Transient analysis of hardware and software systems with warm standbys and switching failures

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Abstract: Redundancy or standby is a provision which plays an important role in improving the reliability of engineering systems. This investigation deals with the reliability and sensitivity analysis of a repairable system (consisting of hardware and software components) with warm standbys and switching failures. The failure and repair rates of the components are assumed to follow exponential distributions. By using Markovian property, the transient state model is developed to establish the system reliability and other performance measures. Numerical technique-based matrix method is used to compute various system performance indices. A numerical example is provided to illustrate the tractability of the proposed method.

Keywords: hardware and software system; Markovian model; repair; transient solution; warm standby; switching failures; reliability.


Biographical notes: Madhu Jain is a Faculty member at the Department of Mathematics, I.I.T. Roorkee. She has been a gold medalist of Agra University at MPhil level. There are more than 280 research publications in refereed international/national journals and more than 20 books to her credit in addition to two reference books. She was recipient of the Young Scientific Award and SERC Visiting Fellowship of DST, and career award of UGC. She has successfully completed six sponsored major research projects of DST, UGC and CSIR. Her current research interest includes performance modelling, soft computing, bio-informatics, reliability engineering and queueing theory. Thirty candidates have received their PhD degrees under her supervision. She has visited more than 30 reputed universities/institutes in the USA, Canada, UK, Germany, France, Holland and Belgium.
1 Introduction

Redundancy is a well established technique which is widely used to improve the system reliability and availability. In the past decades, system reliability models have played an important role in the performance prediction of computer systems, communication networks, power plants, transportation systems, manufacturing systems, etc. Therefore, extensive researches had been carried out on the reliability analysis for the redundant systems. In practice, not only the working units and the standby units but also the switch which is responsible for the switching between the units can fail. The existing researches mainly focused on the reliability analysis of perfect switching of standby units to replace the failed units of the system. It is realised that Markovian models are more natural and suitable to predict the reliability characteristics of many multi-component systems. It is of also vital importance to perform the reliability and sensitivity analysis of such multi-component repairable system with warm standby units by incorporating the concepts of switching failures. The present investigation is concerned with Markovian model for an embedded system which consists of both hardware and software components. There is provision of warm standby hardware units and cold standby software units which are likely to have switching failures when used. The section-wise arrangement of rest of the paper is as follows. Section 2 gives the review of the literature. Section 3 provides the mathematical description of Markov model along with assumptions and notations. In Section 4, we construct the state equations. Section 5 contains governing equations for a particular case to evaluate transient probabilities of the system states. The solution approach based on Laplace transform and matrix method is given. Section 6 is devoted to the performance measures of the system. Numerical results and sensitivity analysis with the help of R-K and matrix method are given in Section 7. The paper is concluded in the final Section 8 which summarises the works done and highlights the important features of present investigation.

2 Review of the literature

In many critical applications of computer systems, fault tolerance has been an essential architectural attribute for achieving high reliability. It is universally accepted that computers without employing redundancy cannot achieve the intended reliability in operating systems, application programmes, control programmes or commercial systems such as nuclear power plant control, space shuttle, etc. The standby redundant systems are frequently employed to obtain high system reliability through hardware redundancy.
There have been several studies dealing with the design and performance analysis of standby redundant systems. Zhang et al. (2006) considered the availability and reliability of \( k \)-out-of-(\( M+N \)): G systems with warm standbys. Yadav (2007) proposed a comprehensive approach for the reliability allocation by comprehending functional dependency and failure criticality for each component or subsystem. The availability of a general repairable system was evaluated by Gamiz and Roman (2008). They evaluated the measures for the effectiveness of repairable system with unknown failure and repair time distributions. Persona et al. (2009) presented a new mathematical function, by introducing the uncertainty of the operating environments as a random variable for predicting the reliability of the systems. Hsu et al. (2009) studied a repairable system with imperfect coverage and reboot with the help of Bayesian and asymptotic estimation. The two unit repairable system was considered with different types of prior assumptions for unknown parameters, in which the coverage factor for an operating unit failure is possible. Ram and Singh (2010) discussed a complex system with common cause failure and two types of repair facilities. Sarkar and Biswas (2010) calculated availability of a one-unit system supported by several spares and repair facilities. They considered a system consisting of one operating unit, \( n-1 \) spares and \( r \) repair facilities. Garg et al. (2010) discussed both time dependent and steady state availability under idealised as well as faulty preventive maintenance. Zheng et al. (2011) defined well-posedness and stability of the repairable system with \( N \) failure modes and one standby unit. Modelling of failure rates as a function of time and influencing factors is proposed by Brissaud et al. (2011). Yang and Meng (2011) discussed the reliability analysis of a warm standby repairable system with priority in use.

One can improve the system reliability either by providing sufficient spare parts. It is also evident that in the most of ‘real world’ multi component machining systems, after any component’s failure, the component is subject to be repaired rather than replaced. The reliability prediction of a repairable system with redundancy has played an increasingly important role in many real time systems. The warm standby component with a lower failure rate than the operating units is recommended due to economic constraints. In many real time systems, the standby might not be able to switch over to act as a primary unit successfully, and it might also need a longer warm-up time. In this paper, we develop Markovian model for a system consisting of primary and warm standby components which are considered as repairable. Subramanian and Venkatakrishnan (1975) investigated reliability of two-unit standby redundant system with repair, maintenance and standby failure. Standby redundant repairable systems have been studied extensively in the past by Kumar and Agarwal (1980), Yearout et al. (1986), etc.; and detailed bibliographies can be found in Osaki and Nakagawa (1976). Wang et al. (2005) suggested the cost benefit analysis of series systems with warm standby components with general repair time. Goel and Shrivastava (1991) analysed the profit of a two unit redundant system with test and correlated failures and repairs. Hsieh and Wang (1995) computed reliability of a repairable system with spares and a removable repairman. Wang and Kuo (2000) gave cost and probabilistic analysis of series systems having mixed standby components. Jain and Baghel (2001) and Bhuyan and Sarmah (2002) estimated reliability of a repairable standby redundant system. Chandrasekhar et al. (2004) studied the two unit standby system and obtained exact confidence limits for the availability of the system, when the time-to-failure of an operative unit is constant and the time-to-repair of the failed unit is governed by a two stage Erlangian distribution. Reliability analysis of a multi-component system with warm standbys and a repairable
service station was done by Wang et al. (2004). They assumed that the life time and repair time of the units are exponentially distributed and the failed units are repaired on FCFS basis. Many mathematicians have contributed significantly towards reliability analysis of redundant system with spares. Xu et al. (2005) established asymptotic stability of a repairable system with imperfect switching mechanism. Wang and Liang (2006) gave cost benefit analysis of the machining systems with warm standby units and imperfect coverage by considering the time-to-repair as general distributed. El-Damcese (2009) analysed warm standby systems subject to common-cause failures with time varying failure and repair rates. Kornecki and Zalewski (2010) studied hardware certification for real-time safety critical systems. Hajeeh and Jabsheh (2009) and Hajeeh (2011) studied the reliability and availability for four series configurations with both warm and cold standby and common cause failure. Jain and Gupta (2011) discussed various key aspects, failure consequences, methodologies of redundant systems along with software and hardware redundancy techniques which have been developed at the reliability engineering level.

The demand for products with more and more functionalities has increased due to the high industrial competition and the advances in embedded hardware and software technologies. To gain and maintain competitive advantage, the system designers require a high level reliability of both hardware as well as software systems. A high or required level of reliability and availability are often essential requisites for embedded computer systems. Lai et al. (2002) analysed the availability of distributed software/hardware systems. Guo and Yang (2007) discussed the methods of simple reliability block diagram for safety integrity verification.

Most studies about the reliability of a system assume that the switchover from warm standby units to primary units is always perfect and there are no failures during switching. But in many real time systems, there is always possibility of failures during the switching from standby state to operating state. Chow (1971) has done studies on an imperfect switching system, which contains two identical components. Alidrisi (1992) gave the recursive formula for the reliability of a dynamic warm standby n components redundant system with imperfect switching and constant failure rate. Pan (1997) predicted the reliability of imperfect switching systems subject to multiple stresses. Wang et al. (2006) compared the availability and reliability analysis for four different system configurations with warm standby components and standby switching failures; the repair time and the failure time for each of the primary and warm standby components are assumed to follow the negative exponential distribution. Jain et al. (2007) gave transient analysis of M/M/R machining system with mixed standbys, switching failures, balkning, reneging and additional removable repairmen. Ke et al. (2007) discussed the reliability of a system which has multiple unreliable service stations and standby switching failures. They considered the time-to-failure, time-to-repair, breakdown time and service time of the failed units governed by the exponentially distribution. Hsu et al. (2008) investigated a problem of a redundant repairable system with switching failure with the help of Bayesian approach. Wang and Chen (2009) provided a comparative analysis of availability between three systems with general repair times, reboot delay and switching failures. In this system, they considered that the time-to-failure and the time-to-repair of the primary and standby units are exponential and general distributed, respectively. Recently, Yun and Cha (2010) and Hsu et al. (2011) considered a general warm standby system with switching time of the standby unit.
3 Model descriptions

In many embedded systems, the operating as well as spares has time dependent failure behaviour. The performance of any hardware/software system is highly influenced by component failures. The component failure may be balanced either by providing spare part support or by facilitating better repair facility so as the system functionality may not suffer. We consider the transient analysis of embedded hardware and software system by incorporating the concepts of:

1. failure due to both hardware and software faults
2. provision of warm hardware standbys and cold software standbys
repair of failed units
switching failure.

There is provision of warm standbys to replace the failed hardware units. All the hardware and software units are subject to failure and repair. The standby units and its associated switching mechanisms are also subject to failures. The system state transition diagram is depicted in Figure 1. The assumptions and notations used to describe the model are as follows:

3.1 Assumptions

- The system consists of M operating and S warm standby hardware components.
- Upon failure of an operative hardware unit, the available warm standby unit becomes operative instantaneously and the failed unit is immediately sent for the repair if the repairman is free, otherwise waits for repair in the queue.
- Once an operating hardware unit fails, a standby unit replaces it and the failure characteristic of the standby unit becomes same as that of the operating unit.
- The repair crew has R repairmen to facilitate the repair of failed unit. Each repairman can repair only one failed unit at a time; the repair discipline is first come first served (FCFS).
- The life times and repair times of all types of units are exponentially distributed.
- When the repair of failed unit is completed, it is as good as new one.
- The switch may fail with probability q during the switching from standby state to operating state.
- The switch over times from failure to repair, from repair to standby and from standby to operating states are negligible.
- The states of all units are mutually independent.

3.2 Notations

\[
\begin{align*}
M & \quad \text{the number of operating units in the system} \\
S & \quad \text{the number of hardware warm standby units in the system} \\
\lambda_h & \quad \text{failure rate of hardware fault} \\
\lambda_s & \quad \text{failure rate of software fault} \\
\alpha & \quad \text{failure rate of standby hardware units} \\
q & \quad \text{failure probability of switching of hardware standby units} \\
R & \quad \text{the number of repairmen} \\
\mu_h(\mu_s) & \quad \text{repair rate of repairmen when he is providing repair of failed hardware (software) units}
\end{align*}
\]
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\[ P_{(i,j)}(t) \] the probability that \( i \) and \( j \) failed units are present at time \( t \), in the system which failed due hardware and software fault, respectively

\[ P_{(i,j)}^*(s) \] Laplace transform of \( P_{(i,j)}(t) \) so that

\[ P_{(i,j)}^*(s) = \int_0^\infty e^{-st} P_{(i,j)}(t) dt. \]

4 Governing equations

For our model, Markovian birth-death process satisfies the following criteria:

- The numbers of failure and repair in a specific time interval depend on the length of the interval but not on its starting point.
- The probability of more than one unit failure and one unit repair in a time interval of length \( \Delta t \) are both \( o(\Delta t) \).

With the help of appropriate failure rates and repair rates for different system states, we construct the Chapman-Kolmogorov equations governing the model as follows:

4.1 System states without software fault

\[
\frac{dP_{(0,0)}(t)}{dt} = -\left[ (M\lambda_h + S\alpha) + M\lambda_S \right] P_{(0,0)}(t) + \mu_h P_{(1,0)}(t) + \mu_S P_{(0,1)}(t)
\]

(1)

\[
\frac{dP_{(1,0)}(t)}{dt} = -\left[ M\lambda_h + (S - 1)\alpha + M\lambda_S + \mu_h \right] P_{(1,0)}(t) + \left[ M\lambda_h (1 - q) + S\alpha \right] P_{(0,0)}(t) + 2\mu_h P_{(2,0)}(t) + \mu_S P_{(1,1)}(t)
\]

(2)

\[
\frac{dP_{(i,0)}(t)}{dt} = -\left[ M\lambda_h + (S - i)\alpha + M\lambda_S + (i \land R)\mu_h \right] P_{(i,0)}(t) + \left[ M\lambda_h (1 - q) + (S - i + 1)\alpha \right] P_{(i-1,0)}(t) + \left[ (i \land R)\mu_h P_{(i+1,0)}(t) + \mu_S P_{(i,1)}(t) \right. \\
\left. + \sum_{n=0}^{i-2} M\lambda_h q^{(i-n-1)}(1-q)P_{(n,0)}(t), \quad 2 \leq i \leq S \right.
\]

(3)

\[
\frac{dP_{(S+1,0)}(t)}{dt} = -\left[ (M + S - i)\lambda_h + M\lambda_S + \mu_h \right] P_{(S+1,0)}(t) + \left[ (M + S - i + 1)\lambda_h \right] P_{(S,0)}(t) + \mu_S P_{(S+1,1)}(t) + R\mu_h P_{(S+2,0)}(t) + \sum_{n=0}^{S-1} M\lambda_h q^{(S-n)}P_{(n,0)}(t)
\]

(4)

\[
\frac{dP_{(i,0)}(t)}{dt} = -\left[ (M + S - i)\lambda_h + M\lambda_S + \mu_h \right] P_{(i,0)}(t) + \left[ (M + S - i + 1)\lambda_h \right] P_{(i-1,0)}(t) + \mu_S P_{(i,1)}(t) + R\mu_h P_{(i+1,0)}(t), \quad S + 2 \leq i \leq N - 1
\]

(5)
\[
\frac{dP_{(N,0)}(t)}{dt} = \lambda_d P_{(N-1,0)}(t) - (R_{\delta}) P_{(N,0)}(t)
\]  \hspace{1cm} (6)

### 4.2 System states without hardware fault

\[
\frac{dP_{(0,j)}(t)}{dt} = -\left[ M\lambda_h + (S - j)\alpha + M\lambda_s + \mu_S \right] P_{(0,j)}(t)
+ M\lambda_h P_{(0,j-1)}(t) + \mu_h P_{(1,j)}(t) + \mu_s P_{(0,j+1)}(t), \hspace{1cm} 1 \leq j \leq S
\]  \hspace{1cm} (7)

\[
\frac{dP_{(0,j)}(t)}{dt} = -\left[ (M + S - j)\lambda_h + (M + S - j)\alpha + \mu_h \right] P_{(0,j)}(t)
+ (M + S - j + 1)\lambda_h P_{(0,j-1)}(t) + \mu_h P_{(1,j)}(t)
+ \mu_s P_{(0,j+1)}(t), \hspace{1cm} S + 1 \leq j \leq N - 1
\]  \hspace{1cm} (8)

\[
\frac{dP_{(0,N)}(t)}{dt} = \lambda_s P_{(0,N-1)}(t) - \mu_s P_{(0,N)}(t)
\]  \hspace{1cm} (9)

### 4.3 System states with both software and hardware faults

\[
\frac{dP_{(i,j)}(t)}{dt} = -\left[ M\lambda_h (1-q) + \left( S - i + j \right)\alpha + M\lambda_s + (i \land R)\mu_h + \mu_S \right] P_{(i,j)}(t)
+ \left[ M\lambda_h (1-q) + (S - i)\alpha \right] P_{(i-1,j)}(t)
+ \sum_{n = 0}^{i+j-2} M\lambda_q \left[ (i+j-(n+\epsilon+1)) \right] (1-q) P_{(n,n+1)}(t)
+ R_{\delta} P_{(i+1,j)}(t) + \left[ M\lambda_h (1-q) + \left( S - i + j \right)\alpha \right] P_{(i,j-1)}(t)
+ \mu_h P_{(i,j+1)}(t), \hspace{1cm} i,j \neq 0, \hspace{1cm} 2 \leq i + j \leq S
\]  \hspace{1cm} (10)

\[
\frac{dP_{(i,j)}(t)}{dt} = -\left[ (M + S - i - j + 1)\lambda_h P_{(i-1,j)}(t) + \mu_h P_{(i-1,j+1)}(t)
+ M\lambda_S P_{(i+1,j-1)}(t) + \sum_{n = 0}^{R-1} M\lambda_q \left[ (i+j-(n+\epsilon)+1) \right] P_{(n,n+1)}(t)
+ (i \land R)\mu_h P_{(i+1,j)}(t) + \mu_S P_{(i,j+1)}(t) \hspace{1cm} i + j = S + 1
\]  \hspace{1cm} (11)

\[
\frac{dP_{(i,j)}(t)}{dt} = -\left[ (M + S - i - j)\lambda_h P_{(i-1,j)}(t) + \mu_h P_{(i-1,j+1)}(t)
+ M\lambda_S P_{(i,j-1)}(t) + \sum_{n = 0}^{R-1} M\lambda_q \left[ (i+j-(n+\epsilon)+1) \right] P_{(n,n+1)}(t)
+ (i \land R)\mu_h P_{(i+1,j)}(t) + \mu_S P_{(i,j+1)}(t) \hspace{1cm} S + 2 \leq i + j \leq N - 1
\]  \hspace{1cm} (12)
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\[
\frac{dP_{i,j}(t)}{dt} = -\left[ M_{i\alpha}(1-q) + (S-j+1)\alpha + M_{iS} + \mu_0 + \mu_S \right] P_{i,j}(t) \\
+ \left[ M_{i\alpha}(1-q) + (S-j)\alpha \right] P_{i-1,j}(t) \\
+ (i+1)\mu_0 P_{i+1,j}(t) \\
+ M_{iS} P_{i,j-1}(t) + \mu_S P_{i,j+1}(t), \quad i = 1, 1 \leq j \leq S - 1
\]  

(13)

\[
\frac{dP_{i,j}(t)}{dt} = \lambda_q P_{i-1,j}(t) + M_{iS} P_{i,j-1}(t) \\
- \left( (R-1)\mu_0 + \mu_S \right) P_{i,j}(t), \quad i + j = N
\]  

(14)

In the construction of above equations, we have used \( iR \) for \( \min (i, R) \).

Taking Laplace transforms of equations (1) to (14) with initial conditions \( P_{(0,0)} = 1 \) and \( P_{(i,j)} = 0 \), for \( i \neq 0, j \neq 0 \), we get:

\[
\left[ (M_{i\alpha} + S\alpha) + M_{iS} + s \right] P^*(0,0)(s) - \mu_0 P^*(0,0)(s) - \mu_SP^*(0,1)(s) = 1
\]  

(15)

\[
\left[ M_{i\alpha} + (S-1)\alpha + M_{iS} + \mu_0 + s \right] P^*(0,0)(s) \\
- \left[ M_{i\alpha}(1-q) + S\alpha \right] P^*(0,0)(s) - 2\mu_0 P^*(2,0)(s) \\
- \mu_SP^*(1,1)(s) = 0
\]  

(16)

\[
\left[ M_{i\alpha} + (S-i)\alpha + M_{iS} + (i \wedge R)\mu_0 + s \right] P^*(0,0)(s) \\
- \left[ M_{i\alpha}(1-q) + (S-i+1)\alpha \right] P^*(0,0)(s) \\
- (i \wedge R)\mu_0 P^*(i+1,0)(s) - \mu_SP^*(1,1)(s) \\
- \sum_{n=0}^{\infty} M_{i\alpha}q^{(i-n-1)}(1-q) P^*(n,0)(s) = 0, \quad 2 \leq i \leq S
\]  

(17)

\[
\left[ (M + S-i)\lambda_q + M_{iS} + R\mu_0 + s \right] P^*(0,1)(s) \\
- \left[ (M + S-i+1)\lambda_q \right] P^*(0,1)(s) - \mu_S P^*(1,1)(s)
\]  

(18)

\[
\left[ (M + S-i)\lambda_q + M_{iS} + R\mu_0 + s \right] P^*(0,0)(s) \\
- \left[ (M + S-i+1)\lambda_q \right] P^*(0,0)(s) - \mu_S P^*(1,1)(s)
\]  

(19)

\[
(R\mu_0 + s) P^*(N,0)(s) - \lambda_q P^*(N-1,0)(s)
\]  

(20)

\[
[M_{i\alpha} + (S-j)\alpha + M_{iS} + \mu_0 + s] P^*(0,j)(s) \\
- \lambda_q P^*(0,j)(s) - \mu_S P^*(0,j+1)(s) = 0, \quad 1 \leq j \leq S
\]  

(21)

\[
[M_{i\alpha} + (M + S-j)\lambda_q + M_{iS} + \mu_0 + s] P^*(0,j)(s) \\
- (M + S-j+1)\lambda_S P^*(0,j)(s) - \mu_S P^*(0,j+1)(s)
\]  

(22)

\[
(R\mu_0 + s) P^*(0,N)(s) - \lambda_q P^*(0,N-1)(s)
\]  

(23)
5 Special case

Due to incorporation of switching failure our model is a general model to deal with more versatile and realistic embedded hardware and computer system. Now, to provide the solution of our model, we consider a special case of general model by considering four hardware components, four software components and three warm standby components for the hardware units. The differential-difference equations governing the model are as follows:

\[
\begin{aligned}
\dot{M} &= (1-q) + \left( S-i+j \right) \alpha \ P_{(i,j)}^s(s) \\
+ M_{i,0} + (i \land R) \mu_i + \mu_S + s \\
- M_{i,0} + (1-q) + (S-i) \alpha \ P_{(i-1,j)}^s(s) \\
- \sum_{n=1}^{i+j-1} M_{i,0} q^{(i+j)-(n+1)}(1-q) P_{(n,n)}^s(s) \\
\end{aligned}
\]

(24)

\[
\begin{aligned}
R_{i,j}^s P_{(i+1,j)}^s(s) - \left[ M_{i,0} + (1-q) + (S-i-j) \alpha \right] P_{(i,j-1)}^s(s) \\
- \mu_S P_{(i,j)}^s(s) = 0, \\
i, j \neq 0, \ 2 \leq i + j \leq S
\end{aligned}
\]

\[
\begin{aligned}
\begin{bmatrix}
(M + S-i+j) \dot{\lambda}_h + M_{i,0} + (i \land R) \mu_i + \mu_S + s
\end{bmatrix} P_{(i,j)}^s(s) \\
- \begin{bmatrix}
M + S-i-j+1
\end{bmatrix} \dot{\lambda}_h P_{(i-1,j)}^s(s) \\
- \lambda_{i,0} P_{(i,j-1)}^s(s) - \sum_{n=1}^{i+j} M_{i,0} q^{(i+j)-(n+1)} P_{(n,n)}^s(s) \\
- \begin{bmatrix}
(i \land R) \mu_i P_{(i+1,j)}^s(s) - \mu_S P_{(i,j+1)}^s(s) = 0,
\end{bmatrix}
\end{aligned}
\]

(25)

\[
\begin{aligned}
\begin{bmatrix}
(M + S-i+j) \dot{\lambda}_h + M_{i,0} + (i \land R) \mu_i + \mu_S + s
\end{bmatrix} P_{(i,j)}^s(s) \\
- \begin{bmatrix}
M + S-i-j+1
\end{bmatrix} \dot{\lambda}_h P_{(i-1,j)}^s(s) \\
- \lambda_{i,0} P_{(i,j-1)}^s(s) - (i \land R) \mu_i P_{(i+1,j)}^s(s) \\
- \mu_S P_{(i,j-1)}^s(s) = 0, \\
S + 2 \leq i + j = N-1
\end{aligned}
\]

(26)

\[
\begin{aligned}
\begin{bmatrix}
M_{i,0} + (1-q) + (S-j) \alpha + M_{i,0} + \mu_i + \mu_S + s
\end{bmatrix} P_{(i,j)}^s(s) \\
- \begin{bmatrix}
(i+1) \mu_i P_{(i+1,j)}^s(s) - M_{i,0} P_{(i,j-1)}^s(s)
\end{bmatrix} \\
- \mu_S P_{(i,j-1)}^s(s) = 0, \\
i = 1, \ 1 \leq j \leq S-1
\end{aligned}
\]

(27)

\[
\begin{aligned}
\begin{bmatrix}
(R-1) \mu_i + \mu_S + s
\end{bmatrix} P_{(i-1,j)}^s(s) \\
- \lambda_{i,j} P_{(i-1,j)}^s(s) - M_{i,0} P_{(i,j-1)}^s(s) = 0, \\
i+j = N
\end{aligned}
\]

(28)
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\[
\frac{dP_{\text{1,0}}(t)}{dt} = -(4\lambda_a + 2\alpha + 4\lambda_S + \mu_S)P_{\text{1,0}}(t) + \left[4\lambda_a(1-q) + 3\alpha\right]P_{\text{0,0}}(t) + 2\mu_aP_{\text{2,0}}(t) + \mu_S P_{\text{1,1}}(t)
\]

(30)

\[
\frac{dP_{\text{2,0}}(t)}{dt} = -(4\lambda_a + \alpha + 4\lambda_S + 2\mu_a)P_{\text{2,0}}(t) + \left[4\lambda_a(1-q) + 2\alpha\right]P_{\text{1,0}}(t) + 3\mu_a P_{\text{3,0}}(t) + \mu_S P_{\text{2,1}}(t) + 4\lambda_a q(1-q)P_{\text{0,0}}(t)
\]

(31)

\[
\frac{dP_{\text{3,0}}(t)}{dt} = -(4\lambda_a + 4\lambda_S + 3\mu_a)P_{\text{3,0}}(t) + \left[4\lambda_a(1-q) + \alpha\right]P_{\text{2,0}}(t) + 3\mu_a P_{\text{5,0}}(t) + 3\mu_a P_{\text{3,0}}(t) + 3\mu_a P_{\text{4,0}}(t) + 4\lambda_a q^2 P_{\text{0,0}}(t) + 4\lambda_a q(1-q)P_{\text{0,1}}(t)
\]

(32)

\[
\frac{dP_{\text{4,0}}(t)}{dt} = -(3\lambda_a + 4\lambda_S + 3\mu_a)P_{\text{4,0}}(t) + 4\lambda_a P_{\text{3,0}}(t) + 3\mu_a P_{\text{5,0}}(t) + \mu_S P_{\text{4,1}}(t) + 4\lambda_a q^2 P_{\text{0,0}}(t) + 4\lambda_a q P_{\text{2,0}}(t)
\]

(33)

\[
\frac{dP_{\text{5,0}}(t)}{dt} = -(2\lambda_a + 4\lambda_S + 3\mu_a)P_{\text{5,0}}(t) + 3\lambda_a P_{\text{4,0}}(t) + 3\mu_a P_{\text{6,0}}(t) + 3\mu_a P_{\text{5,1}}(t)
\]

(34)

\[
\frac{dP_{\text{6,0}}(t)}{dt} = -(\lambda_a + 4\lambda_S + 3\mu_a)P_{\text{6,0}}(t) + 2\lambda_a P_{\text{5,0}}(t) + 3\mu_a P_{\text{6,1}}(t) + \mu_S P_{\text{6,1}}(t)
\]

(35)

\[
\frac{dP_{\text{7,0}}(t)}{dt} = -3\mu_a P_{\text{7,0}}(t) + \lambda_a P_{\text{6,0}}(t)
\]

(36)

\[
\frac{dP_{\text{0,1}}(t)}{dt} = -(4\lambda_a + 2\alpha + 4\lambda_S + \mu_S)P_{\text{0,1}}(t) + 4\lambda_S P_{\text{0,0}}(t) + \mu_a P_{\text{1,1}}(t) + \mu_S P_{\text{0,2}}(t)
\]

(37)

\[
\frac{dP_{\text{1,1}}(t)}{dt} = -(4\lambda_a + \alpha + 4\lambda_S + \mu_S + \mu_a)P_{\text{1,1}}(t) + \left[4\lambda_a(1-q) + 2\alpha\right]P_{\text{0,1}}(t) + 2\mu_a P_{\text{2,1}}(t) + 4\lambda_S P_{\text{1,0}}(t) + \mu_S P_{\text{1,2}}(t)
\]

(38)

\[
\frac{dP_{\text{2,1}}(t)}{dt} = -(4\lambda_a + 4\lambda_S + 2\mu_a + \mu_S)P_{\text{2,1}}(t) + \left[4\lambda_a(1-q) + \alpha\right]P_{\text{1,1}}(t) + 2\mu_a P_{\text{3,1}}(t) + 4\lambda_S P_{\text{2,0}}(t) + \mu_S P_{\text{2,2}}(t) + 4\lambda_a(1-q)P_{\text{0,1}}(t)
\]

(39)

\[
\frac{dP_{\text{3,1}}(t)}{dt} = -(3\lambda_a + 4\lambda_S + 2\mu_a + \mu_S)P_{\text{3,1}}(t) + 2\mu_a P_{\text{4,1}}(t) + 4\lambda_S P_{\text{3,0}}(t) + 4\lambda_a P_{\text{2,1}}(t) + \mu_S P_{\text{3,2}}(t) + 4\lambda_a q^2 P_{\text{0,1}}(t) + 4\lambda_a q P_{\text{1,1}}(t)
\]

(40)
\[
\begin{align*}
\frac{dP_{(1,1)}(t)}{dt} &= -(2\lambda_6 + 4\lambda_5 + 2\mu_6 + \mu_5) P_{(1,1)}(t) + 3\lambda_6 P_{(3,1)}(t) + 2\mu_6 P_{(5,3)}(t) + 4\lambda_5 P_{(4,3)}(t) + \mu_5 P_{(4,2)}(t) \\
\frac{dP_{(5,1)}(t)}{dt} &= -(\lambda_6 + 4\lambda_5 + 2\mu_6 + \mu_5) P_{(5,1)}(t) + 2\lambda_6 P_{(4,3)}(t) + 2\mu_6 P_{(6,3)}(t) + 4\lambda_5 P_{(5,3)}(t) + \mu_5 P_{(5,2)}(t) \\
\frac{dP_{(6,1)}(t)}{dt} &= -(2\mu_6 + \mu_5) P_{(6,1)}(t) + \lambda_6 P_{(5,1)}(t) + 4\lambda_5 P_{(6,0)}(t) \\
\frac{dP_{(0,2)}(t)}{dt} &= -(4\lambda_6 + \mu_6 + \mu_5) P_{(0,2)}(t) + \mu_6 P_{(1,2)}(t) + 4\lambda_5 P_{(0,3)}(t) + \mu_5 P_{(0,2)}(t) \\
\frac{dP_{(2,2)}(t)}{dt} &= -(4\lambda_6 + 4\lambda_5 + \mu_6 + \mu_5) P_{(2,2)}(t) + 4\lambda_6 P_{(1,2)}(t) + 2\mu_6 P_{(2,2)}(t) + 4\lambda_5 P_{(2,3)}(t) + \mu_5 P_{(1,3)}(t) \\
\frac{dP_{(3,2)}(t)}{dt} &= -(3\lambda_6 + 4\lambda_5 + 2\mu_6 + \mu_5) P_{(3,2)}(t) + 3\lambda_6 P_{(2,2)}(t) + 2\mu_6 P_{(3,2)}(t) + 4\lambda_5 P_{(3,3)}(t) + \mu_5 P_{(3,3)}(t) \\
\frac{dP_{(4,2)}(t)}{dt} &= -(\lambda_6 + 4\lambda_5 + 2\mu_6 + \mu_5) P_{(4,2)}(t) + 2\lambda_6 P_{(3,2)}(t) + 2\mu_6 P_{(5,2)}(t) + 4\lambda_5 P_{(4,3)}(t) + \mu_5 P_{(4,3)}(t) \\
\frac{dP_{(5,2)}(t)}{dt} &= -(2\mu_6 + \mu_5) P_{(5,2)}(t) + \lambda_6 P_{(4,2)}(t) + 4\lambda_5 P_{(5,3)}(t) \\
\frac{dP_{(0,3)}(t)}{dt} &= -(4\lambda_6 + 4\lambda_5 + \mu_6 + \mu_5) P_{(0,3)}(t) + \mu_6 P_{(1,3)}(t) + 4\lambda_5 P_{(0,4)}(t) + \mu_5 P_{(0,3)}(t) \\
\frac{dP_{(1,3)}(t)}{dt} &= -(3\lambda_6 + 4\lambda_5 + \mu_6 + \mu_5) P_{(1,3)}(t) + 4\lambda_6 P_{(0,3)}(t) + 2\mu_6 P_{(2,3)}(t) + 4\lambda_5 P_{(1,4)}(t) + \mu_5 P_{(1,3)}(t) \\
\frac{dP_{(2,3)}(t)}{dt} &= -(2\lambda_6 + 4\lambda_5 + 2\mu_6 + \mu_5) P_{(2,3)}(t) + 3\lambda_6 P_{(1,3)}(t) + 2\mu_6 P_{(3,3)}(t) + 4\lambda_5 P_{(2,4)}(t) + \mu_5 P_{(2,3)}(t) \\
\end{align*}
\]
In order to solve the above set of equations (29) to (64), we impose the initial conditions

\[ P_{(0,0)}(0) = 1 \] and \[ P_{(i,j)}(0) = 0, \quad i \neq 0, j \neq 0. \]
For solving the set of equations governing the model, we take Laplace transforms of equations (29) to (64) and solve using eigenvalues approach of matrix method with initial conditions $P_{i(0),j(0)} = 1, P_{i(j),j(k)} = 0$ for $i \neq 0, j = 0$. Now equations (29) to (64) become:

$$
(4\lambda_0 + 3\alpha + 4\lambda_0 + s) P^*_{(0,0)}(s) - \mu_0 P^*_{(1,0)}(s) - \lambda_0 P^*_{(0,1)}(s) = 1
$$

(65)

$$
(4\lambda_0 + 2\alpha + 4\lambda_0 + \mu_0 + s) P^*_{(1,0)}(s)-\left[4\lambda_0 (1-q) + 3\alpha\right]
$$
$$
P^*_{(0,0)}(s) - 2\mu_0 P^*_{(2,0)}(s) - \lambda_0 P^*_{(1,1)}(s) = 0
$$

(66)

$$
(4\lambda_0 + \alpha + 4\lambda_0 + 2\mu_0 + s) P^*_{(2,0)}(s)-\left[4\lambda_0 (1-q) + 2\alpha\right]
$$
$$
P^*_{(1,0)}(s) - 3\mu_0 P^*_{(3,0)}(s) - \lambda_0 P^*_{(2,1)}(s) = 0
$$

(67)

$$
(4\lambda_0 + 4\lambda_0 + 3\mu_0 + s) P^*_{(3,0)}(s)-\left[4\lambda_0 (1-q) + \alpha\right]
$$
$$
P^*_{(2,0)}(s) - 3\mu_0 P^*_{(4,0)}(s) - \lambda_0 P^*_{(3,1)}(s) = 0
$$

(68)

$$
(3\lambda_0 + 4\lambda_0 + 3\mu_0 + s) P^*_{(4,0)}(s)-4\lambda_0 P^*_{(3,0)}(s)
$$
$$
-3\mu_0 P^*_{(5,0)}(s) - \lambda_0 P^*_{(4,1)}(s) = 0
$$

(69)

$$
(2\lambda_0 + 4\lambda_0 + 3\mu_0 + s) P^*_{(5,0)}(s) - 3\lambda_0 P^*_{(4,0)}(s)
$$
$$
-3\mu_0 P^*_{(6,0)}(s) - \lambda_0 P^*_{(5,1)}(s) = 0
$$

(70)

$$
(\lambda_0 + 4\lambda_0 + 3\mu_0 + s) P^*_{(6,0)}(s) - 2\lambda_0 P^*_{(5,0)}(s)
$$
$$
-3\mu_0 P^*_{(7,0)}(s) - \lambda_0 P^*_{(6,1)}(s) = 0
$$

(71)

$$
3\mu_0 P^*_{(7,0)}(s) - \lambda_0 P^*_{(6,0)}(s) = 0
$$

(72)

$$
(4\lambda_0 + 2\alpha + 4\lambda_0 + \mu_0 + s) P^*_{(0,1)}(s) - 4\lambda_0 P^*_{(0,0)}(s)
$$
$$
-\mu_0 P^*_{(1,0)}(s) - \lambda_0 P^*_{(0,2)}(s) = 0
$$

(73)

$$
(4\lambda_0 + \alpha + 4\lambda_0 + \mu_0 + \mu_0 + s) P^*_{(1,1)}(s) - \left[4\lambda_0 (1-q) + 2\alpha\right] P^*_{(0,1)}(s)
$$
$$
-2\mu_0 P^*_{(2,1)}(s) - 4\lambda_0 P^*_{(1,0)}(s) - \lambda_0 P^*_{(1,2)}(s) = 0
$$

(74)

$$
(4\lambda_0 + 4\lambda_0 + 2\mu_0 + \mu_0 + s) P^*_{(2,1)}(s)
$$
$$
-\left[4\lambda_0 (1-q) + \alpha\right] P^*_{(2,0)}(s) - 2\mu_0 P^*_{(3,1)}(s)
$$
$$
-4\lambda_0 P^*_{(2,0)}(s) - \lambda_0 P^*_{(2,2)}(s) = 0
$$

(75)

$$
(3\lambda_0 + 4\lambda_0 + 2\mu_0 + \mu_0 + s) P^*_{(3,1)}(s) - 4\lambda_0 P^*_{(2,1)}(s)
$$
$$
-2\mu_0 P^*_{(4,1)}(s) - 4\lambda_0 P^*_{(3,0)}(s) - \lambda_0 P^*_{(3,2)}(s) = 0
$$

(76)

$$
(2\lambda_0 + 4\lambda_0 + 2\mu_0 + \mu_0 + s) P^*_{(4,1)}(s) - 3\lambda_0 P^*_{(3,1)}(s)
$$
$$
-2\mu_0 P^*_{(5,1)}(s) - 4\lambda_0 P^*_{(4,0)}(s) - \lambda_0 P^*_{(4,2)}(s) = 0
$$

(77)

$$
(\lambda_0 + 4\lambda_0 + 2\mu_0 + \mu_0 + s) P^*_{(5,1)}(s) - 2\lambda_0 P^*_{(4,1)}(s)
$$
$$
-2\mu_0 P^*_{(6,1)}(s) - 4\lambda_0 P^*_{(5,0)}(s) - \lambda_0 P^*_{(5,2)}(s) = 0
$$

(78)
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\[(2\mu_h + \mu_S + s)P_{(0,5)}(s) - \lambda_S P_{(5,5)}(s) - 4\lambda_S P_{(6,0)}(s) = 0 \quad (79)\]

\[(4\lambda + \alpha + 4\lambda_S + \mu_S + s)P_{(0,2)}(s) - \mu_hP_{(1,2)}(s) - \mu_S P_{(0,3)}(s) = 0 \quad (80)\]

\[(4\lambda + 4\lambda_S + \mu_h + \mu_S + s)P_{(1,2)}(s) - [4\lambda(1-q) + \alpha]P_{(0,2)}(s) - 2\mu_hP_{(2,2)}(s) - 4\lambda_S P_{(1,3)}(s) - \mu_S P_{(0,3)}(s) = 0 \quad (81)\]

\[(3\lambda + 4\lambda_S + 2\mu_h + \mu_S + s)P_{(2,2)}(s) - 4\lambda_S P_{(1,2)}(s) - 2\mu_h P_{(3,2)}(s) - \mu_S P_{(1,3)}(s) = 0 \quad (82)\]

\[(2\lambda + 4\lambda_S + 2\mu_h + \mu_S + s)P_{(1,2)}(s) - 3\lambda_P(1-q) + \alpha P_{(0,2)}(s) - 2\mu_h P_{(2,2)}(s) - 4\lambda_S P_{(1,3)}(s) - \mu_S P_{(1,3)}(s) = 0 \quad (83)\]

\[(\lambda + 4\alpha + 2\mu_h + \mu_S + s)P_{(4,2)}(s) - 2\lambda_S P_{(3,2)}(s) - 2\mu_h P_{(5,2)}(s) - 4\lambda_S P_{(4,3)}(s) - \mu_S P_{(3,3)}(s) = 0 \quad (84)\]

\[(2\lambda + \mu_S + s)P_{(5,2)}(s) - \lambda_S P_{(4,2)}(s) - 4\lambda_S P_{(5,3)}(s) = 0 \quad (85)\]

\[(4\lambda + 4\lambda_S + \mu_h + \mu_S + s)P_{(0,3)}(s) - \mu_h P_{(1,3)}(s) - \mu_S P_{(0,4)}(s) = 0 \quad (86)\]

\[(3\lambda + 4\lambda_S + \mu_h + \mu_S + s)P_{(0,5)}(s) - 4\lambda_S P_{(0,3)}(s) - 2\mu_h P_{(2,5)}(s) - \mu_S P_{(0,4)}(s) = 0 \quad (87)\]

\[(2\lambda + 4\lambda_S + 2\mu_h + \mu_S + s)P_{(3,3)}(s) - 3\lambda_P(1-q) + \alpha P_{(0,3)}(s) - 2\mu_h P_{(4,3)}(s) - \mu_S P_{(2,3)}(s) = 0 \quad (88)\]

\[(\lambda + 4\alpha + 2\mu_h + \mu_S + s)P_{(4,3)}(s) - 2\lambda_S P_{(3,3)}(s) - 2\mu_h P_{(5,3)}(s) - 4\lambda_S P_{(4,4)}(s) - \mu_S P_{(3,4)}(s) = 0 \quad (89)\]

\[(2\lambda + \mu_S + s)P_{(4,3)}(s) - \lambda_S P_{(3,3)}(s) - 4\lambda_S P_{(4,4)}(s) = 0 \quad (90)\]

\[(3\lambda + 3\lambda_S + \mu_h + \mu_S + s)P_{(0,4)}(s) - \mu_h P_{(1,4)}(s) - \mu_S P_{(0,5)}(s) = 0 \quad (91)\]

\[(2\lambda + 3\lambda_S + \mu_h + \mu_S + s)P_{(1,4)}(s) - 3\lambda_S P_{(0,4)}(s) - \mu_S P_{(0,5)}(s) - 4\lambda_S P_{(1,3)}(s) = 0 \quad (92)\]

\[(\lambda + 4\lambda_S + 2\mu_h + \mu_S + s)P_{(2,4)}(s) - 2\lambda_S P_{(1,4)}(s) - 4\lambda_S P_{(2,3)}(s) - 2\mu_h P_{(3,4)}(s) - \mu_S P_{(2,5)}(s) = 0 \quad (93)\]

\[(2\lambda + \mu_S + s)P_{(3,4)}(s) - \lambda_S P_{(2,4)}(s) - 4\lambda_S P_{(3,3)}(s) = 0 \quad (94)\]

\[(2\lambda + 2\lambda_S + \mu_h + \mu_S + s)P_{(0,5)}(s) - \mu_h P_{(1,5)}(s) - 3\lambda_S P_{(0,4)}(s) - \mu_S P_{(0,6)}(s) = 0 \quad (95)\]
\( (\lambda_0 + 2\lambda_s + \mu_s + \mu + s)P_{(1,5)}^*(s) - 3\lambda_s P_{(1,4)}^*(s) \)
\(-2\mu S P_{(2,5)}^*(s) - \mu S P_{(1,6)}^*(s) = 0 \)\hspace{1cm} (96)

\( (2\mu + \mu_s + s)P_{(2,5)}^*(s) - \lambda_0 P_{(1,5)}^*(s) - 4\lambda_s P_{(2,4)}^*(s) = 0 \)\hspace{1cm} (97)

\( (\lambda_0 + 2\lambda_s + \mu_s + s)P_{(0,6)}^*(s) - \mu S P_{(1,6)}^*(s) \)
\(-2\lambda_s P_{(0,5)}^*(s) - \mu S P_{(0,7)}^*(s) = 0 \)\hspace{1cm} (98)

\( (\mu_0 + \mu + s)P_{(1,6)}^*(s) - \lambda_0 P_{(1,5)}^*(s) - 2\lambda_s P_{(1,5)}^*(s) = 0 \)\hspace{1cm} (99)

\( (\mu S)P_{(0,7)}^*(s) - \lambda_0 P_{(0,6)}^*(s) = 0 \)\hspace{1cm} (100)

For brevity, we denote the Laplace transform of probabilities \( P_{(i,j)}^*(s) \) with one suffix, i.e., by \( P^*_i(s) \) as shown below:

\[ P^*_0(s) = P_{(i,i)}(s), \quad 0 \leq i \leq 7; \quad P^*_1(s) = P_{(9+i+i)}(s), \quad 0 \leq i \leq 6; \]

\[ P^*_2(s) = P_{(15+i+i)}(s), \quad 0 \leq i \leq 5; \quad P^*_3(s) = P_{(21+i+i)}(s), \quad 0 \leq i \leq 4; \]

\[ P^*_4(s) = P_{(27+i+i)}(s), \quad 0 \leq i \leq 3; \quad P^*_5(s) = P_{(33+i+i)}(s), \quad 0 \leq i \leq 2; \]

\[ P^*_6(s) = P_{(39+i+i)}(s), \quad 0 \leq i \leq 1; \quad P^*_7(s) = P_{(40)}(s). \]

The system of equations (65) to (100) reduces to matrix form as:

\[ Q(s).P^*(s) = P(0) \] (101)

where \( P^*(s) = [P^*_1(s), P^*_2(s), \ldots, P^*_7(s)]^T \) and \( P(0) = [1, 0, 0, \ldots, 0]^T \).

Here, \( Q(s) \) is the 36 \times 36 order matrix and can be written as tri-diagonal block matrix as follows:

\[ Q(s) = \begin{bmatrix} A_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A_j \end{bmatrix} \]

Now, sub matrices \( A_j(i = 1, 2, \ldots, 7 \text{ and } j = 1, 2, \ldots, 7) \) are constructed for particular case as follows:

\[ A_{i2} = \begin{bmatrix} -\mu S I_7, 0_7 \end{bmatrix}^T, \]

\[ A_{i1} = \begin{bmatrix} \Lambda_1 & -\mu_0 & 0 & 0 & 0 & 0 & 0 \\ -[\lambda_0(1-q) + 3\alpha] & \Lambda_2 & -2\mu_0 & 0 & 0 & 0 & 0 \\ -4\lambda_0 q(1-q) -4\lambda_0 q(1-q) + 2\alpha & \Lambda_3 & -3\mu_0 & 0 & 0 & 0 & 0 \\ -4\lambda_0 q^2 & -4\lambda_0 q^2 & -4\lambda_0 q & -4\lambda_0 & -4\lambda_0 & \Lambda_4 & -3\mu_0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3\mu_0 \\ 0 & 0 & 0 & 0 & 0 & -2\lambda_0 & \Lambda_7 & -3\mu_0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\lambda_0 & 3\mu_0 \end{bmatrix} \]

\[ A_{i1} = \begin{bmatrix} -4\lambda_0 I_7, 0_7 \end{bmatrix}, \]

\[ A_{i2} = \begin{bmatrix} -\mu S I_6, 0_6 \end{bmatrix}^T, \]
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\[ A_{22} = \begin{bmatrix}
\Lambda_8 & -\mu_h & 0 & 0 & 0 & 0 \\
-4\lambda_h (1-q + 2\alpha) & \Lambda_9 & -2\mu_h & 0 & 0 & 0 \\
-4\lambda_h q (1-q) & -4\lambda_h q + \alpha & \Lambda_{10} & -2\mu_h & 0 & 0 \\
-4\lambda_h q^2 & -4\lambda_h q & -4\lambda_h & \Lambda_{11} & -2\mu_h & 0 \\
0 & 0 & 0 & -3\lambda_h & \Lambda_{12} & -2\mu_h \\
0 & 0 & 0 & 0 & \Lambda_{13} & 0 \\
0 & 0 & 0 & 0 & 0 & -2\mu_h
\end{bmatrix} \]

\[ A_{12} = [-4\lambda_S I_6, 0_6], \quad A_{14} = [-\mu_S I_5, 0_5]^T, \]

\[ A_{13} = \begin{bmatrix}
\Lambda_{14} & -\mu_h & 0 & 0 & 0 & 0 \\
-4\lambda_h (1-q + \alpha) & \Lambda_{15} & -2\mu_h & 0 & 0 & 0 \\
-4\lambda_h q & -4\lambda_h & \Lambda_{16} & -2\mu_h & 0 & 0 \\
0 & 0 & -3\lambda_h & \Lambda_{17} & -2\mu_h & 0 \\
0 & 0 & 0 & -2\lambda_h & \Lambda_{18} & -2\mu_h \\
0 & 0 & 0 & 0 & -\lambda_h & \Lambda_{19}
\end{bmatrix} \]

\[ A_{43} = [-4\lambda_S I_5, 0_5], \quad A_{45} = [-\mu_S I_4, 0_4]^T, \]

\[ A_{44} = \begin{bmatrix}
\Lambda_{20} & -\mu_h & 0 & 0 & 0 & 0 \\
-4\lambda_h & \Lambda_{21} & -2\mu_h & 0 & 0 & 0 \\
0 & -3\lambda_h & \Lambda_{22} & -2\mu_h & 0 & 0 \\
0 & 0 & -2\lambda_h & \Lambda_{23} & -2\mu_h & 0 \\
0 & 0 & 0 & -\lambda_h & \Lambda_{24}
\end{bmatrix} \]

\[ A_{54} = [-4\lambda_S I_4, 0_4], \quad A_{66} = [-\mu_S I_3, 0_3]^T, \]

\[ A_{55} = \begin{bmatrix}
3\lambda_h + 3\lambda_S + \mu_S + s & -\mu_h & 0 \\
-3\lambda_h & 2\lambda_h + 3\lambda_S + \mu_h + \mu_S + s & -2\mu_h & 0 \\
0 & -2\lambda_h & \lambda_h + 4\lambda_S + 2\mu_h + \mu_S + s & -2\mu_h \\
0 & 0 & -\lambda_h & 2\mu_h + \mu_S + s
\end{bmatrix} \]

\[ A_{67} = [-\mu_S I_2, 0_2]^T \]

\[ A_{65} = \begin{bmatrix}
-3\lambda_S & 0 & 0 & 0 \\
0 & -3\lambda_S & 0 & 0 \\
0 & 0 & -4\lambda_S & 0
\end{bmatrix} \]

\[ A_{66} = \begin{bmatrix}
2\lambda_h + 2\lambda_S + \mu_S + s & -\mu_h & 0 \\
-2\lambda_h & \lambda_h + 2\lambda_S + \mu_S + \mu_h + s & -2\mu_h \\
0 & -\lambda_h & 2\mu_h + \mu_S + s
\end{bmatrix} \]
\[
A_6 = \begin{bmatrix}
\lambda_b + \lambda_s + \mu_s + s & -\mu_b \\
-\lambda_b & \mu_b + \mu_s + s \\
-\lambda_s & 0
\end{bmatrix},
A_7 = \begin{bmatrix}
-\mu_b \\
0 \\
-\mu_s
\end{bmatrix}
\]

Other sub matrices \(A_{ij}(1 \leq i, j \leq 7)\) are zero matrices of appropriate size.

In sub matrices, we have used the following notations

\[
A_1 = 4\lambda_b + 3\alpha + 4\lambda_s + s,
A_9 = 4\lambda_b + \alpha + 4\lambda_s + \mu_b + \mu_s + s,
A_{17} = 2\lambda_b + 4\lambda_s + 2\mu_b + \mu_s + s
\]

\[
A_2 = 4\lambda_b + 2\alpha + 4\lambda_s + \mu_b + s,
A_{10} = 4\lambda_b + 4\lambda_s + 2\mu_b + \mu_s + s,
A_{18} = \lambda_b + 4\lambda_s + 2\mu_b + \mu_s + s
\]

\[
A_3 = 4\lambda_b + \alpha + 4\lambda_s + 2\mu_b + s,
A_{11} = 3\lambda_b + 4\lambda_s + 2\mu_b + \mu_s + s,
A_{19} = 2\mu_b + \mu_s + s
\]

\[
A_4 = 4\lambda_b + 4\lambda_s + 3\mu_b + s,
A_{12} = 2\lambda_b + 4\lambda_s + 2\mu_b + \mu_s + s,
A_{20} = 4\lambda_b + 4\lambda_s + \mu_s + s
\]

\[
A_5 = 3\lambda_b + 4\lambda_s + 3\mu_b + s,
A_{13} = \lambda_b + 4\lambda_s + 2\mu_b + \mu_s + s,
A_{21} = 3\lambda_b + 4\lambda_s + \mu_b + \mu_s + s
\]

\[
A_6 = 2\lambda_b + 4\lambda_s + 3\mu_b + s,
A_{14} = 4\lambda_b + \alpha + 4\lambda_s + \mu_s + s,
A_{22} = 2\lambda_b + 4\lambda_s + 2\mu_b + \mu_s + s
\]

\[
A_7 = \lambda_b + 4\lambda_s + 3\mu_b + s,
A_{15} = 4\lambda_b + 4\lambda_s + \mu_b + \mu_s + s,
A_{23} = \lambda_b + 4\lambda_s + 2\mu_b + \mu_s + s
\]

\[
A_8 = 4\lambda_b + 2\alpha + 4\lambda_s + \mu_s + s,
A_{16} = 3\lambda_b + 4\lambda_s + 2\mu_b + \mu_s + s,
A_{24} = 2\mu_b + \mu_s + s
\]

Using Cramer’s rule, the probabilities \(P'_s(s)\) can be obtained as:

\[
P'_s(s) = \frac{|Q_{s+1}(s)|}{|Q(s)|}, \quad 0 \leq k \leq L
\]

where \(|Q_{s+1}(s)|\) is the determinant obtained by replacing the \((k + 1)\)th column of determinant \(|Q(s)|\) by RHS vector \(P(0)\).

For calculating the characteristic roots of the matrix \(Q(s)\), we note that \(s = 0\) is one of the roots. Let \(s = -d\), so that we get:

\[
Q(-d) = (Q - dl)
\]

Now, equation (101) becomes:
Transient analysis of hardware and software systems

\[ Q(-d).P'(s) = (Q - dI)P'(s) = P(l) \]  

(104)

It is observed that the eigenvalues of \( Q \) are real and distinct and it is also observed that \( Q \) is positive definite. So, all eigenvalues of \( Q \) are positive. Let \( \nu_k \) denote the eigenvalues of \( Q \), then we get:

\[ Q(s) = s \prod_{k=1}^L (s + \nu_k) \]  

(105)

\[ P_l(s) = \frac{|Q_{l+1}(s)|}{s \prod_{k=1}^L (s + \nu_k)}, \quad 1 \leq l \leq 36 \]  

(106)

We may write \( P_l(s) \) in partial fractions form as:

\[ P'_l(s) = \frac{a_0}{s} + \sum_{k=1}^L \frac{a_{lk}}{s + \nu_k} \]  

(107)

\[ P''_l(s) = \sum_{k=1}^L \frac{a_{lk}}{s + \nu_k} \]  

(108)

where \( a_0 \) and \( a_{lk} \) are real numbers calculated as:

\[ a_0 = \frac{|Q_l(0)|}{\prod_{j=1}^L \nu_j} \]  

(109)

and

\[ a_{lk} = -\frac{|Q_{l+1}(-\nu_k)|}{\nu_k \prod_{j=1}^L (\nu_j - \nu_k)}, \quad 1 \leq l \leq L, \quad 2 \leq k \leq L \]  

(110)

On taking inverse Laplace transform of equations (107) and (108), we get:

\[ P_l(t) = \frac{|Q_l(0)|}{\prod_{k=1}^L \nu_k} \sum_{k=1}^L \frac{|Q_l(-\nu_k)| \exp(-\nu_k t)}{\nu_k \prod_{j=1}^L (\nu_j - \nu_k)} \]  

(111)

\[ P_l(t) = -\sum_{k=1}^L \frac{|Q_{l+1}(-\nu_k)| \exp(-\nu_k t)}{\nu_k \prod_{j=1}^L (\nu_j - \nu_k)}, \quad \text{where} \ 2 \leq l \leq L \]  

(112)
6 Performance measures

To evaluate the measures of performance of the concerned system is the main objective of developing a mathematical model of real time system. In this section, we discuss various performance measures which are often needed for investigating the behaviour of a hardware and software system. Now we obtain some performance indices in terms of transient probabilities obtained in previous section as follows:

- Expected number of failed components at time $t$ due to hardware failure is:
  \[ F_{Hi}(t) = \sum_{i=1}^{M+S} \sum_{j=0}^{M-S-i} R_{i,j}(t) \]  
  \( (113) \)

- Expected number of failed components at time $t$ due to software failure is:
  \[ F_{Si}(t) = \sum_{j=1}^{M+S} \sum_{i=0}^{M+S-j} R_{i,j}(t) \]  
  \( (114) \)

- Expected number of standby components in the system at time $t$ is:
  \[ S_C(t) = \sum_{i+j=0}^{S} (S-i-j)P_{i,j}(t) \]  
  \( (115) \)

- Component availability at time $t$ is:
  \[ A_C(t) = 1 - \left[ \frac{F_{Hi}(t) + F_{Si}(t)}{M+S} \right] \]  
  \( (116) \)

- Failure frequency at time $t$ is:
  \[ \omega_F(t) = \lambda_d P_{M+S-1}(t) \]  
  \( (117) \)

- Reliability of the system is:
  \[ R(t) = \sum_{i=0}^{M+S-1} \sum_{j=0}^{M+S-1} P_{i,j}(t) \]  
  \( (118) \)

7 Numerical result

In this section, we are interested in exploring the effect of various parameters on the system reliability and other performance measures by taking the numerical example. We check the validity of the proposed model by employing matrix method to solve the system of differential equations by computing the eigenvalues using MATLAB software. We consider a time span with equal intervals. For different values of $\lambda_h$, $\lambda_s$, $\alpha$, $\mu_h$ and $\mu_s$, Table 1(a) to Table 1(f) and Figure 2(a) to Figure 2(f) depict various performance
measures and reliability of the system. For illustration purpose, we choose default parameters as \( \lambda = 0.9, \lambda_s = 0.1, C = 0.3, \theta = 0.6, \beta = 1 \) and \( \mu = 3 \).

Numerical results are displayed graphically and summarised in the tables for various measures of performance which may be useful for improving the system reliability. In Table 1(a) to Table 1(f), various performance measures such as expected number of failed components due to hardware and software failure, expected number of standby components, availability and failure frequency of the system are summarised. From tables it is noticed that the expected number of failed components due to software failures is increasing with respect to time but due to failures of hardware components, it initially increases and after some time decreases gradually. Expected number of standby components and availability are decreasing function of time but failure frequency is increasing with respect to time. From Table 1(a) to Table 1(f), it is noticed that \( F_H(t) \) increases as \( \lambda, \alpha, \mu_S, q \) increase but decreases as \( \lambda_s \) and \( \mu_h \) increase. It is seen that \( F_S(t) \) increases on increasing \( \lambda_s \) and \( \mu_h \) but decreases on increasing the values of \( \lambda_h, \mu_s \). By increasing the probability of perfect switching \( q \), it is found that \( S_C(t) \) and \( A_C(t) \) are decreasing while \( \omega_F(t) \) is increasing. With respect to parameter \( \alpha \), \( S_C(t) \) decreases but \( A_C(t) \) and \( \omega_F(t) \) remain almost constant.

### Table 1(a) Performance indices for different values of \( \lambda_h \)

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<th>( \lambda_h = 0.9 )</th>
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<td>( S_C(t) )</td>
<td>( A_C(t) )</td>
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<td>0.00 0.00 3.00 1.00 0.000</td>
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<td>1.77 1.94 0.35 0.47 0.028</td>
</tr>
<tr>
<td>4</td>
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<td>1.28 2.97 0.21 0.39 0.054</td>
</tr>
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<td>6</td>
<td>0.45 3.61 0.30 0.42 0.016</td>
<td>0.76 3.58 0.21 0.38 0.042</td>
<td>1.03 3.55 0.15 0.35 0.079</td>
</tr>
<tr>
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<td>0.90 3.86 0.11 0.32 0.096</td>
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### Table 1(b) Performance indices for different values of \( \lambda_s \)

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Table 1(c) Performance indices for different values of $\alpha$

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Table 1(d) Performance indices for different values of $\mu_h$

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Table 1(e) Performance indices for different values of $\mu_s$

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Table 1(f) Performance indices for different values of $q$

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In Figure 2(a) to Figure 2(f), we compute the reliability with respect to time $t$ for different system parameters. In Figure 2(a) and Figure 2(b), reliability decreases as time increases. Also as $\lambda_h$ increases, the reliability decreases; however we observe the reverse effect on increasing $\mu_h$. Reliability with respect to time, initially decreases and after some time it becomes almost constant as seen in Figure 2(c) and Figure 2(d). We also notice that reliability decreases sharply on increasing the values of $\lambda_s$; it increases as $\mu_s$ increases. From Figure 2(e) and Figure 2(f), it can be observed that initially reliability decreases sharply, then after decreases slowly but finally reliability becomes asymptotically constant for the higher values of $\alpha$, $q$ and time $t$.

**Figure 2(a)** Reliability vs. time by varying $\lambda_h$ (see online version for colours)

![Figure 2(a)](image1)

**Figure 2(b)** Reliability vs. time by varying $\mu_h$ (see online version for colours)

![Figure 2(b)](image2)
Figure 2(c) Reliability vs. time by varying $\lambda_s$ (see online version for colours)

Figure 2(d) Reliability vs. time by varying $\mu_s$ (see online version for colours)

Figure 2(e) Reliability vs. time by varying $\alpha$ (see online version for colours)
Overall from the tables and figures we can conclude that the availability $A_c(t)$ and reliability $R(t)$ decrease with time. It is quite obvious to notice that as failure rate of the hardware (software) components increases, the expected number of failed hardware (software) components increases. Overall, on the basis of numerical results, it can be concluded that a system would work more effectively with the adequate support of standbys and repair facility.

8 Conclusions

With the advancement of technology, performance modelling based on reliability assessment can play an important role to improve the real time embedded computer systems. In this paper, we demonstrated how system reliability can be enhanced with the provision of standbys and repair facility for an embedded system which contains both hardware and software components. Explicit expressions for the reliability, availability and other performance measures provided may be helpful to prepare ready reckoner. Our numerical results indicate that the switching failure has a significant adverse effect on the system reliability as such incorporation of switching failure in the model makes our model more realistic and versatile to deal with real time system. The study done may provide an insight for determining the optimal number of standbys and repairmen to the system designers and production engineers while dealing with embedded engineering computer systems. The performance indices obtained may be helpful to the decision makers for improving the reliability/availability of the system as for as embedded hardware and computer systems are concerned.

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References


Transient analysis of hardware and software systems


