New Approaches of Implementing STBC Technique and MIMO-OFDM Channel Estimation

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Abstract

The fundamental detection problem in fading channels involves the correct estimation of transmitted symbols at the receiver in the presence of Additive White Gaussian Noise (AWGN). This project adopts a different view to estimator performance, by evaluating the accuracy of CSI. The superior performance promised by the MIMO-OFDM and OFDM technologies rely on the availability of accurate Channel State Information (CSI) at the receiver by transmitting pilots along with data symbols. Pilot symbol assisted channel estimation is especially attractive for wireless links, where the channel is time-varying.

In this project we investigate and compare various efficient pilot based channel estimation schemes; Least Square Error (LSE) and Minimum Mean Square Error (MMSE) channel estimators has been employed for OFDM system. Then conclude that LS algorithm gives less complexity but MMSE algorithm provides comparatively better results.

Also in this project, performance analysis of channel estimation for Multiple-Input Multiple-Output (MIMO) communication system combined with the Orthogonal Frequency Division Multiplexing (OFDM) through different algorithms for estimating channel using different modulation scheme are investigated. The estimation implemented here of channel at pilot frequencies is based on Least Square, Weiner Filter Estimator and Orthogonal Training Sequence Estimator algorithms. We have compared the performances of channel estimation algorithm by measuring bit error rate vs. SNR. Weiner Filter Estimator has been shown to perform much better than LS but is more complex than other channel estimation algorithm. Also compared the actual and the estimated channel in Orthogonal Training Sequence channel estimation and notice that errors in the CSI estimation are as a result of AWGN in the received symbols.

Finally, new methods for implementing QO-STBC and DHSTBC over OFDM for four, eight and sixteen transmitter antennas are presented. The QO-STBC and DHSTBC over OFDM scheme eliminates the interference from the detection matrix and improved the performance by increasing the diversity order in the transmitter side. The proposed code promo the diversity gain when compared with the real STBC scheme and DHSTBC gives the best performance; both techniques reduce the effect of Inter Symbol Interference (ISI) due to the existence of OFDM.
Acknowledgment

First and foremost, we would like to thank Almighty God, to reconcile us at each step and for giving us everything to make our work continuous without laziness and foil.

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Disclaimer Statement

This report was written by students at the Telecommunication Engineering Department, Faculty of Engineering, An-Najah National University. It has not been altered or corrected, other than editorial corrections, as a result of assessment and it may contain language as well as content errors. The views expressed in it together with any outcomes and recommendations are solely those of the students. An-Najah National University accepts no responsibility or liability for the consequences of this report being used for a purpose other than the purpose for which it was commissioned.
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<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>MIMO</td>
<td>Multiple Input Multiple Output</td>
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<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>STBC</td>
<td>Space Time Block Coding</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal To Noise Ratio</td>
</tr>
<tr>
<td>SISO</td>
<td>Single Input Single Output</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
</tr>
<tr>
<td>SV</td>
<td>Saleh-Valenzuela</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary phase Shift Keying</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter-Symbol Interference</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximum Ratio Combining</td>
</tr>
<tr>
<td>LS</td>
<td>Least Square</td>
</tr>
<tr>
<td>MMSR</td>
<td>Minimum Mean Square error</td>
</tr>
<tr>
<td>Mbps</td>
<td>Mega bit per second</td>
</tr>
<tr>
<td>QAM</td>
<td>Quadrature Aperture Modulation</td>
</tr>
<tr>
<td>PSK</td>
<td>Phase Shift Keying</td>
</tr>
<tr>
<td>TX</td>
<td>Transmitter</td>
</tr>
<tr>
<td>RX</td>
<td>Receiver</td>
</tr>
<tr>
<td>SCs</td>
<td>Sub Carriers</td>
</tr>
<tr>
<td>DAC</td>
<td>Digital to Analog Converter</td>
</tr>
<tr>
<td>CP</td>
<td>Cyclic Prefix</td>
</tr>
<tr>
<td>ZP</td>
<td>Zero Padding</td>
</tr>
<tr>
<td>GI</td>
<td>Guard Interval</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
</tr>
<tr>
<td>LTE</td>
<td>Long Term Evolution</td>
</tr>
<tr>
<td>LSE</td>
<td>Least Square Error</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>SER</td>
<td>Symbol Error Rate</td>
</tr>
<tr>
<td>LLR</td>
<td>Log-Likelihood Ratio</td>
</tr>
<tr>
<td>CIR</td>
<td>Channel Impulse Response</td>
</tr>
<tr>
<td>PDP</td>
<td>Power Delay Profile</td>
</tr>
<tr>
<td>WSS</td>
<td>Wide Sense Stationary</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>AOA</td>
<td>Angle Of Arrival</td>
</tr>
<tr>
<td>QO-STBC</td>
<td>Quasi Orthogonal Space Time Block Coding</td>
</tr>
<tr>
<td>DHSTBC</td>
<td>Diagonalized Hadamard Space Time Block Coding</td>
</tr>
<tr>
<td>EVCM</td>
<td>Equivalent Virtual Channel Matrix</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Channel Gains Parameter</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Interference from neighboring signals</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Variance</td>
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Chapter One: Introduction

1.1 Overview

It is a well-known fact that the amount of information transported over communication systems grows rapidly. Not only the file sizes increase, but also large bandwidth-required applications such as video on demand and video conferencing require increasing data rates to transfer the information in a reasonable amount of time or to establish real-time connections. To support this kind of services, broadband communication systems are required. Large-scale penetration of wireless systems into our daily lives will require significant reductions in cost and increases in bit rate and/or system capacity.

Recent information theoretical studies have revealed that the multipath wireless channel is capable of huge capacities, provided that multipath scattering is sufficiently rich and is properly exploited through the use of the spatial dimension. Appropriate solutions for exploiting the multipath properly, could be based on new techniques that recently appeared in literature, which are based on Multiple Input Multiple Output (MIMO) technology. Basically, these techniques transmit different data streams on different transmit antennas simultaneously. By designing an appropriate processing architecture to handle these parallel streams of data, the data rate and/or the Signal-to-Noise Ratio (SNR) performance can be increased. Multiple Input Multiple Output (MIMO) systems are often combined with a spectrally efficient transmission technique called Orthogonal Frequency Division Multiplexing (OFDM) to avoid Inter Symbol Interference (ISI)[1].

Channel estimation is a crucial and challenging issue in coherent demodulation. Its accuracy has significant impact on the overall performance of the MIMO-OFDM system. The digital source is usually protected by channel coding and interleaved against fading phenomenon, after which the binary signal is modulated and transmitted over multipath fading channel. Additive noise is added and the sum signal is received. Due to the multipath channel there is some intersymbol interference (ISI) in the received signal. Therefore a signal detector needs to know channel impulse response (CIR) characteristics to ensure successful removal of ISI.

The channel estimation in MIMO-OFDM system is more complicated in comparison with SISO system due to simultaneous transmission of signal from different antennas that cause co-channel interference. This issue highlights that developing channel algorithm with high accuracy is an essential requirement to achieve full potential performance of the MIMO-OFDM system. A number of channel estimation methods have been introduced for MIMO-OFDM systems which are Wiener channel estimation and the orthogonal training sequence channel estimation [2].
Another issue taken into consideration in this project is that in present days wireless communication systems are in great quest for efficient communications. Wi-Fi and terrestrial base stations are increasingly deploying the multi-antenna system for seamless communications. For instance, the multiple input multiple output (MIMO) antenna configuration is useful in achieving higher throughput in these wireless communication systems. Space Time block coding (STBC) is one of interesting methods for deploying this technique. The advantage of using, for example, the orthogonal STBC (OSTBC) over OFDM is that it exploits full power transmission for orthogonal codes so long as the transmitter diversity order is no more than two [3,4]. For more than two transmit diversity, it has been shown that full rate power is not possible [5]. Meanwhile, it is possible to deploy the STBC technology in way that full rate power transmission can be achieved.

In such case, the codes are rather formed in a special orthogonal way. This is usually discussed as the quasi-orthogonal STBC over OFDM, hereinafter QO-STBC. The QO-STBC offers the advantage of improved channel capacity and also improved bit error ratio (BER) statistics for a multi-antenna transmission [2]. Also full rate and full diversity order Diagonalized Hadamard Space Time code (DHSTBC) over OFDM for 4, 8 and 16 transmitter antennas is presented.

1.2 Motivation

Designing high-speed wireless systems can be very complex with a lot of variable to test and analyze, so using physical prototypes to analyze these systems can be a very slow and expensive process. The incorporation of computer simulation into the modeling of dynamic systems is one of the most important fusions in engineering design. The best way to design and test complex systems like the channel estimation for MIMO-OFDM systems, also the design of QO-STBC and DHSTBC over OFDM are to develop a computer simulation that can mirror as closely as possible the behavior of the real-life systems and then use it to test and compare results for different configurations and scenarios. This approach is not only cost effective but it is also faster and more reliable.

The use of MATLAB to simulate this MIMO-OFDM channel estimation system and QO-STBC, DHSTBC over OFDM will allow for the deliverables of this project to be used for testing and analyzing future ideas that may come up as regards to the MIMO-OFDM technology. Also, MATLAB is a tool used by many engineers and designers around the world so it will be easily understood and appreciated.
1.3 Aims and objectives

The aims of this research include a comprehensive study of MIMO-OFDM channel estimation systems and the use of diversity techniques. Also discussed the application of Quasi Orthogonal Space Time Block Coding (QO-STBC) and Diagonalized Hadamard Space Time Block Coding (DHSTBC) over OFDM for future wireless communications systems. A MIMO-OFDM channel estimation systems using two transmit antennas and two receive antennas configuration are implemented and analyzed by wiener estimation and orthogonal training sequence estimation techniques. In addition, the QO-STBC and DHSTBC over OFDM are implemented and discussed for four, eight and sixteen transmitter antennas over different modulation schemes.

To achieve the above aims the following objectives have been set for this work

• To provide a general theoretical overview of channel estimation for OFDM.
• To provide a general overview of channel estimation for MIMO-OFDM systems.
• To build a simulated OFDM and MIMO-OFDM channel estimation systems using MATLAB.
• To compute and discuss the results using two transmit antennas and two receive antennas configuration for different channel estimation techniques.
• To provide a general theoretical overview for QO-STBC and DHSTBC.
• To combine OFDM system with QO-STBC and DHSTBC to provide full diversity.
• To build a simulated QO-STBC and DHSTBC over OFDM using MATLAB.
• To compute and discuss the results for different antenna configurations and modulation schemes for QO-STBC and DHSTBC over OFDM.
• To write the graduation project report.

1.4 Dissertation structure

This project exploited the OFDM and MIMO-OFDM channel estimation techniques with the view of bit error rate versus signal to noise ratio to measure the performance of these two systems for two antennas at the transmitter and two antennas at the receiver, also this project implements two methods to increase the diversity gain which are QO-STBC and DHSTBC over OFDM for different antenna configurations and modulation schemes then compare the performance at these different scenarios.
Chapter one is an introduction to all subjects presented in this project and the motivation and aims of this project work. Chapter two looks at some of the published works in the MIMO-OFDM channel estimation technology and new STBC approaches over OFDM and the direction of research in this field and earlier course work in addition to the standards that used during this project.

Chapter three will introduce theory and methodology for channel estimation that is used in our project.

Chapter four will then discuss the new STBC approaches, how it works and the implementation of MIMO system over OFDM using four, eight and sixteen transmitter antennas. Chapter five explore the results from applying the codes and analyze the results of BER vs. SNR.

Finally, chapter six will conclude the report with the recommendation for future work.
Chapter Two: Constraints, Standards and Earlier course work

2.1 Constraints and Limitation

In the first part of this project, many constraints faced us due to modernity of this topic in wireless communication world. First of all, blind channel estimation was ambiguous to work on it so we moved toward non-blind channel estimation. Although, this was difficult to deal with but we overcome this problem.

In the second part which was implemented for the first time with eight and sixteen transmitter antennas over OFDM because of increasing the number of antennas, the cost of equipment will increase. Also computational complexity appears in the run time of symbolic Matlab code due to huge matrices size.

2.2 The 802.11n Standards

The IEEE 802.11 standard defines the standards for the physical layer and media access control (MAC) for local Area networks (LANs) that share the same logical link control (LLC) layer. The 802.11 family has so far have the (a, b, g) version and the latest being (n) which defines the MIMO-OFDM in the WLAN environment [6,7].

802.11 {a, b, g} had some fundamental issues that necessitated the introduction of 802.11n into the 802.11 family [8]. For instance they had limited capacity in terms of end-to-end throughput and data rate, and the way that the channel was coded was prone to inefficiency [9]. Too much overhead was wasted in contention issues given that 802.11{a, b, g} were design as hub type network, and as the number of devices increased so did contention issues thereby wasting a lot of air time [10]. This made 802.11 {a, b, g} only give about 50-60% efficiency, delivering only about 20 Mbps throughput for a 54 Mbps peak data rate [11]. The introduction of 802.11n was to improve the 802.11 {a, b, g} technology in three key areas,

i) The RF layer: One of the basic improvements of 802.11n is based on the concept of splitting the information to be transmitted into multiple lower-rate parallel streams with different parts of the information encoded in the each stream instead of transmitting in one stream at higher and higher data rates (OFDM).
Also to improve the RF layer, the direction of transmission from the transmitter to the receiver is no longer Omni directional but directed towards the receiver. This is referred to as transmit Beam Forming [12,13]. For this technique to be effective it must be dynamic because an access point does not communicate with only one receiver at a time but many so the transmitter must figure out a way to push maximum energy in the directions of necessity and little everywhere else.

**ii) The Physical Layer:** At the physical layer, 802.11{a, b, g} had approximately 20MHZ for every channel but 802.11n gives the option of setting a 40MHZ channels, potentially increasing the capacity [14]. Also 802.11n allows for better encoding to be used, the 64-QAM encoding of bits to hertz. This encoding tries to pack in a higher number of bits per hertz but this inherently makes the package to be lossy. However, this effect is made less apparent by the fact that the channel is already made cleaner at the RF layer.

**iii) The MAC Layer:** At the MAC layer 802.11n tries to minimize the amount of contention overhead. The basic idea is that the transmitter compresses a bunch of frames into one and once it wins a transmission time from a contention round, it transmits a bunch of frames instead of one and receives a block acknowledgement instead of just a frame acknowledgement [14]. His technique increase channel utilization by reducing time wasted in contention and contention resolution.

**2.2.1 Issues with 802.11n**

802.11n brings significant improvements to the WLAN environment, providing about 6 x increases in peak data rate and about 10 x increases in peak throughput with about 2x in link range [14]. However there are some issues in designing enterprise networks using 802.11n. For instance 802.11n show significantly spikier and unpredictable pattern in terms of the transmission profile that is received such that a movement of just few feet could result in a significantly different profile.
Also, the loss of a block acknowledgement could result in the transmitter assuming that the entire block is lost, necessitating a complete retransmission of the entire block even though the initial block transmission was successful [11].

2.3 Earlier coursework

Modeling, Digital Signal Processing (DSP), Mobile, Random Variables, Information Theory, Signals and Systems and Numerical courses are utilized in our project.

2.4 Related Work

The multiple-input–multiple-output–orthogonal frequency-division multiplexing (MIMO–OFDM) technology has been considered as a strong candidate for the next generation wireless communication systems. Using multiple transmit as well as receive antennas, a MIMO–OFDM system gives a high data rate without increasing the total transmission power or bandwidth compared to a single antenna system. Further, the frequency-selective problem that exists in a conventional wireless system can be well solved by the OFDM technique in the MIMO–OFDM system. On the other hand, the performance of MIMO–OFDM systems depends largely upon the availability of the knowledge of the channel.

It has been proved [15] that when the channel is Rayleigh fading and perfectly known to the receiver, the capacity of a MIMO–OFDM system grows linearly with the number of transmit or receive antennas, whichever is less. Therefore, an accurate estimation of the wireless channel is of crucial importance to MIMO–OFDM systems. A considerable number of channel estimation methods have already been proposed for MIMO–OFDM systems. They can broadly be categorized into three classes, namely, the training- based method, the blind method, and the semiblind one as a
combination of the first two methods. First, the training-based methods employ known training signals to render an accurate channel estimation [16]–[19].

One of the most efficient training-based methods is the least squares (LS) algorithm, for which an optimum pilot design scheme has been given in [16]. When the full or partial information of the channel correlation is known, a better channel estimation performance can be achieved via some minimum mean square error (MMSE) methods [17]. By using decision feedback symbols, the Takagi–Sugeno–Kang (TSK) fuzzy approach proposed in [18] can achieve a performance similar to the MMSE methods while with a low complexity. In contrast to training-based methods, blind MIMO–OFDM channel estimation algorithms, such as those proposed in [20], often exploit the second-order stationary statistics, correlative coding, or other properties to give a better spectral efficiency.

With a small number of training symbols, a semiblind method has been proposed in [22] to estimate the channel ambiguity matrix for space-time coded OFDM systems. It is worth pointing out that most of the existing blind and semiblind MIMO–OFDM channel estimation methods are based on the second-order statistics of a long vector whose size is equal to or larger than the number of subcarriers. To estimate the correlation matrix reliably, they need a large number of OFDM symbols, which is not suitable for fast time-varying channels. In addition, because the matrices involved in these algorithms are of huge size, their computational complexity is extremely high. In contrast, a linear prediction-based semiblind algorithm that is based on the second-order statistics of a short vector with a size only slightly larger the channel length has been found much more efficient than the conventional LS methods for the estimation of frequency-selective MIMO channels [23]–[25].

An OSTBC scheme was first proposed by Alamouti for achieving maximum diversity gain for two transmit antennas [26]. It can achieve the full rate and full diversity gain. Subsequently, Tarokh proposed OSTBC schemes to achieve full diversity for more than two transmit antennas. In 2013, Y.A.S Dama et al.[27] proposed a new approach for Quasi-Orthogonal Space Time Block Coding (QO-STBC) that eliminate the interference from the detection matrix to improve the diversity gain compared with the conventional QO-STBC scheme, it also reduces the decoding complexity. Diagonalized Hadamard Space Time Block Coding (DHSTBC) which provide full rate full diversity order was presented.
Chapter Three: Channel Estimation Theory and Methodology

Channel estimation plays an important role in a communication receiver. In order to mitigate hostile channel effects on the received signal, precise channel estimation is required to provide information for further processing of the received signal. Channel estimators can be categorized as non-data-aided or data-aided. Non-data-aided or blind channel estimators estimate channel response from the statistics of the received signals. No specialized reference (training) signals are needed and the transmission efficiency is retained for systems using such channel estimation schemes. However, without precise knowledge of the transmitted signals, a large number of data must be collected in order to obtain reliable estimation. On the other hand, data aided channel estimators require known reference (training) signals to be transmitted. Rapid and accurate channel estimation can be achieved by comparing the received and transmitted reference signals. A sufficient number of such reference signals must be inserted according to the degree of channel variation, namely coherence time and coherence bandwidth of the channel under estimation[32].

Channel estimation provides information about distortion of the transmission signal when it propagates through the channel. This information is then used by equalizers so that the fading effect and/or co-channel interference can be removed and the original transmitted signal can be restored.

3.1 Theory: OFDM channel estimation

Orthogonal Frequency Division Multiplexing (OFDM) is most commonly employed in wireless communication systems because of the high rate of data transmission potential with efficiency for high bandwidth and its ability to combat against multi-path delay. It has been used in wireless standards particularly for broadband multimedia wireless services.

An important factor in the transmission of data is the estimation of channel which is essential before the demodulation of OFDM signals since the channel suffers from frequency selective fading and time varying factors for a particular mobile communication system[28]. The estimation channel is mostly done by inserting pilot symbols into all of the subcarriers of an OFDM symbol or inserting pilot symbols into some of the sub-carriers of each OFDM symbol.
The first method is called as the pilot based block type channel estimation and it has been discussed for a slow fading channel. The second method is the comb-type based channel estimation in which pilot symbols are transmitted on some of the sub carriers of each OFDM symbol. This method usually uses different interpolation schemes such as linear, low-pass, spline cubic, and time domain interpolation.

The idea behind these methods is to exploit knowledge of transmitted pilot symbols at the receiver to estimate the channel. For a block fading channel, where the channel is constant over a few OFDM symbols, the pilots are transmitted on all subcarriers in periodic intervals of OFDM blocks. This type of pilot arrangement, depicted in Fig.(3.a), is called the block type arrangement. For a fast fading channel, where the channel changes between adjacent OFDM symbols, the pilots are transmitted at all times but with an even spacing on the subcarriers, representing a comb type pilot placement, Fig. (3.b) The channel estimates from the pilot subcarriers are interpolated to estimate the channel at the data subcarriers.

![Figure 3.1. Block Pilot and Comb Pilot](image)

This section discusses the estimation of the channel for the block type pilot arrangement which is based on Least Square (LS) Estimator and Minimum Mean-Square Error (MMSE) Estimator[29].

### 3.1.1 System Description For OFDM

![Figure 3.2: OFDM transmission system.](image)
In this section, we will present the signal model, and analyze the SER performance of OFDM in the presence of channel estimation errors.

First of all, the OFDM transmission system under consideration is depicted in Fig.(3.2). Information and pilot symbols are modulated on a set of subcarriers, and transmitted over a frequency-selective fading channel through a single transmitter antenna. After demodulation at the receiver end, where we allow for multiple antennas, the channel per receive-antenna is estimated using pilots. Based on the estimated channels.

After demodulation, the received signal at the mth receive-antenna on the mth subcarrier corresponding to pilot symbols can be written as

\[ y_m[n] = \sqrt{\varepsilon_p} H_m(n)s(n) + w_m(n), m = 1, \ldots, M, n \in I_p \] (3.1)

Where \( I_p \) denotes the set of subcarriers on which pilot symbols are transmitted, \( \varepsilon_p \) is the transmitted power per pilot symbol, \( H_m(n) \) is the channel frequency response of the mth antenna at the mth subcarrier, \( S(n), n \in I_p \) is the pilot symbol, and \( w_m(n) \) is complex additive white Gaussian noise (AWGN) with zero-mean and variance \( N_0/2 \) per dimension; and \( M \) is the number of receive-antennas. We select pilot symbols of constant modulus.

The received samples corresponding to information symbols can be expressed as

\[ y_m[n] = \sqrt{\varepsilon_i} H_m(n)s(n) + w_m(n), m = 1, \ldots, M, n \in I_s \] (3.2)

Where \( \varepsilon_i \) is the transmitted power per information symbol, and \( I_s \) denotes the set of subcarriers on which information symbols are transmitted. Suppose that the total number of subcarriers is \( N \) and the size of \( I_p \) is \( |I_p| = P \) For simplicity, we assume that the size of \( I_s \) is \( |I_s| = N - P \), although it is possible that \( |I_s| < N - P \) when null subcarriers are inserted for spectrum shaping.

Selecting information symbols from QPSK constellations, we have also that \( |S_n| = 1 \). The frequency-selective channel is assumed to be Rayleigh-fading, with channel impulse response \( h_m = [h_m(0), \ldots, h_m(L - 1)]^T \) corresponding to the mth receive-antenna, and \( L \) denoting the number of taps, i.e. \( h_m(l), \forall m \in [1, M], \forall l \in [0, L - 1] \) are uncorrelated complex Gaussian random variables with zero-mean. We assume that channels associated with different antennas have identical power delay profiles specified by the variance \( \sigma_h^2(l) \) the same \( \forall m \in [1, M] \) Channels are normalized so that \( \sum_{l=0}^{L-1} \sigma_h^2(l) = 1 \). Define the \( L \times N \) matrix \( [F_{l,n}] = \exp(j2\pi(l - 1)(n - 1)/N) \). And let \( f_n \) be the nth column of \( F \). Then \( H_m(n) = f_n^H h_m \) is a complex Gaussian random variable with zero-mean and unit variance. The average signal-to-noise ratio (SNR)
per pilot (information) symbol at each antenna is $\varepsilon_p / N_0 (\varepsilon_s / N_0)$. The AWGN variables $w_m(n)$ are assumed to be uncorrelated.

Suppose that the set of pilot subcarriers is given by $I_p = \{i_t\}^{I_p}_{i=1}$.

Letting $\tilde{h}_m := [H_m(n_1), ..., H_m(n_{I_p})]^T$ contain the channel frequency response on pilot subcarriers, and defining $F_p = [f_{n_1}, ..., f_{n_{I_p}}]$, we can relate the fast Fourier transform (FFT) pair via:

$$\tilde{h}_m = F_p^H h_m$$

Let the $P \times 1$ vector $y_m = [y_m(n_1), ..., y_m(n_{I_p})]^T$ consist of the received pilot samples per block, and define $s_p = [s(n_1), ..., s(n_{I_p})]^T$, and

$$w_m = [w(n_1), ..., w(n_{I_p})]^T$$

From (3.1), we have

$$y_m = \sqrt{\varepsilon_p} D(s_p) \tilde{h}_m + w_m = \sqrt{\varepsilon_p} D(s_p) F_p^H h_m + w_m \quad (3.3)$$

Given $s_p$ and $y_m$ we wish to estimate $h_m$ based on (3.3). While it may be possible to use pilot samples from different OFDM blocks to estimate the channel as advocated in [31], we will rely on pilots from only one block to estimate the channel on a per block basis as in [32] and [33]. This is particularly suitable for packet data transmission, where the receiver may receive different blocks with unknown delays.

### 3.1.2 SER With LSE Channel Estimation

If we define $G = (\varepsilon_p F_p D^H(s_p)D(s_p) F_p^H)^{-1}(\sqrt{\varepsilon_p} D(s_p) F_p^H)^H \quad (3.4)$, then the least-squares error (LSE) estimate of the channel impulse response is given by

$$\hat{h}_m = G y_m$$

Where $\eta_m = G w_m$ Using the fact that $D^H(s_p)D(s_p) = I_p$.

The estimated channel frequency response on the $t$th subcarrier can then be obtained from (4.5) as

$$\tilde{h}_m = \sqrt{\varepsilon_p} D(s_p) \hat{h}_m + w_m = \sqrt{\varepsilon_p} D(s_p) F_p^H h_m + w_m$$

With $\sigma_v^2(n) = f_n^H (F_p F_p^H)^{-1} f_n N_0 / \varepsilon_p$. Since the variance of $v_m(n)$ does not depend on the antenna index $m$ we omitted the index $m$ in $\sigma_v^2(n)$. For notational brevity, we also define

$$a_n = f_n^H (F_p F_p^H)^{-1} f_n \quad (3.7)$$
And then $\sigma_{H_m(n)}^2 + \sigma_{v(n)}^2 = 1 + \sigma_{v(n)}^2$. Since $H_m(n)$ and $v_m(n)$ are uncorrelated Gaussian random variables with zero-mean, $H_m(n)$ is Gaussian distributed with zero-mean, and variance $\sigma_{v(n)}^2 = a_n \frac{N_0}{\varepsilon_p}$.

From (3.7), we see that $H_m(n)$ is correlated with $v_m(n)$. Hence, $v_m(n)$ can be written as $v_m(n) = \overline{v_m(n)} + v_m(n)$, where $\overline{v_m(n)} = E[v_m(n)H_m(n)]$ and $\overline{v_m(n)}$ is a complex Gaussian random variable with zero mean, which is uncorrelated with $\overline{v_m(n)}$. Clearly, $\overline{v_m(n)}$ is the linear minimum mean-square error (MMSE) estimate of $v_m(n)$

$$\overline{v_m(n)} = \frac{1}{1 + \sigma_{v(n)}^2} \overline{v_m(n)}$$

and the variance of $\overline{v_m(n)}$ is the corresponding MMSE, which can be found as

$$\sigma_{\overline{v_m(n)}}^2 = \frac{\sigma_{v(n)}^2}{1 + \sigma_{v(n)}^2}$$

The output of the $m$th MRC branch for $s(n)$, can be expressed as

$$z_m(n) = H_m^*(n)y_m(n)$$

Substituting $v_m(n) = \overline{v_m(n)} + v_m(n)$, into (3.9), we obtain

$$z_m(n) = \frac{\varepsilon_s |H_m(n)|^2}{1 + \sigma_{v(n)}^2} s(n) - \sqrt{\varepsilon_s H_m^*(n)} s(n) v_m(n) + H_m^*(n) w_m(n)$$

Since $w_m(n)$, $\overline{v_m(n)}$ are uncorrelated, the instantaneous SNR of the MRC output

$$z(n) = \sum_{m=1}^{M} z_m(n)$$
can be found from (3.10) as

$$\gamma(n) = \frac{1 + \sigma_{v(n)}^2}{\gamma(n)}$$

To quantify the performance degradation caused by channel estimation errors, we define an SNR as

$$\gamma(n) = \frac{\varepsilon_s |H_m(n)|^2}{N_0 (1 + \sigma_{v(n)}^2)}$$

Since $\sigma_{H_m(n)}^2 = 1 + \sigma_{v(n)}^2$, and $\sigma_{H_m(n)}^2 = 1$, $\hat{\gamma}(n)$ in (3.12) is equivalent to $\gamma(n)$ in (3.11) in the sense that the average SER calculated from $\hat{\gamma}(n)$ is equal to that calculated from $\gamma(n)$. If $N_0$ denotes the total transmitted power per block, then
\( N \varepsilon = (N - P) \varepsilon_s + P \varepsilon_p \). Accounting for pilot power, the average power per information symbol is
\( \bar{\varepsilon}_s = \frac{N \varepsilon}{(N-P)} \), and (3.12) can be written as

\[
\tilde{y}(n) = G_L(n) \bar{\varepsilon}_s \sum_{m=1}^{M} H_m(n)^2 / N_0 \quad (3.13).
\]

where

\( G_L(n) \) in (3.12) quantifies the performance degradation caused by channel estimation errors, and by the power reduction needed for channel estimation. Substituting \( \sigma_y^2(n) \) into (3.14), we have

\[
\sigma_y^2(n) = \sigma_v^2 \sum_{m=1}^{M} H_m(n)^2 / N_0
\]

Where \( \alpha_n \) is defined in (3.7). In the ideal case where no pilot symbols are transmitted, and the receiver has perfect CSI, the transmitted power per symbol is \( \varepsilon \), and the SNR at the MRC output is \( \varepsilon \sum_{m=1}^{M} H_m(n)^2 / N_0 \). Compared to this ideal case, the performance degradation is

\[
G_{L1}(n) = \frac{G_L(n)}{G_L(n) + 1}
\]

While \( G_L(n) \) in (3.14) reflects the performance degradation caused by channel estimation errors, and accounts for the power reduction allocated to pilots, \( G_{L1}(n) \) in (3.15) captures the performance loss only due to channel estimation errors. Since \( \bar{\varepsilon}_s > \varepsilon_s \), we see from (3.14) that \( G_L(n) < 1 \), which implies that there is always performance loss. On the other hand, it may be interesting to compare the SER performance of pilot symbol assisted channel estimation with that of the ideal case. If equal power is allocated to pilot and information symbols \( \varepsilon_s = \varepsilon_p = \varepsilon \), then we can increase \( P \) to decrease the variance of channel estimation error \( G_{L1}(n) \) However, with this equal power allocation, we see from (3.15) that \( G_{L1}(n) < 1 \). If on the other hand, power is optimally distributed between pilots and information symbols, it will be shown later that \( G_{L1}(n) \) can be greater than one, which implies that performance may improve relative to the ideal case. Because \( G_L(n) \) depends on this power allocation, but also on the number and placement of pilot symbols.
3.1.3 SER With MMSE Channel Estimation

The LSE channel estimator does not depend on the fading channel’s power delay profile. If this knowledge is available, we can use the MMSE channel estimator to further improve SER performance.

From (3.3), the covariance matrix of $y_m$ is given by

$$R_{yy} := \mathbb{E}[y_my_m^H] = \varepsilon_p D(s_p) F_p^H R_{hh} F_p D(s_p)^H + N_0 I_p \quad (3.17)$$

Where

$$R_{hh} := \mathbb{E}[h_m h_m^H] = \text{diag} \left( \sigma^2_h(0), \ldots, \sigma^2_h(L-1) \right) \quad (3.18)$$

The cross-correlation between $y_m$ and $h_m$ is

$$R_{yh} = \mathbb{E}[y_m h_m^H] = \sqrt{\varepsilon_p D(s_p) F_p^H} R_{hh} \quad (3.19)$$

Then, the MMSE estimator of $h_m$ is given by $\widehat{h}_m = R_{yh}^{-1} R_{yy}^{-1} y_m$. The channel estimation error is given by $\epsilon_m = h_m - \widehat{h}_m$ which is Gaussian distributed with zero mean, and covariance [34]

$$R_\varepsilon = \mathbb{E}[\epsilon_m \epsilon_m^T]$$

Where $\sigma^2_h(l) \neq 0, \forall l$, so that $R_{hh}$ is invertible. When there are zero taps in $h_m$, we can remove these taps from (3.3), to guarantee invariability of $R_{hh}$. The estimated channel frequency response on the $m$th subcarrier can be obtained as $\widehat{H}_m(n) = f^H_n \widehat{h}_m = H_m(n) - \epsilon_m(n)$, where $\epsilon_m(n) := f^H_n \epsilon_m$.

With $\epsilon_m(n) \sim \mathcal{CN}(0, \sigma^2_{\epsilon_m(n)})$. The estimator $\widehat{H}_m(n)$ is Gaussian distributed with zero mean. Since the orthogonality principle renders $\epsilon_m$ uncorrelated with $\widehat{h}_m$, $\epsilon_m(n)$ and $\widehat{H}_m(n)$ are also uncorrelated. Thus, the variance of $\widehat{H}_m(n)$ can be found as

$$\sigma^2_{\widehat{H}_m(n)} = \sigma^2_{H_m(n)} - \sigma^2_{\epsilon_m(n)} = 1 - \sigma^2_{\epsilon_m(n)}$$

The output of the $m$th MRC branch for $s(n)$, can be written as

$$z_m(n) = \sqrt{\varepsilon_s} |\widehat{h}_m(n)|^2 s(n) + \sqrt{\varepsilon_s} |\widehat{H}_m(n)|^* s(n) \epsilon_m(n) + [\widehat{H}_m(n)]^* w_m(n) \quad (3.21)$$

Using the fact that $\epsilon_m(n)$ and $\widehat{H}_m(n)$ are uncorrelated, the instantaneous SNR at the MRC output can be found from (3.21) as

$$\gamma(n) = \frac{\varepsilon_s \sum_{m=1}^M |\widehat{H}_m(n)|^2}{N_0 + \varepsilon_s \sigma^2_{\epsilon_m(n)}} \quad (3.22)$$
Similar to (3.12), we define an SNR equivalent to $\gamma(n)$ in (3.22) as

$$\bar{\gamma}(n) = \frac{\varepsilon_s \sum_{m=1}^{M}|\hat{H}_m(n)|^2 (1 - \sigma^2_{\varepsilon(n)})}{N_0 + \varepsilon_s \sigma^2_{\varepsilon(n)}} \quad (3.23)$$

From the SNR in (3.23), and the independent and identical Gaussian distributions of $\{H_m(n)\}_{m=1}^{M}$ we can calculate the average SER[35].

Similar to (3.14), the performance degradation caused by MMSE channel estimation can be found from (3.23) as

$$G_M(n) = \frac{\varepsilon_s N_0 \left( 1 - \sigma^2_{\varepsilon(n)} \right)}{\varepsilon_s \left( N_0 + \varepsilon_s \sigma^2_{\varepsilon(n)} \right)} \quad (3.24)$$

Compared to the ideal case, the performance loss is given by

$$G_{M,I}(n) = \frac{\varepsilon_s N_0 \left( 1 - \sigma^2_{\varepsilon(n)} \right)}{\varepsilon \left( N_0 + \varepsilon_s \sigma^2_{\varepsilon(n)} \right)} \quad (3.25)$$

In the ensuing section, we will optimize pilot symbol parameters to maximize $G_M(n)$, and thus minimize the average SER.
3.2 Methodology of OFDM channel estimation

In this section the methodology of the channel estimation over OFDM will be introduced for the two methods used. First of all, the maximum ratio combiner (MRC) is employed to yield decision statistics. Also suppose that the frequency-selective channels remain invariant over an OFDM block, and the length of the cyclic prefix exceeds the channel order.

![Diagram of OFDM Channel Estimation]

- OFDM Channel Estimation
  - OFDM simulation parameter is chosen depends on chapter(5)
  - Pilot interval is obtained and the location of Pilots is specified
  - OFDM Modulation
    - Channel $H + \text{Noise}$
  - OFDM Demodulation
    - Brought transmitted and received pilots and sorted them in the diagonal.

To be Count.
Find the noise variance

Find auto correlation for $y$ and $H$

Find cross correlation

Find the estimated channel for the pilots

Find the estimated data by divide the received ones on $H_{est}$

$MSE = \left| \frac{|H| - |h_{est \ pilot}|}{|H|} \right|^2$

Find $G$ from Equ. (3.4)

Find the estimated channel for the pilots

Applied FFT for the previous step to fined $H_{est}$

Find the estimated data by divide the received ones on $H_{est}$

$MSE = \left| \frac{H_{est}}{H} \right|^2$

200 iteration

End
3.3 MIMO-OFDM Channel estimation

This section will focus mainly on data-aided channel estimation algorithms for MIMO antenna configurations over OFDM. Once channel estimates at data subcarriers are derived, the receiver performs equalization to compensate for signal distortion. Typically, a one-tap equalizer is often employed in MIMO OFDM systems to deal with flat-faded signals on each subcarrier. As opposed to the hard-output equalizer, the soft-output equalizer that generates the log-likelihood ratio (LLR) provides more information to the channel decoder, resulting in better error rate performance. However, there are times at which the multipath channel varies so rapidly that the channel state cannot be regarded as unchanged within one symbol period.

In such cases, interference among subcarriers, also known as inter-carrier interference (ICI), is induced and must be eliminated in the receiver. Moreover, synchronization, channel estimation, equalization, and channel decoding can be connected in an iterative loop structure at the receiver, called an iterative receiver. The error rate performance can be significantly improved, at the expense of increasing latency and complexity. Several popular pilot (reference signal) arrangements in OFDM systems introduced in section(3.1). The channel estimation algorithms based on different pilot patterns will be addressed.

3.3.1 The MIMO-OFDM System Model

Fig.(3.2) depicts a generic MIMO-OFDM system where a sequence of bits is coded for space-frequency communication, transmitted via the wireless channel, and subsequently decoded at the receiver. It can be noted that the MIMO-OFDM system derives data from a single application (e.g. a video frame) on the mobile device, for which each sample (e.g. a pixel), is encoded as a binary number. The sample is digitally encoded to increase the security of a transmission, minimize errors at the receiver, or maximize the rate at which data is sent [36].

The binary data corresponding to several contiguous samples forms a serial bit stream which constitutes the input bit sequence $b[n]$ in Fig.1. The input bit sequence is converted into a sequence of complex symbols (each with real and imaginary components) through the process of In phase and Quadrature (IQ) constellation mapping. IQ constellation mapping is an intermediate step in Quadrature Amplitude Modulation (QAM) which is usually followed by quantization of the complex symbols (QAM symbols), Digital to Analogue Conversion (DAC) and carrier modulation. However, for a system implementing Orthogonal Frequency Division Multiplexing (OFDM) modulation, an IFFT process is implemented after the IQ constellation mapping. In order to implemented the IFFT, $N$ QAM symbols are
arranged in a column vector which is then pre-multiplying by the inverse of the Fourier transformation matrix.

For the remainder of this section, the column vector of $N$ symbols will be referred to as the OFDM symbol which is in the frequency domain before the IFFT and in the time domain after the IFFT. In addition, the elements of the OFDM symbol will be referred to as QAM symbols before the IFFT, whilst the elements of the OFDM symbol will be referred to as OFDM samples after the IFFT. The OFDM modulation process is repeated $nt$ times resulting in a stack of OFDM symbols as depicted at the transmitter in Fig.(3.2).
Figure 3.3: A generic MIMO-OFDM Communication Systems.
The stack of OFDM symbols can then be mapped onto the \( n_t \) antenna elements at the transmitter array using spatial diversity. The OFDM samples then go through the process of quantization, pulse shaping for spectral efficiency, digital to analogue conversion and carrier modulation. At the receiver, various schemes can be implemented to detect the transmitted symbols. The main functions within the MIMO-OFDM system are the MIMO-OFDM air interface, MIMO-OFDM mapping/de-mapping and the MIMO-OFDM channel. Perhaps the most significant function in the MIMO-OFDM wireless system is the wireless channel/link, a snapshot of which is depicted in Fig.(3.3). The Channel Impulse Response (CIR) is a description of the output of a wireless channel when the input is an impulse, or typically, a wideband signal representing the maximum communications system bandwidth.

An ideal channel will reproduce the input signal (in this case an impulse) exactly at the output. Such ideal channels are called flat fading channels because the frequency response of the channel (the Fourier transform of the channel impulse response) is constant flat across all frequencies [36, 37]. A flat fading channel represents a wireless channel where there is effectively only one propagation path between the transmitter and the receiver. A more realistic channel will however have several paths by which the transmitted impulse signal can propagate to the receiver due to several mechanisms.

The power delay profile (PDP), is a plot of the received power against time when an impulse is transmitted. Channels that are characterized by multipath have a frequency response that varies depending on the frequency and are called frequency selective channels [36, 37].

The MIMO-OFDM air interface can be defined as the protocol that allows for the exchange of information between transmitter and receiver stations for the MIMO OFDM system. Alternatively, the air interface can be defined as the radio-frequency portion of the system. The MIMO-OFDM air interface consists of a combination of Quadrature Amplitude Modulation (QAM) constellation mapping and Orthogonal Frequency Division Multiplexing (OFDM).

QAM constellation mapping is used to generate symbols with real and imaginary components for the FFT process used in OFDM modulation. On the one hand, the implementation of M-QAM in MIMO is motivated by the realization of greater spectral efficiency for the overall digital modulation scheme [29, 38]. On the other hand, OFDM modulation effectively divides a wideband frequency selective channel into numerous narrowband channels that are, as a result, flat fading [31]. The combination of the two modulation schemes is used to convey data by changing the phase of a carrier signal that is then transmitted as an electromagnetic wave via an antenna.
The MIMO-OFDM mapping/de-mapping function determines how the transmit vector of $nt$ symbols is formed and how the receive vector of $nr$ symbols can be manipulated in order to detect the transmitted vector. Depending on the mapping/de-mapping function specified for the MIMO system, data communications can be improved in terms of increased data throughput or data detection reliability.

Link reliability can be improved by sending correlated data streams from the transmitter antenna array and exploiting these correlations at the receiver to improve data detection [39, 40]. Data throughput may be increase by transmitting $nt$ uncorrelated data streams from the transmitter antenna array [41 - 43]. The receiver has then to be specially designed in order to detect the transmitted data as each received symbol at a given antenna is a weighted sum of the $nt$ transmitted symbols. It can be shown that at a particular Signal to Noise Ratio (SNR), the data detection error rates can be reduced for particular transmission schemes using MIMO antenna [36].

### 3.3.2 Space-Frequency Coding

The source QAM symbols to be transmitted can be correlated in space and frequency using MIMO antennas. A space-frequency coding technique for a ($nt = 2; nr = 2$) MIMO-OFDM system based on Alamouti codes [39] is depicted in Fig.(4.4). The Alamouti scheme is generalized to orthogonal designs in the literature [40].

The stacking of OFDM symbols in Fig.(3.4) would therefore consist of a single OFDM symbol that has been arranged into two OFDM symbols as depicted in Fig.(3.4). At the receiver, the unknown data in the transmit vector can be deduced from two successive received symbols.

\[
R_1[k] \\
R_2[k] \\
R_1[k + 1] = H_{1,1} \\
R_2[k + 1] = H_{1,2}
\]

The Alamouti scheme assumes that the channel parameters in adjacent subcarriers are highly correlated so that the channel parameters for subcarrier $k$ are equal to the channel parameters of sub-carrier $k+1$ for the MIMO-OFDM system.
3.3.3 Wireless Channel Models – Saleh-Valenzuela and Clarcke channel Model

The Saleh-Valenzuela (SV) model [44] can be used to generate the PDP of an indoor environment which is then used to simulate the CIR with Rayleigh distributed amplitudes and uniform distributed phase. It is assumed that the transmitter and receiver links in the MIMO-OFDM systems are uncorrelated and the condition under which this assumption can be made are stated in the discussion in section (3.3.4). The SV model is used to generate that uncorrelated channel taps for wireless channels in an indoor environment by generating independent power delay profiles.
A single PDP generated by the Saleh-Valenzuela model can be used to simulate numerous random realizations of a CIR. However, because we wish to evaluate the performance of the channel estimation algorithm for varying maximum delay spread $\tau_{\text{max}}$. We generate random PDPs using the Saleh-Valenzuela model for each CIR realization. Because the user is stationary, we can assume that the CIR in our simulation corresponds to channel measurements that are performed in different locations within the indoor environment.

In order to simulate the multipath component gain, the relationship between the Power Delay Profile (PDP) and the variance of the Rayleigh distributed channel amplitude is exploited. Because the multipath component gain is assumed to be a wide sense stationary (WSS) process, average power measurements are sufficient for describing the channel in any location with a room when the user is stationary. The Saleh-Valenzuela model is used to simulate the PDP using the exponential decay of the multi-path component power with increasing delay, and the Poisson process to predict the number of multipath components and their inter-arrival times[45].

And then Clarke's model described, which is used to model the variations of the multipath channel gain with measurement time.

The frequency selective channel model (3.30) considered thus far for CSI channel estimation is extended to include the effects of doppler frequency change. The doubly selective channel model implemented, which is commonly referred to as Clarke's Model, is described in the literature [37] and [46]. In this model, the phase associated with the nth path is considered independent from the phase due to the Doppler frequency change. As it can be noted from the discussion below, the phase change due to path length is much greater than the phase change due to the Doppler frequency change which necessitates a distinction of the two quantities.

Clarke's Model can be derived from the equation for the received complex envelope for a signal transmitted at a carrier frequency $f_c$ as (3.31).

$$\hat{h}_n(t) = \sum_{n=0}^{N-1} y_n(t) e^{-j2n\pi f_c \tau_n(t)} S_{\text{m}}(t - \tau_n(t)) \quad (3.31)$$

When the receiver is stationary, the complex channel gain can be modeled. In order to separate the effect path length to those associated with the motion of the receiver, we shall start by expressing the phase of the multipath gain in the frequency selective model as (3.30) as a function of path length. The phase of the multipath gain can be expressed as a function of path length $\ell_n$ by writing $\phi_n = \frac{2\pi}{\lambda_n} \ell_n$ where $\lambda_n$ is the wavelength of the RF carrier frequency. When the receiver is in motion at a constant
velocity $v$, the phase of the multipath gain will change because of changes in the path lengths $\ell_n$. In addition, the frequency of the signal arriving via the $n$th path will experience a Doppler frequency shift which we denote as a variable $f_n$. The Doppler frequency shift $f_n$ for each multipath component is modified according to the azimuth Angle of Arrival (AoA) which we shall denote as $\theta_n$.

$$f_n = f_d \cos(\theta_n) = \frac{f_c v}{c} \cos(\theta_n)$$  \hspace{1cm} (3.32)

$\frac{f_c v}{c}$ is the maximum Doppler frequency shift (Doppler bandwidth), which is attributed to the LOS multipath components. The phase of the multipath gain can be modeled as the summation of the path length induced and Doppler frequency induced phases when the receiver is moving.

$$s_{m}^{rx}(t) = \sum_{n=0}^{N-1} y_n(t)e^{j(2\pi f_c \tau_n(t) - 2\pi \frac{\ell_n}{\lambda_m} \tau_n(t + \tau_n(t))}$$  \hspace{1cm} (3.33)

We now introduce an alternative view to the signal received in a multipath environment in order to determine the measurement time variations in the complex channel gain $\overline{y_n}(\tau_n(t), t)$. Consider that at some measurement time instant $t$, $N$ multipath components arrive with the same delay $\tau_n(t) = t$. The channel gain affecting the signal $s_{m}^{rx}(t)$ in (3.33) at this measurement time is given by:

$$\overline{y_n}(t) = \sum_{n=0}^{N-1} y_n(t)e^{j(2\pi f_c \tau_n(t) - 2\pi \frac{\ell_n}{\lambda_m} \tau_n(t) + \alpha_n)}$$  \hspace{1cm} (3.34)

$\alpha_n$ is a random phase associated with the $n$th path. The phase $\phi_n = \frac{2\pi}{\lambda_m - \alpha_n} \ell_n$ is independent of the measurement time and can be modeled using uniform distribution. This assumption generalizes the geometry of the communications system in terms of location of the transmitter, receiver and multipath mechanisms.

The azimuth AoA ($\theta_n$) determines the Doppler frequency shift of the $n'th$ path as $\frac{f_c v}{c} \cos(\theta_n)$ and can also be modeled using uniform distribution. The amplitudes $y_n(t)$ can be modeled using Gaussian distribution. It is assumed that as the measurement time $t$ elapses, the amplitude of the multipath gain remains constant ($y_n(t) = y_n$). This model is Clarke's flat fading model [47]. Note that, in Clarke's model, path length induced phase for the different multipath components will be the same whilst the AoA-dependent Doppler induced phase will differ depending on the path. This is due to the fact that the $N$ multipath components arriving at the measurement time $t$ have the same delay $\tau_n(t) = t$ and hence the same path lengths $\ell_n$ but may have different AoA's. Clark's model evaluates the gain of the channel $\overline{y_n}(\tau_n(t), t)$ when $\tau_n(t) = t$ so that we are effectively considering a single multipath delay $\tau_n(t)$ as time elapses.
3.3.4 One\Two Dimensional Channel Estimation

Receiver complexity is reduced when one-dimensional channel parameter estimation is implemented in OFDM systems, see section (3.1) because time and frequency correlations may be exploited separately. The SNR performance of 1D channel estimators is however inferior to that of 2D channel estimators [48] and the literature indicates that fewer pilots are required for 2D estimation leading to spectral efficiency [51]. In this section, the frequency correlation of the channel parameters are used to develop a low complexity 1D estimators for the OFDM wireless system.

Temporal correlations may also be used based on the observation that the channel parameters in the time domain are a band-limited stochastic process. The simplest channel estimator based on the frequency correlations can be implemented by simply dividing the received QAM symbol by the transmitted QAM symbol. For a single OFDM symbol the flat fading channel for each sub-carrier can be estimated as follows

This estimator is referred to as the Least Squares Estimator in the literature [48] and [52] and has the major disadvantage of having an over simplified channel model i.e., the absence of AWGN and perfect equalization are assumed [48].

The frequency correlations of the channel gain for the OFDM symbol are linked to the finite maximum delay spread of the channel. For a well designed OFDM system, the duration of the OFDM symbol NTs is much longer the maximum channel delay LTs, where Ts is the QAM symbol period. Channel estimation can be performed in the time domain where there are fewer parameters. This leads to a low complexity solution with improved SNR performance.

Considering without loss of generality that

\[
\bar{x} = [1, 1, ..., 1]^T \in \mathbb{R}^{N \times 1}
\]  

(3.36)

The received OFDM symbol can be written as

0 is an \((N - L) \times 1\) null vector, and \(N = K\) is the length of the column vector \(\bar{x}\). \(F\) is a \(N \times N\) matrix that can be separated into the "signal subspace" and the "noise subspace", and the received OFDM symbol can be re-written with the partitioning of the \(F\) matrix.
Relying on this model, the reduced space estimates of the channel can be calculated as follows

\[ \hat{h} = F_h^\dagger \tilde{r} = h + F_h^\dagger \tilde{n} \]  \hspace{1cm} (3.39)

\( F_h^\dagger \) is the pseudo inverse of the signal subspace FFT matrix.

From Fig.(3.5) the shaded subcarriers contain training symbols. In 2-D channel estimation, the time and frequency correlation of the training sub-carriers are used to estimate the channel.

To explain why we moved toward two dimensional channel estimation; in OFDM system not all the sub-carriers are required for channel estimation because of the strong frequency correlations and the pilot QAM symbols can be spaced at interval in frequency to estimate the channels. The performance of the channel estimator can also benefit from the rather strong time correlations when pilot QAM symbols are spaced at interval in time (Figure 3.5)
Exploiting both time and frequency correlations can significantly reduce the spectral inefficiency due pilot symbol placement whilst providing the functions of filtering, smoothing and prediction [48] [19].

In order to explain the aforementioned functions, it is necessary to understand the process of 2D channel estimation. At the pilot sub-carrier time-frequency locations, an a posteriori least squares estimate of the channel parameters corrupted by Additive White Gaussian Noise (AWGN) is given by

$$
\hat{H}[k, m] = \frac{\hat{R}[k, m]}{X[k, m]} = \frac{R[k, m] + N[k, m]}{X[k, m]} \\
= H[k, m] + \frac{N[k, m]}{X[k, m]}
$$

Note that for the flat fading OFDM sub-carrier channel, the received QAM symbol is given by the product $R[k, m] = H[k, m]X[k, m]$ , where $H[k, m]$ is the sub-carrier channel gain and $X[k, m]$ is a transmitted QAM symbol (data or pilot). An estimate of the sub-carrier channel gain at any given time-frequency location $\hat{H}[k, m]$ is given by a linear combination of the estimates $\hat{H}[k, m]$ at the pilot locations.

$$
\hat{H}[k, m] = \sum_{[k, m]} \omega[k, m] \hat{R}[k, m] = \omega^H \hat{R}
$$

The total number of pilots in the OFDM frame can be denoted by $N_{frame}$, where an OFDM frame refers to $M$ received OFDM symbols each containing $K$ QAM symbols.

The OFDM frame is used for both channel estimation and data detection in the 2-D estimator (Figure 4.4). $\hat{h} \in \mathbb{C}^{N_{frame} \times 1}$ is a vector formed from some arrangement of the least square channel parameter estimates $\hat{H}[k, m]$ for the OFDM frame. This arrangement can be for example a collection of the estimates $\hat{H}[k, m]$ for increasing frequency index from the first to the last OFDM symbol in the OFDM frame. The optimal weights $w \in \mathbb{C}^{N_{frame} \times 1}$, in the sense of minimizing the MSE across all the time-frequency sub-carrier locations (the so-called 2-D Wiener filter coefficients), are given by

$$
\omega^H = \theta^H \Phi^{-1}
$$

$\theta \in \mathbb{C}^{N_{frame} \times 1}$ is a cross-correlation vector for the correlation between the estimated parameter and the least squares channel parameter estimates, $\phi^{N_{frame} \times N_{frame}}$ is a covariance matrix for the least squares channel parameter estimates. However since these channel statistics are not known at the receiver, the elements of the cross correlation vector $\theta$ can be approximated as follows [49]
In the above formulation, $\tau_{max}$ is the maximum delay spread of the multipath channel, $F_s$ is the bandwidth of each sub-carrier, $f_D = \frac{\nu f_c}{c}$ is the Doppler frequency for a receiver traveling at a velocity $\nu$, for a carrier frequency $f_c$ and $c = 3 \times 10^8$ m/s is the velocity of Electromagnetic waves in free space. $K$ is the number of QAM symbols in the OFDM symbol, $L$ is the maximum number of non-zero elements in the CIR vector and $T_s$ is the QAM symbol period. The correlation of CSI in the frequency domain can be related to the power delay profile (PDP).

Assuming that the correlations can be approximated by sinc functions is equivalent to assuming a rectangular power delay profile, and despite the fact the PDP has been modeled as exponentially decaying, the results obtained in this thesis and in the literature [49] are compelling. Similarly, the elements of the covariance matrix $\phi$ can be approximated by the formulation

$$E[\hat{h}[k]\hat{h}^*[k]]$$

$$E[\hat{h}[m]\hat{h}^*[m]]$$

$$E[\hat{h}[k,\hat{m}]\hat{h}^*[k]]$$

$\delta_{km}$ is the kronecker delta function and $\delta_{n}^2$ is the noise variance at pilot sub-carrier locations. The assumption of sinc correlations for the OFDM CSI is equivalent to assuming a rectangular power spectral density.

In terms of 2-D channel estimation, filtering refers to the channel parameter estimates at the data carrying frequency indices which are in a sense an interpolated estimate due to Wiener filtering. Prediction refers to the channel parameters estimates at data carrying time indices which are procured through a process of time projection using the Wiener filter. Smoothing refers to a refinement of the initial 'noisy' least squares channel parameter estimates at the pilot locations. 2-D Wiener Filter estimators, also called Minimum Mean Square Error (MMSE) estimators, have greatly increased computational complexity for the improved.
3.3.5 Least Squares Solution

The forward problem \( r = Xh \) can easily be formulated for the MISO-OFDM system using the convolution channel model. The forward solution predicts the outcome \( r \) as a function of known system inputs matrix \( X \) and channel vector \( h \). The channel vector \( h \) has a minimum length \( L \) and the MISO system employs \((nt, 1)\) antennas with \( K \) sub-carriers for each transmit/receive antenna link. MISO-OFDM estimators can be generalized to MIMO-OFDM estimators by repeating the estimation process at each receive antenna at a time.

In the inverse problem, \( N \) measured values of the system output \( r \) are used to estimate \( n_t L \) unknown channel parameters [50]. Both the system output \( r \) and the system inputs matrix \( X \) are known at the receiver. In general \( X \) in non-invertible and a pseudo inverse must be used to solve the inverse problem.

\[
\hat{h} = X^+ r \tag{3.50}
\]

For a length \( N \) CIR vector \( h \), only \( L \) non-zero elements need to be estimated. This reduces the number of channel gain parameters to be estimated per MIMO-OFDM link from \( N \) to \( L \), and the number of channel gain parameters per receive antenna from \( n_t N \) to \( n_t L \) where \( L \ll N \). Each received OFDM symbol of length \( N \) (in the time domain) is used to estimate \( n_t N \) channel gain parameters where \( n_t L \leq N \). After channel gain estimation in the time domain, the relationship \( \tilde{h} = F\hat{h} \) is used to obtain frequency domain estimates.

The length \( L \) CIR vectors of \( n_t \). Multiple Input Single Output (MISO) links for the \( j^{th} \) receiver can be written as a vector

\[
h_{\text{MISO}} = [h_{1,j}^T, h_{2,j}^T, ..., h_{nt,j}^T]^T \tag{3.51}
\]

\( h_{i,j} \) is the SISO CIR vector \( h \) for the \((i,j)\)th MISO link that has been truncated to a length \( L \). The transmitter sends unique training sequences from the \( i^{th} \) antenna \( x_i \).

A circulant matrix of the training symbols may be observed at the receiver due to the convolution channel model based on the transmitted training sequence for antenna \( i \).

\[
X_i = \begin{bmatrix}
x_i[L-1] & \ldots & x_i[1] & x_i[0] \\
x_i[L] & \ldots & x_i[2] & x_i[1] \\
\vdots & \ddots & \ddots & \vdots \\
x_i[N+L-2] & \ldots & x_i[N]x_i[N-1]
\end{bmatrix} \in \mathbb{C}^{N \times L} \tag{3.52}
\]

The first and last \( L - 1 \) received QAM symbol of each burst are ignored in the formulation of the circulant matrix above. The circulant training sequence matrix in
are concatenated to form a larger matrix $X$ which can be used together with equation (4.46) to describe a received symbol vector.

$$ r = Xh_{MISO} + n $$  \hspace{1cm} (3.53)

$n$ is Additive White Gaussian Noise (AWGN) vector. When referring to equation (4.48), the subscript MISO will be omitted to simplify the notation.

The vector $r$ is the received symbol vector for the OFDM system, before the FFT operator, and is therefore considered a time domain vector. The Least Squares channel estimate can be found for equation (3.53) by pre-multiplying both sides of the equation by the Moore-Penrose inverse. Because the OFDM frame is designed such that $N \geq n_tL$, the LS solution is given by

$$ \hat{h} = X^T r \approx h $$  \hspace{1cm} (3.54)

There is a small error in the estimated channel because the Saleh-Valenzuela model maximum delay of the Channel Impulse Response (CIR) is 200ns but the estimation considers a CIR with a maximum duration $L \times T_s \approx 160 \text{ ns}$, where $L = 16$ and $T_s = 11 \text{ ns}$.

**3.3.6 Orthogonal Training Sequence for Channel Estimation**

This section describes an effective MIMO-OFDM channel estimator that has been implemented in MIMO-OFDM in the literature [51]. The estimator is based on the use of an orthogonal training sequence such as the Hadamard sequence and the correlations of MIMO-OFDM channels over OFDM subsymbols ($K_{coh}$ sub-carriers) within a length $K$ OFDM symbol, where ($K_{coh} \ll K$).

It was noted in section (4.3.4) that the correlation of the OFDM CSI in frequency can be approximated by a sinc function, where the first null is related to the maximum delay spread ($\tau_{rms}$) of the channel. This result was used to develop the Wiener filter which was found to improve the MSE performance of OFDM estimator at the cost of increased computational complexity at the receiver.

The concept of coherence bandwidth is particularly useful when describing the wireless channel for a multi-carrier system such as OFDM. To reiterate, the main advantage of OFDM is to eliminate Inter-Symbol Interference (ISI), which results when the duration of the transmitted symbol is shorter than the maximum delay of the wireless channel. ISI is eliminated by sending several symbols in parallel using evenly spaced carriers (referred to as sub-carriers), so that each symbol is transmitted for a longer duration.
However, the channel is frequency selective, meaning that the Fourier transform of the Channel Impulse Response (CIR) is not flat. This in turn implies that the gain experienced by different sub-carriers varies as has been observed in section (3.3.4) 1-D and 2-D channel estimation. The relationship between the maximum delay of the channel and correlation of the channel gain at different frequencies can be intuitively understood as follows: if the maximum delay is zero, the FFT of the CIR is unity for all frequencies from Fourier transform theory [38].

A rectangular CIR can be shown to result in a sinc correlation function. If the power delay profile is the rectangular function

\[
p(\tau) = \begin{cases} 
\frac{1}{\tau_{\text{max}}} & \text{if } |\tau| < \frac{1}{\tau_{\text{max}}} \\
0 & \text{otherwise}
\end{cases}  \tag{3.55}
\]

Then the autocorrelation function is the sinc function.

\[
R(\Delta f) = R((k - \hat{k})F_s) = \frac{\sin(\pi \tau_{\text{max}}(k - \hat{k})F_s)}{\pi \tau_{\text{max}}(k - \hat{k})F_s} \tag{3.56}
\]

are well known Fourier transform duals [38],[46]. This observation motivates the OFDM sub-symbol based MIMO-OFDM channel estimators. The idea is that if the CSI is invariant for \(K_{\text{coh}}\) sub-carriers, then a reduction in the number of CSI unknowns is possible leading to an accurate estimate of the CSI over \(K_{\text{coh}}\) subcarriers.

If the MIMO-OFDM system is equipped with \(nt\) transmit antennas and the channel estimation is performed at single receive antenna, the CSI \(H_i[k]\) corresponding to the \(i^{th}\) transmit antenna at the sub-carrier \(k = k_0\) can be approximated by

\[
\hat{H}_i[k_0] = \sum_{i=1}^{k_{\text{coh}}} \sum_{i=1}^{K_{\text{coh}}} R[k_0 + i - 1]T_i^*[k_0 + i - 1] \quad \forall \ k_0 = 0, k_{\text{coh}}, ..., K - 1 \tag{3.57}
\]

\(z^*\) is the complex conjugate of a complex number \(z\). For the MISO-OFDM system, the received QAM symbol is given by \(R[K] = \sum_{i=1}^{nt} y_i[k]T_i[k]\) The Hadarmard training sequence is an orthogonal training sequence such that

\[
\sum_{k=k_0}^{k_0+K_{\text{coh}}-1} T_i[k]T_j^*[k] = \begin{cases} 
1 & \text{if } i = j \\
0 & \text{otherwise}
\end{cases} \tag{3.58}
\]

If the difference between the CSI for the \(k^{th}\) sub-carrier and the \(k_0\) sub-carrier is denoted by \(\Delta H_i^{k,k_0} = H_i[k] - H_i[k_0]\) the estimated CSI as equation (3.57) becomes
The error in the estimated CSI $\tilde{H}_i[k_0]$ is the difference between the actual and estimated CSI. A simple rearrangement of equation (4.54) shows that the error in the estimated CSI is a function of the gradients $\Delta H^k_{i}^{k_0} = H_i[k] - H_i[k_0]$.

$$\delta H^k_{i}[k_0] = H_i[k_0] - H_i[k_0] = \sum_{m=0}^{n_t-1} \sum_{n=0}^{K_{coh}-1} \Delta H^k_{i+m+1} T_i[k_0 + n] T_i^*[k_0 + n] (3.60)$$

Noise free transmission is assumed in equation (3.59). If coherence is assumed over the coherence bandwidth so that $\Delta H^k_{i}^{k_0} i \to 0$, then the error in the CSI estimate $\tilde{H}_i[k_0]$ tends towards zero. The advantage of the OFDM sub-symbol based channel estimator is that the strong correlations of the CSI over a few sub-carriers are used to form channel estimates. As such, the performance of the estimator for a large number of transmitting antennas is only limited by the knowledge of the change in CSI over $K_{coh} = n_t$ sub-carriers. The OFDM sub-symbol estimators can be used to train a large number of antennas by differentiating each antenna using a unique Hardarmard sequences. However, the more the number of antennas in the MIMO-OFDM system, the fewer the number of estimated CSI as indicated in equation (3.57). The performance of the estimator is then limited by the interpolation requirements[45].

Using OFDM sub-symbol based estimators, the CSI over $K_{coh}$ sub-carriers is assumed to be invariant. This assumption is based on the sinc function model for the correlations between the CSI with increasing frequency index, where the first null is inversely proportional to the maximum delay spread $\tau_{rms}$ of the channel.

$$R(\Delta f) = R((k - \tilde{k}) F_s) = \frac{\sin(\pi \tau_{max}(k - \tilde{k}) F_s)}{\pi \tau_{max}(k - \tilde{k}) F_s} \quad (3.61)$$

As was noted previously, the spaced-frequency correlation function can be determined by Fourier transform of the correlation function. The function $P(\Delta f)$ represents the correlation between the channels response to two narrowband sub-carriers with the frequencies $f_1$ and $f_2$ as a function of the difference $\Delta f$ [41]. Because the Fourier transform of the correlation function is the rectangular function [38] with a bandwidth $B_{coh} = 1/\tau_{max}$, the channel gain is assumed to be constant for the coherence bandwidth.

For a maximum delay spread of $\tau_{max} = 200$ ns and an RF channel bandwidth of 200MHz, the coherence bandwidth is $B_{coh} = 1/\tau_{max} = 5$MHz and approximately
128 × 5MHz/200MHz ≈ 3 OFDM sub-carriers have the same gain for \( K = 128 \). In order to accurately train \( n_t \) transmit antennas, the coherence assumption must hold for \( Kcoh \geq n_t \) sub-carriers and therefore a maximum of \( n_t = 3 \) antennas can be trained in the example given. In this thesis, it is argued that the CSI varies within the coherence bandwidth causing a significant error in CSI estimates. It is also shown that if such variation of CSI within the coherence bandwidth are taken into account, CCSI can be achieved even when the coherence is assumed over \( Kcoh < n_t \) sub-carriers at high SNR.

The estimator noted previously is now reformulated to indicate how the number of transmit antennas \( n_t \) affects the error in CSI estimation Fig.(3.5). Given that the received QAM symbol at a given receiver of a MIMO-OFDM system is given by

\[
R[K] = \sum_{i=1}^{n_t} H_i[k] T_i[k]
\]

the initial estimate of the CSI at a sub-carrier \( k = k_0 \) is given by

\[
\hat{H}_i[k_0] = \sum_{i=1}^{n_t} R[k_0 + i - 1] T_i[k_0 + i - 1] \quad \forall \ k_0 = 0, n_t, ..., K - 1
\]  

(3.62)

**Figure 3.6 :** Training symbol placement for a QAM symbol based channel estimator for a (4,1) MISO-OFDM system[45].
Each transmit antenna transmits a row of Walsh code (Hadamard) matrix which is used to uniquely identify the antenna at the receiver. \( W_4(m, n) \) is the element in the \( m \)th row and \( n \)th column of the Walsh matrix.

\[
W_4 = \begin{bmatrix}
1 & 1 & 1 \\
1 & -1 & 1 \\
1 & 1 & -1 \\
-1 & -1 & 1
\end{bmatrix}
\] (3.63)

In the traditional sense, it is usual to assume that the coherence in CSI is observed over at least \( K_{coh} = n_t \) number of sub-carriers. In this case, orthogonal Hadamard training sequences of length \( n_t \) can be arranged within the OFDM symbol so as to determine the CSI after every \( n_t \) sub-carriers as Fig.(3.6). However it is known that the CSI will vary within the coherence bandwidth so that if the difference between the CSI for the \( k \)th sub-carrier and the \( k_0 \) sub-carrier is denoted by \( \Delta H_i^{k, k_0} = H_i[k] - H_i[k_0] \).

Then an error is incurred in estimating the CSI, which is given by

\[
\bar{H}_i[k_0] = H_i[k_0] + \sum_{m=0}^{n_t-1} \sum_{n=0}^{n_t-1} \Delta H_i^{k_0+n, k_0} T_{i+m}[k_0 + n] T_i^*[k_0 + n] 
\] (3.64)

An iterative algorithm for reducing the error \( \delta H_i[k_0] \) is described. The algorithm is based on the notion that the gradients \( \Delta H_i^{k, k_0} = H_i[k] - H_i[k_0] \), can be accurately predicted through interpolation. The information on the gradients can be used to improve the a posteriori estimates \( \bar{H}_i[k_0] \) and the gradients recalculated. The estimated and interpolated channels are depicted in the methodology part. If this process is repeated iteratively, it is expected that the estimated CSI will approach the actual CSI, providing C-CSI.

Also in this section an iterative algorithm is devised to improve CSI estimates when a large number of antennas is to be trained. It is assumed that the mobile station is equipped with \( n_t = 2 \) antennas and that two receive antenna at a time. The main idea is to exploit the channel correlation in frequency so that we could in a sense convert the underdetermined system into a determined system. To illustrate this, without loss of generality (WLOG), we consider only the first four sub-carriers, i.e., \( k = 0; 1; 2; 3 \), and assume there is no noise.
Figure 3.8: An example of the partitioning of 128 CSI estimates for the OFDM symbol into sub-symbols for a (2,1) MIMO-OFDM system.

Firstly, given that \( \{ t_i[k] \} \) are orthogonal pilot training sequences spanning two subcarriers so that

\[
|t_1[0]|^2 + |t_1[1]|^2 = |t_2[0]|^2 + |t_2[1]|^2 = 1, \quad (3.65)
\]

\( t_1[0]t_2^*[0] \)

We can have a coarse estimate for \( \hat{H}_1[0], \hat{H}_1[1], \hat{H}_2[0], \hat{H}_2[1] \) by linear combining the received signals

Where,

\[
\Delta H_1^{1,0} \triangleq H_1[1] - H_1[0] \quad (3.68)
\]

\( \Delta H_2^{1,0} \triangleq H_2[1] - H_2[0] \)

\( H_2^{\text{est}}[0] = t_2^*[0] \)

Similarly, we also have:
3.4 Methodology of MIMO-OFDM channel estimation

3.4.1 Wiener channel estimation

- Clarke's model is used to generate a 2D channel
  - Simulation parameter is chosen depends on table (5.2).
  - Generate some random data and pilots
  - Stack the pilots indices down the frequency index first then across time index
  - Clarke's model is used to generate a 2D channel
  - FFT for the Channel Gain
  - Add noise to the received symbols

To be Count.
Calculate the channel estimate at the data location

Calculate wiener filter coefficients

Initial channel estimate at the pilot symbol

Find the auto correlation matrix

Find the correlation matrix
3.4.2 Orthogonal training sequence channel estimation

Orthogonal training sequence

This method is applied on $2 \times 2$ MIMO-OFDM system

Simulation parameter is chosen depends on table (5.3)

Generate the data, then split the data into two antennas

OFDM modulation

Generate orthogonal training sequence, sort them in odd indices, and in even indices put the data

Generate a PDP and channel gain from Saleh-Model.

Then FFT for channel gain and add noise

To be Count.
Interpolation to estimate the channel at data indices as seen in section (3.3.6)

Estimate channel at each orthogonal training symbol indices

Receive data and orthogonal training sequence at receiver side

Combined the data from the two antennas in one antenna

OFDM Demodulation

End
Chapter Four: QO-STBC and DHSTBC Theory and Methodology

High date rate wireless systems with very small symbol periods usually face unacceptable Inter-Symbol Interference (ISI) originated from multipath propagation and their inherent delay spread. Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier-based technique for mitigating ISI to improve capacity in the wireless system with spectral efficiency (bps/Hz). On the other hand, Multiple-Input Multiple-Output (MIMO) systems have rising attention of the wireless academic community and industry because their promise to increase capacity and performance with acceptable BER proportionally with the number of antennas[53].

In communication systems there is only one antenna at both the transmitter and the receiver. This antenna system is known as Single Input Single Output (SISO). SISO systems have a major drawback in terms of the capacity. In order to increase the capacity of SISO systems to meet the high bit rate transmission demanded by modern communications, the bandwidth and the power have to increase significantly. Fortunately, using the Multiple-Input Multiple-Output (MIMO) system could increase the capacity and improve performance with acceptable BER proportionally with the number of antennas of the wireless system without the need to increase the transmission power or the bandwidth, also it decreases the error rates compared to single-antenna system by sending multiple redundant versions of the same data sequence and perform appropriate combining. Another drawback in wireless systems is the Inter-Symbol Interference (ISI) originated from multipath propagation and their inherent delay spread. Orthogonal Frequency Division Multiplexing (OFDM) system is a multicarrier-based technique for mitigating ISI to improve capacity in the wireless system with spectral efficiency.[26]

The structure of MIMO-OFDM system is described in Fig. (4.1)

*Figure 4.1: MIMO-OFDM block diagram*
In 2013, Y.A.S Dama et al. [27] proposed a new approach for Quasi-Orthogonal Space Time Block Coding (QO-STBC) that eliminate the interference from the detection matrix to improve the diversity gain compared with the conventional QO-STBC scheme, it also reduces the decoding complexity. Diagonalized Hadamard Space Time Block Coding (DHSTBC) which provide full rate full diversity order was presented.

These new two approaches were implemented for MIMO system with three and four transmitter antennas. Based on Y.A.S Dama et al. paper [27], QO-STBC and DHSTBC are implemented and introduced in this chapter over OFDM for four, eight and sixteen transmitter antennas.

4.1 Theory: QO-STBC over OFDM for four, eight and sixteen transmitter antennas

In Quasi-orthogonal code structure, the columns of the transmission matrix are divided into groups. The columns within each group are not orthogonal to each other but those from different groups are orthogonal to each other [54]. By using quasi-orthogonal design, pairs of transmitted symbols can be decoded independently; the loss of diversity in QOSTBC is due to some coupling terms between the estimated symbols [55].

The encoding matrix for two (2 × 2) Alamouti codes $X_{12}$ and $X_{34}$ to form $X_{ABBA}$

$$X_{12} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \quad (4.1)$$

$$X_{34} = \begin{bmatrix} x_3 & x_4 \\ -x_4^* & x_3^* \end{bmatrix} \quad (4.2)$$

$$X_{ABBA} = \begin{bmatrix} X_{12} & X_{34} \\ X_{34} & X_{12} \end{bmatrix} \quad (4.3)$$

The Equivalent Virtual Channel Matrix (EVCM) $H_v$ can be written as:

$$H_v = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3 & h_4 & h_1 & h_2 \\ h_4^* & -h_3^* & h_2^* & -h_1^* \end{bmatrix} \quad (4.4)$$

A simple method to decode the QO-STBC over OFDM is by applying the maximum ratio combining (MRC) technique. MRC can be done by multiplying the received vector $Y$ with $H_v^H$ thus:

$$X = H_v^H Y = H_v X_{ABBA} + H_v^H \quad (4.5)$$

$$= D_4 X_{ABBA} + H_v^H n$$

Where $D_4 = H_v^H H_v$ is a non diagonal detection matrix used to decode the received signal.
The diagonal elements; $\alpha$ represent the channel gains, and $\beta$ in equation (4.7) represent the interference from the neighboring signals.

For four transmit antennas

$$\alpha = |h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2$$

(4.7)

$$\beta = h_1^* h_3 + h_2^* h_4^* + h_3^* h_1 + h_4^* h_2$$

The interference term $\beta$ will cause performance degradation, more complex decoding methods where introduced to detect the estimate $\hat{X}$. Therefore QO-STBC scheme for three and four transmitter antennas to diagonalize the detection matrix using eigenvalue eigenfunction where proposed by Y.A.S Dama et al [27].

The solution of the eigenvalue problem of matrix $D_4$ defined in Equ. (4.6) is

$$D_4 V_{4\text{QO--STBC}} = V_{4\text{QO--STBC}} D_4 D_{4\text{QO--STBC}} = 0$$

Where $D_{4\text{QO--STBC}}$ is eigenvalue and $V_{4\text{QO--STBC}}$ is eigenvector for $D_4$

$$D_{4\text{QO--STBC}} = \begin{bmatrix} \alpha + \beta & 0 & 0 & 0 \\ 0 & \alpha + \beta & 0 & 0 \\ 0 & 0 & \alpha - \beta & 0 \\ 0 & 0 & 0 & \alpha - \beta \end{bmatrix}$$

(4.8)

$$V_{4\text{QO--STBC}} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

(4.9)

From this property, the channel matrix for four, eight and sixteen transmit antennas over OFDM can be derived.

So a new channel matrix can be defined as:

$$H_{4\text{QO--STBC}} = H_v V_{4\text{QO--STBC}}$$

(4.10)

Where $H_{4\text{QO--STBC}}$ is given by matrix (4.11):

$$H_{4\text{QO--STBC}} = \begin{bmatrix} h_1 + h_3 h_2 + h_4 h_3 - h_1 h_4 - h_2 \\ h_2^* + h_4^* - h_1^* h_3^* h_4^* - h_2^* h_1^* - h_3^* \\ h_1 + h_3 h_2 + h_4 h_1 - h_3 h_2 - h_4 \\ h_2^* + h_4^* - h_1^* h_3^* h_2^* - h_4^* h_3^* - h_1^* \end{bmatrix}$$

(4.11)
$H^H_{4 \text{QO–STBC}} \cdot H_{4 \text{QO–STBC}}$ is diagonal matrix which can achieve simple linear decoding, because of the orthogonal characteristics of the channel matrix $H_{4 \text{QO–STBC}}$ in equation (4.11).

The encoding matrix $X_{4 \text{QO–STBC}}$ over OFDM can be derived corresponding to the channel matrix $H_{4 \text{QO–STBC}}$ as follows in Equ. (4.12):

$$X_{4 \text{QO–STBC}} = \begin{bmatrix} x_1 - x_3 & x_2 - x_4 & x_1 + x_3 & x_2 + x_4 \\ x_1^* - x_3^* & -x_2^* + x_4^* - x_2 - x_1^* & x_1^* - x_3 & x_2 + x_4^* - x_2 - x_1 \\ x_1 + x_3 & x_2 + x_4 & x_3 - x_1 & x_4 - x_2 \\ -x_1^* - x_2^* & x_3^* + x_1^* & x_4 - x_2^* - x_3 + x_1^* \end{bmatrix}$$

(4.12)

The QO-STBC over OFDM for four transmit antennas can be expanded to eight and sixteen transmitter antennas, as in following:

In four transmitter antennas $\alpha$ which are the channel gains were in the diagonal and $\beta$ were the interference term. In eight and sixteen transmitter antennas; $\alpha_8, \alpha_{16}$ the channel gains described in Equ. (4.13,14) respectively are in the diagonal, and all other terms are the interference from the neighboring signals.

For eight transmitter antennas

$$\alpha_8 = \alpha + |h_5|^2 + |h_6|^2 + |h_7|^2 + |h_8|^2$$

(4.13)

$$\beta_8 = \beta + h_5^* h_7 + h_8 h_5^* + h_7^* h_5 + h_8 h_6^*$$

$$\gamma = h_1^* h_5 + h_2 h_6^* + h_3^* h_7 + h_4 h_6^* + h_5^* h_1 + h_6 h_2 + h_7^* h_3 + h_8^* h_4$$

$$\sigma = h_1^* h_7 + h_2 h_8^* + h_3^* h_5 + h_4 h_6^* + h_5^* h_3 + h_6 h_4^* + h_7^* h_1 + h_8^* h_2$$

In the same way these terms are derived for sixteen transmitter antennas

$$\alpha_{16} = \alpha_8 + |h_9|^2 + |h_{10}|^2 + |h_{11}|^2 + |h_{12}|^2 + |h_{13}|^2 + |h_{14}|^2 + |h_{15}|^2 + |h_{16}|^2$$

$$\beta_{16} = \beta_8 + h_9^* h_{11} + h_{10} h_{12} + h_{11}^* h_9 + h_{12} h_{10} + h_{13}^* h_{15} + h_{14} h_{16} + h_{15}^* h_{13} + h_{16} h_{14}$$

(4.14)

$$\gamma_{16} = \gamma + h_9^* h_{13} + h_{10} h_{14} + h_{11}^* h_{15} + h_{12} h_{16} + h_{13}^* h_9 + h_{14} h_{10} + h_{15}^* h_{11} + h_{16} h_{12}$$

$$\sigma_{16} = \sigma + h_9^* h_{15} + h_{10} h_{16} + h_{11}^* h_{13} + h_{12} h_{14} + h_{13}^* h_{11} + h_{14} h_{12} + h_{15}^* h_9 + h_{16} h_{10}$$

$$\omega = h_1^* h_9 + h_2 h_{10} + h_3^* h_{11} + h_4 h_{12} + h_5^* h_{13} + h_6 h_{14} + h_7^* h_{15} + h_8 h_{16} + h_9^* h_1 + h_{10} h_2 + h_{11}^* h_3 + h_{12} h_{14} + h_{13}^* h_{11} + h_{14} h_{12} + h_{15}^* h_9 + h_{16} h_{10}$$
The eigenvalues matrix and the corresponding eigenvectors for eight transmitter antennas giving by Equ. (4.15) and Equ (A.4) in appendix A

\[\eta = h_1^* h_{13} + h_2 h_{14} + h_3^* h_{15} + h_4 h_{16} + h_5^* h_9 + h_6 h_{10} + h_7^* h_1 + h_8 h_2 + h_9^* h_3 + h_{16} h_4\]

\[\varphi = h_1^* h_{15} + h_2^* h_{16} + h_3^* h_{13} + h_4^* h_{14} + h_5^* h_{11} + h_6 h_{12} + h_7^* h_9 + h_8 h_{10} + h_9^* h_7 + h_{10} h_6 + h_1^* h_5 + h_{12} h_6^* + h_1^* h_3 + h_4 h_4^* + h_1^* h_4 + h_{16} h_2^*\]

The eigenvalue matrix \(D_8\) and the corresponding eigenvectors \(V_8\) for eight transmitter antennas giving by Equ. (4.15) and Equ (A.4) in appendix A

\[
V_{8 \text{ QO- } STBC} = \begin{bmatrix}
-1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 \\
0 & -1 & 0 & -1 & 0 & -1 & 0 & 1 \\
-1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 \\
0 & -1 & 0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\
0 & 1 & 0 & -1 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & -1 & 0 & 1
\end{bmatrix}
\tag{4.15}
\]

From the eigenvectors matrix \(V_8\) in Equ. (4.15), the new channel matrix is derived based on the virtual channel matrix as shown in Equ. (4.16).

\[H_{8 \text{ QO- } STBC} = H_{vb} V_{8 \text{ QO- } STBC}\]

\[
H_{vb} = \begin{bmatrix}
h_1 & h_2 & h_3 & h_4 & h_5 & h_6 & h_7 & h_8 \\
h_2^* & -h_1^* & h_4^* & -h_3^* & h_6^* & -h_5^* & h_7^* & h_8^* \\
h_3 & h_4 & h_1 & h_2 & h_7 & h_8 & h_5 & h_6 \\
h_4^* & -h_3^* & h_2^* & -h_1^* & h_8^* & -h_7^* & h_6^* & -h_5^* \\
h_5 & h_6 & h_7 & h_8 & h_1 & h_2 & h_3 & h_4 \\
h_6^* & -h_5^* & h_8^* & -h_7^* & h_2^* & -h_1^* & h_4^* & -h_3^* \\
h_7 & h_8 & h_5 & h_6 & h_3 & h_4 & h_1 & h_2 \\
h_8^* & -h_7^* & h_6^* & -h_5^* & h_4^* & -h_3^* & h_2^* & -h_1^*
\end{bmatrix}
\tag{4.17}
\]

Then the encoding matrix \(X_{8 \text{ QO- } STBC}\) is derived corresponding to the channel matrix \(H_{8 \text{ QO- } STBC}\) see appendix A Equ. (A.1,2).

Similarly, the detection matrix for sixteen transmitter antennas scheme can be derived using identically the above method to eliminate the interference terms from matrix (A.5) in Appendix A, using matrix (A.7). The resultant channel and Quasi-Orthogonal coding matrices result in a free interference detection matrix as matrix (A.6).
4.2 DHSTBC over OFDM for 4, 8 and 16 Transmit Antennas

In this section a full rate full diversity order Diagonalized Hadamard Space Time code (DHSTBC) over OFDM for 4, 8 and 16 transmitter antennas is implemented; based on what Y.A.S Dama et al. [27] have been presented. The codes generated using this method is orthogonal space time codes, $XX^H = D$ in Equ. (4.33), where $D$ is a diagonal matrix[56].

The generated codes are able to provide full rate and full diversity when the number of the receiver antennas are at least equal to the number of transmitter antennas, the code matrices for DHSTBC over OFDM are limited to the Hadamard matrixes size $N = 2^n$ where $n \geq 1$.

Let $s_1, s_2, ..., s_N$ are the transmitted symbols. These symbols are sorted to form a cyclic matrices which are $S_4, S_8$ described by Equ. (4.20, 22) respectively as follows:

$$S_{12} = \begin{bmatrix} s_1 & s_2 \\ s_2 & s_1 \end{bmatrix} \quad (4.18)$$

$$S_{34} = \begin{bmatrix} s_3 & s_4 \\ s_4 & s_3 \end{bmatrix} \quad (4.19)$$

$$S_4 = \begin{bmatrix} S_{12} & S_{34} \\ S_{34} & S_{12} \end{bmatrix} \quad (4.20)$$

$$S_5 = \begin{bmatrix} S_{56} & S_{78} \\ S_{78} & S_{56} \end{bmatrix} \quad (4.21)$$

$$S_8 = \begin{bmatrix} S_4 & S_5 \\ S_5 & S_4 \end{bmatrix} \quad (4.22)$$

The same procedure is applied to form $S_{16}$ in Equ. (4.26).

$$S_9 = \begin{bmatrix} S_{9-10} & S_{11-12} \\ S_{11-12} & S_{9-10} \end{bmatrix} \quad (4.23)$$

$$S_{10} = \begin{bmatrix} S_{13-14} & S_{15-16} \\ S_{15-16} & S_{13-14} \end{bmatrix} \quad (4.24)$$

$$S_{11} = \begin{bmatrix} S_9 & S_{10} \\ S_{10} & S_9 \end{bmatrix} \quad (4.25)$$

$$S_{16} = \begin{bmatrix} S_8 & S_{11} \\ S_{11} & S_8 \end{bmatrix} \quad (4.26)$$

The Hadamard matrices of order four, eight and sixteen which are used to form the new channel matrix are given in Equ. (4.27, 28, 29) respectively,
The resultant matrix \( X \) is a DHSTBC over OFDM and hence, the overall expression is given by

\[
X = H.S
\]  

(4.30)

If applying equation (4.30) on four transmitter antennas then the matrix shown in Equ.(4.31) is produced.

Equation (4.31)

\[
X_4 = \begin{bmatrix}
  s_1 + s_2 + s_3 + s_4 & s_1 + s_2 + s_3 + s_4 & s_1 + s_2 + s_3 + s_4 \\
  s_1 - s_2 + s_3 - s_4 & s_1 - s_2 + s_3 - s_4 & s_1 - s_2 + s_3 - s_4 \\
  s_1 + s_2 - s_3 - s_4 & s_1 + s_2 - s_3 - s_4 & s_1 + s_2 - s_3 - s_4 \\
  s_1 - s_2 + s_3 + s_4 & s_1 - s_2 + s_3 + s_4 & s_1 - s_2 + s_3 + s_4
\end{bmatrix}
\]

We can notice that \( X_4X_4^H \) is diagonal matrix which can achieve simple linear decoding as the shown in matrix (4.32).
Equation (4.32)

\[ X_4X_4^H = \begin{bmatrix} 4(x_1 + x_2 + x_3) & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \]

The same sequence to have both \( X_8 \) as in Appendix A, Equ. (A.3) and \( X_{16} \) as shown in appendix A.

4.3 Methodology of QO-STBC and DHSTBC for four, eight and sixteen transmitter antennas over OFDM

In this part of our graduation project, a symbolic toolbox is used to derive the equations to be used for four, eight and sixteen transmitter antennas.

4.3.1 Methodology of QO-STBC over OFDM

![Diagram of QO-STBC over OFDM]

- Simulation parameter is chosen based on section(5.3)
- Generate the data with OFDM modulation
- Generate the channel from symbolic toolbox as seen in section(4.1) and add noise.
- Received MISO-OFDM data
4.3.1 Methodology of DHSTBC over OFDM

Multiply the received data by the inverse of the channel, then OFDM demodulation to fined the estimated data.

DHSTBC over OFDM

Simulation parameter is chosen based on section (5.3)

Generate the data with OFDM modulation

Generate the channel from symbolic toolbox as seen in section (4.2).

To be Count.
Multiply the generated channel by the Hadamard matrix for four, eight and sixteen as in section (4.2).

Multiply the previous step by data and then add noise

Received MISO-OFDM data

Multiply the received data by the inverse of the channel, then OFDM demodulation to fined the estimated

End
Chapter Five: Results and Analysis

This chapter discusses the results of the simulation that were performed based on the information and mathematics discussed in the Chapter three and Chapter four respectively.

A simulation results of Matlab code is presented for channel estimation in OFDM and MIMO-OFDM systems, also implementing two methods which are QO-STBC and DHSTBC over OFDM for four, eight and sixteen transmitter antennas.

5.1 OFDM channel estimation
In this section, a channel estimation for OFDM system using pilot based channel estimation techniques such as lest square estimation (LS) and maximum mean squared estimation (MMSE) algorithms is implemented.

For the simulation of basic OFDM system, we used the following parameters as shown in Table

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFT size</td>
<td>128</td>
</tr>
<tr>
<td>Pilot number</td>
<td>16</td>
</tr>
<tr>
<td>Cyclic prefix</td>
<td>32</td>
</tr>
<tr>
<td>Modulation scheme</td>
<td>BPSK</td>
</tr>
<tr>
<td>Channel model</td>
<td>AWGN</td>
</tr>
<tr>
<td>Number of used sub-carrier</td>
<td>112</td>
</tr>
<tr>
<td>Iteration number</td>
<td>200</td>
</tr>
</tbody>
</table>

Table (5.1): Simulation parameter for channel estimation of OFDM system
Figure(5.1): the Mean Square Error MSE versus SNR for the LS and MMSE and ZF Estimators.

It can be noticed from Fig (5.1) that LS method gives the same performance as zero forcing method and MMSE gives the best performance but its complexity is higher because MMSE estimators assume a priori knowledge of noise variance and channel covariance. So its complexity is large compared to LS estimators.

6.2 MIMO-OFDM channel estimation

In this section the results of the channel estimation for MIMO-OFDM system with Space Frequency Block codes(SFBC) is introduced.

In MIMO-OFDM system, 2-D channel estimation have been used where the time and frequency correlation of the training sub-carriers are used to estimate the channel.

Weiner filter which implemented is an example of 2-D channel estimation, also called Minimum Mean Square Error (MMSE) estimators, have greatly increased computational complexity for the improved SNR performance.

Wiener filter filtering, Prediction and Smoothing. Where the Filtering refers to the channel parameter estimates at the data carrying frequency indices which are in a sense an interpolated estimate due to Wiener filtering, and Prediction refers to the channel parameters estimates at data carrying time indices which are procured through a process of time projection using the Wiener filter, finally Smoothing refers
to a refinement of the initial 'noisy' least squares channel parameter estimates at the pilot locations.

Table (5.2): Simulation parameter for wiener channel estimation of MIMO-OFDM system.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency</td>
<td>$1.8 \times 10^9 Hz$</td>
</tr>
<tr>
<td>OFDM symbol length</td>
<td>16</td>
</tr>
<tr>
<td>OFDM frame length</td>
<td>32</td>
</tr>
<tr>
<td>QAM symbol period ($T_s$)</td>
<td>$10 \times 10^{-6} sec$</td>
</tr>
<tr>
<td>Maximum delay spread</td>
<td>$4 \times T_s sec$</td>
</tr>
<tr>
<td>RMS delay spread</td>
<td>$3.5 \times 10^{-6} sec$</td>
</tr>
<tr>
<td>Pilot frequency spacing</td>
<td>4</td>
</tr>
<tr>
<td>Pilot time spacing</td>
<td>8</td>
</tr>
<tr>
<td>RF channel Bandwidth</td>
<td>$200 \times 10^3 Hz$</td>
</tr>
</tbody>
</table>

Figure (5.2): BER Vs. SNR of wiener filter channel estimation method compared with ZF.

It can be noticed from Fig (5.2) that the wiener estimation shown a better performance and improve BER compared with the zero-forcing method.
Also a channel estimation approach for MIMO-OFDM system using orthogonal training sequence is introduced in this section, where the transmitter sends a known and orthogonal sequence of QAM symbols which are used to derive knowledge of the channel parameters at the receiver.

The correlation of the channel parameters for successive sub-carrier channels, the so called frequency correlations, can be exploited to reduce the number of channel estimation parameters. Alternatively, correlations of the channel parameters for successive OFDM symbols, time correlations, can be exploited for the same purpose.

Table (5.3): Simulation parameter for the analysis of orthogonal training sequence channel estimation of MIMO-OFDM system. Refer to the Saleh-Valenzuela model in Section (3.3.3).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency</td>
<td>$2.4 \times 10^9 Hz$</td>
</tr>
<tr>
<td>OFDM symbol length</td>
<td>128</td>
</tr>
<tr>
<td>QAM symbol period ($T_s$)</td>
<td>$11 \times 10^{-9} \text{ sec}$</td>
</tr>
<tr>
<td>Cluster arrival rate</td>
<td>$\frac{1}{200} \times 10^{-9} \text{ sec}^{-1}$</td>
</tr>
<tr>
<td>Ray arrival rate</td>
<td>$\frac{1}{5} \times 10^{-9} \text{ sec}^{-1}$</td>
</tr>
<tr>
<td>Tx antenna gain</td>
<td>3 $dB$</td>
</tr>
<tr>
<td>Rx antenna gain</td>
<td>3 $dB$</td>
</tr>
<tr>
<td>Cluster power Delay Time Constant</td>
<td>$60 \times 10^{-9}$</td>
</tr>
<tr>
<td>Ray power Delay Time Constant</td>
<td>$20 \times 10^{-9}$</td>
</tr>
</tbody>
</table>
Figure (5.3): Orthogonal training sequence channel estimation for $H_{11}$

Figure (5.4): Orthogonal training sequence channel estimation for $H_{12}$
Figure (5.5): Orthogonal training sequence channel estimation for $H_{21}$

Figure (5.6): Orthogonal training sequence channel estimation for $H_{22}$
Figures (5.3-5.6) shows the absolute value of the sub-carrier gain for the actual and estimated channel using orthogonal training sequence for $2 \times 2$ MIMO-OFDM systems.

Noting that perfect recovery of $\{H_i[k]\}$ is not possible even without noise because of the underdetermined structure, this conventional method incurs an irreducible error in the estimate for the channel pairs $(H_1[0], H_1[1])$ and $(H_2[0], H_2[1])$, which is inversely proportional to the degree of correlation for the channel pairs. That is to say, if the difference in the channel pairs, $\Delta H_1^{1,0}$ and $\Delta H_2^{1,0}$, is small, then the error in the estimate will be small. In an environment where there is a great degree of multipath (i.e., large $\tau_{rms}$), the channels are less correlated, and $\Delta H_1^{1,0}$ and $\Delta H_2^{1,0}$ are significant, moreover it's noticed that there is a small difference between them but they have the same envelope.

### 5.3 QO-STBC and DHSTBC over OFDM for four, eight and sixteen transmitter antennas

The performance of QO-STBC and DHSTBC over OFDM was evaluated over Rayleigh fading channel is introduced in this section. The signals were modulated using 16-QAM, and the total transmit power was divided equally among the number of transmitter antennas. The fading was assumed to be constant over four, eight and sixteen consecutive symbol periods for four, eight and sixteen transmitter antennas respectively and the channel was known at the receiver. Finally compared the results of these methods with real STBC.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency (MHz)</td>
<td>5.8</td>
</tr>
<tr>
<td>Sample frequency (MHz)</td>
<td>40</td>
</tr>
<tr>
<td>Bandwidth (MHz)</td>
<td>40</td>
</tr>
<tr>
<td>FFT (size)</td>
<td>128</td>
</tr>
<tr>
<td>Cyclic prefix ratio</td>
<td>0.25</td>
</tr>
<tr>
<td>Constellation</td>
<td>16-QAM</td>
</tr>
<tr>
<td>Data subcarrier /Pilots</td>
<td>108/6</td>
</tr>
<tr>
<td>Virtual carrier</td>
<td>14</td>
</tr>
</tbody>
</table>

*Table 5.4: OFDM Simulation Parameter when used in implementing QO-STBC and DHSTBC*
It should be noted that the same data bits and the same mechanism process were used to evaluate the performance of the implemented methods and the real STBC scheme.

Fig. (5.7) shows BER performance of QO-STBC over OFDM for four, eight and sixteen transmit antennas. One can notice that the best BER is achieved by using sixteen transmitter antennas, since the diversity order increases by increasing the number of the transmitted antenna, due to increase in the transmitter diversity.

![Figure 5.7: BER performance of the QO-STBC over OFDM for Four, Eight and Sixteen transmitter antennas.](image)

Fig.(5.8) shows BER performance of DHSTBC over OFDM for four, eight and sixteen transmitter antennas. As it is shown from Fig.(5.8) the best BER performance is achieved by using sixteen transmitter antennas, due to increase in the transmitter diversity.
Figure 5.8: BER performance of the DHSTBC over OFDM for Four, Eight and Sixteen transmitter antennas.

Now comparing the implemented methods which are QO-STBC and DHSTBC with the real STBC method with the same number of transmitter antennas to evaluate BER performance. From Fig. (5.9) to Fig. (5.11) it’s clearly indicated that the DHSTBC achieves best performance compared with QO-STBC and real STBC. Also QO-STBC has better performance than the real STBC.

Figure 5.9: BER performance Real STBC, QO-STBC and DHSTBC over OFDM for Four transmitter antennas.
Figure 5.10: BER performance Real STBC, QO-STBC and DHSTBC over OFDM for Eight transmitter antennas.

Figure 5.11: BER performance Real STBC, QO-STBC and DHSTBC over OFDM for Sixteen transmitter antennas.

Now, the performance analysis is done with different Modulation schemes for QO-STBC and DHSTBC over OFDM for four, eight and sixteen transmitter antennas. The
effects of varying different modulation schemes were observed while doing BER (Bit Error) analysis.

Figure 5.12: BER Vs. SNR for MISO-OFDM 4 Tx x 1 Rx QO-STBC for different modulation schemes

Figure 5.13: BER Vs. SNR for MISO-OFDM 8 Tx x 1 Rx QO-STBC for different modulation schemes
Figure 5.14: BER Vs. SNR for MISO-OFDM $16 \times 1$ QO-STBC for different modulation schemes

Figure 5.15: BER Vs. SNR for MISO-OFDM $4 \times 1$ DHSTBC for different modulation schemes
Figure 5.16: BER Vs. SNR for MISO-OFDM 8Tx x 1Rx DHSTBC for different modulation schemes

Figure 5.17: BER Vs. SNR for MISO-OFDM 16Tx x 1Rx DHSTBC for different modulation schemes
From figures (5.12-5.17) it is noticed that, as the order of the modulation scheme increased, the complexity in recovering the original data increased and the BER also increased. So as it appears the BPSK has the smallest BER, so the arrangement is as follows BPSK < QPSK < 16-QAM < 32-QAM < 64-QAM, in terms of BER.

This can be explained as: in any modulation scheme the symbols are spread at a certain distance, this distance represents the energy of that symbol according to the constellation diagram. As the order of modulation scheme increasing; this distance will decrease causing increase in the error. This is why the BER increases by increasing the order of modulation scheme.

On the other hand; the BPSK has larger Bandwidth and lower data rate than the higher modulation schemes, also it has low spectral efficiency. This is not preferred because BPSK symbols will consume the whole BW. And so on for any modulation scheme with respect to the next one.

The engineer must make trade-off between the order of modulation scheme in order of its complexity, BER, the required Bandwidth and data rate.
Chapter Six: Conclusions and Recommendation

6.1 Conclusion

This project highlights the channel estimation technique based on pilot aided block type training symbols using LS and MMSE algorithm. The Channel estimation is one of the fundamental issues of OFDM system design. The transmitted signal under goes many effects such reflection, refraction and diffraction. Also due to the mobility, the channel response can change rapidly over time. At the receiver these channel effects must be canceled to recover the original signal.

Also it covers the channel estimation in MIMO-OFDM systems using more than one techniques, these were used to improve the performance. Firstly the Least Square (Zero-Forcing) estimation is used which is very simple to implement and widely used. Secondly, the wiener filter estimator uses the weighted matrix to improve the performance but this system is complex. With the help of these techniques the ICI and ISI problems are reduced and the results of Wiener estimator shows a better performance compared with LS method.

Finally the orthogonal training sequence channel estimation were implemented based on the coherence of the CSI over a few sub-carriers. If the flat fading channel is assumed to be constant over a few sub carriers, an orthogonal training sequence can be used to estimate the channel at a given sub-carrier. However, there are variations in the channel parameters within the coherence bandwidth which lead to errors in the estimated channel. The key to accurate CSI estimation for the OFDM sub-symbol based estimator is to have the knowledge of the variations in CSI.

In this project new methods for QO-STBC and DHSTBC over OFDM for four, eight and sixteen transmitter antenna were implemented by deriving the orthogonal channel matrix that results in simple decoding scheme. The performance of QO-STBC and DHSTBC over OFDM was evaluated by varying the number of transmitter antennas and tested with different modulation schemes. When these compared with real STBC it shows a better performance.

6.2 Recommendation for Future Works

Back to work presented in this project ones can noticed that there is a percent of error between the actual channel and the estimated one, so to reduce the error either implement a noise cancellation method as a feature inside the presented techniques in
this project, or to move toward new methods which guarantee more error reduction. Another drawback was when worked on the non-blind channel estimation, because it requires transmission of known training sequence to both transmitter and receiver which is not practical situation in real communication systems. So we recommend to go through blind and semi-blind techniques that assumes the receivers don't know anything about the transmitted symbols.

A recommendation for the last topic in this project is to implement QO-STBC and DHSTBC over CDMA or LTE instead of OFDM.
References


[38] F. Stremler, Introduction to Communication Systems, Addison-Wesley, 1990


Appendix A

In this appendix the equation of eight and sixteen transmitter antenna for QO-STBC and DHSTBC over OFDM will introduced.

\[ H_{\text{QO-STBC}} = \]

\[
\text{A.1}
\]

\[ X_{\text{seno}} =
\]

\[
\text{A.2}
\]

\[ X_0 =
\]

\[
\text{A.3}
\]

\[ D_{\text{QO-STBC}} =
\]

\[
\text{A.4}
\]
\[ D_{16} = \begin{bmatrix}
\sigma_{16} & 0 & \beta_{16} & 0 & \gamma_{16} & 0 & \sigma_{16} & 0 & \omega & 0 & \xi & 0 & \eta & 0 & \phi & 0 \\
0 & a_{16} & 0 & \beta_{16} & 0 & \gamma_{16} & 0 & \sigma_{16} & 0 & \omega & 0 & \xi & 0 & \eta & 0 & \phi \\
\beta_{16} & 0 & a_{16} & 0 & \sigma_{16} & 0 & \gamma_{16} & 0 & \xi & 0 & \omega & 0 & \phi & 0 & \eta & 0 \\
0 & \beta_{16} & 0 & a_{16} & 0 & \sigma_{16} & 0 & \gamma_{16} & 0 & \xi & 0 & \omega & 0 & \phi & 0 & \eta \\
\gamma_{16} & 0 & a_{16} & 0 & \beta_{16} & 0 & \eta & 0 & \phi & 0 & \omega & 0 & \xi & 0 & \sigma_{16} & 0 \\
0 & a_{16} & 0 & a_{16} & 0 & \beta_{16} & 0 & \eta & 0 & \phi & 0 & \omega & 0 & \xi & 0 & \sigma_{16} & 0 \\
\sigma_{16} & 0 & \gamma_{16} & 0 & \beta_{16} & 0 & \sigma_{16} & 0 & \phi & 0 & \eta & 0 & \xi & 0 & \omega & 0 \\
0 & \sigma_{16} & 0 & \gamma_{16} & 0 & \beta_{16} & 0 & \sigma_{16} & 0 & \phi & 0 & \eta & 0 & \xi & 0 & \omega \\
\omega & 0 & \xi & 0 & \eta & 0 & \phi & 0 & a_{16} & 0 & \beta_{16} & 0 & \gamma_{16} & 0 & \sigma_{16} & 0 \\
0 & \omega & 0 & \xi & 0 & \eta & 0 & \phi & 0 & a_{16} & 0 & \beta_{16} & 0 & \gamma_{16} & 0 & \sigma_{16} & 0 \\
\xi & 0 & \omega & 0 & \phi & 0 & \eta & 0 & \beta_{16} & 0 & a_{16} & 0 & \sigma_{16} & 0 & \gamma_{16} & 0 \\
0 & \xi & 0 & \omega & 0 & \phi & 0 & \eta & 0 & \beta_{16} & 0 & a_{16} & 0 & \sigma_{16} & 0 & \gamma_{16} & 0 \\
\eta & 0 & \phi & 0 & \omega & 0 & \xi & 0 & \gamma_{16} & 0 & \sigma_{16} & 0 & a_{16} & 0 & \beta_{16} & 0 \\
0 & \eta & 0 & \phi & 0 & \omega & 0 & \xi & 0 & \gamma_{16} & 0 & \sigma_{16} & 0 & a_{16} & 0 & \beta_{16} & 0 \\
\phi & 0 & \eta & 0 & \xi & 0 & \omega & \sigma_{16} & 0 & \gamma_{16} & 0 & \beta_{16} & 0 & a_{16} & 0 & \sigma_{16} & 0 \\
0 & \phi & 0 & \eta & 0 & \xi & 0 & \omega & \sigma_{16} & 0 & \gamma_{16} & 0 & \beta_{16} & 0 & a_{16} & 0 \\
\end{bmatrix} \]

\[ D_{16,\omega-\sigma_{16}} = \begin{bmatrix}
\phi & \zeta & \beta_{16} & a_{16} \\
0 & 0 & 0 & 0 \\
\end{bmatrix} \]
\[
H_{hs} = \begin{bmatrix}
    a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} \\
    b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & b_8 & b_9 & b_{10} & b_{11} & b_{12} & b_{13} \\
    c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} & c_{11} & c_{12} & c_{13} \\
    d_1 & d_2 & d_3 & d_4 & d_5 & d_6 & d_7 & d_8 & d_9 & d_{10} & d_{11} & d_{12} & d_{13} \\
    e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} & e_{11} & e_{12} & e_{13} \\
    f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & f_7 & f_8 & f_9 & f_{10} & f_{11} & f_{12} & f_{13} \\
    g_1 & g_2 & g_3 & g_4 & g_5 & g_6 & g_7 & g_8 & g_9 & g_{10} & g_{11} & g_{12} & g_{13} \\
    h_1 & h_2 & h_3 & h_4 & h_5 & h_6 & h_7 & h_8 & h_9 & h_{10} & h_{11} & h_{12} & h_{13} \\
    i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 & i_8 & i_9 & i_{10} & i_{11} & i_{12} & i_{13} \\
    j_1 & j_2 & j_3 & j_4 & j_5 & j_6 & j_7 & j_8 & j_9 & j_{10} & j_{11} & j_{12} & j_{13} \\
    k_1 & k_2 & k_3 & k_4 & k_5 & k_6 & k_7 & k_8 & k_9 & k_{10} & k_{11} & k_{12} & k_{13} \\
    l_1 & l_2 & l_3 & l_4 & l_5 & l_6 & l_7 & l_8 & l_9 & l_{10} & l_{11} & l_{12} & l_{13} \\
    m_1 & m_2 & m_3 & m_4 & m_5 & m_6 & m_7 & m_8 & m_9 & m_{10} & m_{11} & m_{12} & m_{13} \\
    n_1 & n_2 & n_3 & n_4 & n_5 & n_6 & n_7 & n_8 & n_9 & n_{10} & n_{11} & n_{12} & n_{13} \\
    o_1 & o_2 & o_3 & o_4 & o_5 & o_6 & o_7 & o_8 & o_9 & o_{10} & o_{11} & o_{12} & o_{13} \\
    p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 & p_9 & p_{10} & p_{11} & p_{12} & p_{13} \\
    q_1 & q_2 & q_3 & q_4 & q_5 & q_6 & q_7 & q_8 & q_9 & q_{10} & q_{11} & q_{12} & q_{13} \\
    r_1 & r_2 & r_3 & r_4 & r_5 & r_6 & r_7 & r_8 & r_9 & r_{10} & r_{11} & r_{12} & r_{13} \\
    s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 & s_{10} & s_{11} & s_{12} & s_{13} \\
    t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_8 & t_9 & t_{10} & t_{11} & t_{12} & t_{13} \\
    u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 & u_9 & u_{10} & u_{11} & u_{12} & u_{13} \\
    v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} & v_{11} & v_{12} & v_{13} \\
    w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 & w_9 & w_{10} & w_{11} & w_{12} & w_{13} \\
    x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} & x_{12} & x_{13} \\
    y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8 & y_9 & y_{10} & y_{11} & y_{12} & y_{13} \\
    z_1 & z_2 & z_3 & z_4 & z_5 & z_6 & z_7 & z_8 & z_9 & z_{10} & z_{11} & z_{12} & z_{13} \\
\end{bmatrix}
\]