
<th>$D = 2$</th>
<th>$D = 3$</th>
<th>$D = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTMC</td>
<td>$C_{\text{div}}$</td>
<td>0.2308</td>
<td>0.0718</td>
</tr>
<tr>
<td></td>
<td>$C_{\text{div}}$</td>
<td>0.1875</td>
<td>0.0432</td>
</tr>
<tr>
<td></td>
<td>decrease(%)</td>
<td>18.8</td>
<td>39.8</td>
</tr>
<tr>
<td>Exp(1)</td>
<td>$C_{\text{div}}$</td>
<td>0.2308</td>
<td>0.0718</td>
</tr>
<tr>
<td></td>
<td>$C_{\text{div}}$</td>
<td>0.1714</td>
<td>0.0349</td>
</tr>
<tr>
<td></td>
<td>decrease(%)</td>
<td>25.7</td>
<td>51.3</td>
</tr>
<tr>
<td>EXACT</td>
<td>$C_{\text{div}}$</td>
<td>0.2107</td>
<td>0.0573</td>
</tr>
<tr>
<td></td>
<td>$C_{\text{div}}$</td>
<td>0.1397</td>
<td>0.0217</td>
</tr>
<tr>
<td></td>
<td>decrease(%)</td>
<td>33.7</td>
<td>62.1</td>
</tr>
<tr>
<td>Gamma(1/2,2)</td>
<td>$C_{\text{div}}$</td>
<td>0.2468</td>
<td>0.0849</td>
</tr>
<tr>
<td></td>
<td>$C_{\text{div}}$</td>
<td>0.1989</td>
<td>0.0498</td>
</tr>
<tr>
<td></td>
<td>decrease(%)</td>
<td>19.4</td>
<td>41.3</td>
</tr>
</tbody>
</table>

Table 2  Exact registration cost and its reduction ratio.

<table>
<thead>
<tr>
<th></th>
<th>$D = 2$</th>
<th>$D = 3$</th>
<th>$D = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DBIR(CTMC)</td>
<td>0.2308</td>
<td>0.0718</td>
<td>0.0242</td>
</tr>
<tr>
<td>DBIR(Exact)</td>
<td>0.1875</td>
<td>0.0432</td>
<td>0.0102</td>
</tr>
<tr>
<td>(A-B) / A × 100 (%)</td>
<td>18.8</td>
<td>39.8</td>
<td>57.8</td>
</tr>
<tr>
<td>(A-C) / A × 100 (%)</td>
<td>25.7</td>
<td>51.3</td>
<td>70.4</td>
</tr>
</tbody>
</table>

by substituting $P[T_c \geq T_m]$ with $P[T_c \geq T_m]P[T_{oc} \geq T_m]$ from each state, $i$, to each state $j$ ($i, j = 0, 1, 2, \cdots$), of the state transition diagram in Fig. 4, and by substituting $1 - P[T_c \geq T_m]$ with $1 - P[T_c \geq T_m]P[T_{oc} \geq T_m]$ from each state, $i$ ($i = 0, 1, 2, \cdots, D-1$), to state 0 of the state transition diagram in Fig. 4. This indicates that, compared with DBR, the probability that an MS is in state 0 in the DBIR increases, and the probability that an MS is in ring $D-1$ (the farthest state from state 0) in the DBIR decreases. Therefore, the probability that an MS is in ring $D-1$ in the DBIR (Fig. 3), $\pi_{D-1}$, is less than or equal to the probability that an MS is in ring $D-1$ in the DBR (Fig. 2), $\pi_{D-1}$.

To obtain numerical results, it is assumed that $U=1.0$, $V=0.1$, $\lambda_m = 1$, and $\lambda_c = \lambda_{oc} = 0.5$ [1], [6].

Table 1 shows the registration cost for various cell sojourn time distributions [1], [3], [4]. It is evident in Table 1 that, even when the cell sojourn time follows an exponential distribution, the registration cost by CTMC modeling is significantly different from the exact value. It is also apparent that, in general, analyzing DBIR using the new modeling scheme decreases the registration cost to a greater degree than does the use of CTMC modeling.

Table 2 shows the registration cost when $D$ changes, assuming that the cell sojourn time follows an exponential distribution. Note that, in this case, the registration cost of the DBIR is the same when both modeling methods are applied. Table 2 shows that the registration cost of the DBIR using the new modeling scheme is significantly less than that of the previous CTMC modeling for every $D$. This means that DBIR performs better than originally thought for every $D$. For example, when $D=4$, DBIR’s cost using the CTMC modeling is reduced by 57.8%, but DBIR’s cost using the exact modeling is reduced by 70.4%.

Figure 5 shows the exact signaling cost using the new modeling when $D$ changes. Figure 5 shows that the registration cost of the DBIR is less than that of DBR for every $D$, and therefore the total signaling cost of the DBIR is less than that of DBR for every $D$.

Figure 6 shows the signaling cost when the CMR (Call-to-Mobility Ratio) changes, assuming that $D=2$ and that the cell sojourn time follows an exponential distribution. It is apparent that the registration cost of the DBIR is less than that of DBR for every CMR, and therefore the total signaling cost of the DBIR is less than that of DBR for every CMR.

5. Conclusion

In this study, we considered distance-based registration. We pointed out some problems of the performance analysis on the DBIR in the previous studies, and proposed a new modeling of the DBIR to analyze its exact performance. Using the new analysis, we showed that DBIR is always superior to DBR, and the extent of the improvement is generally greater than what was shown in the previous study. Consequently, DBR should be considered along with implicit registration in order to achieve the best performance.

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References