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Dynamic strength of rocks and physical nature of rock strength

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Abstract: Time-dependence of rock deformation and fracturing is often ignored. However, the consideration of the time-dependence is essential to the study of the deformation and fracturing processes of materials, especially for those subject to strong dynamic loadings. In this paper, we investigate the deformation and fracturing of rocks, its physical origin at the microscopic scale, as well as the mechanisms of the time-dependence of rock strength. Using the thermo-activated and macro-viscous mechanisms, we explained the sensitivity of rock strength to strain rate. These mechanisms dominate the rock strength in different ranges of strain rates. It is also shown that a strain-rate dependent Mohr-Coulomb-type constitutive relationship can be used to describe the influence of strain rate on dynamic rock fragmentation. A relationship between the particle sizes of fractured rocks and the strain rate is also proposed. Several time-dependent fracture criteria are discussed, and their intrinsic relations are discussed. Finally, the application of dynamic strength theories is discussed.

Key words: rock dynamics; deformation and fracturing; time-dependence; dynamic strength; criteria of fracturing

1 Introduction

Traditional strength theories are mainly concerned with the macroscopic deformation and fracturing of continuum materials. The time-dependence of the material strength is usually neglected. In those theories, the failure takes place when the combination of stresses or strains at one point in a solid reaches a limit value. The selection of stress and strain combinations and the determination of their limit values are the basis of particular strength theories.

Actually, failure processes of material generally take place over some time. As the failure of rock is resulted from the nucleation, growth and coalescence of inherent microcracks progressively at a limited velocity, the macroscopic deformation and fracturing of materials are time-dependent. The strain rate sensitivity of strength and the incubation time of fracturing for rock material when the strength limit is reached are the typical examples of the time-dependence of material response.

Hence, the careful consideration of the time-dependence is essential to study the deformation and fracturing processes of materials, especially for those subject to strong dynamic loadings. Therefore, the time-dependence of the deformation and fracturing of rocks, its origin at the microscopic scale, and the mechanisms of rock dynamic strength are investigated in this paper.

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2 Traditional strength theories

Traditional strength theories (or criteria) may be divided into five classes: (1) the maximum normal stress theory, (2) the maximum normal strain theory, (3) the maximum shear stress theory, (4) the maximum specific strain energy theory (Von Mises criterion), and (5) the Mohr-Coulomb (M-C) criterion. Among these strength theories, the M-C criterion, modified from the shear stress theory, is widely used in geotechnical engineering practice. It is noted that the Hoek-Brown criterion and the Drucker-Prager (D-P) criterion, which are also widely used in geotechnical engineering, are the modifications of the M-C criterion and the Von Mises criterion [1–3].

The above-mentioned criteria are applicable to special failure modes under a certain stress state. For example, the M-C criterion mathematically does not consider the influence of intermediate principal stress on the strength of material. It takes only shear and normal stresses into account on one shear plane. Therefore, it may be called a single-shear stress theory.

Further development of single-shear stress theory produces the twin-shear stress theory, which in turn is the basis of the unified stress theory [4, 5]. The single-shear stress theory, the twin-shear stress theory and other strength theories apply to particular cases, or linear approximations of the unified strength theory.

The unified strength theory represents advancement in the development of a more general strength theory. However, the above-mentioned strength theories are far from being perfect or mature. The main shortcoming of these strength theories includes the neglect of time-dependence of the deformation and the internal structure of solids.

3 Kinetic nature of solid strength

Investigations of the microscopic physical nature and failure mechanisms of solids fall into two categories: static methods and kinetic methods.

Static methods are characterized by the transition from viewing solids as elastic or viscoelastic media to viewing solids as atom or molecule systems. In these systems, atoms or molecules are connected by cohesion, and the external forces applied to the solids are distributed on the links between atoms or molecules. In this way, the internal forces are induced. Therefore, the stability of solids before failure is determined by the relationship between (1) the cohesions between atoms or molecules and (2) the internal forces in bonds induced by external forces. If the internal forces are less than cohesions, elastic deformation will be induced, otherwise irreversible deformation and fracturing will occur.

In microscopic static theories, the strength property of solids is described by the concept of limit strength, and the failure of materials is considered to be a critical event that takes place instantaneously when the internal force in any bond of atoms reaches its critical value. Based on an understanding of atomic structure of solids, a theoretical strength of a solid can be determined.

However, there are two contradictions between the static microscopic failure mechanism and experimental observations for materials. The first is that the actual strength of materials is much less (1–3 orders lower) than the theoretical strength ($\sigma_{\text{th}} \approx 0.1E$, where E is the Young's modulus). According to previous investigations, the remarkable difference between the theoretical and the actual strengths may be attributed to the existence of defects near which significant stress concentration takes place.

The second contradiction is that the static microscopic failure concept assumes that the failure is of instantaneous event, but experiments show that the failure of materials is a time-dependent process. The duration of failure may be determined by Zhurkov's formula.

The attempts to solve the second contradiction give rise to kinetic theories, the second category of theories describing the deformation and fracturing of materials. In kinetic theories, the atomic system is under thermal vibration, and it interacts with the external loads. The atom vibration changes the distances between atoms and consequently changes the forces in the bonds of atoms. Rough estimations show that the frequency of thermal vibration of molecules is approximately 10^{12}–10^{13} s^{-1}, and the average kinetic energy distributed to every degree of freedom for an atom is $KT/2$ (where K is Boltzmann's constant and T is absolute temperature). When $T = 300$ K, the resultant average force in atomic bonds is of the order of 9 800 MPa, and the force for the breakage of atomic bonds has the same order, 14 700–29 400 MPa. The difference between the two energies is called the energy barrier.

The problem is also related to the non-uniformity of atom vibration, called the thermodynamic energy fluctuation, resulting from the chaotic thermal motions of atoms. This means that the kinetic energy distributed to individual degrees of freedom within individual atom may be much higher than the average vibration energy of the atom. As a result, the forces in atomic bonds in individual atom may exceed the limit forces for the breakage of atomic bonds. The breakage of atomic links thus will occur, leading to fracturing.

It is clear from the foregoing analysis that thermal fluctuation plays a fundamental role in the breakage of atomic bonds.

The roles of external forces applied to solids are two-fold. First, the external forces are smaller than the energy barrier U for breakage of atomic bonds defined by $\Delta U(f) = f\Delta r$, where f is the force induced by external forces in every atomic bond and Δr is the change in distance between atoms induced by external forces.

Second, the force f reduces the probability of the restoration of broken atomic bonds because the action of f increases the distance between atoms. Therefore, a mutual compensation between external forces and thermal fluctuation exists: the thermodynamic energy fluctuation makes the breakage of atomic bonds possible, but external forces exclude the possibility of the restoration of broken atomic bonds (some chemical processes may restore broken atomic bonds, e.g. by sealing micro-fractures in clays).

The foregoing discussion deals with the kinetic nature of the breakage of bonds at atomic scales. However, the development of fracturing in a material should be treated as the accumulation of breakages of atomic bonds in a solid, leading to the initiation of fractures (micro-cracks and micro-voids). This process is called fracturing localization.

Thermal fluctuation is a time-dependent stochastic process. Furthermore, the force *f* needs a certain time to overcome the resistance provided by the energy barrier and to increase the distances between atoms. The process of fracturing localization also needs a certain time to activate and develop. All these facts indicate that the failure of a material founded on thermal fluctuation at the atomic level is a time-dependent process, which presumably needs time to be activated and to develope. The larger the external force is, the shorter the time needed for overcoming the energy barrier will be, i.e. fracturing will happen more quickly.

There are still many problems to be solved regarding kinetic theories of deformation and fracturing in solids. Such theories are under development.

4 Dynamic strength theories

From the above discussion, we can conclude that material strengths are not physical constants and fracturing of solids needs time to be activated, to develop and to complete. These conclusions are also based on experimental data. Indeed, many solids show the strain rate sensitivity of strength. In this case, new parameters, e.g. strain rate or stress rate, should be taken into account in the description of deformation and fracturing of solids. Dynamic strength theories expand upon traditional and kinetic strength theories of solids by considering the dynamic effects induced by high strain rate loading.

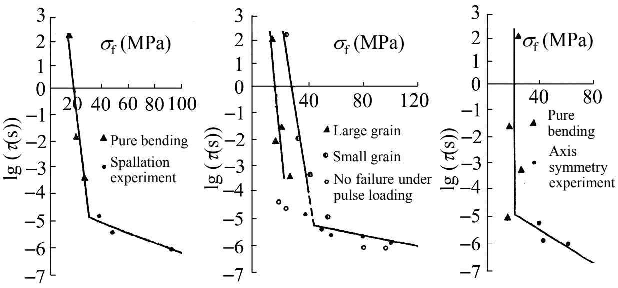
4.1 Experimental observation

The fracturing strengths of rocks increase significantly under intensive dynamic loading. Some experimental data are presented in Table 1 [6].

Figures 1 and 2 represent laboratory experimental data collected under a constant loading rate [7], where τ is the loading time from initial application of the load to failure, σ_f is the failure stress, $\dot{\varepsilon}$ is the strain rate, and σ_{dyn} and σ_{st} are the dynamic and static failure stresses.

Table 1 The fracturing strengths of rocks [6].

Rocks	Static strength (MPa)	Dynamic strength (MPa)	The ratio of dynamic strength to static strength
Limestone	42.56	276.62	6.5
Marble (normal to the deposit)	21.28	191.50	9.0
Marble (parallel to the deposit)	63.84	496.49	7.8
Granite	70.93	405.30	5.7



(a) Granite (b) Marble (c) Sandstone

Fig.1 Relationship between fracture time and load amplitudes [7].

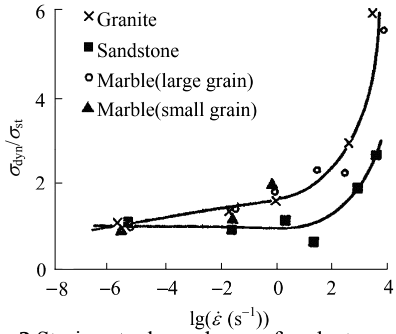


Fig.2 Strain rate dependence of rock strength [8].

It can be observed from Fig.1 that when $\tau > 10^{-5}$ s, the loading is quasi-static and a weak time-dependence of failure stress is observed. When $\tau < 10^{-5}$ s, failure stress increases remarkably with a decrease in loading time.

It can be observed from Figs.2 and 3 that dynamic strength rapidly increases with the strain rate when strain rate $\dot{\varepsilon} > 10$ s^{-1} [8].

To establish deformation and fracturing models for surrounding rocks of tunnels, it is necessary to apply dynamic strength theories and failure criteria.

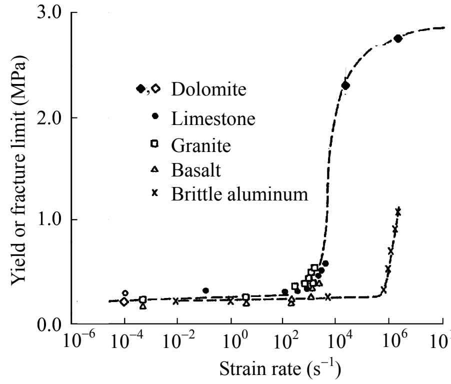


Fig.3 Strain rate dependence of yield or fracture limit [8].

4.2 Stain rate sensitivity of rock strength

Under moderate uniaxial tension, the expected life time (instigation of loading to failure) τ may be determined by Zhurkov's formula:

$$\tau = \tau_0 \exp\left(\frac{U_0 - \gamma\sigma_t}{KT}\right) \tag{1}$$

where U_0 is the activation energy, σ_t is the uniaxial tensile stress, γ is the activation volume, and τ_0 is a temporal parameter which is in the order of the thermal vibration period of atoms [9].

Zhurkov's formula shows the thermo-activated nature of deformation and fracturing of solids, which gives the dependence of the strength on the life time as

$$\sigma = Y = \frac{1}{\gamma}\left(U_0 + KT\ln\frac{\tau_0}{\tau}\right) \tag{2}$$

where ε_0 is the limit deformation at failure, and $\dot{\varepsilon}$ is the constant strain rate of loading process.

When $\tau = \varepsilon_0 / \dot{\varepsilon}$, the Eq.(2) becomes

$$\sigma = Y = \frac{1}{\gamma}\left[U_0 + KT\left(\ln\dot{\varepsilon} - \ln\frac{\varepsilon_0}{\tau_0}\right)\right] \tag{3}$$

i.e.

$$\sigma = Y = \frac{1}{\gamma}\left(U_0 + KT\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right) \tag{4}$$

where $\dot{\varepsilon}_0 = \varepsilon_0 / \tau_0$ represents the maximum possible tensile strain rate in the material.

A similar formula holds true for the dynamic shear strength Y_τ:

$$Y_\tau = \frac{1}{\gamma_\tau}\left(G_0 + KT\ln\frac{\dot{\gamma}}{\dot{\gamma}_0}\right) \tag{5}$$

where γ_τ is the activation volume under shear deformation, G_0 is the activation energy, $\dot{\gamma}$ is the shear strain rate, and γ_0 is the limit shear strain, $\dot{\gamma}_0 = \gamma_0 / \tau_0$.

In principle, the dependence of the compressive strengths of solids on strain rate is similar to that of the tensile strengths, but the values of the parameters in the formulae are different. Only compressive and shear strengths will be examined in this paper. Qi and Qian [10] have re-derived Zhurkov's formula on the basis of microscopic theory.

Experiments by Stavrogin and Protosenja [11] showed that Eqs.(4) and (5) can describe the strain rate sensitivity of compressive, shear and tensile strengths of solids at relatively low strain rates. Their results indicate that a thermo-activated mechanism dominates the strain rate sensitivity of strength. When the strain rate exceeds a certain threshold value, the strain rate sensitivity of strength moves into a new regime, where the strength increases rapidly with increases in the strain rate, and the deformation and fracturing of solids are more adiabatic. In this case, according to current knowledge, phonon damping (macroscopic viscosity) plays a predominant role.

The investigation shows the general features of dynamic strength of solids as illustrated in Fig.4 [11]. In a low strain rate regime, the strength of materials increases slowly as the strain rate increases. This regime is provisionally named Regime 1. When the strain rate exceeds a threshold value, the strength increases rapidly with the increase of the strain rate. This regime is named Regime 2. When the strain rate is very high, the dependence of strength on strain rate becomes weak again and is somewhat similar to that in Regime 1. This regime is named Regime 3 (Fig.4).

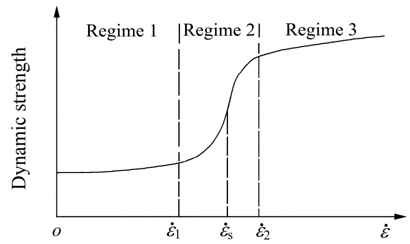


Fig.4 Dependence of dynamic strength on strain rates of brittle materials ($\dot{\varepsilon}_1 \approx 10^0$–$10^2\ s^{-1}$, $\dot{\varepsilon}_s \approx 10^3\ s^{-1}$, $\dot{\varepsilon}_2 \approx 10^4\ s^{-1}$).

The smooth transition from Regime 1 to Regime 2 represents a gradual change of the deformation and failure mechanisms during the transition, i.e. the thermo-activated mechanism gradually loses its predominance and phonon damping (macroscopic viscosity) gradually emerges as the dominant mechanism. However, the two mechanisms do coexist.

In Regime 2, the material behavior is closely related to its viscosity. Generally, viscosity can be defined as the transport of momentum along a velocity gradient. In a steady shock wave process, viscosity can be viewed as the diffusion of momentum along the axis of wave propagation [12]. The viscosity is commonly considered to be a material property, which describes proportionality between a viscous stress component and the velocity gradient or strain rate, and depends on temperature. However, more complex constitutive behaviors can appear under shock loading.

The transition from Regime 2 to Regime 3 is accompanied by weaker dependence of strength on strain rate. Kipp et al. [13] determined the fracturing stresses of penny-shaped cracks at different strain rates. They showed that when the strain rate grows, the fracturing stress of cracks increases in magnitude and becomes effectively independent of crack size at very high strain rates. At very high strain rates, a wide range of crack sizes is initiated simultaneously and the failure grows by multiple cracks growth and coalescence. An experimental demonstration of this effect was reported by Kalthoff and Shockey [14] using short pulse loading on cracks with finite lengths. The results imply that in the transition process from Regime 2 to Regime 3, the effect of locality (or localization) of deformation and fracturing is reduced gradually, and homogeneity of deformation and fracturing gradually emerges.

At high strain rates, the number of crack nuclei grows rapidly due to the thermo-fluctuation rupture of inter-molecular bonds in intact regions, in addition to the athermal growth of cracks. In other words, the thermo-activated mechanism is reactivated in the absence of a significant stress concentration. The presence of defects only results in an increase in the rate of these ruptures due to the specific local features of energy dissipation in the deformation and fracturing process of solids. The last situation should cause a weaker dependence on temperature of solid strength in sub-microsecond lifetime intervals. At very high stain rates, the material fragments after fracturing are very small due to the simultaneous initiation of damage throughout the solid volume.

Therefore, based on analysis of the available experimental data, another conclusion about the dependence of solid strength on strain rate can be drawn. At low strain rates, the deformation and fracturing of rocks are controlled by the thermo-activated mechanism and the strength sensitivity to strain rate can be expressed by Eqs.(4) and (5). When the strain rate increases, the phonon damping (macroscopic viscosity) mechanism emerges, and gradually plays the dominant role. Since the crack propagation velocity in a solid is limited by the Rayleigh wave velocity, the viscosity coefficient decreases with the strain rate. At the structural level, the decrease of viscosity with the strain rate activates internal degrees of freedom and the related motion of meso-particles. At very high strain rates, the stress attained in solids approaches the theoretical limit of the strengths. In this case, a wide range of crack sizes is simultaneously initiated. In intact areas, the inter-molecular bonds are broken. These broken bonds serve as the growing athermal nuclei of damage, and the thermo-activated mechanism is reactivated. This means that the localization of the deformation and damage is gradually lost. Thus, the thermo-activated mechanism again emerges as the dominant mechanism of deformation and fracture at high strain rates. Also, the strength sensitivity to strain rate can be considered as the result of competition between the thermo-activated and the macroscopic viscous mechanisms. The viscous mechanism and its mathematic formula are examined below.

The divergence of viscosity within a rock, even one that is experiencing a constant deformation rate, is very large. This is obviously related to the fact that deformation and fracturing take place at different scales.

Rocks have multi-level structures. This observation is critical for the determination of their physical and mechanical properties. For example, the viscosity of a rock is directly related to its multi-level structure. In engineering practice, viscosity can be grouped into three scales, i.e. macro-, meso-, and microscopic levels.

Mathematically, viscosity η is expressed by the following equation:

$$\eta = G\tau' \tag{6}$$

where G is the shear modulus and τ' is the relaxation time [15].

The relaxation of materials is due to not only the relative sliding between structural elements, but also the reorganization of these elements and their internal structural changes. Thus, rock relaxation is accompanied by dilatancy. When a rock structure is fractured, stress concentrations arise, which then diminish with time. This relaxation time is proportional to the structural element size and inversely proportional to the growth velocity of the induced flaws. During the process of dilatancy, structural flaws tend to occur somewhat uniformly in the rock. The growth velocity of the induced flaws (e.g. dislocation and micro- and macro-cracks) is limited. Also, it depends on the applied external stresses and therefore on stress relaxation.

From a phenomenological point of view, the growth velocity of flaws is assumed to be a function of strain rate, i.e.

$$v = v(\dot{\varepsilon}) \tag{7}$$

Expanding Eq.(7) into Taylor's series, we obtain

$$v(\dot{\varepsilon}) = v_0 + \alpha\dot{\varepsilon} + \cdots \tag{8}$$

where $v_0 = v_0(0)$ may be regarded as the growth velocity of flaws at a fixed magnitude of deformation.

It is necessary here to point out that the thermo-activated mechanism also contributes to the velocity of the growth of flaws. Experiments show that the propagation velocity of cracks (growth velocity of flaws) is limited to the range from 0.2 to 0.5 of the shear wave velocity. Taking this into consideration, we choose the following formula to approximate the change of growth velocity with strain rate due only to macro-viscosity:

$$v = b\left(\frac{1 + \lambda\dot{\varepsilon}^n}{\xi\dot{\varepsilon}^n}\right) \tag{9}$$

where b, ξ, λ and n are experimental constants.

On the other hand, according to the Maxwell model of Radionov et al. [23], a rock with an internal element size L can not withstand the deformation when the deformation rate is more than $\dot{\varepsilon}^* = \sigma_f \upsilon / (GL)$, where G is the shear modulus and $\upsilon \approx 2\times10^{-6}$ cm/s is a parameter characterizing the relaxation rate of stress, because of the stress concentration due to the heterogeneity of the rock. Therefore, the resulted strain rate is inversely proportional to the size L, i.e. $\dot{\varepsilon} \propto 1/L$ [23]. Hence, the viscosity can be given as

$$\eta = A\frac{L}{v} = Ab\left(\frac{\xi\dot{\varepsilon}^n}{1+\lambda\dot{\varepsilon}^n}\right)\frac{1}{\dot{\varepsilon}} = \frac{C_0\dot{\varepsilon}^{n-1}}{B+B_1\dot{\varepsilon}^N} \tag{10}$$

where A, B, B_1 and C_0 are experimental constants.

Equation (10) can be re-written as

$$\eta = C_1\frac{\dot{\varepsilon}^{n-1}}{1+\dot{\varepsilon}^n} \tag{11}$$

where C_1 is an experimental constant.

Equation (11) is similar to the second term of the following expression:

$$\eta = A\frac{\ln(\dot{\varepsilon}/\dot{\varepsilon}_0)}{\dot{\varepsilon}} + C\frac{(\dot{\varepsilon}/\dot{\varepsilon}_s)^{n-1}}{[(\dot{\varepsilon}/\dot{\varepsilon}_s)^n+1]} \quad (n \geq 1) \tag{12}$$

where $\dot{\varepsilon}_0$ is a deformation rate on the order of 10^{12} s^{-1}, $\dot{\varepsilon}_s$ is an approximation parameter, and C is an experimental constant [16].

In this case, the first term on the right-hand side of Eq.(12) may be regarded as the contribution of the thermo-activated mechanism of deformation [16], and the second term is the contribution of the macro-viscosity mechanism.

Hence, the macro-viscosity can be expressed as

$$\eta_{\text{macv}} = C\frac{(\dot{\varepsilon}/\dot{\varepsilon}_s)^{n-1}}{[(\dot{\varepsilon}/\dot{\varepsilon}_s)^n+1]} \tag{13}$$

From the above analysis, it is shown that an increase of the deformation rate leads to a decrease of viscosity, which means that the deformation and fracturing of rocks gradually converge at the macro- and micro-scopic scales.

The compressive strength sensitivity to the strain rate then can be expressed as the summation of the following two terms (Y_D is the compressive strength, and $Y_{\tau D}$ is the shear strength):

$$Y_D = Y_T(\dot{\varepsilon}) + Y_V(\dot{\varepsilon}) \tag{14}$$

$$Y_{\tau D} = Y_{\tau T}(\dot{\gamma}) + Y_{\tau V}(\dot{\gamma}) \tag{15}$$

The first term on the right-hand side of Eqs.(14) and (15) represents the contributions of the thermo-activated mechanism, and the second term represents those of the viscosity mechanisms.

According to Eq.(13), the contributions of the macro-viscosity may be expressed as

$$Y_V(\dot{\varepsilon}) = \dot{\varepsilon}\eta = \frac{b(\dot{\varepsilon}/\dot{\varepsilon}_s)^n}{(\dot{\varepsilon}/\dot{\varepsilon}_s)^n+1} \tag{16}$$

and

$$Y_{\tau V}(\dot{\gamma}) = \dot{\gamma}\eta = \frac{b_1(\dot{\gamma}/\gamma_S)^n}{(\dot{\gamma}/\gamma_S)^n+1} \tag{17}$$

where b and b_1 may be interpreted as the maximum values of the contributions from the macro-viscosity mechanism, and $\dot{\gamma}_S$ is the experimental parameter. The effect of temperature is implicitly included in these formulae.

Finally, a unified relationship between strength and strain rate is obtained. It includes the thermo-activation and the viscosity mechanisms as two coexisting and competing mechanisms, i.e.

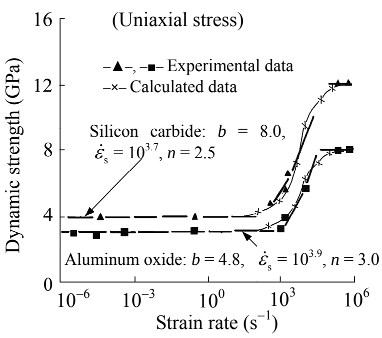
$$Y_D = \frac{1}{\gamma}\left(U_0 + KT\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right) + \frac{b(\dot{\varepsilon}/\dot{\varepsilon}_s)^n}{(\dot{\varepsilon}/\dot{\varepsilon}_s)^n+1} \tag{18}$$

$$Y_{\tau D} = \frac{1}{\gamma_\tau}\left(G_0 + KT\ln\frac{\dot{\gamma}}{\dot{\gamma}_0}\right) + \frac{b_1(\dot{\gamma}/\gamma_S)^n}{(\dot{\gamma}/\gamma_S)^n+1} \tag{19}$$

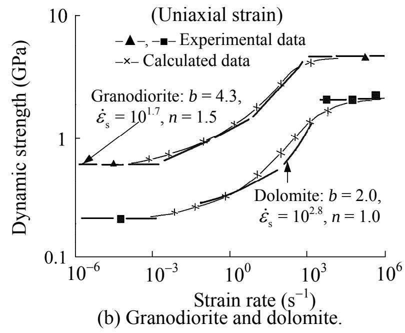
The temperature rise caused by impact affects the strength of a solid. Generally, the strength of a metal decreases significantly when the temperature reaches 85%–90% of its melting temperature, and is different from the results calculated by Eqs.(18) and (19). However, the melting temperatures of rocks are significantly higher than those of metals. Generally the temperature caused by impact in rock is not very close to the melting temperature. Therefore, Eqs.(18) and (19) are applicable to rocks.

The thermo-activated mechanism acts more significantly near stress concentration areas and along grain boundaries at low strain rates. At very high strain rates, the thermo-activated mechanism is activated again in the intact areas of rocks, but the parameters in Eqs.(4) and (5) in these regimes should be different, with γ, γ_τ being less at high strain rates.

The experimental data for silicon carbide, aluminum oxide, granodiorite and dolomite are shown in Fig.5. The left parts of the experimental curves are almost straight horizontal lines (Fig.5(a)). Thus, it is very easy to determine items U_0/γ, K/γ, G/γ_S and K/γ_S in Eqs.(18) and (19) by data fitting.



(a) Aluminum oxide and silicon carbide.



(b) Granodiorite and dolomite.

Fig.5 Comparison of Eq.(18) with the experimental data.

Considering that the contribution of macro-viscosity is very small at low strain rates, it may be argued that the contribution of the thermo-activation mechanism calculated by Eqs.(4) and (5) is also small, and that the material strength is weakly dependent on the strain rate. On the other hand, at high strain rates, the contribution of macro-viscosity, i.e. the second terms of the right-hand side of Eqs.(18) and (19), is dominant. For convenience, the left (horizontal) parts of the curves in Fig.5 may be prolonged to the right. The straight lines are chosen as the basis for superimposing the contribution of macro-viscosity.

According to the above descriptions, example calculations have been carried out using Eqs.(18) and (19). The results of these calculations are compared with the experimental results reported by Grady [12]. The calculated and experimental results agree well (Fig.5). This indicates that the given model has a sound physical basis, and it is applicable to a wide range of strain rates, and is simple and convenient for practical use.

For step-type loading and continuously changing load $\sigma(t)$, Bailey's damage accumulation criterion can be used:

$$\sum \frac{\Delta t_i}{\tau(\sigma_i)} = 1 \quad \text{or} \quad \int_0^{t_p} \frac{dt}{\tau[\sigma(t)]} = 1 \tag{20}$$

where t is time, the subscript "i" denotes the loading ordering, t_p is the loading time to failure, and $\tau(\sigma_i)$ is the life time of material under stress σ_i.

4.3 Other strength theories that consider temporal factors

4.3.1 *Nikiforovsky-Shemyakin impulse criterion*

According to the Nikiforovsky-Shemyakin impulse criterion [17], when the total pulse J_0 reaches its limit value, i.e. when $\int_0^{t_p} \sigma(t)dt = J_0$, failure will take place. In a one-dimensional case, the relationship between the stress in solids σ and the particle velocity V can be expressed as $\sigma = \rho DV$, where ρ is the solid's density and D is the shock wave propagation velocity. Substituting this relationship into the Nikiforovsky-Shemyakin impulse criterion gives

$$\int_0^{t_p} \sigma(t)dt = \int_0^{t_p} \rho DV dt = \rho Du = J_0 \tag{21}$$

where u is the displacement at failure.

The impulse criterion indicates the damage accumulation nature of the fracturing processes, which coincides with Zhukov's criterion.

On the other hand, if the characteristic length of the shock wave is L', then $u = L'\varepsilon$, and Eq.(21) becomes

$$\int_0^{t_p} \sigma(t)dt = \rho DL'\varepsilon = J_0 \tag{22}$$

which shows that fracturing takes place when strain reaches a critical value. Therefore, the second strength theory can be applied to dynamic fracturing problems. The creep phenomenon and strength-strain rate sensitivity show the temporal effect of fracturing; their physical origin is identical. By multiplying Zhukov's formula with the Aleksandro creep formula $\dot{\varepsilon} = \dot{\varepsilon}_0 \exp[-(U_0 - \gamma\sigma)/KT]$, then $\tau\dot{\varepsilon} = \tau_0\dot{\varepsilon}_0 = \dot{\varepsilon}_{st}$ is obtained. Thus, the same conclusion is drawn, i.e. the critical failure strain is the same no matter what strain rate is applied.

Experiments show that, under shear, triaxial compression and other complex loading conditions, in a wide range of strain rates covering 9–10 orders of magnitude, the critical strain rate $\dot{\varepsilon}_{st}$ is only weakly dependent on temperature, stresses and strain rate. Thus, it can be considered as a constant [18]. This situation indicates a close relationship between deformation and fracturing. Therefore, the second strength theory may be considered as a quasi-temporal criterion.

4.3.2 *Failure criterion based on damage evolution*

According to Kachanov [19], the evolution of the damage parameter ψ may be described by the following equation:

$$\frac{d\psi}{dt} = f(\sigma, \psi) = \begin{cases} A\left(\dfrac{\sigma}{1-\psi}\right)^n & (\sigma \ge 0) \\ 0 \quad (\sigma \le 0) \end{cases} \tag{23}$$

Failure takes place when the damage parameter reaches its critical value.

Integrating Eq.(23) yields

$$\int_0^{t_p} \sigma^n dt = \frac{1-(1-\psi_p)^{n+1}}{A(n+1)} = J_0 \tag{24}$$

which coincides with the criterion of Eq.(22) when $n = 1$.

4.3.3 *Structural-temporal criterion*

According to the principles of the fracture mechanics of solids, when the average stress $\sigma(t, x)$ over spatial-temporal cells $[t-\tau, t]\times[0, d]$ reaches its static strength σ_c, failure takes place, i.e.

$$\frac{1}{\tau}\int_{t-\tau}^{t} dt' \int_0^d \sigma(t', r)dr \le \sigma_c \tag{25}$$

where r is the spatial coordinate. This criterion is called Morozov-Petrov's structural-temporal criterion [20].

If we introduce a new parameter $J_c = \sigma_c \tau_c$, then Eq.(25) becomes

$$\int_{t-\tau}^{t} \mathrm{d}t' \int_0^d \sigma(t',\ r)\mathrm{d}r = J_c(t) \tag{26}$$

where τ_c is the fracture incubation time corresponding to the characteristic time for the energy transfer between two neighboring cells d/v', where v' is the elastic wave velocity and d is the structural element size.

Therefore, Morozov-Petrov's structural-temporal criterion is physically manifested as a critical structural impulse.

4.3.4 *Mohr-Coulomb-type constitutive models of strain rate dependence*

The M-C criterion is a simple and practical criterion for geological materials. The strengths of geological materials show a significant strain rate dependency (sensitivity). Therefore, when analyzing geomechanical problems, it is necessary to consider the dependence of strength on strain rates. Under general stress states expressed in terms of principal stresses, the Mohr-Coulomb failure criterion can be written

$$\frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1 + \sigma_3}{2} \sin\phi + c\cos\phi \tag{27}$$

where σ_1 and σ_3 are the maximum and minimum principal stresses, respectively, ϕ is the internal friction angle, and c is the internal cohesion of the material.

With a uniaxial compression test, the internal cohesion c can be determined by

$$c = \frac{\sigma_Y^C (1 - \sin\phi)}{2\cos\phi} \tag{28}$$

where σ_Y^C is the uniaxial compressive strength.

Substituting Eq.(18) into Eq.(28), we obtain

$$c = \frac{1 - \sin\phi}{2\cos\phi} \left[\frac{1}{\gamma} \left(U_0 + KT \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) + \frac{b(\dot{\varepsilon}/\dot{\varepsilon}_s)^n}{(\dot{\varepsilon}/\dot{\varepsilon}_s)^n + 1} \right] \mathrm{e}^{AC_0} \tag{29}$$

The last term in Eq.(29), e^{AC_0}, expresses the influence of strain on internal cohesion, and $C_0 = \sigma_1 / \sigma_3$ is a parameter of stress state.

Substituting Eq.(29) into Eq.(27), a Mohr-Coulomb-type failure (strength) criterion with strain rate dependence can be obtained.

For loading conditions with high strain rates, the thermo-activated term may be replaced by static uniaxial compressional strength σ_{YS}^C because of the weak influence of the thermo-activated term on strength:

$$\sigma_1 - \sigma_3 = (\sigma_1 + \sigma_3)\sin\phi + (1 - \sin\phi) \cdot \left[\sigma_{YS}^C + \frac{b(\dot{\varepsilon}/\dot{\varepsilon}_s)^n}{(\dot{\varepsilon}/\dot{\varepsilon}_s)^n + 1} \right] \mathrm{e}^{AC_0} \tag{30}$$

or

$$\sigma_1 = \frac{1 + \sin\phi}{1 - \sin\phi} \sigma_3 + \left[\sigma_{YS}^C + \frac{b(\dot{\varepsilon}/\dot{\varepsilon}_s)^n}{(\dot{\varepsilon}/\dot{\varepsilon}_s)^n + 1} \right] \mathrm{e}^{AC_0} \tag{31}$$

For underground explosions, explosion-induced fractures occur in proximity to the center of the explosion by a shear mechanism. The problem may be simplified furthermore, because $\varepsilon_r >> \varepsilon_\theta$, where ε_r is the radial strain and ε_θ is the tangential strain. Therefore, the shear strain $\varepsilon_\gamma = \varepsilon_r - \varepsilon_\theta \approx \varepsilon_r$, and the volumetric strain are approximated as $\varepsilon_v = \varepsilon_r + 2\varepsilon_\theta \approx \varepsilon_r$. Furthermore, it can be taken $\dot{\varepsilon} = \dot{\varepsilon}_r$. The relationship between the two principal stresses is $\sigma_1 = \alpha\sigma_3$, where $\alpha = C_0 = \mu/(1-\mu)$ and μ is Poisson's ratio.

In this case, the M-C criterion in the vicinity of explosion may be written as

$$\sigma_1 = \frac{1 + \sin\phi}{1 - \sin\phi} \sigma_3 + \left[\sigma_{YS}^C + \frac{b(\dot{\varepsilon}/\dot{\varepsilon}_s)^n}{(\dot{\varepsilon}/\dot{\varepsilon}_s)^n + 1} \right] \mathrm{e}^{A\alpha} \tag{32}$$

4.3.5 *Fragment size of fractured rock mass under dynamic loading*

The strength of a fractured rock mass depends on the sample size. Generally, the compressive strength of materials σ_D can be expressed as a function of the sample size D as follows [21, 22]:

$$\sigma_D = \sigma_0 (1 + D/D_0)^{-1/2} \tag{33}$$

where σ_0 and D_0 are constants.

If D_0 in Eq.(33) is replaced by Δ_i/D_0, where Δ_i is the size of blocks of i-th rank, then Eq.(33) becomes

$$\sigma_D = \sigma_0 (1 + \Delta_i / D_0)^{-1/2} \tag{34}$$

which can be rewritten as

$$\Delta_i = D_0[(\sigma_0/\sigma)^2 - 1] \tag{35}$$

where parameter σ_D is replaced by σ representing the applied load.

Replacing σ in Eq.(35) by the strength of rock mass, the following formula is obtained to determine the fragment size of fractured rock mass:

$$\Delta_i = D_0 \left[\left(\frac{\sigma_0}{Y_\tau} \right)^2 - 1 \right] \tag{36}$$

Equation (36) shows that the mean fragment sizes of fractured rock mass decrease with the growth of external loads.

This conclusion is confirmed by quasi-static and dynamic experiments. In the case of a one-fold fracture, under both dynamic and quasi-static conditions, Fig.6 shows the relationship between the specific shear deformation energy E_τ and mean fragment size D given by the same curve [23]. This relationship applies to both shear fracture and cleavage fracture, which can be approximated by the following equation:

$$D \sim \frac{1}{E_\tau} = \frac{3}{1+\mu} \frac{E}{(\sigma_r - \sigma_\phi)^2} \tag{37}$$

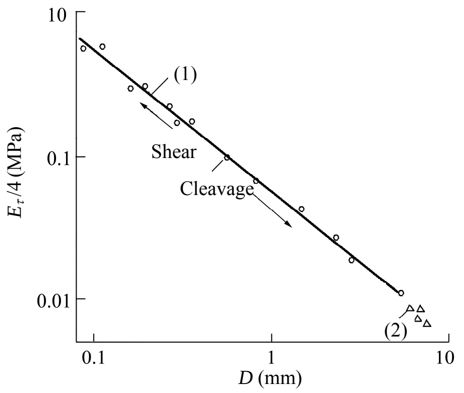


Fig.6 Dependence of specific shear deformation energy on rock fragment size based on the results of (1) a chemical explosion blast (0.4 g) and (2) quasi-static experiments when $\sigma_{22} = 0$ MPa.

where σ_r and σ_ϕ are the radial and tangential stresses respectively.

Replacing $\sigma_r - \sigma_\phi$ in Eq.(37) by $2Y_\tau$, the following result is obtained:

$$D \sim \frac{1}{E_\tau} = \frac{3}{4(1+\mu)} \frac{E}{Y_\tau^2} \tag{38}$$

which has the same D-Y_τ dependence as Eq.(36).

To predict the mean fragment size under uniaxial dynamic loading, Y_τ should be replaced in Eq.(36) by Y_{D} or $Y_{\tau\mathrm{D}}$, which are determined by Eq.(18) and (19) respectively.

For predicting the mean fragment size near the center of the explosion, Eq.(36) is used. But, the dynamic shear strength of rock mass should be determined by the following formula:

$$Y_{\tau\mathrm{D}} = \left[\frac{1}{\gamma_\tau} \left(G_0 + KT \ln \frac{\dot{\gamma}}{\dot{\gamma}_0} \right) + \frac{b_1 (\dot{\gamma}/\dot{\gamma}_{\mathrm{S}})^n}{(\dot{\gamma}/\dot{\gamma}_{\mathrm{S}})^n + 1} \right] \mathrm{e}^{AC} \tag{39}$$

Under external loads, fracturing takes place on the structural surfaces of the largest scale fragments in the medium. The fragment size is the characteristic size of the structural elements at this scale. A further increase in the stress intensity results in fracturing at the next lower scale, and the size of fragments is the characteristic size of the structural elements at that scale. The increases of strain rate, confining pressure and plastic strain hardening may enhance the strength of the medium. Consequently, the deformation and fracturing may cover the small-scale levels of a rock mass and the fragment size will decrease.

5 Application of modern strength theories

5.1 Internal structure of a spallation plane

The introduction of temporal factors into strength criteria improves our understanding of failure mechanisms, and may produce results that are significantly different from those of traditional strength theories. As an example, the spallation problem for the propagation of a triangular impulse stress wave without rise time is considered below.

According to the traditional strength theories, tensile stress occurs when a stress wave reflects from free surfaces. When the resultant tensile stresses at some distance from the free surfaces reach a critical value σ_{t}, spalling takes place at that distance $x = \sigma_{\mathrm{t}}/(2\sigma_{\mathrm{m}})$, where σ_{m} is the amplitude of the stress wave. However, according to the new strength theories, every point in the rock covered by the reflected wave is under tensile stress and prone to tensile fracturing, i.e. spalling.

In a rock section where tensile stresses are small at a particular time, it is reasonable to assume that the rock will need a longer time to be fractured. Such a section may fracture simultaneously with other sections where tensile stress is greater and incubation time for fracture is longer, i.e. connected rock within a definite width may fracture simultaneously. Experiments validate this hypothesis. Therefore, spalling zones generally are defined to have certain widths (or thickness), which in turn means that spalling zones have internal structures. It is apparent that the simulation of such an event will not be realistic if we use traditional static strength theories to simulate dynamic fracture.

5.2 Safety thresholds for explosive detonations

Ground vibration induced by explosions may damage surface infrastructure and underground facilities. The decisive parameters for assessing the likely degree of damage and the safety of proximal infrastructure are the seismic vibration parameters: acceleration, particle velocity, and displacement. At present, consensus on the issue of which parameters should be used for safety evaluations of structures by explosion-induced seismicity has not been completely resolved. Most jurisdictions around the world take the ground surface velocity as the control parameter. The use of such a parameter agrees with in-situ investigation, i.e. it is the ground surface velocity or displacement, not internal forces, that controls the damage to buildings and facilities. This also agrees with the modern strength theories.

According to the modern strength theories, the damage to infrastructure under explosion-induced ground vibration is caused by dynamic failures. The control parameter for dynamic failure is displacement or particle velocity. Considering that displacement is the integration of velocity over time, the introduction of failure criteria involving vibration velocity and frequency as controlling parameters, as proposed by American Mining Bureau, and authorities in Germany and Finland, is more reasonable. Large numbers of observations show that, under the same geological conditions, at the same site and for the same type of structures, the degree of damage to the structures is the same when the vibration velocity exceeds a characteristic value for the particular kind of construction.

6 Conclusions

Usually, the attention is focused on the spatial aspect of rock engineering problems, and the time-dependence is often ignored. However, the dependence of deformation and fracturing processes on time lies in the fact that the fracturing of rocks requires time to be mobilized, to develope and to complete, and rock strengths depend on strain rates. The consideration of the time-dependence of material deformation improves our understanding of material deformation and fracturing.

At low strain rates, the deformation and fracturing of rock are controlled by the thermally activated mechanisms. With an increase of strain rate, the phonon damping (macroscopic viscosity) mechanism emerges and gradually dominates. At very high strain rates, deformation and fracturing occur gradually at the microscopic scale, under which conditions the thermally activated mechanism is reactivated. In this case, a wide range of crack sizes is initiated simultaneously in the rock and inter-molecular bonds in previously intact regions are broken. These broken bonds serve as the nuclei from which damage (micro-cracks) grows. This means that the localization of deformation and damage will be gradually reduced and finally disappear.

At high strain rates, thermal activation emerges as the dominant mechanism of deformation and fracturing. Thus, the dependence of strength on strain rate may be considered as the result of competition between two coexisting mechanisms, thermally activated and macro-viscous mechanisms, which take turns playing the leading role over different ranges of strain rate. The dependence of rock strength on strain rate may be expressed as the summation of the contributions from these two mechanisms. A comparison between experimental and calculated data has shown that the proposed model describes the strength dependence (sensitivity) on strain rate very well over a wide range of strain rates. The proposed model has a sound physical foundation, is applicable to a wide range of strain rates, and is simple and convenient for practical applications.

The influence of dynamic loading on the fragment size of rock has also been shown to depend on the accumulation of increased shear deformation energy at the moment of fracturing due to the strength enhancement originating from the change in stress states, the accumulation of plastic deformation, and strain rate. The suggested relationship describes the fragmentation size well.

A Mohr-Coulomb-type constitutive relationship has been proposed, and the intrinsic relations between different temporal failure criteria have been expressed and explained. The applicability of these modern strength theories has been shown by explaining some unusual phenomena that cannot be easily explained with the traditional strength theories.

References

[1] Hoek E, Brown E T. Empirical strength criterion for rock masses. Journal of Geotechnical and Geoenvironment Engineering, ASCE, 1980, 106 (9): 1 013–1 019.

[2] Hoek E, Brown E T. The Hoek-Brown failure criterion–a 1988 update. In: Proc. 15th Can. Rock Mech. Symp.. Toronto: Press of University of Toronto, 1988: 31–36.

[3] Drucker D C, Prager W. Soil mechanics and plastic analysis or limit design. Quart of Applied Mathematics, 1952, 10: 157–162.

[4] Yu M H, He L N, Song L Y. Twin shear stress strength theory and its generalization. Science in China (serials A), 1985, 28 (11): 1 113–1 120.

[5] Yu M H, He L N. A new model and theory on yield and failure of materials under complex stress state. In: Mechanical Behavior of Materials (VI). Oxford: Pergamon Press, 1991: 851–856.

[6] Rinehart J S, Dynamic fracture strength of rocks. In: Proceedings of the 7th Symposium on Rock Mechanics. New York: American Institute of Mining, 1965: 205–208.

[7] Bellendir E N, Klyadchenko V F, Kazachuk A I. Strength of rock mass under the intervals of loading of 10^2–10^{-6}s. Journal Mining Science, 1991, 27 (2): 46–49 (in Russian).

[8] Nikolaevsky V N. Mechanics of porous and fractured media. Singapore: World Scientific, 1990.

[9] Kuksenko V S, Betechtin V I, Ryskin V S, et al. Nucleation of submicroscopic cracks in stressed solid. International Journal Fracture, 1975, 11 (5): 829–840.

[10] Qi Chengzhi, Qian Qihu. The microscopic physical mechanism of deformation and damage of materials. Chinese Journal of Solid Mechanics, 2002, 23 (3): 312–317 (in Chinese).

[11] Stavrogin A N, Protosenja A G. Mechanics of deformation and fracture of rocks. Moscow: Nedra, 1992 (in Russian).

[12] Grady D E. Shock wave properties of brittle solids, in: Shock Compression of Condensed Matters. New York: AIP Press, 1995: 9–20.

[13] Kipp M E, Grady D E, Chen E P. Strain-rate dependent fracture initiation. International Journal Fracture 1980, 16 (3): 471–478.

[14] Kalthoff J F, Shockey D A. Instability of cracks under impulse loading. Journal of Applied Physics, 1977, 48 (3): 986–993.

[15] Landau L D, Lifshits E M. Theory of elasticity. New York: Pergamon Press, 1959.

[16] Qi Chengzhi, Qian Qihu. Physical mechanism of brittle material strength-strain rate sensitivity. Chinese Journal of Rock Mechnics and Engineering, 2003, 21 (2): 177–181 (in Chinese).

[17] Nikiforovsky V S, Shemyakin E I. Dynamic fracture of solid. Nauka: Novosibirck, 1979 (in Russian).

[18] Regel V P, Slutsker A N, Tomashevsky E E. Kinetic nature of strength of solids. Moscow: Nedra, 1974 (in Russian).

[19] Kachanov L M. Fundamentals of fracture mechanics. Moscow: Nedra, 1974 (in Russian).

[20] Morozov N, Petrov Y. Dynamics of fracture. Berlin: Springer-Verlag, 2000.

[21] Bažant Z P, Ožbolt J, Eligehausen R. Fracture size effect: review of evidence for concrete structures. Journal of Structural Engineering, 1994, 120: 2 377–2 398.

[22] Bažant Z P. Scaling in nonlinear fracture mechanics. In: IUTAM symposium on nonlinear analysis of fracture. Dordrecht: Kluwer Academic Publishers, 1997: 1–12.

[23] Radionov V N, Sizov I A, Tsvetkov V M. Fundamentals of geomechanics. Moscow: Nedra, 1986 (in Russian).